Vuza Canons into the Museum

Georges Bloch

Abstract. A remarkable feature of OpenMusic is, of course, the ability to experiment with musical structures that, otherwise, would be impossible to construct by hand. An example is "canons of maximal categories", that is, non commonplace rhythmical figures that, when played in canon, form a continuum without any voice playing simultaneously with another. A mathematical theory of these canons has been developed by the Rumanian mathematician Dan T. Vuza and expanded by Moreno Andreatta at IRCAM. This is an intuitively evident structure that has interested numerous composers, among them Messiaen. But the construction of such a canon was beyond the means of composers working without computers, and research on the characteristics of such objects requires a musical representation tool such as OpenMusic.

Another possibility provided by OM stems from the fact it is an extension of the LISP programming language. This allows the use of specific research (in this case, the research on constraint programming carried out by Charlotte Truchet) to be applied to musical structures.

1 Introduction

1.1 The Beyeler Project

In July 2001, a piece of mine was performed, in the framework of the "composers of the week" series of the Europäischer Musikmonat. Entitled *Fondation Beyeler: une empreinte sonore*, this project basically consisted of a musical visit of the collections of the Fondation Beyeler. The music for this project made extensive use of the so-called "Vuza canons".

Fondation Beyeler is a museum designed by architect Renzo Piano to house the private collection of the art dealer Ernst Beyeler. It is located in Riehen, a small town east of Basel (Switzerland). Containing mainly pictures and sculptures from the 20th century, it is one of the most impressive private collections in the world. There are also special exhibitions, for the most part these come from well-known relatively provocative periods of modern art. The performance took place at the same time as a special exhibition entitled *Ornament and Abstraction*. Organized by Markus Bruederlin, the Fondation Beyeler curator, its main idea was that ornament was the "stowaway" of abstract art.¹

This special exhibition was divided into ten sections. In a Prologue, 18th Century Moroccan windows were presented side by side with Rothko and Taafe paintings. Entitled In the Beginning was Ornament, the first chapter displayed the long history of the line (especially in the form of Arab Calligraphy) before it was used by abstract artists such

¹See the catalog for the exhibition: *Ornament and Abstraction*, Dumont / Fondation Beyeler, 2001, which has been published in German and in English.

as Paul Klee. The next two chapters show how, in Munich and Vienna at the turn of the 20th Century, Art Nouveau and Secession artists had already achieved a transition from folk ornament or architectural shapes to Abstraction. The fifth chapter shows how Abstraction eventually spread to the mural form, as in Matisse's *papiers collés* or, more recently, through fractal murals like those by Sol LeWitt. The next sections were devoted to the relationship between ornament and signs. The two last sections showed the ornamentalisation of modernism and the birth of digital mass-ornament (including video installations by Peter Kogler and Shirin Neshat).

It is important to emphasize that *Ornament and Abstraction* made wide use of the Beyeler collection paintings, to which were added paintings borrowed from other museums as well as creations (from various artists such as Kara Walker, Paul Buren or Sol LeWitt) created especially for this exhibition.

1.2 The musical setup

The musical project consisted of five musical visits, each one being led by a musician (violin, clarinet, tenor saxophone, double bass and percussion). There was no question of "setting the pictures to music", since there is absolutely no need to add music to a Picasso or Sam Francis picture. In this respect, it was really a visit in the teaching sense of the term, since the music primarily demonstrated existing relationships between pictures and space or between the works of art themselves.

One visit was special in that, contrary to the other musicians, the clarinettist was accompanied by a (speaking) guide. Both followed the "*Tour Fixe*", an extensive visit of the foundation (including *Ornament and Abstraction*). This tour, which looked like any regular tour, was the "easy-access" visit and was followed by a large number of the German-speaking audience (at least at the beginning of the performance, since many people moved from one visit to another). Two tours (saxophone and double bass) were devoted to the special exhibition *Ornament and Abstraction*. The last two visits were thematic. One was about "Picasso and the human Face" (violin). Another was entitled "Imaginary Landscapes" (percussion). The first explored the fabulous Picasso collection belonging to Ernst Beyeler, as well as a large number of African and Melanesian masks, and representations of the human figure. The second one focused mainly on abstract landscape representations (Claude Monet, Frank Stella, Sam Francis, Francis Bacon, ...) actually passing through the Stella part of *Ornament and Abstraction*.

There were several reasons for this idea of simultaneous visits. First, a museum, even a relatively small one, is a place in which it is impossible to see everything. Therefore, the presence of music in another room demonstrated the impossibility of hearing the whole "concert" and, consequently, the impossibility of grasping all the pictures that were being seen, had been or were to be seen. There were pictures that would never be seen. Also, when two guided tours meet in a room, the listener's attention was immediately drawn—in a somewhat rebellious fashion—to the guide of the other group, especially if that person was talking about a work already explained by the listener's own guide. Sometimes guides collaborated. A number of similar techniques were explored. The musical work was performed by four large ensembles: a trio, two duets and a finale, with all five players present (there were other smaller ensembles, with lesser structural impact). The entire event lasted approximately 70 minutes.

1.3 Some reasons for using Vuza canons

Outside of my personal interest, Vuza canons were relevant to one important idea developed in *Ornament and Abstraction*: the disappearance of the ground, or at least an ambiguity existing between ground and figure. In Matisse's *Acanthes* or *Océanie*, for example, the figures (leaves) become part of the huge wallpaper that is the picture. This applied also to the huge *Sol LeWitt* mural that marked the beginning (and the end) of the visits.

Let us recall that these canons are non-trivial solutions to a basic rhythmical problem: having identical rhythmical voices which, when they are all present, fill up a rhythmical grid without overlapping. We say non-trivial, since there are obvious but trivial solutions: when the grid step is a sixteenth note, for example, a single voice playing each quarter note could be followed by a similar one a sixteenth later, a second one an eighth note later, and finally a third one three sixteenths later. This would be a valid solution since the grid is full, with no voice overlaps.

An interesting aspect of these canons is that they provide a relevant metaphor to the premises of *Ornament and Abstraction*. The final result (when all voices are present) is a rhythmical drone, like an ostinato: the dialectic between form and content is immediately audible, since the final result of such a contrapuntal display is a single continuous line.

The canons were used in many ways: for instance, the clarinet player gave a signal every five minutes. This signal was itself the development of a canon. As with a tiling canon, these calls ended up sounding like a continuum, but each call was also a more complete version of the preceding one. The canon characteristic of these pieces became less important than their tiling aspect.

The ensembles, as we mentioned, structured the performance. Three of those (corresponding roughly to the first third, the second third and the end of the visit) are Vuza canons, entitled *Canon 1*, *Canon 2* and *Canon Final. Canon 1* is scored for clarinet, saxophone and double bass. *Canon 2* for violin and berimbao (a Brazilian percussion instrument which is a kind of musical bow). The last Canon starts with a repeat of *Canon 1*, a different version of *Canon 2* for violin, double bass and vibraphone and ends after a bridge with a third canon with everybody playing.

Some other versions of *Canon 2* appear at various moments. One of them, for solo percussion, is of particular interest: it is scored for solo percussion but it is not perceived as a canon anymore, although when *Canon 2* performed later on, it becomes obvious that it is the same music. Each instance of the *Canon 2* corresponded to a frame of the Matisse pictures entitled *Océanie*.

1.4 Three practical problems

Although we will come back to the aesthetic reasons for using the canons in this project, they are not the subject of this paper. When using the Vuza canons, several questions arise, linked to aesthetic choices, but which end up being very practical problems indeed.

The number of voices.

The simplest solution given for Vuza canons is a six-voice canon on a period of 72 units. What are the practical solutions when there are five players (or less) playing a six (or more) voice counterpoint with a relatively perceptible result? More especially, is the canon character (the rhythmical unity of the voices) lost?

The relationship between canons.

Is it possible to generate a large scale form from several canons? More precisely, how to make them relate, especially if they have neither the same period nor the same number of voices?

Continuum or texture?

When the voices are played by different instruments, with diverse attack times and resonance characteristics, even a very precise execution will fail to create the impression of a continuous line, except at very slow tempi. It is perhaps more useful to think in terms of a texture of equal weight and equal distribution. Working out the harmonic evolution in such dense textures can be difficult, especially with notes changing at every division of the beat. That is why a global texture evaluation can be helpful.

2 Basic strategies for choosing canonic values

The choice for the values of Vuza canons was obviously aesthetic. However, the selection process is interesting, since it is based on an examination of the existing values.

2.1 Examining the rhythmical characteristics of the values

Let's take the period N = 144 as an example. This is a twelve voice canon, each composed of twelve note-events in one period. Below are the values given by the Vuza reconstruction algorithm. R is the time interval between the individual note-event of one voice, and S the time interval between the entrances of the voices. They are interchangeable.

R	S
$(1 \ 5 \ 3 \ 21 \ 19 \ 8 \ 15 \ 6 \ 19 \ 5 \ 3 \ 39)$	$(2\ 2\ 14\ 2\ 2\ 14\ 2\ 16\ 58\ 16\ 2\ 14)$
(1 5 24 11 8 23 6 11 8 5 35 7)	$(2\ 14\ 2\ 16\ 4\ 16\ 16\ 22\ 16\ 16\ 4\ 16)$
$(1\ 6\ 1\ 40\ 1\ 7\ 17\ 6\ 17\ 8\ 17\ 23)$	$(2\ 16\ 2\ 16\ 2\ 16\ 10\ 18\ 18\ 18\ 10\ 16)$
$(1 \ 6 \ 33 \ 8 \ 1 \ 24 \ 6 \ 9 \ 8 \ 25 \ 15 \ 8)$	$(2 \ 16 \ 10 \ 10 \ 16 \ 10 \ 18 \ 10 \ 16 \ 10 \ 10 \ 16)$
$(1 \ 7 \ 5 \ 12 \ 23 \ 8 \ 17 \ 12 \ 11 \ 1 \ 7 \ 40)$	$(4\ 10\ 4\ 14\ 4\ 14\ 18\ 26\ 18\ 14\ 4\ 14)$
$(1 \ 7 \ 5 \ 35 \ 8 \ 5 \ 12 \ 12 \ 11 \ 8 \ 29 \ 11)$	$(4\ 10\ 18\ 4\ 22\ 10\ 22\ 4\ 18\ 10\ 4\ 18)$
$(1\ 7\ 17\ 23\ 8\ 11\ 6\ 23\ 1\ 7\ 35\ 5)$	
$(1 \ 8 \ 33 \ 7 \ 8 \ 9 \ 6 \ 25 \ 8 \ 9 \ 24 \ 6)$	
$(1 \ 12 \ 12 \ 15 \ 8 \ 25 \ 12 \ 3 \ 8 \ 1 \ 39 \ 8)$	
$(1 \ 12 \ 27 \ 8 \ 13 \ 12 \ 12 \ 3 \ 8 \ 37 \ 3 \ 8)$	
$(1 \ 24 \ 15 \ 8 \ 19 \ 6 \ 15 \ 8 \ 1 \ 39 \ 3 \ 5)$	
$(3\ 3\ 5\ 37\ 3\ 8\ 13\ 6\ 21\ 8\ 13\ 24)$	
$(3\ 3\ 37\ 5\ 3\ 21\ 6\ 13\ 8\ 21\ 19\ 5)$	
$(3\ 6\ 24\ 7\ 8\ 27\ 6\ 7\ 8\ 9\ 31\ 8)$	
$(3 \ 8 \ 13 \ 27 \ 8 \ 7 \ 6 \ 24 \ 3 \ 8 \ 31 \ 6)$	
$(3 \ 9 \ 12 \ 19 \ 8 \ 21 \ 12 \ 7 \ 5 \ 3 \ 40 \ 5)$	
$(3 \ 9 \ 31 \ 8 \ 9 \ 12 \ 12 \ 7 \ 8 \ 33 \ 7 \ 5)$	
$(5\ 6\ 29\ 8\ 5\ 24\ 6\ 5\ 8\ 29\ 11\ 8)$	

Again these values can be freely combined. Clearly, in this case, most R values display multiples of 3 (or even 9), and S values favor multiples of 2 (or even 4). This would imply

a ternary rhythm for the subject of the canon, and a binary one for the voice entrance points (or the opposite, if we take R as entrance points and S as subjects). This result is not at all surprising, since $144 = 2^4 \times 3^2$. This is not a general result: in the case of $N = 108 \ (2^2 \times 3^3)$, we can no longer assert that values for R are "ternary", and that Svalues are rather ternary than binary.

2.2 The value of finding redundant rhythms

One of the interesting features of the canons is the way the music, starting out as a recognizable rhythmical shape (at first emphasized by the canon entries), gets transformed into a continuum. Thus, finding redundant rhythms inside the individual voices can be of value.

By implementing a short OpenMusic program that analyses R time-point values, we can find those that are most divisible by 9. Of course, we take the time-points into account, because they give rhythmical values relative to a given beat: this means that (1 5 3 21 19 8 15 6 19 5 3 39), for example, will be transformed, into (0 1 6 9 30 49 57 72 78 97 102 105 [144]). Taking the permutations into account is important: indeed, if we consider a list of values such as (1 8 9 9 9 9 9 9 ...), it would provide points perfectly on the beat of a time-division by 9; but this is not true of (8 9 9 9 9 9 9 ... 1), where all points other than the first will fall off-beat.

In the case of N = 144, the program computes a sum of the reminders by 9 of the time-points of all R elements (as well as all their permutations). For the *Canon 1* (played in front of Matisse's *Acanthes*) the third best solution (3 6 24 7 8 27 6 7 8 9 31 8) was chosen. There are several (mostly subjective) musical explanations for this choice: in order to have a clear rhythm, we wanted to avoid a "scotch snap" (short value followed by a long one, like 1 8) at the onset, and this rhythm was present in the other solutions. The third note, long, accented and clearly on the first beat makes for a clear beginning. In addition, all values with the accent on every 12^{th} unit were eliminated so as to avoid a binary feel.

One has to keep in mind that all this rhythm clarity rapidly disappears as each voice enters. That is the reason for choosing an almost "cliché" pattern. We actually selected a ternary rhythm for the voice patterns of all canons. The reason for this was that we desired to create a link between all the canons. This is the topic of our "second problem", namely the construction of a relationship between canons of different periods.

2.3 Ordering the voices

In order to make the redundancy more obvious, and therefore to delay the appearance of the continuum-like texture, we had the voices which start "on the beat" appear first. If we take 9 as the measure division, a good result for nine yields also a good one for 3, therefore implying a ternary division of the beat (for example, a measure in 9/16, where the sixteenth note is the rhythmical unit of the canon). So the voices starting on 0, 3, 6 will also reinforce the sensation of triple time.

3 Reducing the number of voices

Vuza's theory yields in many-voices structures. The smallest one (not used in *Une Empreinte sonore*) has a period of 72 rhythmical units and is divided into six voices. Six is the smallest possible number of voices. This can become a problem when one is dealing with a small number of instruments or, more simply, with a more limited number of streams in the polyphony.

The problem becomes particularly obvious when we reflect that the end result of the process is a continuum, implying a monophonic texture (whether this is unfounded or not).

3.1 The basic solution: compound voices

In some respects, the problem of the number of voices is a false one: there are countless works in the history of western music written for a monophonic instrument (for example Bach's flute pieces, etc.) The classical solution is to compound the voices, that is, to merge two lines into one by playing the notes when they are supposed to appear and stop them when another one appears, whether or not it is the same voice. This is a common procedure in baroque music.

Naturally, the same technique is perfectly applicable to Vuza canons. In our project, the clarinet made a "call" every five minutes. These were canons, with one voice being added every ten minutes.

3.2 Sub-canons

In the case of a compound solution with several instruments, another problem obscures the canon characteristics: a given value of S sets the distance between the entrances of the voices. There is very little chance that the distance between the voices of each compound will be the same. As an example, let us imagine a period of 144 with $S=(2\ 2\ 14\ 2\ 2\ 14\ 2\ 16\ 58\ 16\ 2\ 14)$. We can use permutation and put 58 in the end, which gives (16 2 14 2 2 14 2 2 14 2 16 58). If we try to generate four voices from these twelve, the starting points will be as follows:

 $V_1 = (0, 0+16, 0+16+2),$ $V_2 = (32, 32+2, 32+2+2),$ $V_3 = (50, 50+2, 50+2+2),$ $V_4 = (68, 68+2, 68+2+16).$

Only voices two and three display similar compound values (start, start+2, start+4). The other voices are different. This gives us an interesting lead in selecting the subcanon voices: if we can find a case where all values are the same, we will have similar sub-canons.

Getting a smaller number of voices

In our search for a smaller number of voices, we will therefore mix (compound) some voices together. The result will be called a *metavoice*.

Using the "on the beat - offbeat" strategy

For example, with a ternary triple time (that is three beats divided into three subdivisions) we can easily distinguish when the voices start. We have already emphasized how we can make the basic voice rhythm (here ternary) more obvious by mixing the voices starting on the beat. Here is an example: with a period of 144 and $S = (18\ 10\ 16\ 10\ 10\ 16\ 2\ 16\ 10\ 10\ 16\ 10)$, the attack time points are (0 18 28 44 54 64 80 82 98 108 118 134). A simple modulo 9 operation shows that 0, 18, 54 and 108 start "on the beat". 28, 64, 82 and 118 start one unit later. And that 44, 80, 98 and 134 start just before the beat. We would therefore build our metavoices with these three rhythmic groups. One starting on the beat $M_1=(0\ 18\ 54\ 108)$, one one sixteenth later $M_2=(28\ 64\ 82\ 118)$ and one one sixteenth before $M_3=(44\ 80\ 98\ 134)$.

However these metavoices are not themselves similar. This is because the distance between the first and the second subvoices is 18 in the first metavoice and 36 in the second and third metavoices: they cannot yield the same global result. We cannot speak of canons between metavoices, since the compound rhythmical patterns are different.

Looking for real self-similarities

But the subvoices of M_1 , as we have seen, start on 0, 18, 54 and 108. Therefore, the distances separating the respective subvoices are 18, 36 and 54. For M_2 , the subvoices start on 28, 64, 82 and 110. We find 18 between 64 and 82, 36 between 82 and 118, and... 54 between 118 and 28 on the next period (the period is 144, 144+28 = 172, that is 118+54). We find the same result for the third voice, if we take 80, 98, 134 and 44 (see figure 1).

The basic time unit is the sixteenth note, and $R = (3\ 6\ 24\ 7\ 8\ 27\ 6\ 7\ 8\ 9\ 31\ 8)$. The pitches displayed here carry no value whatsoever, they simply allow a further distinction to be made between the various voices. The first four voices (first metavoice) start on the beat. The next four voices start a sixteenth just after the beat, and the last four just before the beat. The reordering shows how the voices could be self-similar by folding some of them to the next period. In other words, by delaying the entrance of voices 7 and 12 until the next period.

Examining the first voices, we notice that the distance between voices is two bars (=18 units), then four bars, then six. But this is also the case for the next four lines and the last four lines. For voices 4 to 8, the distance between voices is two bars, then four, then by folding to the beginning, six. We can generate a completely similar voice if we delay these last voices by one period.

We can therefore change the S to emphasize the self-similar aspect, pushing the start of metavoices 2 and 3 to their proper place. Instead of (0 18 28 44 54 64 80 82 98 108 118 134) as starting points, we get (0 18 54 64 80 82 98 108 118 134 144+28 144+44). Let us use different typographical characters to distinguish our metavoices: (0 18 54 **64** 80 **82** 98 108 **118** 134 **144+28** 144+44), which gives a transformed S, (18 36 10 16 2 10 10 16 38 16 -44). There is actually another way of constructing metavoices: we can generate four metavoices by using voices of the same rank (0 18 **54** 64 80 82 98 <u>108</u> **118 134** <u>144+28</u> <u>144+44</u>).



Figure 1. The first period of a canon of period 144, with the voices reordered in order to display rhythmical similarities.

Conclusion: a slower start and the loss of the maximal category

At this point we have transformed a twelve-voice canon in a three-voice canon, each voice of which is made of a four-voice canon. This is an impressive self-similar structure, a canon made of canonic voices.

However, as we know from theory, it is an illusion that there are three-voice tiling canons in a 144 period. And here theory is borne out by practice. A careful examination of the metavoices shows that they actually have, as the theory would predict, limited transposition characteristics. For example, the metavoices in *Canon 1* repeat three times the same rhythmical pattern (see figure 2).

However, it can be taken as a virtue, because the period (48, that is a third of 144) still is relatively long, and the repetition can be taken as a characteristic of the theme.



Figure 2. The same Canon 1, in its third period, as all voices have entered. The "maximal category" characteristic is completed, and we note that there is a continuum every sixteenth note with no voices sounding simultaneously. We note (relatively easily in the clarinet part) that, as the theory predicts, the metavoices are actually repetitive patterns. The rhythm of the clarinet on the first five bars get repeated, starting on the second beat (E dotted eight) of the sixth bar, and again on the third beat of the eleventh bar of the excerpt (sung Ab). The change of beat at the end of the example prepares the transition to Canon 2.

4 Relationship between different canons

The question of the relationship between canons is even more important, if one wishes to create Vuza canons with a form that is larger than or different from the simple repetition of the periodic pattern. We noticed a similarity between the subcanons of different periods, which led to the idea of *canonic modulation*.

This is like any rhythmic modulation. As we know, there are two types of these: either the unit stays the same, or the whole group stays the same. We explored the latter case, by constructing canonic modulations that maintain the same duration for one period, but that have canons of different periods.

4.1 Canonic modulations

The idea of canonic modulation arose when we noticed similarities between the subcanons. We have seen that we had:

Canon 1	12 voices	Period 144	divided into	3 subcanons	0	64	80
Canon 2	6 voices	Period 108	divided into	3 subcanons	0	48	60

This creates a simple but remarkable numerical relationship:

$$144 = 108 \times (4/3)$$

 $64 = 48 \times (4/3)$
 $80 = 60 \times (4/3)$ to which we can add
 $0 = 0 \times (4/3)$

We can choose to have the same span of time for the periods of both canons (that is, the canon of 144 elements goes 4/3 faster than the canon of 108) and, more remarkably, each corresponding metavoice will start at the same moment in the canon.

A convenient notation is *Canon 1* in 9/16 (the unit being the sixteenth note), and *Canon 2* in 3/4 (the unit being the eighth triplet). 144 sixteenth notes are equivalent to 36 quarter notes, as are 108 eighth triplets.

In this case, not only do the whole canons have the same duration, but the three metavoices start at the same time in the canon. M_1 starts after 64 sixteenth notes = 48 triplets = 16 quarter notes, and M_2 starts after 80 sixteenth = 60 triplets = 20 quarter notes. The same logic is used in the last part of the final canon. The period 216 (= 108×2) is divided into sextuplets.

This gives us a canonic modulation from *Canon 1* to *Canon 2*. The entrances of the voices are tiled.

4.2 Analysis of an example

The figure 3 shows the passage between Canon 1 and Canon 2 in the Canon Final. This page exactly follows the music shown in the preceding figure. This example is worth careful analysis, as it demonstrates most of the processes examined in this article.

Let us first examine Canon 1. As we said, it is a canon of period 144 sixteenth notes, with $R = (3\ 6\ 24\ 7\ 8\ 27\ 6\ 7\ 8\ 9\ 31\ 8)$ and the voices starting on (0 18 54 **64** 80 **82** 98 108 **118** 134 **144+28** 144+44). The clarinet was the first to start, and is the first to end. The saxophone was the last to start and is the last to end. The cycle lasted for 16 bars in 9/16 time, but the beat has changed to 3/4 (at the same speed for the sixteenth note). So the cycle now only lasts twelve measures. It is significant that the last note of the saxophone comes more than 144 sixteenths after the beginning of the last cycle, since the exposition of the voices lasted more than one cycle.

Because the last voice of each subvoice is relatively isolated, the last notes are heard with their full values (8 9 31 8). This is clearly seen in the four final notes of the clarinet (first system) or the saxophone (second system).

Twelve bars is also the length of *Canon 2*. The rhythmical modulation is calculated so that the period of *Canon 1* is equal to the period of *Canon 2*. Since the *Canon 2* has only 108 notes in its period, it is somewhat slower, the unit value being the triplet.

The *R* value for *Canon 2* is $(9\ 5\ 1\ 1\ 5\ 25\ 4\ 1\ 1\ 5\ 2\ 9\ 14\ 5\ 1\ 5\ 11)$, and we recognize the values and even the notes of the clarinet "call". The time points for the entrances of the voices are $(0\ 48\ 60\ 81\ 129\ 141)$. The violin enters with the first voice, then the vibraphone, then the contrabass in pizzicato on the last beat of the seventh bar of



Figure 3. The modulation between Canon 1 and Canon 2 in Canon Final. This page just follows the preceding example. We are in 3/4 time, but the clarinet and the saxophone round off the Canon 1 in 9/8 (although they are notated in 3/4 for practicality). The violin enters with the first metavoice of Canon 2, then the vibraphone, then the contrabass in pizz. The latter instrument goes directly from Canon 1 (in sixteenths, arco) to Canon 2 (in triplets, pizzicato). The first cycle is completed four bars before the end of the example (we see the violin starting again on high C).

the excerpt. The second voice of the violin happens only 81 triplets later, i.e. at the beginning of the second system (it is mostly played in harmonics).

The contrabass alternates between *arco* and pizzicato until its part in *Canon 1* is completed. As for *Canon 1*, one cycle is insufficient for the entire the voice, and therefore we have to wait for the last bar of the example to hear a measure in which all triplets are played.

5 Texture and constraints

A very interesting passage was the last part of the *Canon Final*. This is a 216-period structure and, with the process of modulation, the events happened twice as fast. The duration of a whole cycle being the same, the unit was now the sixteenth note sextuplet. The quantity of notes and the fact that they (supposedly) did not occur at the same time helped to create a very dense texture. In this case, the ambiguity was not between line and contrapuntal construction, but between counterpoint and texture.

Even with a very precise performance, it was impossible to hear it as a continuum, since the tempo was not slow and the instruments had very different types of attack and response time (a vibraphone, a double-bass and a clarinet are clearly very different). What was perceptible, however, was the relatively equal distribution of events, enforced by the principle of maximal category canons: it was perceptible that the vibraphone, the only instrument playing two voices at a time, contained a denser passage.

However, it was difficult to devise a means of controlling the harmonic structure while keeping something like a canonic melodic relationship between the voices. That is where constraint programming was useful.

Charlotte Truchet developed, at IRCAM, a constraint programming environment using OpenMusic. This made it possible to test several melodic solutions subject to constraints: the first constraint was that the voices should be melodically in canon, within certain limits. The second constraint was to tend towards a harmonic reference texture.

Because the program operates by random selection, it was even possible to give the melodic constraint less and less importance, while giving the harmonic one more and more. Thus the final canon converged towards a unified texture, as the original melody progressively disappeared.

6 Conclusion

Only OpenMusic allows research of this scope. We encountered two types of problems: one, the maximal category canons, presents an obvious long known musical problem. The solution, however, depends on the computer tools used for musical representation. The devices found while using them stem from the very concept of the canons: using voice mixing for example, to find sub-voices that have self-similar characteristics. Rhythmic modulation between two canons is another very interesting possibility. The best solutions can easily be computed automatically with OpenMusic. A more general consequence is that many canonic structures with varying periods can be grouped.

The second problem is solved in a more empirical way. It turned out that constraint research could be used as a development process.

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Georges Bloch



Georges Bloch began studying composition relatively late, at UC San Diego, after graduating in Engineering at the Ecole Centrale de Lille. He also performs as a singer. Born in Paris, he lives now in Strasbourg, where he teachs at the University. Georges Bloch's music is based on three different but nevertheless perfectly compatible centers of interest:

- Music and space: *Palmipèdes d'agrément* was the first piece using the IRCAM spatializer; *Palmipèdes salins* takes advantage of the particular acoustics of the Salt factory conceived by Nicolas Ledoux in the 18th cen-

tury; *Fondation Beyeler: une empreinte sonore* offers five simultaneous musical visits to the Foundation Beyeler in Riehen, Switzerland.

- Interaction, mainly based on the paradox of composed improvisation (*Jimmy Durante Boulevard*, *Palm Sax*); more generally, computer-assisted composition, in real time or otherwise. He is presently associated with the Omax project (computer assisted improvisation).

- Collaboration with other artists—mostly sculptors or painters (Souvenirs et moments is based on pictures by Jean-Michel Albérola, inserted into the score). A piece such as Palmipèdes corbuséens palmés combines the three characteristics: a mezzo-soprano wanders into the strange acoustic space of a water tower built by Le Corbusier, and the building itself is made to resonate by an interactive sound sculpture.