Topological Spaces of Motives of Brahms Op. 51 No1
(preliminary results)

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Séminaire MaMuX: mathématiques, musique, et relations avec d’autres disciplines
IRCAM, Paris
April 6, 2008
Content

- Topological Model & Implementation
- Application to Brahms Op.51 No.1
- Two additional Space Representations
- Final Remarks
Topological Model

- Set of all possible motives (segmentation)
- Contours (Pitch interval vector, COM, ...)
- Classes (gestalt)
- \( \varepsilon \)-Neighborhood of a motif
- Similarity - when same cardinality

Motivic Structure of a Score

Understand the geometry of the space
Neighborhood of a Motif

The $\varepsilon$-neighborhood of the motif $M$ is

$$V_{\varepsilon}(M) := \{ N \in \text{MOT} \mid \exists N^* \subset N \text{ s.t. } gd_t^P(M, N^*) < \varepsilon \}$$

Motif $N$ is in the $\varepsilon$-neighborhood of motif $M$ if:

$$gd_t^P(M, N^*) = \left( \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 \right)^{1/2} < \varepsilon$$

proximity structure for all motives of any cardinality

proximity structure for motives with same cardinality
Weight Function

M a motif and 𝜖 > 0 a radius (similarity threshold)
N a motif with \( \text{card}(N) \geq \text{card}(M) \)
\[ P_{M,N,\epsilon} = \#\{ N^* \subset N \mid \text{gd}_{t,m}(M,N^*) < \epsilon \} \]

The presence of M at radius 𝜖 is
\[ \text{presence}(M,\epsilon) := \sum_{N \text{ in score}} \frac{1}{2^{n-m}} \cdot P_{M,N,\epsilon} \]

The content of M at radius 𝜖 is
\[ \text{content}(M,\epsilon) := \sum_{N \text{ in score}} \frac{1}{2^{m-n}} \cdot C_{M,N,\epsilon} \]

Weight(M,\epsilon) := Presence(M,\epsilon) \cdot Content(M,\epsilon)
Weight Graph: the higher the more significant
Our Approach: Motivic Topologies

Introduced by Guerino Mazzola (1994)
*(MeloRubette: Zahorka & Mazzola)*

Further developed by
- Buteau (since 2003)
*(OM-Melos: Buteau & Vipperman with Agon)*
Implement: OM-Melos
Implement: OM-Melos

Some characteristics:
• Polyphonic music
• Automatic or semi-automatic segmentation
• Motives with non-necessarily consecutive notes
• Many model parameters: contour, par. groups, weight functions, ...
• Many analyses each with a fixed motive representation
• Output Visualization:
  Space Representations (in OpenMusic and Maple):
  • Weight Functions
  • Motivic Evolution Trees
  • Clustering (dynamic tables)
Motives and Gestalt representations in score (in OpenMusic)
Topological Space of Motives of Brahms Op.51 No.1

(preliminary results)

- Opening bars (1 - 10)
  - All
  - 1st Violin
- Exposition (bars 1 - 82)
  - Melody (in 3 voices)
Motivic Spaces of the Opening Bars

(1-10): 1st Violon vs All

Dia = (0,0,-1), (0,-1,0), and (0,0,-5)

Undesired germinal motives!
This segmentation is not appropriate when using our approach

First Violin
362 motives
219 gestalts

All
16053 motives
359 gestalts
Motivic Space of Opening Bars (1-10) for First violin with Pitch Intervals

<table>
<thead>
<tr>
<th>Motif Cardinality</th>
<th>Number of Motives</th>
<th>Number of Gestalts</th>
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</thead>
<tbody>
<tr>
<td>2</td>
<td>80</td>
<td>15</td>
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<tr>
<td>3</td>
<td>98</td>
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<tr>
<td>4</td>
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<td>66</td>
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<tr>
<td>5</td>
<td>59</td>
<td>44</td>
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<tr>
<td>6</td>
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<td>7</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>1</td>
</tr>
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</table>

TOTAL = 362  219
Forte’s Motives in the Topological Space

alpha beta gamma
Motivic Space of Opening Bars (1-10) for First violin with COM matrices

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TOTAL = 362 38
Exposition - First Results
(limitation of visualization implementation)

with pitch intervals (Dia, Id, REd)

with pitch intervals (Dia, Id, Abs)
Two Additional Visualizations
(exemplified with Schumann’s Träumerei)
Motivic Evolution Tree (Maple)

“This is the signature melodic gesture of the piece…”
Repp (1992)
Schumann’s Träumerei Melodic Clustering
(dynamic visualization in OpenMusic)
Final Remarks

- Results depend on the segmentation (e.g. repetitions in viola section should be excluded)
- Tedious manual segmentation
- Tedious manipulations for visualization (3 programs)
- Limitation of computations (depend on the number of motives & gestalts)
- Musical Significance of results
- Limitation of visualization (of ‘large’ spaces) - next week...
- Preliminary analysis of the quartet