

Topological Spaces of Motives of Brahms Op. 51 No1

(preliminary results)

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Content

- ★ Topological Model & Implementation
- ★ Application to Brahms Op.51 No.1
- ★ Two additional Space Representations
- ★ Final Remarks

Topological Model

Set of all possible motives

(segmentation)

Contours

(Pitch interval vector, COM, ...)

Classes

(gestalt)

Similarity

- when same cardinality

ϵ -Neighborhood of a motif

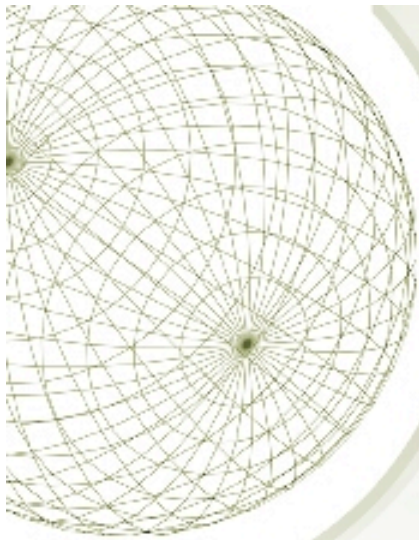
Topological Space for **all** Motives of a Score

Understand the geometry of the space

(motif, ϵ)

\Downarrow

weight _{ϵ} (motif)

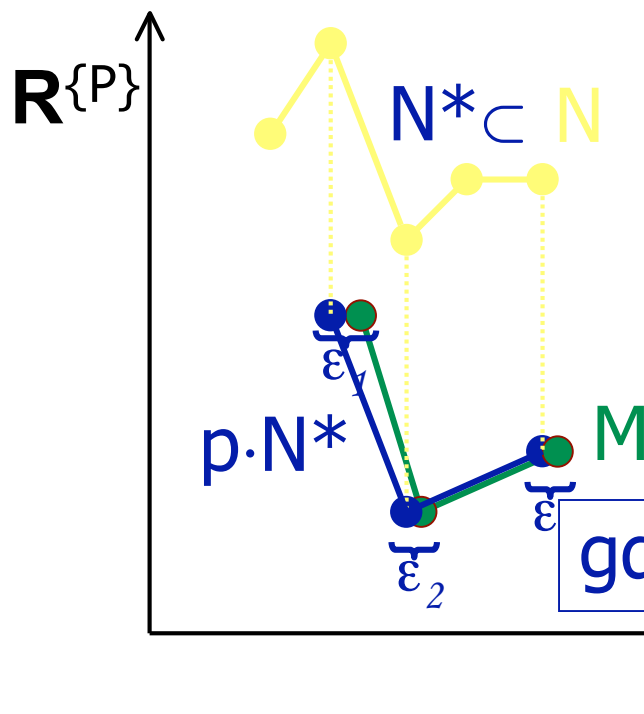


Neighborhood of a Motif

The ε -neighborhood of the motif M is

$$V_\varepsilon(M) := \{N \in \text{MOT} \mid \exists N^* \subset N \text{ s.t. } \text{gd}_t^P(M, N^*) < \varepsilon\}$$

Motif N is in the ε -neighborhood of motif M if:



proximity structure for
all motives of any cardinality

$$\text{gd}_t^P(M, N^*) = (\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2)^{1/2} < \varepsilon$$

proximity structure for
motives with same cardinality

Weight Function

M a motif and $\varepsilon > 0$ a radius (similarity threshold)

N a motif with $\text{card}(N) \geq \text{card}(M)$

$$P_{M,N,\varepsilon} = \#\{ N^* \subset N \mid \text{gd}_{t,m}(M, N^*) < \varepsilon \}$$

The **presence of M at radius ε** is

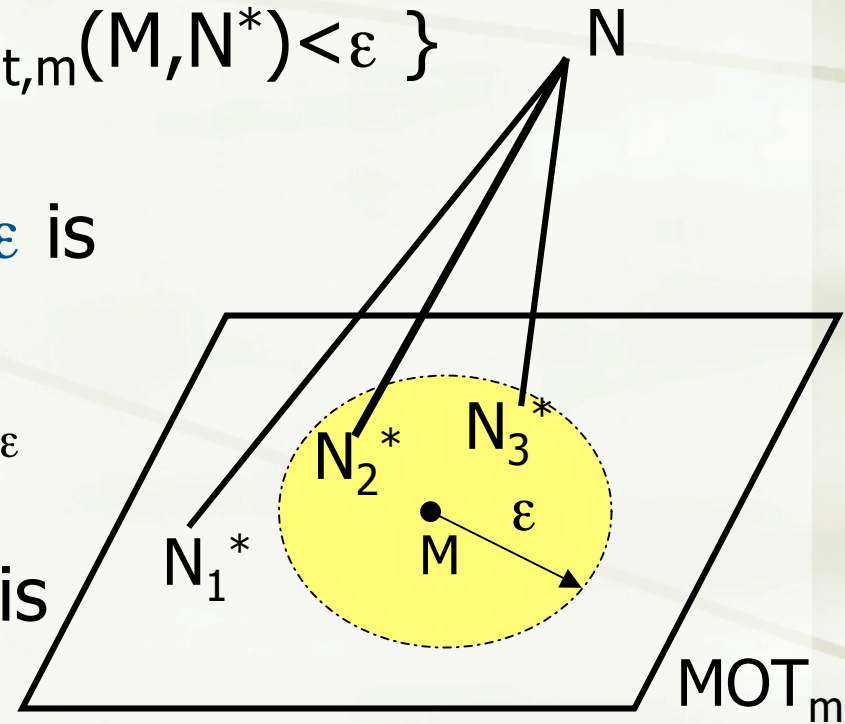
$\text{presence}(M, \varepsilon) :=$

$$\sum_{N \text{ in score}} 1/2^{n-m} \cdot P_{M,N,\varepsilon}$$

The **content of M at radius ε** is

$\text{content}(M, \varepsilon) :=$

$$\sum_{N \text{ in score}} 1/2^{m-n} \cdot C_{M,N,\varepsilon}$$



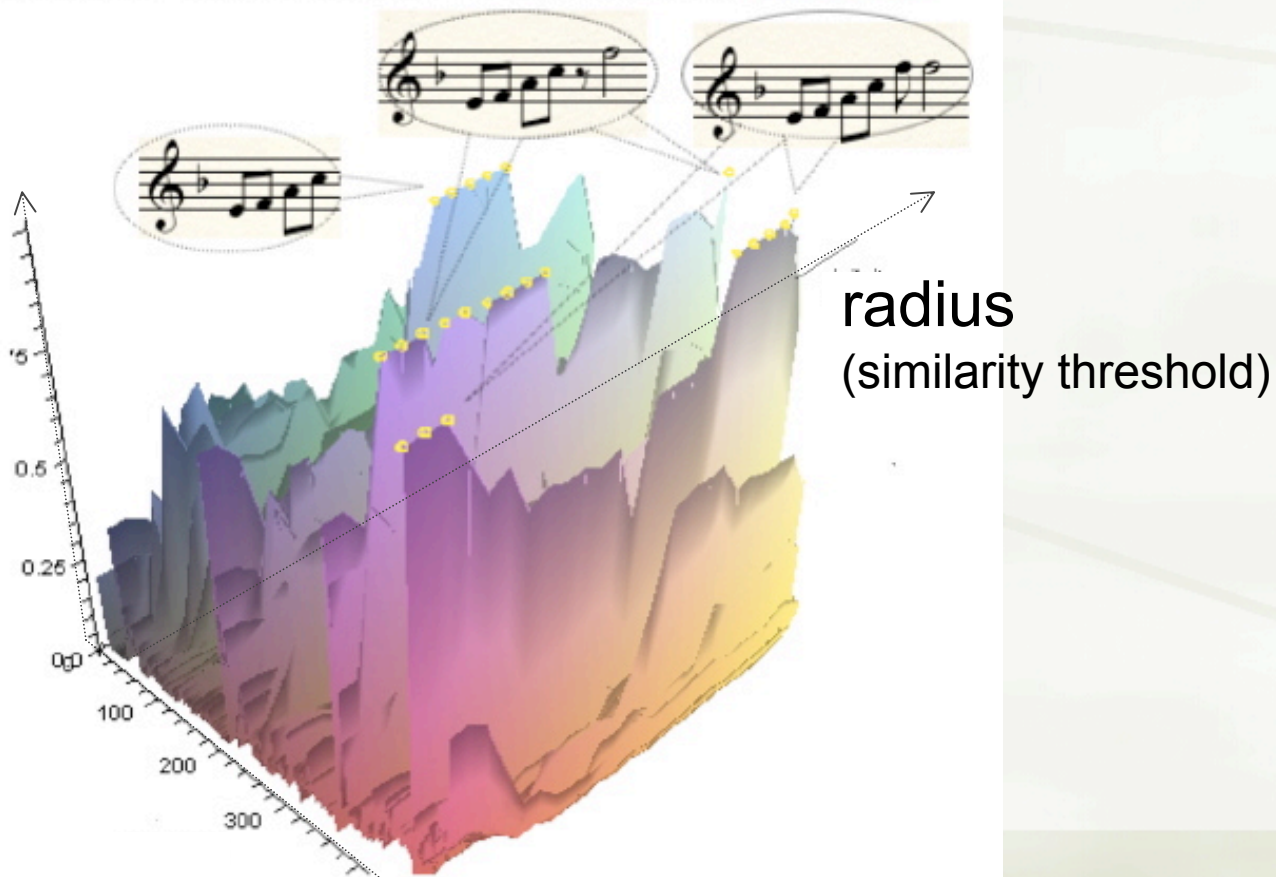
$$\text{Weight}(M, \varepsilon) := \text{Presence}(M, \varepsilon) \cdot \text{Content}(M, \varepsilon)$$

Weight Graph: the higher the more significant



OM-Maple 3D-Weight Function Graph of Schumann's Traumerei

weight



radius
(similarity threshold)

gestalt

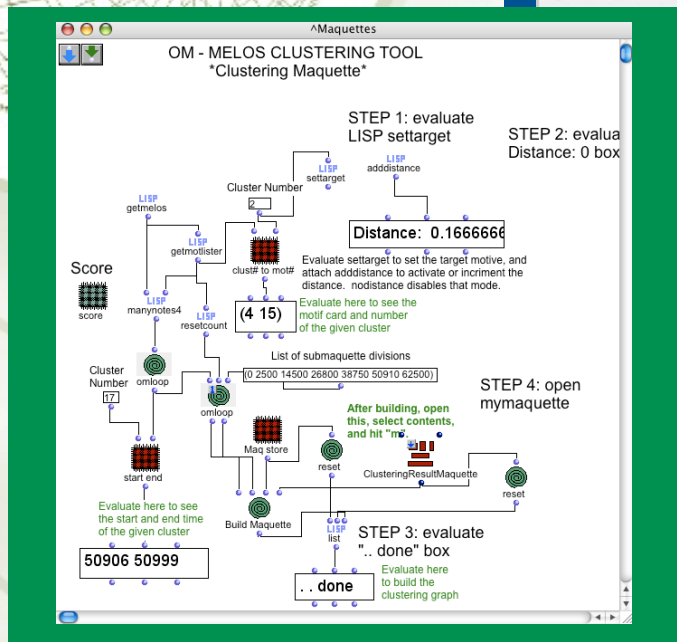


Our Approach: Motivic Topologies

Introduced by Guerino Mazzola (1994)
(MeloRubette: Zahorka & Mazzola)

Further developed by
Buteau & Mazzola (1998-2003)
Buteau (since 2003)
(OM-Melos: Buteau & Vipperman with Agon)

Implement: OM-Melos



Melos
MELOS PROGRAM Complete

Username: Buteau Score File: /melos/scores/traumerei/Traum-Ex-ALL-final.txt Browse

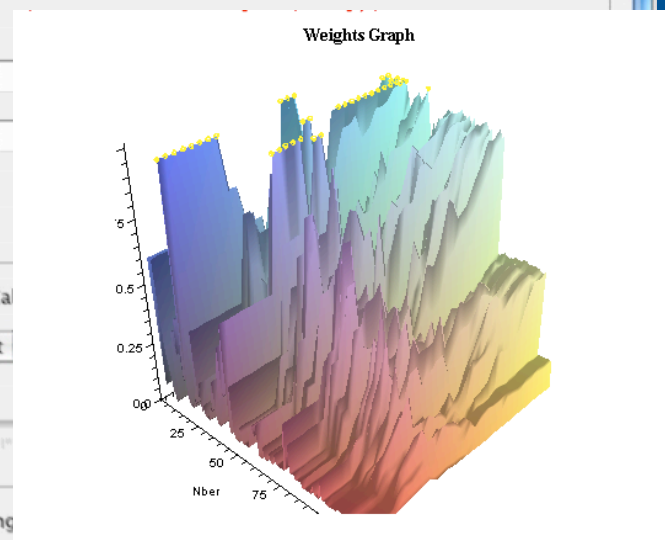
Topological Parameters
Diastematic
Group: Identity (dropdown menu: Identity, Inverse, Retrograd, Counterpoint)
Distance: Euclidean (dropdown menu: Euclidean, Relative Standard)
Form: Relative Standard
Coefficients

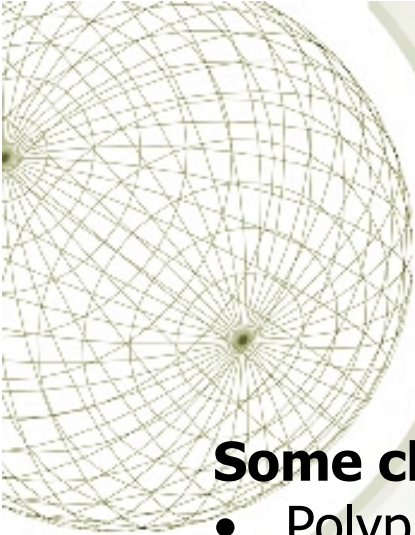
Weight Functions
Coefficient: 1.0 Presence Exponent: 1.0
Coefficient: 1.0 Content Exponent: 2.0
Addition Multiplication

Weight Calculation
Changing Epsilons Radii Form: 6 First

Steps: 6 Step Interval: [slider]

Include Clustering Analysis
Motif Set File: s/research/melos/scores/segmentation/segTraum-Ex-NoArrow-ALLsimplified-final.txt Browse
Cardinality Restriction: +/- Cardinality Clustering Radii: All
Steps: 1 Radius Step: [slider]





Implement: OM-Melos

Some characteristics:

- Polyphonic music
- Automatic or semi-automatic segmentation
- Motives with non-necessarily consecutive notes
- Many model parameters: contour, par. groups, weight functions, ...
- Many analyses each with a fixed motive representation
- Output Visualization:

Space Representations (*in OpenMusic and Maple*):

- Weight Functions
- Motivic Evolution Trees
- Clustering (dynamic tables)

Motives and Gestalt representations in score (*in OpenMusic*)

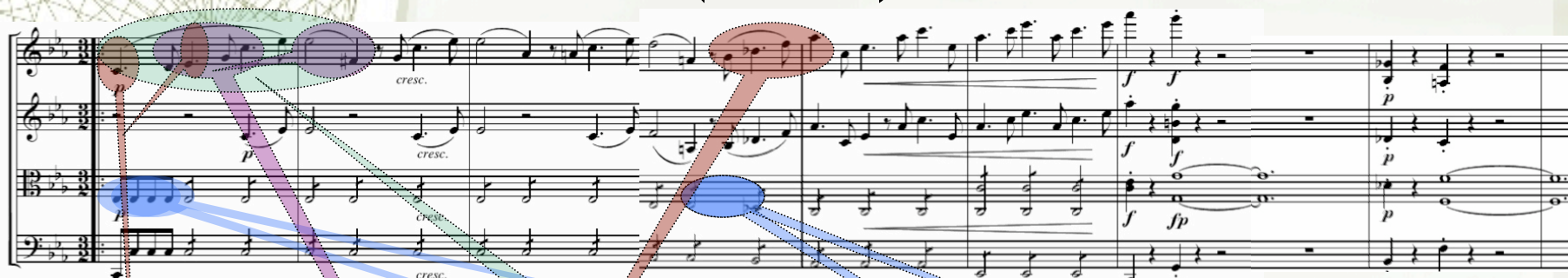


Topological Space of Motives of Brahms Op.51 No.1

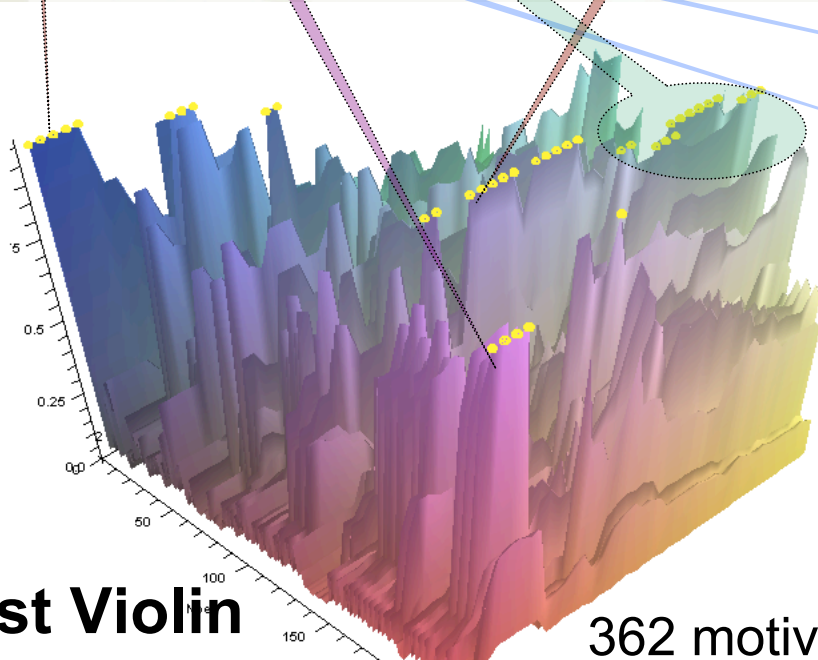
(preliminary results)

- ◆ Opening bars (1 - 10)
 - ◆ All
 - ◆ 1st Violin
- ◆ Exposition (bars 1 - 82)
 - ◆ Melody (in 3 voices)

Motivic Spaces of the Opening Bars (1-10): 1st Violon vs All

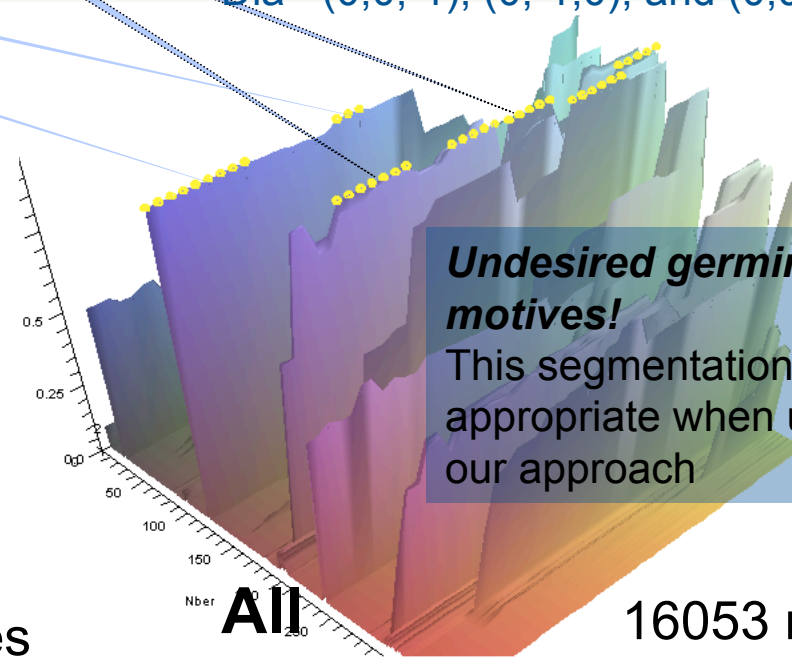


Dia= (0,0,-1), (0,-1,0), and (0,0,-5)



First Violin

362 motives
219 gestalts



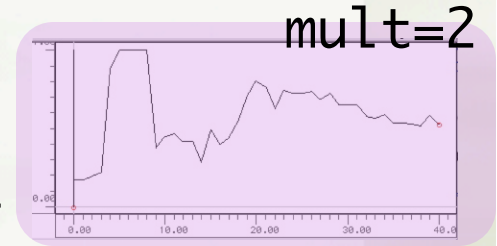
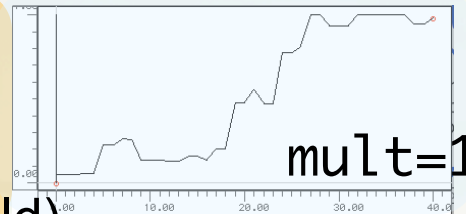
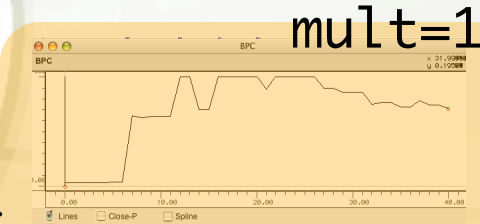
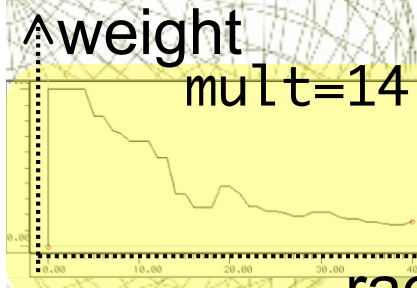
All

16053 motives
359 gestalts

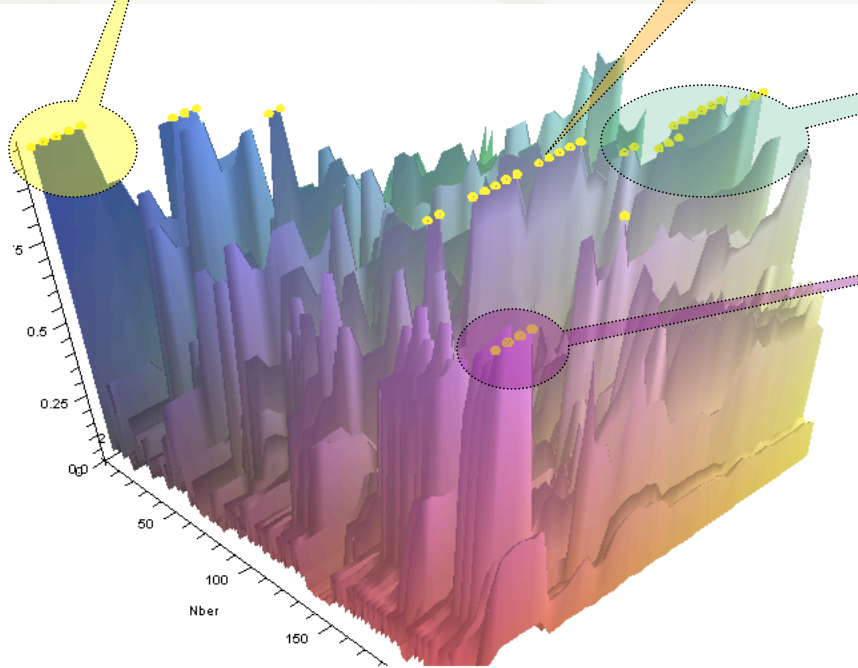
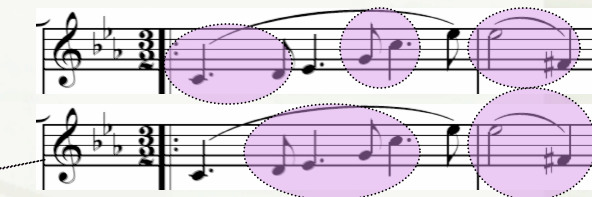
Undesired germinal motives!

This segmentation is not appropriate when using our approach

Motivic Space of Opening Bars (1-10) for First violin with Pitch Intervals

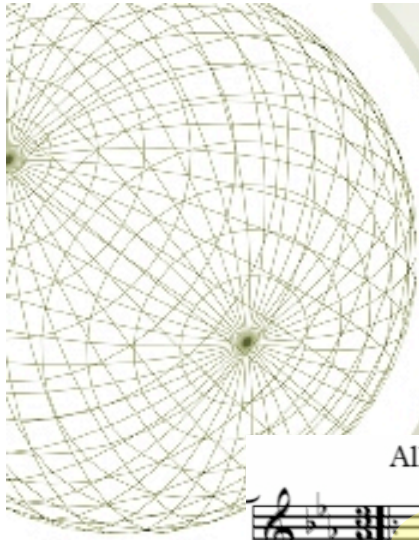


radius (similarity threshold)



Motif Cardinality	Number of Motives	Number of Gestalts
2	80	15
3	98	64
4	88	66
5	59	44
6	28	22
7	8	7
8	1	1
TOTAL =	362	219

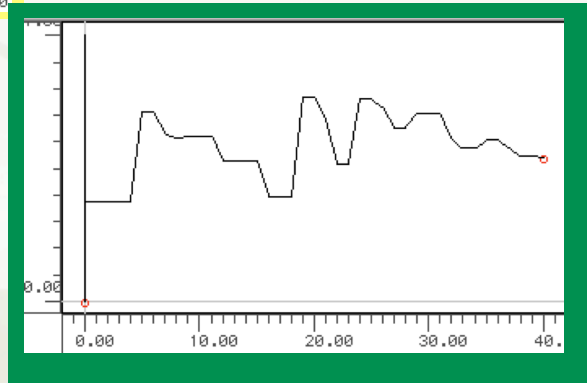
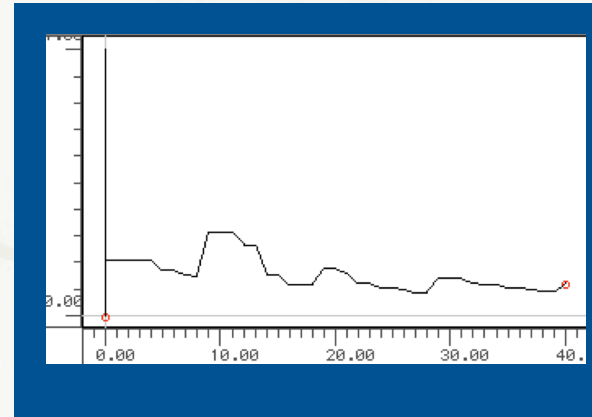
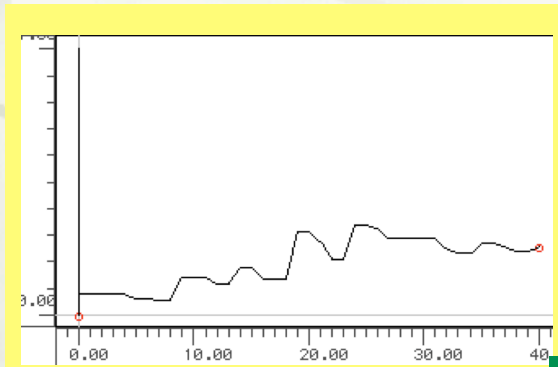
Forte's Motives in the Topological Space



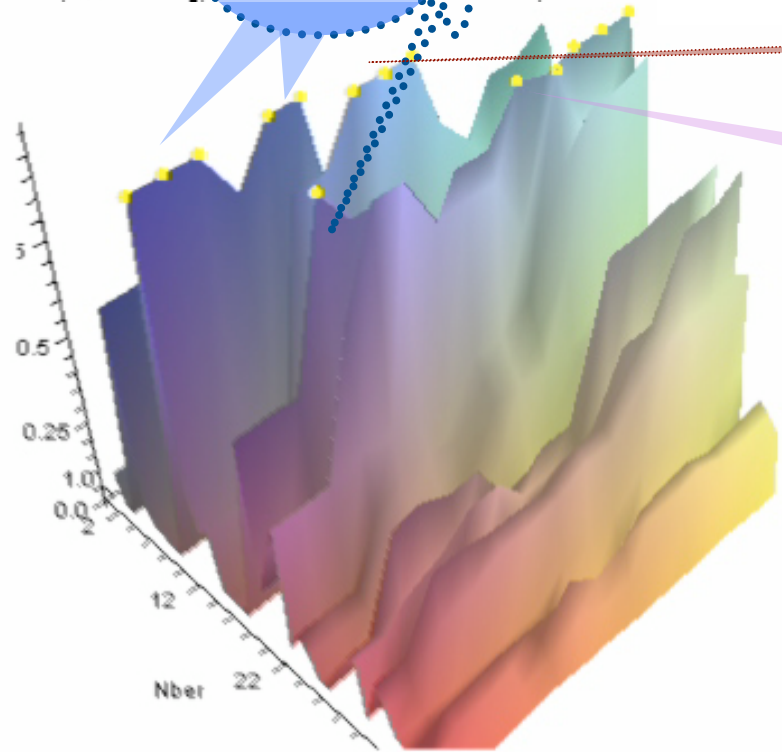
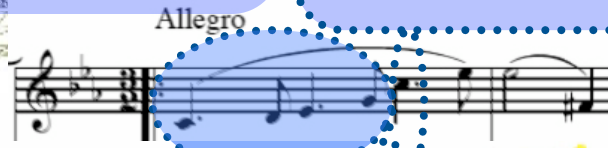
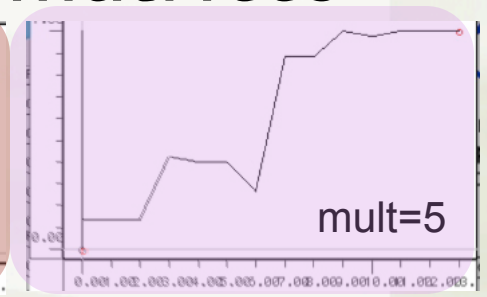
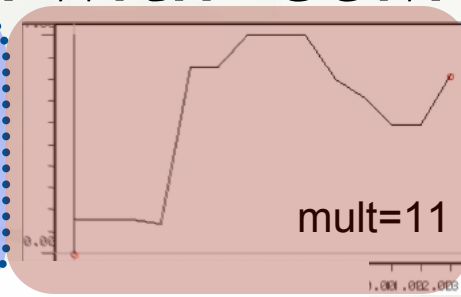
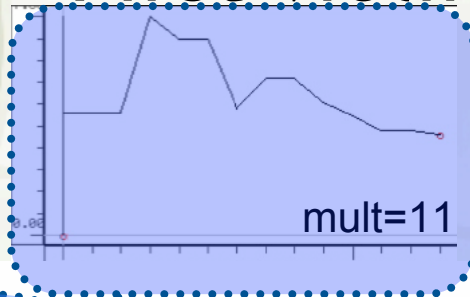
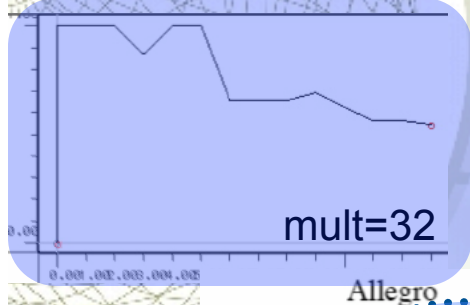
Allegro



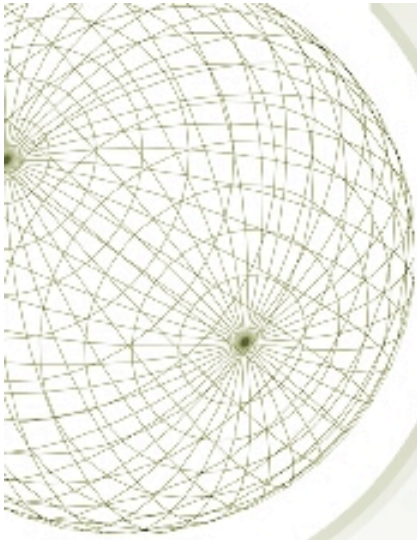
alpha beta gamma



Motivic Space of Opening Bars (1-10) for First violin with COM matrices



Motif Cardinality	Number of Motives	Number of Gestalts
2	80	3
3	98	6
4	88	8
5	59	9
6	28	7
7	8	4
8	1	1
TOTAL =	362	38



Exposition - First Results

(limitation of visualization implementation)

Allegro

1 2

with pitch intervals (Dia, Id, REd)

Allegro

with pitch intervals (Dia, Id, Abs)

A wireframe sphere is positioned in the top-left corner of the slide. It is composed of a grid of thin, light-colored lines that form a spherical shape, with a central point from which the lines radiate outwards. The sphere is partially obscured by a white circular shape that frames the text.

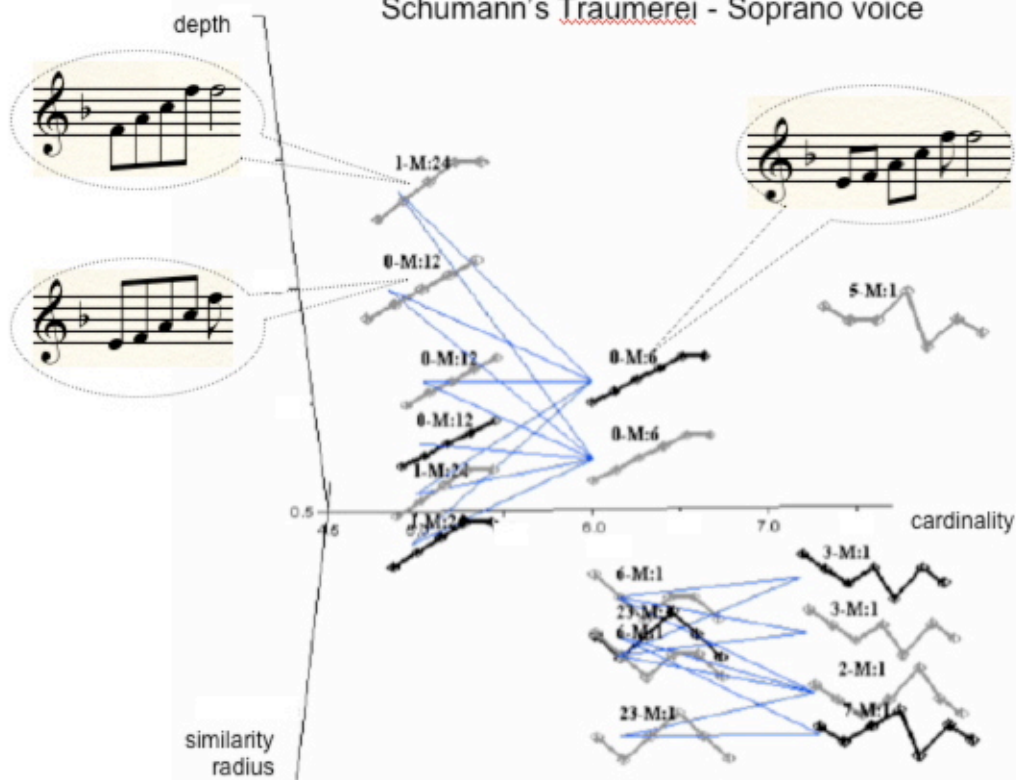
Two Additional Visualizations

(exemplified with Schumann's Träumerei)

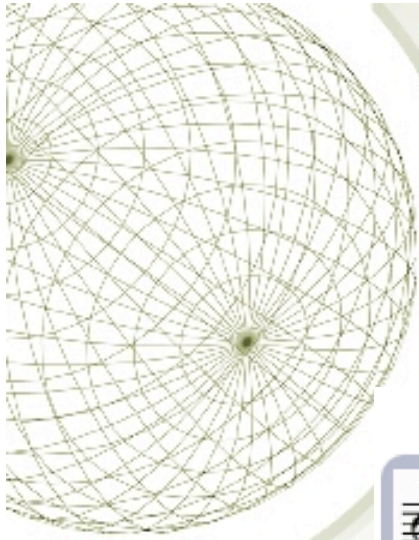
Motivic Evolution Tree (Maple)



Motivic Evolution Tree of Schumann's Traumerei - Soprano voice

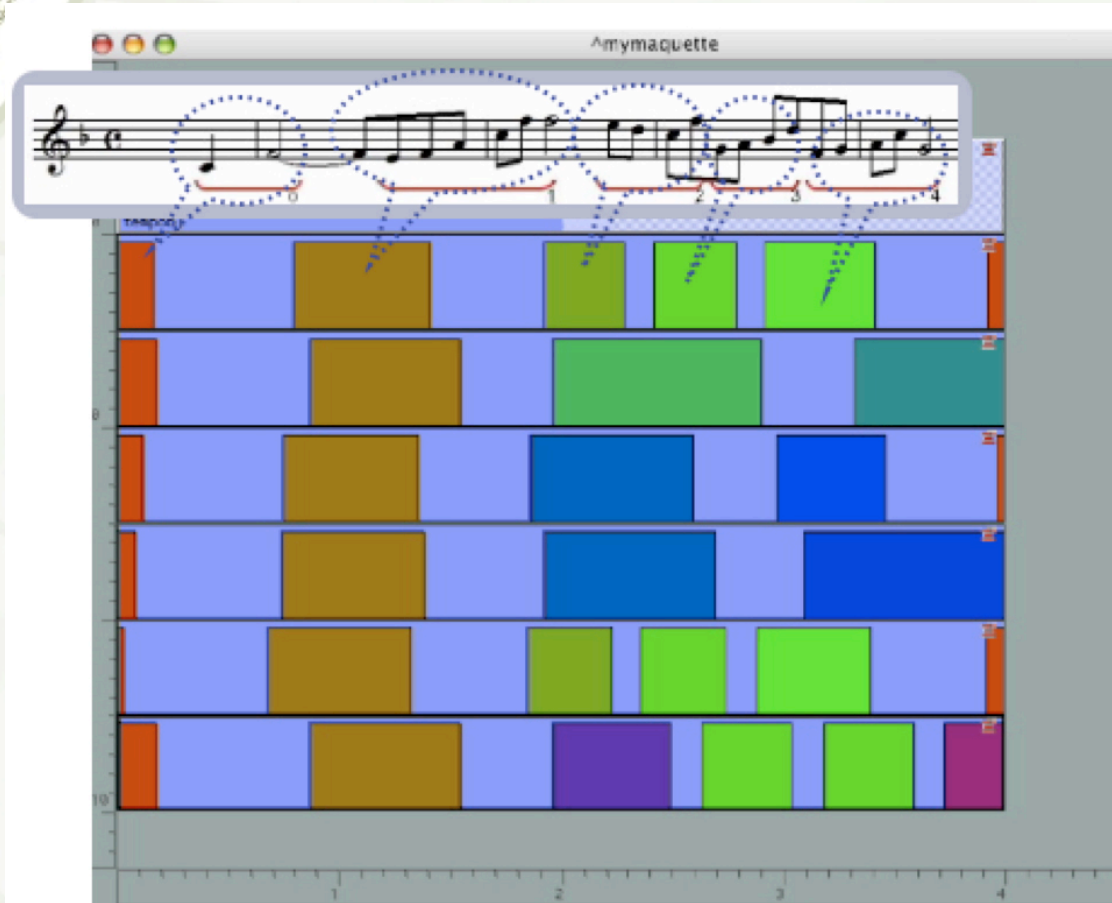


“This is the signature melodic gesture of the piece...”
Repp (1992)



Schumann's Träumerei Melodic Clustering

(dynamic visualization in OpenMusic)





Final Remarks

- ★ Results depend on the segmentation
(e.g. repetitions in viola section should be excluded)
- ★ Tedious manual segmentation
- ★ Tedious manipulations for visualization
(3 programs)
- ★ Limitation of computations
(depend on the number of motives & gestalts)
- ★ Musical Significance of results
- ★ Limitation of visualization
(of 'large' spaces) - *next week...*
- ★ *Preliminary* analysis of the quartet