USING LINEAR PREDICTION TO ENHANCE THE TRACKING OF PARTIALS

Mathieu Lagrange†, Sylvain Marchand‡, and Jean-Bernard Rault†

†France Telecom R&D
4, rue du Clos Courtel, BP 59
F-35512 Cesson Sevigné cedex, France
firstname.name@rd.francetelecom.com

‡SCRIME – LaBRI, Université Bordeaux 1
351, cours de la Libération,
F-33405 Talence cedex, France
sylvain.marchand@labri.fr

ABSTRACT

In this article, we present an enhanced algorithm, of low complexity, for the tracking of partials in the context of sinusoidal modeling. By considering past evolution of each partial in the time/frequency and time/amplitude planes to predict its future evolutions, this algorithm allows a better discrimination between sinusoidal and noisy components and an easier cancellation of sudden change in the evolution of the partials.

1. INTRODUCTION

The sinusoidal model presented in Section 2 provides a high-quality representation of pseudo-stationary sounds. Therefore, this model is widely used for many musical audio processing purposes such as musical source separation, transcription or coding. One of the most challenging parts of the analysis chain presented in Figure 1 is known as partial tracking.

Given a discrete time/frequency representation, a partial tracker should be able to extract continuous informations by linking frequency components of successive frames. Most tracking methods [1, 2, 3] use heuristics such as distance in frequency, amplitude, and phase between two successive frequency components to achieve such a task. While the use of such heuristics is successful in handling monophonic sound, they generally fail when extended to polyphonic sounds because of spectral degradations "blurring" the time/frequency representation. In such a case, a model for the evolutions of the partials is clearly needed. A partial should have an evolution which is slow time-varying and predictable and for specific purposes, the model can be more rigorous [4]. In Section 3 we present a tracking method that exploits these constraints. It is based on the well-known linear prediction model presented in Section 4, but this time applied to the spectral parameters of the sounds. Its implementation is detailed in Section 5. Discussion about the problem of the validation and comparison of the different tracking methods, as well as results, follow in Section 6.

2. SINUSOIDAL MODELING

Additive synthesis is the original spectrum modeling technique. It is rooted in Fourier's theorem, which states that any periodic function can be modeled as a sum of sinusoids at various amplitudes and harmonic frequencies. For stationary pseudo-periodic sounds, these amplitudes and frequencies continuously evolve slowly with time, controlling a set of pseudo-sinusoidal oscillators commonly called *partials*. The audio signal s can be calculated from the additive parameters using Equations 1 and 2, where P is the number of partials and the functions f_p, a_p, and ϕ_p are the instantaneous frequency, amplitude, and phase of the p-th partial, respectively. The P pairs (f_p, a_p) are the parameters of the additive model and
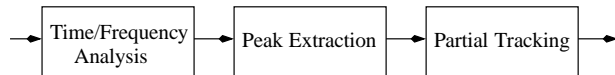

Fig. 1. Sinusoidal analysis procedure.

represent points in the frequency-amplitude plane at time t. This representation is used in many analysis / synthesis programs such as Lemur [5], SMS [3], or InSpect [6].

$$s(t) = \sum_{p=1}^{P} a_p(t)\, \cos(\phi_p(t)) \qquad (1)$$

$$\phi_p(t) = \phi_p(0) + 2\pi \int_0^t f_p(u)\, du \qquad (2)$$

2.1. Analysis Procedure

Sinusoidal analysis is generally made of three steps as shown in Figure 1. Instantaneous parameters of partials are estimated by picking some local maxima commonly called *peaks* from a short-term time/frequency analysis. Partials are then formed by tracking peaks over time, from frame to frame.

In our system, spectral estimation is done using a short time Fourier transform. The instantaneous frequency, amplitude, and phase of the peaks are estimated using the derivative method [7]. For various reasons, peaks can be missing or erroneous. The peak-picking process can remove some erroneous ones [8], but is not able to recover missing spectral information. In order to form partials of reasonable length, the partial tracking step should be able to interpolate missing peaks.

2.2. Constraints on the Evolutions of Partial Parameters

We assume that the partials have a minimum length and that their evolutions in frequency and amplitude are predictable since repeated "birth"/"death" and sudden changes in the evolutions of the partials will generate noisy "clicks" at the resynthesis stage and offer a poor representation of the spectral contents.

3. ALGORITHM OVERVIEW

The first partial tracking algorithm was introduced in [1] by McAulay and Quatieri in the field of the sinusoidal modeling of voice. The algorithm is based on the assumption that partials composing a voiced signal have stationary frequencies. Given a set of partials ending at frame k, it is proposed to consider frequency differences below a given threshold Δ_f between the last inserted peak of each
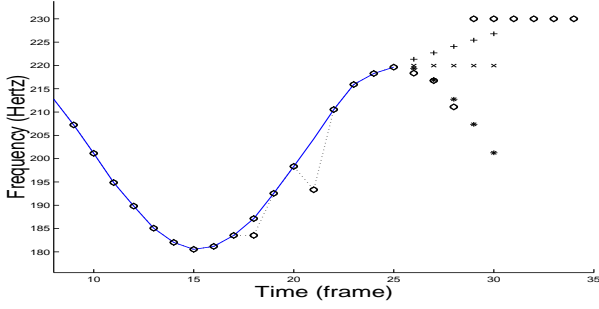
Fig. 2. Prediction capability of constant (x), linear (+) and LP (*) predictors on a synthetic vibrato. Peaks are represented with circles (o). The already-tracked partial using the MAQ algorithm and the proposed method are plotted, respectively, with dotted and solid lines.

partial and the peaks of frames $k + n_i + 1$. The n_i parameter is the number of interpolated peaks allowed to be used at once. If no peak satisfying this constraint can be found the partial is "dead". If a peak remains unlinked at frame $k + 1$, a partial is "born". Considering $n_i = 0$ gives the original McAulay and Quatieri (MAQ) algorithm.

To select the next peak, the frequency and amplitude evolutions of the partials are considered constant (see Figure 2). Yet, frequencies and amplitudes of partials are not stationary, but their evolutions are often predictable. A better approximation has been proposed in [2] by considering the continuity of the slope. However, without a model for the evolutions of partials it seems impossible to overcome spectral artifacts. It is proposed in [4] to model the evolution of partials evolutions of instrumental sounds of the brass family by means of Kalman filtering using pre-extracted statistical informations. In order to gain generality, we showed in [9] that linear prediction (LP) can be used to model and predict the frequency and amplitude evolutions with parameters computed from the past evolution of the partials. Therefore, we propose to use both the predicted frequency and amplitude to select peak candidates in the next frames and to interpolate missing peaks.

Using linear prediction has two advantages that can be seen in Figure 2. The predicted frequency is closest to those of the next peak to be linked, so that the algorithm is more precise (we choose the right peak at frame 18). Since the prediction error is lower than using a constant predictor, the algorithm can be much more selective by using a lower Δ_f. We can then discard the peak at frame 21 and use an interpolated one instead.

4. LINEAR PREDICTION

In the linear prediction (LP) model, also known as the autoregressive (AR) model, the current sample $x(n)$ is approximated by a linear combination of k past samples of the input signal. We are then looking for a vector a of k coefficients, k being the order of the LP model. Provided that the a vector is estimated, the predicted value $\hat{x}$ is computed simply by FIR filtering of the k past samples with the coefficients using Equation 3:

$$\hat{x}(n) = \sum_{i=1}^{k} a_i x(n - i) \qquad (3)$$

The challenge in linear prediction modeling is to choose the model order k, the number of samples and type of method to estimate the

coefficients that suit specific needs.

For frequency and amplitude evolutions, we want to model exponentially increasing or decreasing evolutions (portamento) and sinusoidal evolutions (vibrato), the order of the LP model should not be below 2. Experimental testing showed that a model order in the $[2, 8]$ range is suitable.

The number of samples used has to be large enough to be able to extract the signal periodicity, and short enough not to be too constrained by the past evolution. The short-term analysis module uses a sliding time / frequency transform with a hop size of 512 samples on sound signals sampled at CD quality (44.1 kHz). This means that the frequency and amplitude trajectories are sampled at ≈ 86 Hz. Since we want to handle natural vibrato with a frequency about 4 Hz, we need at least 20 samples to get the period of the vibrato. Experimental testing showed that for most cases a number of samples in the $[4, 32]$ range is suitable.

On one hand spectral data suffer from imprecision, so the method has to be resistant to noise. On the other hand, the evolutions of the partials in frequency and amplitude are sampled at a low sample rate, so the method to estimate the coefficients has to be reactive. Among the three methods tested in [9], the Burg method was the most satisfactory. It minimizes the average of the power of the forward e^f and backward e^b errors calculated using Equations 4 and 5, it then leads to stable filter coefficients, see [10, 11] for details. Moreover, the minimization is done on a finite support, so the method requires only few samples to be effective.

$$e_k^f(n) = \frac{1}{(N-k)} \sum_{n=k}^{N-1} |x(n) + \sum_{i=1}^{k} a(i)x(n-i)|^2 \qquad (4)$$

$$e_k^b(n) = \frac{1}{(N-k)} \sum_{n=0}^{N-1-k} |x(n-k) + \sum_{i=1}^{k} a(i)x(n+k+i)|^2 \qquad (5)$$

5. ENHANCED ALGORITHM

Linear prediction requires at least a few samples to be effective, all starting partials then begin using a selection mode similar to one used in the MAQ algorithm for a fixed number of frames n_s. Partials are processed in decreasing amplitude order and the non-starting ones are processed first.

At each frame, each partial predicts its ideal evolution in frequency and amplitude using linear prediction and selects n_c peak candidates, in the next frame, with a difference between its frequency and the predicted one below a given threshold Δ_f. it then picks from these n_c peaks the one that has the smallest difference between its amplitude and the predicted one. If no peak can be found, the predicted frequency and amplitude are used to create an interpolated peak. As a first approximation, its phase is interpolated using the maximally smooth polynomial interpolation detailed in [1].

If two partials cross, a peak corruption occurs in the crossing region (see Figure 3). In this case, interpolated peaks will be used for the partial having the lower amplitude since there is no peak satisfying the prediction constraints. When this partial links to an extracted peak, the crossing is detected (at frame 272 in Figure 3). If the amplitude difference between the two partials is below a given threshold, the highest amplitude partial will also use interpolated peaks in the crossing region because the extracted peaks in the crossing region are considered too corrupted to be reliable.
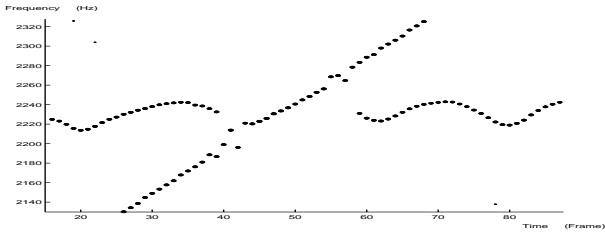
Fig. 3. Peak extraction of the crossing of an harmonic of a saxophone tone and a synthetic frequency ramped sinusoid sound.

6. RESULTS

Due to the several possible applications of a partial tracking module, it seems hard to give a complete evaluation of partial tracking algorithms by a straightforward quality measure. As a consequence, we left this task for further research and give an intuitive comparison of the MAQ algorithm and the proposed method using several criteria.

We can roughly set two requirements concerning the results of a tracking algorithm: the resynthesis quality concerning coding applications and the identification of onset/offset and evolution of partials concerning indexing applications. Both requirements are needed for source separation. The resynthesis quality will be discussed in the first part using a synthetic tone. Crossing partials management is studied in a second part. The identification of onset/offset and evolution of partials is then studied in a third part using natural violin tones.

6.1. Resynthesis Quality

In the processing chains of hybrid sound models such as sinusoids-+noise or sinusoids+transients+noise, the sinusoidal components are first extracted and then synthesized to be subtracted from the original signal to give a residual that will be considered as a random process (noise). Ideally, the partials should efficiently represent all sinusoidal components and only them. Otherwise, if the partial set represents only a few of the sinusoidal components, there will be some sinusoidal components left in the residual. If the partial tracking algorithm has no discrimination capability, it will wrongly identify partials in noise and the random process will be modeled as slow-time varying sinusoids, forcing the residual module to handle those synthesized sinusoids that were not present in the original sound.

Efficiency and discriminating capabilities of the two algorithms are tested using a synthetic constant amplitude 4 Hz vibrato sinusoid with frequency ranges from 1950 to 2050 Hz, embedded in a white noise of growing level. Concerning the parameterization of the tracking algorithms, n_i was set to 4. For the MAQ algorithms, we used the smallest Δ_f such that the vibrato could be tracked, that is 20 Hz. For the proposed method, Δ_f was set to 12 Hz, the prediction order to 6, the prediction length to 20 frames, and n_s to 10. All the partials whose length was below 15 peaks were discarded. The peak extraction method proposed in [7] and the original partial resynthesis method introduced in [1] were used for the two compared methods. The quality of the tracking algorithm is measured with the reconstruction-SNR in function of the degradation-SNR.

In the first experiment, in order to evaluate the efficiency, only the partial having the highest mean amplitude was synthesized to compute the reconstruction-SNR. At degradation-SNR below -7 dB (see Figure 4.1), the MAQ algorithm produces partials that
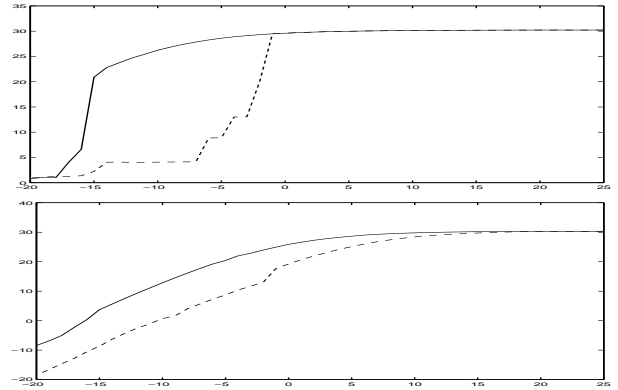

Fig. 4. Evaluation of the efficiency and discrimination capabilities of the two methods: proposed method (solid line) and MAQ method (dashed line) using a synthetic vibrato tone embedded in white noise. The plot shows reconstruction-SNR versus degradation-SNR on the dB scale. To evaluate the efficiency, on the top, only the partial with the highest amplitude is synthesized. At the bottom, to evaluate the discrimination capability, all extracted partials are synthesized to compute the reconstruction-SNR.

are a mix of noisy peaks and tonal peaks so that the tone is split into several partials. Whereas the proposed method, by having a smaller Δ_f, is able to track correctly the tone with vibrato by not choosing noisy peaks. The slow decay is due to errors in the estimation of the spectral peaks. In the second experiment, to evaluate the discriminating capability of the two algorithms, all retained partials that have frequencies in the [1900, 2100] Hz band are synthesized to compute the degradation-SNR (see Figure 4.2). Because the proposed algorithm makes it possible to use a lower Δ_f, it has a better discrimination capability.

6.2. Crossing of Partials

The problem of partial crossing arises when we have to deal with a mixture of non-stationary sounds. The tracking has to be able to identify the evolution of the partials and to interpolate correctly missing spectral data (see Figure 3). In order to test the management of crossing, a natural A-440 Hz saxophone tone was corrupted by a synthetic constant-amplitude sinusoid beginning 20 frames later and having frequency going from 200 Hz to 4 kHz. Only the extracted partials starting before the frame 20 were synthesized to compute the reconstruction-SNR. Results for the MAQ method using a Δ_f of 80 Hz and the proposed method using a Δ_f of 25 Hz are plotted in Figure 5. Having a model of the evolutions of parameters leads to an easier management of crossing partials, by being more selective and by having a better interpolation capability. Furthermore, the presented algorithm schedules the partials in decreasing amplitude, so that the partial with the slower degradation is processed first. It reduces the probability of handling the crossing incorrectly.

6.3. Identification of Partials

In applications such as indexing or source separation of stationary pseudo-periodic sounds, a good representation of the partials provides a higher level of description, useful to extract robustly high
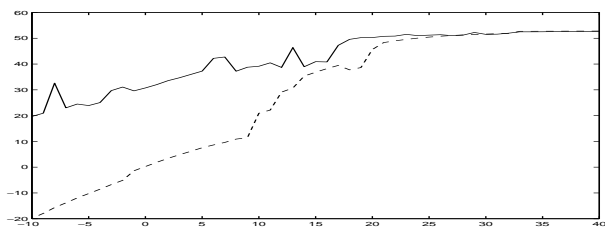
Fig. 5. Evaluation of the crossing management capability of the two methods: the proposed method (solid line) and the MAQ algorithm (dashed line). A natural A-440 Hz saxophone tone is corrupted by a synthetic constant-amplitude sinusoid beginning 20 frames later and having frequency going from 200 Hz to 4 kHz. The plot shows reconstruction-SNR versus degradation-SNR on the dB scale. Only the extracted partials starting before the frame 20 were synthesized to compute the reconstruction-SNR.

level informations such as note onset/offset, pitch detection and source identification.

In order to easily detect the note onset/offset, one would like to have a good time separation, meaning that a partial should belong to only one source. And in order to detect the pitch and to identify the sources, the partials should show "clear" time/frequency and time/amplitude evolutions in order to be able to cluster partials. As can be seen in Figure 6.1, the partial set extracted by the MAQ algorithm is not satisfactory. A lot of partials belong to two or three tones and it would be very difficult to detect the vibrato frequency of the second tone. The proposed method shows better results in time separation and the vibrato of the second tone is clearer (see Figure 6.2). The implementation described in Section 5 is not perfect, some partials belong to two tones and the vibrato is still not perfectly handled in noisy conditions.

7. CONCLUSION

In this article, we proposed to use linear prediction to replace the classic stationary assumption to better track and to interpolate partials in the context of sinusoidal modeling. However the presented implementation has to be compared with tracking algorithm explicitly dedicated to musical recording analysis [2, 4], this new approach leads to promising results in terms of resynthesis and representation quality, two important requirements for an approach of source separation from mono recording based on sinusoidal modeling.

8. REFERENCES

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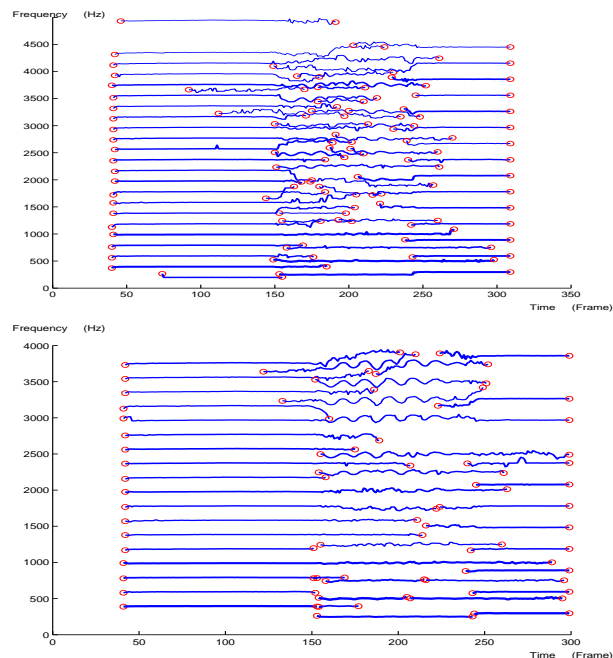
Fig. 6. Partials extracted from three successive violin tones (second with vibrato) by the MAQ algorithm (up) and the proposed method (bottom). The partials are represented by solid lines, starting and ending with circles matching the birth and the death of the partials.

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