Motivic Pattern Extraction in Music

And Application to the Study of Tunisian Modal Music

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ABSTRACT. A new methodology for automated extraction of repeated patterns in time series data is presented, aimed in particular at the analysis of musical sequences. The basic principles consists in a search for closed pattern paradigm in a multi-dimensional parametric space. It is shown that this basic mechanism needs to be articulated with a periodic pattern discovery system, implying therefore a strict chronological scanning of the time series data. Thanks to this modelling global pattern filtering may be avoided and rich and highly pertinent results can be obtained. The modelling has been integrated in a collaborative project between ethnomusicology, cognitive sciences and computer science, aimed at the study of Tunisian Modal Music.

RÉSUMÉ. Une méthodologie d'extraction automatique de motifs répétés dans des séquences temporelles est présentée, dédiée en particulier á l'analyse de séquences musicales. L'approche initiale consiste en une recherche de motifs fermés dans un espace paramétrique multidimensionnel. Il est montré que ce premier mécanisme doit tre articulé avec un systéme de découverte de motifs périodiques, ce qui implique un parcours strictement chronologique de la séquence. Cette modélisation permet d'éviter un filtrage global des patterns, et donc d'obtenir des résultats présentant une richesse et une pertinence élevée. La modélisation a été intégrée au sein d'un projet collaboratif entre éthnomusicologie, sciences cognitives et informatique, dédié á l'étude de la musique modale tunisienne.

KEYWORDS : pattern extraction, time series data, closed pattern, periodic pattern, music analysis, tunisian modal music

MOTS-CLÉS : extraction de motifs, séquences temporelles, motifs fermés, motifs périodiques, analyse musicale, musique modale tunisienne
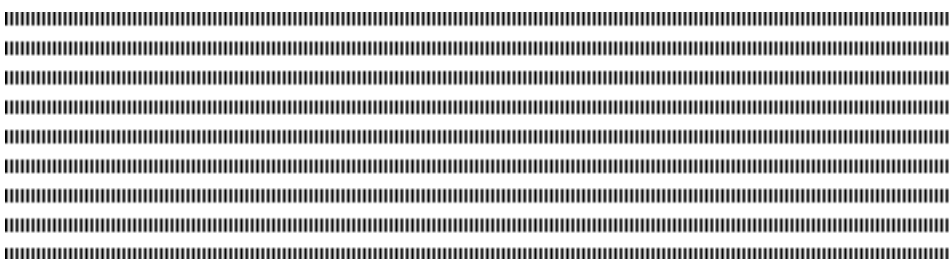
1. Introduction

This paper introduces a new methodology for repeated pattern (or motif) extraction in symbolic sequences, and is applied particularly to the analysis of musical scores. Among the different approaches that can be considered for time-series data analysis, one domain of research that has received much attention is the problem of extraction of motives, i.e. the discovery of patterns appearing frequently in time-series data [Tanaka, Lin]. Indeed, motives may characterise important aspects of the data, and help discovering new association rules. In music too, repeated sequences of notes are easily perceived by listeners as important structures, forming the "words" of the musical structure.

Lots of research have been carried out in this domain and numerous interesting solutions have been proposed. One major problem stems from the structural redundancies logically resulting from this task, which, if not carefully controlled, may provoke combinatorial explosion and infringe the quality of the results. Few researches have considered the pattern discovery problem within a general and difficult context. The approach presented in this paper follows this idea of closed pattern, which is defined here in a multidimensional parametric space. Another combinatorial redundancy problem, provoked by immediate succession of same patterns, is solved by introducing the concept of cyclic pattern. The model has been applied to the automated motivic analysis of musical scores, and in particular to the study of Arabic improvisations played by Tunisian masters.

Most music databases contain sound files of performance recordings, which correspond to the way music is commonly experienced. The underlying structure of music, on the other hand, is represented in a symbolic form – the score – that describes musical pieces regardless of the way they are performed. There exist numerous digital formats of symbolic music representation (MIDI, MusicXML, Humdrum, etc.). The pattern discovery system described in this paper is applied uniquely to symbolic representation. A direct analysis on the signal level would arouse tremendous difficulties. A pattern extraction task on the symbolic level, although theoretically simpler, remains extremely difficult to carry out, and its automation has not been achieved up to now. Indeed, computer researches on this subject hardly offer results close to listeners' or musicologists' expectations. Hence the pattern discovery task is too complex to be undertaken directly at the audio signal, and needs rather a prior transcription from the audio to the symbolic representations, in order to carry out the analysis on a conceptual level.

2. An incremental multidimensional motivic identification

2.1. Definitions

Music is expressed along multiple parametric dimensions. This paper will focus on two main dimensions (Figure 1):

– Melodic dimension (melo) defined by pitch differences between successive notes. (In scores, pitches are represented by the vertical position of the notes.)

– Rhythmic dimension (rhyt) defined by durations between successive notes, and expressed with respect to metrical unit. For instance, in a 6/8 metric (whose metrical unit is the 8th note) a dotted 8th note correspond to the value 1.5.

A repeated succession of descriptions forms a pattern, whose occurrences are these repetitions. The pattern can be modeled as a chain of states, each successive state representing each successive note of occurrences, and each successive transition describing each successive intervals between successive notes (see figure 1). The set of all motives can be represented as a prefix tree, since two motives with same prefix can be considered as two different continuations of this prefix.
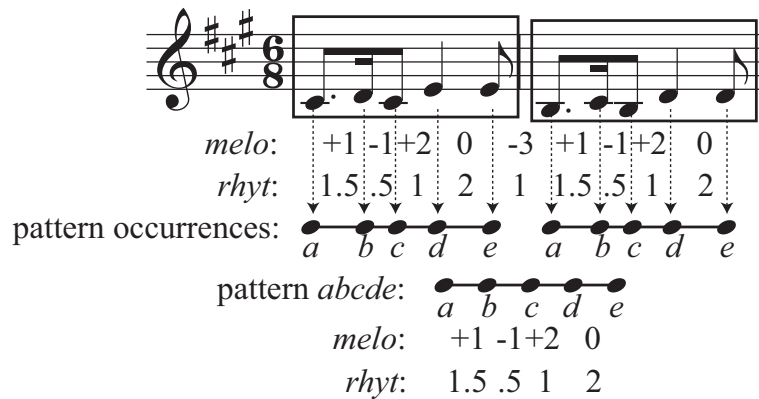

Figure 1. *Multi-dimensional description of a musical sequence.*

2.2. Identification of similarities

Patterns are generally not exactly repeated but transformed in multiple ways. These patterns should therefore be detected through an identification of their different occurrences beyond their apparent diversities. Current approaches follow two different strategies. One is based on numerical similarity, and tolerates a certain amount of dissimilar-

ity between compared parameters [Cope, Rolland]. The main drawback of this strategy arises from the impossibility of fixing precisely similarity thresholds, on which identification decision are based, and hence insuring relevant analyses. Reference cognitive studies [Dowling], on the other hand, assert that similarity does not come from numerical distance minimization, and propose instead an alternative strategy based on exact identification along multiple musical dimensions of various specificity levels. Several approaches to pattern discovery follow this second strategy of identification along different musical dimensions [Cambouropoulos, Meredith] and search for repetitions along each different dimension and product of dimensions. Nonetheless there exist patterns that are progressively constructed along variable successive musical dimensions. These heterogeneous patterns cannot be identified by traditional approaches. For instance, each line of the score in Figure 2 contains a repetition of a same pattern: in the first half, both melodic and rhythmic dimensions are repeated whereas, in the second half, only the rhythmic dimension is repeated. The model presented in this paper is able to discover heterogeneous patterns.
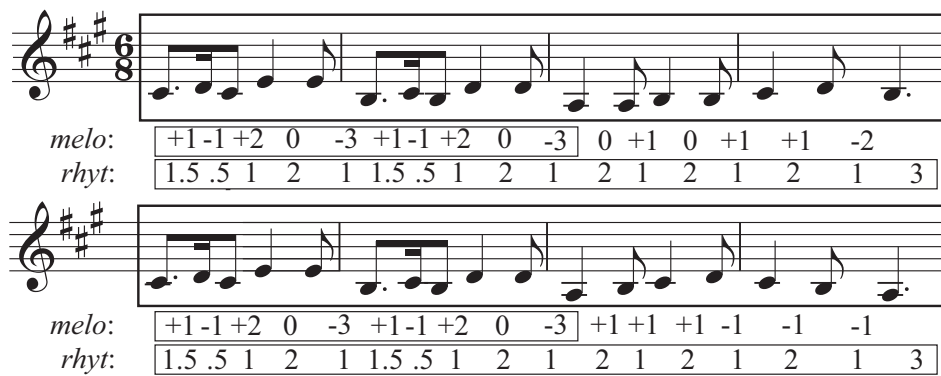

Figure 2. *Repetition of a heterogeneous pattern.*

2.3. Incremental pattern construction

The basic principle of our algorithm, aimed at an exhaustive discovery of repeated patterns, refers to associative memory, i.e. the capacity of relating items that feature similar properties. The associative memory is modeled through hash tables related to the different musical parameters (i.e. melodic and rhythmic dimensions). A first set of hash tables store the intervals of the piece with respect to their values along each different musical dimension. For instance, two tables (Figure 3, line a) store the intervals of the score according to their melodic and rhythmic values. The melodic table shows that the

first interval of each bar shares same melodic value melo = +1, and, the rhythmic table indicates another identity rhyt = 1.5.
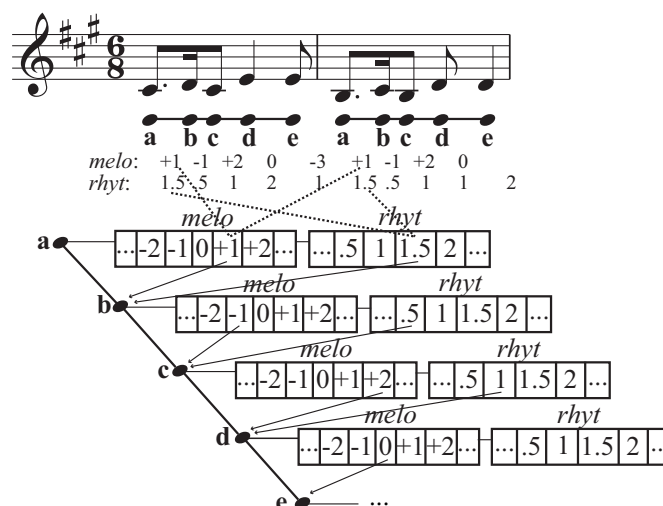
Figure 3. *Progressive construction of pattern abcde.*

Intervals sharing a same value form occurrences of an elementary pattern that simply represents this particular interval parameter. The elementary pattern is represented as a child (here b) of the root of the pattern tree (a). Each time a new pattern is created, new tables (at the right of node b) store all the possible intervals that immediately follow the occurrences of the new pattern (b). When any identity is detected in these new tables, a new pattern is created as an extension of the previous one (c, as an extension of b), and is represented as a child in the pattern tree, and so on. This algorithm enables a progressive discovery of the successive extensions of each pattern, either homogeneous or heterogeneous: the selection of musical dimensions defining each successive extension of a pattern may vary. For instance, in Figure 3, the last extension of pattern abcde is simply melodic since the rhythm of the last interval in each occurrence is different. Besides, additional constraints have been integrated in order to insure a minimal continuity along these variable successive musical dimensions.

3. Combinatorial redundancy filtering

A running of the basic algorithm on musical examples, even simple, produces a huge number of patterns that do not correspond, for most of them, to actual perceived structures,

and implies a combinatorial explosion. The complexity is commonly reduced through a filtering of the results following global criteria, such as a selection of longest or most frequent patterns [Cambouropoulos, Meredith, Lin]. However, this filtering does not improve the perceptive relevance of the results,and may arbitrarily discard interesting patterns.

3.1. Multi-dimensional closed patterns

In fact, the pattern discovery task implicitly leads to combinatorial explosion. Indeed when a pattern of length l is discovered, all its $2l - 1$ subpatterns may be considered as patterns too and would then be discovered explicitly by the algorithm [Zaki]. One way to avoid this redundancy consists in focusing only on maximal patterns, that is: patterns that are not subpatterns of other patterns. This heuristics enables a significant reduction of redundancy, but leads also to important loss of information. Pattern aij (which can simply be denoted by its last state j) in figure 4 is a simple prefix of pattern abcde (or e). It does not need to be explicitly represented, since the set of its occurrences (or pattern class) can be directly deduced from the class of its superpattern e.
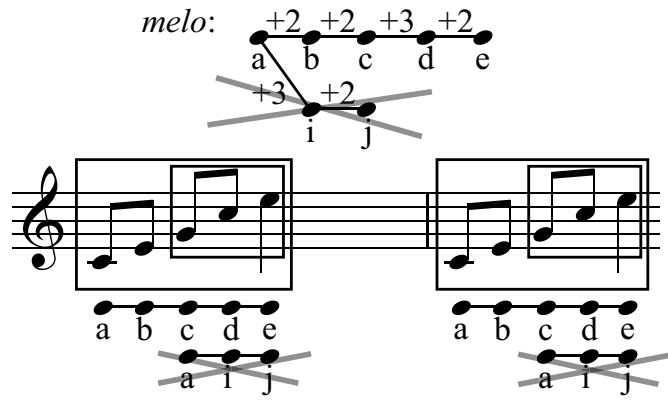
Figure 4. *Pattern aij, suffix of abcde with same support, is not explicitly represented.*

In figure 5, on the contrary, the pattern class of j cannot be directly deduced from the pattern class of e, and should therefore be explicitly represented in the final analysis. This principle corresponds exactly to the notion of close pattern, which are patterns whose number of occurrences (or support) is not equal to the support of their superpatterns.

The model presented in this paper looks for closed pattern in musical sequences. For this purpose, the notion of inclusion relation between patterns founding the definition of closed patterns needs to be generalized to the multi-dimensional parametric space of
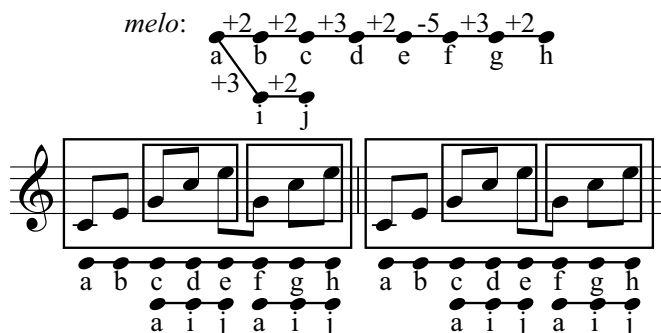
Figure 5. *Pattern aij, whose support is now greater than the support of abcde or abcdefgh. is explicitly represented.*

music, defined in previous section. This problem can be solved using the Gallois correspondence between pattern classes and pattern description, as studied in Formal Concept Analysis theory in particular [Ganter]: each pattern may be considered as a concept C=(G,M), where G is the pattern class, or set of objects of the concept, and M is the pattern description. A notion of subconcept-superconcept relation between concepts is defined: C1=(G1,M1) is a subconcept and C2=(G2,M2) is the corresponding superconcept, if the description M-1 is included into the description of M2, and, reversely, the pattern class of G2 is included into the pattern class of G1. A subconcept will be called less specific than its superconcept [Zaki]. For instance, pattern abcde (in Figure 6) features melodic and rhythmical descriptions, whereas pattern afghi only features its rhythmic part. Hence pattern abcde can be considered as more specific than pattern afghi, since its description contains more information. When only the two first occurrences are analyzed, both patterns having same support, only the more specific pattern abcde should be explicitly represented. But the less specific pattern afghi will be represented once the last occurrence is discovered, as it is not an occurrence of the more specific pattern abcde.

3.2. Avoiding redundant description of pattern occurrences

We have prolonged this attempt to optimize pattern descriptions by adding a principle of maximally specific descriptions of pattern occurrences: when a pattern occurrence is discovered (pattern e in Figure 6), all the occurrences of less specific patterns (pattern i) are not superposed on it, since they do not bring additional information, and can be directly deduced from the most specific pattern occurrence (e) and from the specificity relation (between e and i). The less specific description should be taken into account implicitly though, because their extensions may sometimes lead to specific descriptions. For instance (Figure 7), groups 1 and 3 are occurrences of pattern h, and groups 3 and 4 are occurrences of pattern d. Since pattern d is more specific, the less specific pattern h
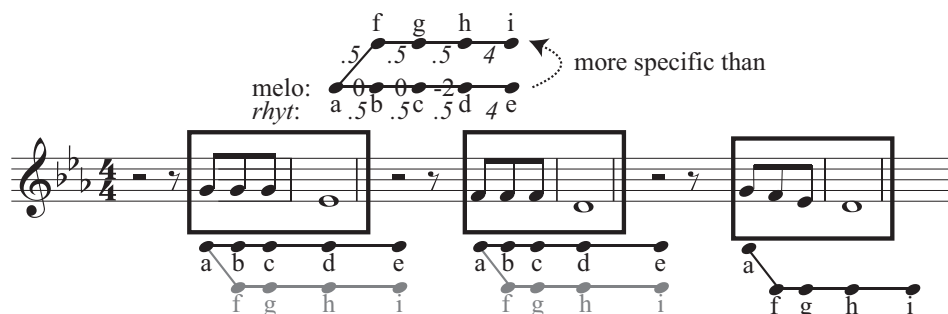
Figure 6. *The rhythmic pattern afghi is less specific than the melodico-rhythmic pattern abcde.*

does not need to be associated with group 4. However in order to detect groups 2 and 5 as occurrences of pattern l, it is necessary to implicitly consider group 4 as an occurrence of pattern h. Hence, even if pattern h, since less specific than d, was not explicitly associated with group 4, it had to be considered implicitly in order to construct pattern l. Implicit information is reconstituted through a traversal of the pattern network along specificity relations.
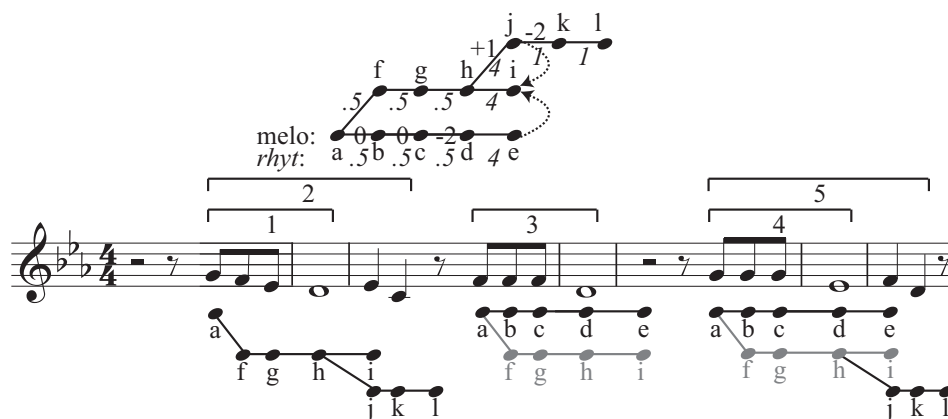
Figure 7. *Group 4 can be simply considered as occurrence of pattern d. However, in order to detect group 5 as occurrence of pattern l, it is necessary to implicitly infer group 4 as occurrence of pattern h too.*

3.3. Generalization of patterns

New patterns can be discovered as simple generalizations of already known patterns. In bar 7 of Figure 8, the two first notes form an occurrence of pattern h. The third note cannot however fulfill the known extension of pattern h into pattern i, because the melodic description melo = 0 does not match here. However, as the rhythmic description rhyt = 2 matches, a new extension j is discovered as a generalization of pattern i. The less specific patterns, although usually not explicitly represented in the analysis, should be updated if necessary. In particular, when a generalization of a pattern is discovered, the generalization of all its more general patterns should also be considered. For instance, as i has been generalized into j, it should also be inferred that c is generalized into k in the same way. Hence the analysis of the next bar (8) consists simply in recognizing this general pattern k already known.

4. Cyclic Patterns

In this section, we present another important factor of redundancy that, contrary to closed patterns, has not been studied in current general algorithmic researches.

4.1. Periodic Sequences

Periodicity in sequence descriptions leads to other kinds of combinatory explosion. Indeed, as can be seen in figure 8, all possible periods (i.e. all the possible rotations of one period) can be considered as patterns, as well as all possible concatenations of periods and their different prefixes. These redundant structural artefacts should be replaced by a compact representation that explicitly describe the structural properties of such configuration. For this purpose, we propose to model periodic sequence through cyclic graphs. A cyclic pattern chain (CPC) is constructed from an originally acyclic pattern chain (APC) representing one period of the cycle, where a transition is added from the last state to the first state. In this way, the whole local periodicity can be represented by a single POC where each successive is uniquely linked to one phase of the CPC.

Each successive state of a pattern chain is related to each successive prefix of the pattern occurrence. For this reason, concerning the long chain representing a local periodicity, the first states that represent the first period should not be associated with the CPC since they are already associated with the APC of that period. On the contrary, the states of the long POC following the first period will be associated to the CPC, since they represent a configuration that is actually specific to the periodicity. In this way, the CPC may be considered as a child of the APC, as can be seen in the figure.

This additional concept immediately solves the redundancy problem. Indeed, each type of redundant structure considered previously are non-closed suffix of prefix of the
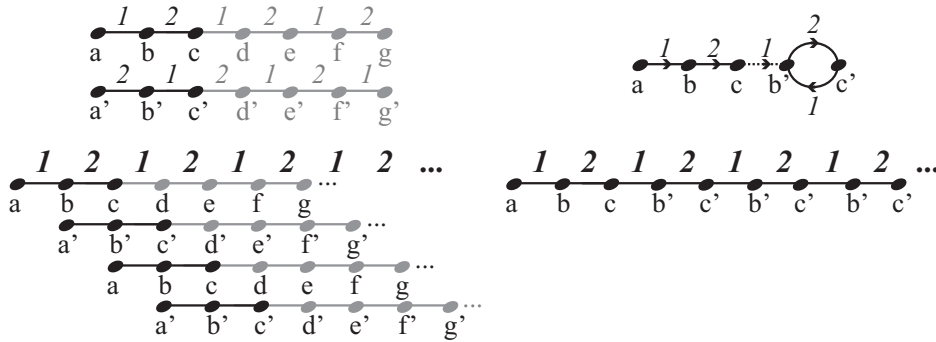
Figure 8. *Multiple successive repetitions of pattern* abc *form a complex intertwining of non-perceived structures.*

long pattern chain, and will therefore not be represented any more. But this compact representation will be possible only if the initial period (corresponding to the APC) is considered and extended before the other possible periods. That is to say, in figure 8, the APC abc should be considered before $a'b'c'$. This shows therefore that *the sequence needs to be scanned in a chronological way*. This justifies therefore the incremental approach followed by the algorithmic realisation of the modelling.

For instance, figure 6 represents in fact two progressive step of the analysis of a score. When only the first two occurrences are considered, pattern $afghi$ is considered as redundant and therefore not represented, then once the third occurrence is discovered, pattern $afghi$ is inferred as the most specific description of this new occurrence. Moreover, the new pattern $afghi$ can be constructed immediately as a direct generalisation of pattern $abcde$ through a discard of some of the parameters. This generalisation mechanism enables the inference of less specific patterns that does not follow the classical (and more expensive) process based on associative memory.

4.2. Related Works

Researches have been dedicated to the automated discovery of periodic patterns in time series data [Han98, Han99, Ma, Yang]. But as the search is focused on periodic patterns only, no interaction is proposed with acyclic pattern discovery. Hence, although offering interesting descriptions of time series data, they cannot be used in order to solve the combinatory problem presented in the previous paragraph. In our approach, on the other hand, the periodic pattern problem is deeply articulated with the acyclic pattern discovery process, insuring the compactness of the results.

A simpler solution to the combinatory problem consists in forbidding overlapping between patterns [Tanaka]. But this heuristics presupposes that time-series data are seg-

mented into one-dimensional series of successive segments. Time-series data do not all fulfil this requirement: musical sequences, in particular, may sometimes be composed of multi-levelled hierarchy of structures. Another solution is to control the combinatorial explosion by selecting, once the analyses completed, patterns featuring minimal temporal overlapping between occurrences [Cambouropoulos]. But as the selection is inferred globally, relevant patterns may be discarded. Besides combinatorial redundancy remains problematic since the filtering is carried out after the actual analysis phase. Our focus on local configurations enables a more precise filtering.

4.3. General and Specific Cycles

The integration of the concept of cyclic pattern in the multidimensional musical space requires a generalisation of specificity relations, defined in previous section, to cyclic patterns. A cyclic pattern C is considered as more specific than another cyclic pattern D when the sequence of description of pattern D is included in the sequence of description of pattern C. For instance, figure 9 displays four different cycles, the less specific cycle $d'' \circlearrowleft f$ describes the alternation of 1 and 2, the most specific cycle $b'' \circlearrowleft g'$ describes the alternation of 1A and 2B, and the two other cycles $b' \circlearrowleft c'$ and $d' \circlearrowleft e'$ are in-between in the specificy graph. All these four cycles forms therefore an oriented graph called *specific graph* (SG) whose root is the less specific cycle $d'' \circlearrowleft f$.
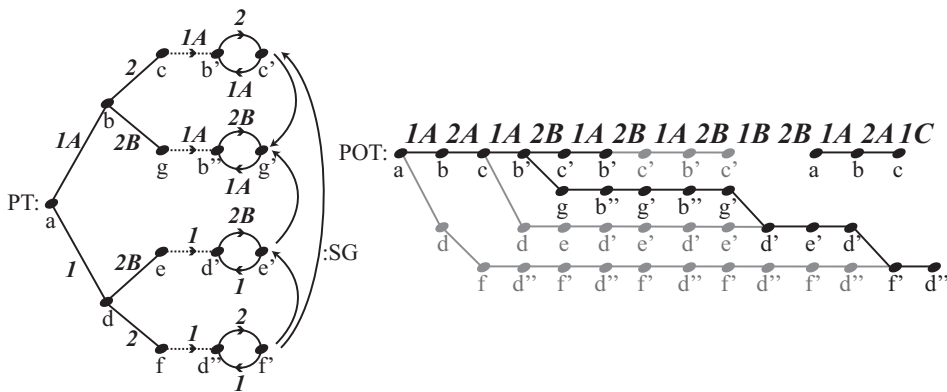

Figure 9. *More detailed analysis of the perceived cyclic configurations.*

As for acyclic patterns, in order to avoid combinatory explosion and to improve the compactness of the representation, cyclic patterns need to be filtered using the closure heuristics: i.e., only closed cyclic patterns should be selected. As seen previously, the different possible patterns are considered in a chronological way, and new general patterns are constructed through generalisation, and specific patterns through specialisation. See

figure. Moreover, the pattern tree is constructed in a most compact way, by discarding chains that are less specific than others chains.

4.4. The Figure/Ground Rule

Another kind of redundancy appears when occurrences of a pattern – such as pattern acd <1A 2A> in figure 10 – are superposed to a cyclic pattern (b'), such that the pattern acd is more specific than the cycle period (b' simply representing the successive repetition of As). In this case, the intervals that follow these occurrences are identical, since they are related to the same state (b') of the cyclic pattern. Logically the pattern could be extended following the successive extensions of the cyclic patterns (leading to pattern e, and so on). This phenomenon, which frequently appears, leads to another combinatorial proliferation of redundant structures if not correctly controlled by relevant mechanisms. On the contrary, following the *Gestalt* Figure/Ground rule, the pattern acd can be considered as a specific figure that emerges above the periodic background. Following the *Gestalt* rule, the figure cannot be extended (into d) by a description that can be simply identified with the background extension. This rules shows the interest of integrating cognitive rules into the model, as these rules concern as much the perceptive adequacy of the results than the computational efficiency of the process.
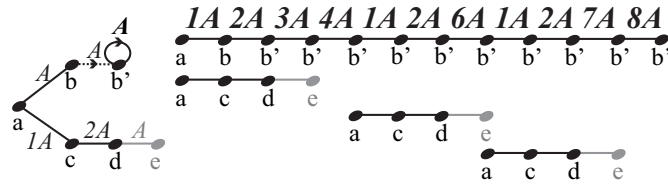
Figure 10. *Pattern c is a specific figure, above a background generated by the cyclic pattern b'.*

5. General results

This model was first developed as a library of *OpenMusic* [Assayag], called *OMkanthus*. A new version will be included in the next version 2.0 of *MIDItoolbox* [Eerola-Toiviainen], a *Matlab* toolbox dedicated to music analysis. The model can analyse monodic musical pieces (i.e., constituted by a series of non-superposed notes) and highlight the discovered patterns on a score.

5.1. Experiments

The model has been tested with different musical sequences taken from several musical genres (classical music, pop, jazz, etc.). Table 1 shows some results. The experiment has been undertaken with version 0.6.8 of *OMkanthus* on a 1-GHz PowerMac G4. A musicologist expert has validated the analyses. The proportion of patterns considered as relevant is displayed in the table.

Table 1. *Results of analyses, either melodic (M) or melodico-rythmic (M+R), performed by* OMkanthus *0.6.8.*

Musical sequence		Anal.	Pattern classes			Comp.
Name	**Notes**	**type**	**Disc.**	**Relv.**	**Succ.**	**time**
Geisslerlied medieval song	108	M	6	5	83%	2.2 sec.
Au clair de la lune folk song	44	M+R	21	5	24%	5.6 sec.
Bach, *Invention in D minor* BWV 775	283	M	49	34	69%	37.6 sec.
Mozart, *Sonata in A* K331 1st theme, 1st half, melody	36	M+R	14	10	71%	0.8 sec.
The Beatles *Obla Di Obla Da*	390	M	14	10	71%	28.1 sec.

The analysis of a medieval song called *Geisslerlied* – sometimes used as a reference test for formalised motivic analysis – gave quite relevant results. The analysis has been actually carried out on a slight simplification of the actual piece presented in [Ruwet], excluding local motivic variations out of reach of the current modelling.

The melodico-rhythmic analysis of the French song *Au clair de la lune* posed problems: 21 patterns were discovered from a 44-note long sequence. This is due to the fact that the successive steps of progressive generalisation or specification of cycles are currently modelled using distinct intermediary cyclic patterns. The inference of these redundant cyclic patterns will be avoided in further works.

The algorithm has been successfully applied on a melodic analysis of a complete two-voice *Invention* by J.S. Bach. Figure 11 shows the analysis of the 21 first bars. The repetition of ascending quarter notes in bars 3 and 4 has not been detected because the contour dimension was not considered in the experiment. The cyclic patterns are represented by graduated lines, the graduation representing each return of one possible phase. Due to the nature of the cyclic patterns, no preference is given by the model between different possible phases of the same cycle. The rhythmic analysis of the piece, on the contrary, failed, due to the alternation of sequences of either quarter notes or 8th notes,

which will require a formalisation through hierarchical pattern chains (where successive states of higher-level patterns are linked to distinct lower-level patterns).
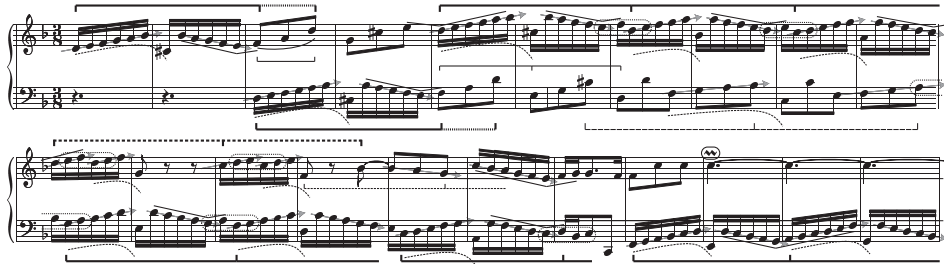
Figure 11. *Automated motivic analysis of J.S. Bach's* Invention in D minor *BWV 775, 21 first bars. The occurrences of each pattern class are designated in a distinct way.*

The analysis of The Beatles' *Obla Di Obla Da* melody shows 14 relevant pattern classes, representing the chorus, verses, phrases and motives inside each of these structures. The 4 irrelevant patterns are redundant patterns subsumed by the 14 relevant ones.

In all these pieces, some patterns are considered as irrelevant because they cannot be perceived as such by listeners. Additional mechanisms should be added to prevent these irrelevant inferences, based on short-term memory, top-down mechanisms, etc.

5.2. About Algorithm Complexity

The algorithm complexity may be expressed first in terms of discovered structures: proliferation of redundant patterns, for instance, would lead to combinatorial explosion, since each new structure needs proper processes assessing its interrelationships with other structures, and inferring possible extensions. Hence a maximally compact description insures in the same time the clarity and relevance of the results and the limitation of combinatorial explosion. Concerning technical implementation, the prototype needs further optimisations. Yet the modelling has been conceived with a view to minimising computational costs. Hence the identification of similar descriptions is based on hash tables, which reduce time complexity.

The overall computational modelling results in a complex system formed by a large number of highly dependent mechanisms. Without a real synthetic vision of the whole system, no general assessment of the global complexity of the modelling has been achieved yet. The complete rebuilding of the modelling currently undertaken should enable a better awareness and control of complexity.

6. Cognitive Study of Tunisian Modal Music

The project, presented in this paper, of modelling of musical pattern discovery processes has been integrated into a more general collaborative project between computer science, ethnomusicology and cognitive sciences. The main objective of this project is to design a cognitive modelling, using a complex system, of the processes of music perception and understanding, in order to understand the perceptive, musical and computational aspects of sequence segmentation and patterns recognition.

This study has been focused on Tunisian Modal Music, and particularly on *Tba'*, a modal system that presents interesting configurations. A *Tba'* is based on a *musical scale* – i.e. a set of pitches, such as (C, D, E, F, G, A, B) for instance – subdivided into two or several sub-scales called *genres*. Each *genre* is characterised by pivotal notes (more important than others) and melodic profile. Hence in a specific genre, the pitches that it contains are played in a specific order. Genres themselves are hierarchically connected one with the others. In the musical scale of the *Tba'*, some of the notes play particular role in the mode: some are mostly played at the beginning of the improvisation, or at the end of phrases. Finally, to each *Tba'* is also associated a set of characteristic melodic patterns. Figure 12 presents an example of *Tba'* modal structure.

Figure 12. *Description of the* Tba' Mhayyer Sika D*(*Ré*), in terms of a sets of Genres and pivotal notes.*

The study has been focused on one particular improvisation by the *Nay* flute player Mohamed Saada, along the *Tba' Istikhbar Mhayyer Sika*. First, the improvisation has

been transcribed from an audio record into a musical score. Then the resulting symbolic sequence has been analyzed by the modeling.

6.1. Psychological experiments

Psychological experiments have been carried out in order to obtain a detailed description of listening strategies, and to assess the role of cultural schemes in particular. This study has been focused in particular on the determination of the patterns that form the basic structures of the musical *genres*. In order to understand the impact of cultural knowledge on this particular task, two groups of subjects have been considered: one group formed by European subjects unfamiliar to Arabic modal music, and another group formed by Arabic subjects of various degree of expertise in this music. Subjects have been asked to performed several tasks successively: First of all, after hearing the musical piece, they have to recognise the most salient musical structures and, using these structures, to reduce the whole improvisation in order to exhibit the dynamic macro-structure of the piece.

The experiments have been first carried out in Paris on European subjects, and will be extended to Arabic subjects in Tunisia in the end of this year. The first results shows the relative variability and divergence of the judgements of European subjects. This is due to the fact that they cannot follow their own cultural scheme when analysing Arabic modal improvisations. They have to rely instead on the structural characteristics of the musical discourse, and in particular the discovery of repeated patterns.

The collaboration between experimental psychology and computational modelling is twofold. First of all, the experimental results have been used in order to improve the model presented in the previous sections in order to take into account the stylistic characteristics of Arabic modal improvisations. Then, the results offered by the improved version of the model (as presented in the next paragraph), will be validated throughout a second listening test.

6.2. Improvement of the modelling

One major limitation of the first version of the modelling, as presented in previous sections, is that only repetition of sequences of notes that are immediately successive could be detected. In music in general, and in modal improvisation in particular, repeated patterns are often ornamented: secondary notes can be added whose purpose is to emphasise the primary notes of the initial pattern. Figure 13 displays, for instance, a melodic phrase, and one possible ornamentation. To some of the notes of the original phrase are added secondary notes (displayed with smaller size in the score) that are located in the neighbourhood of the primary notes, both in time and pitch dimensions.

Figure 13. *A melodic phrase and an ornamented version of it.*

In order to take into account these ornamentation, a set of mechanisms have been added to the modelling. Solutions have been proposed [Rolland] based on optimal alignments between approximate repetitions using dynamic programming and edit distances. We have developed algorithms that automatically discover, from the rough surface level of musical sequences, musical transformations revealing the sequence of pivotal notes forming the deep structure of these sequences. These mechanisms induce new connections between non-successive notes, transforming the *syntagmatic chain* of the original musical sequence into a complex *syntagmatic graph*. The direct application of the pattern discovery algorithm on this syntagmatic graph enables the detection of ornamented repetitions.

6.3. Results of the computational modelling

The analysis of Mohamed Saada's improvisation of *Istikhbar Mhayyer Sika* is displayed in figure 14. The discovered structures are represented below each line of the score. Each line represents an occurrence of the pattern, designated by a sign (1, 2, 3, 4, 5, + and -) on the left of the line. The notes actually considered by each pattern occurrence are represented by squares vertically aligned to the notes. These squares represents therefore the successive states along the pattern occurrence chain, as shown in Figure 1.

Pattern '-' represents a simple sequence of notes of continuously decreasing pitch heights, and pattern '+' represents a sequence of notes of continuously increasing pitch heights. Patterns 1 to 5 are sequences repeated several times in the improvisation. Each black square represents the beginning of a new occurrence, and each white square one successive state along the pattern chain. Grey squares corresponds to optional states that are not found in all the occurrences of the pattern. Finally, multiple branches designates multiple possible paths for one same pattern occurrence.

The improvisation is built on the specific mode *Tba' Mhayyer Sika D*, characterised by the use of a specific set of notes (D, E, F, G, A, Bb, C) and a specific melodic figure, which corresponds exactly to the pattern 2. The beginning of the improvisation is also based on the successive repetition of pattern 1, which corresponds to a periodic melodic curve starting from note F and ending to the same note F, which is therefore a pivotal note of the improvisation. This pattern correspond to the mode *Mazmoum F* shown in figure 12. The second line of the improvisation is characterised by the successive repetition of pattern 3, which is a little melodic line progressively transposed. Pattern 4 corresponds

Figure 14. *Analysis of the beginning of* Istikhbar Mhayyer Sika *improvised at the* Nay *flute by Mohamed Saada.*

to another important melodic profile associated to pattern 2. Finally the two last lines of the improvisation are characterised by the repetition of pattern 5. Patterns 2 and 4 may be considered as stylistic characterisations of the mode *Istikhbar Mhayyer Sika* whereas patterns 1, 3 and 5 shows the characteristics of the individual style of the improviser.

The integration of these new mechanisms is not completely achieved. The application of the pattern discovery algorithm in the general syntagmatic graph leads to combinatorial explosion of redundant patterns not fully controlled yet, which will need further works.

7. Current Researches

7.1. Addition of Segmentation Principles.

The structures currently found are based solely on pattern repetitions. Segmentation rules based on *Gestalt* principles of proximity and similarity [Lerdahl,Cambouropoulos] need to be added. Although this rule plays a significant role in the perception of large-scale musical structures, there is no common agreement on its application to detailed structure, because it highly depends on the subjective choice of musical parameters used for the segmentations. The study will focus in particular on the competitive/collaborative interrelations between the two mechanisms, in particular the masking effect of local disjunction on pattern discovery.

7.2. From Monody to Polyphony.

Our approach is limited to the detection of repeated monodic patterns. Music in general is polyphonic, where simultaneous notes form chords and parallel voices. Researches have been carried out in this domain [Meredith], focused on the discovery of exact repetitions along different separate dimensions. Our model will be generalised to polyphony following the syntagmatic graph principle. We are developing algorithms that construct, from polyphonies, syntagmatic chains representing distinct monodic streams. These chains may be intertwined, forming complex graphs along which the pattern discovery algorithm will be applied. Pattern of chords may also be considered in future works.

7.3. Applications to Musical Databases.

The automated discovery of repeated patterns can be applied to automated indexing of musical content in symbolic music databases. This approach may be generalised later to audio databases, once robust and general tools for automated transcription of musical sound into symbolic scores will be available. A new kind of similarity distance between musical pieces may be defined, based on these pattern descriptions, offering new ways of browsing inside a music database using pattern-based similarity distance.

8. References

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