Journée d'étude

Mathématiques/Musique & Cognition : théories diatoniques

Vendredi 25 April 2008
Ircam (1, place I. Stravinsky, Paris 4e)
Salle Igor Stravinsky

[entrée libre dans la limite des places disponibles]

La théorie diatonique [diatonic theory] est une approche désormais bien établie au sein de la tradition set-théorique américaine ainsi que des diverses orientations de la musicologie systématique européenne. Cette journée d'étude, organisée dans le cadre des activités du nouveau groupe de travail "Mathématiques/Musique et Cognition", a pour objectif de présenter quelques aspects récents d'une approche mathématique du diatonisme, tout en essayant d'ouvrir des passerelles entre théories mathématiques de la musique et musicologie cognitive.

PROGRAMME

- 10h00-10h30 Moreno Andreatta (Ircam/CNRS) : A short introduction to the mathematical theory of diatonicity
- 10h30-11h30 Eytan Agmon (Dept. of Music, Bar-Ilan University, Israel) : The Generic Function and Its Significance in Western Music
- 11h45-12h30 Emmanuel Amiot (mathematician, Perpignan) : DFT vs JSB
- Discussion

Pause

- 14h30-15h30 Thomas Noll (ESMuC, Barcelona) : Generalizing the Diatonic System
- 15h45-16h45 Julien Junod (Ircam/ Université de Paris VI) : Measuring the diatonicity in the 7-notes scales space (in collaboration with Pierre Audétat, Conservatoire de Lausanne)
- Discussion finale

Résumés [Toutes les communications seront en anglais]

An introduction to the mathematical theory of diatonicity

In this introductory talk we give a survey of some mathematical models that have been proposed for the study of diatonic theory. After briefly reviewing some seminal contributions to this field (Euler, Longuet-Higgins, Balzano, Clough, Agmon, Amiot…), we will discuss the notion of Maximally Even Sets by showing some surprising connections between diatonic theory, Fourier analysis and physics (as suggested by Jack Douthett and Richard Krantz in their « music-theoretical » formalization of the one-dimensional antiferromagnetic Ising model). We will end by discussing some cognitive and perceptual ramifications of the mathematical theory of diatonicism and, more generally, to mathematical models in music theory, analysis and composition.

The Generic Function and Its Significance in Western Music

The generic function is a rounding function that sends integers x=0, 1, . . ., 5, 7, . . . , 11 into integers y satisfying y is the nearest integer to the product x times 7/12. Two musical interpretations of the function are offered, a musical example is provided, and the relation of the

function to the "J-function" of maximal evenness is discussed. Finally, the function's significance is considered from a cognitive-scientific perspective.

DFT vs JSB

Major scales in a given temperament are approximations of a regular heptagon. A measure of the quality of this approximation is the first Fourier coefficient of the scale, considered as a map from the cyclic group **Z/7Z** into the unit circle.
The closeness of these Fourier coefficients computed for all 12 major scales is in turn an indication of the sameness of the these scales. As an unforeseeenable application, the relevance of the temperament proposed by David Lehman as the one used by J.S. Bach himself is discussed.

Generalizing the Diatonic System

I will present recent work by Franck Jedrzejewski as well as some components of my joint work with David Clampitt and Karst de Jong. I will discuss the underlying motivation for generalizing the diatonic system on mathematical grounds and - aside from touching the primary musical challenges of such research - I will explore some of the theoretical insights that can be drawn from the activity of generalization.
In a forthcoming article "Generalized Diatonic Scales" (*Journal of Mathematics and Music* 2/1) Franck Jedrzejewski connects early theoretical ideas of the micro-tonal composer Ivan Wyschnegradski with later work by Eytan Agmon. He extends the search for a generalized diatonic scale to all chromatic systems and presents his proposal within an elegant mathematical framework which he calls the Shuffled Stern-Brocot Tree.
Aside from generalizing the diatonic to larger scales (John Clough and Jack Douthett termed them "hyperdiatonic scales") there is another direction of generalization, namely to smaller scales. The interpretation - for example - of the diatonic triads and seventh-chords as well-formed scales within the generic diatonic has interesting consequences, to which Eytan Agmon contributed seminal insights. Norman Carey and David Clampitt as well as Yves Hellegouarch highlighted another central fact, namely the hierarchical organization of prominent scales (authentic or plagal division, structural, pentatonic, diatonic, chromatic, ...) along the semi-convergents of Log(2, 3/2) on the Stern-Brocot Tree. Which theoretical insights can be drawn from the mathematical connection between these scales? I will discuss some possible answers to this question on the background of the theory of Sturmian words and morphisms.

Measuring the diatonicity in the 7-notes scales space

We propose a way of generalizing the construction of the diatonic scale as a sequence of perfect fifths. By altering part or all of the fifths, we can generate every seven-note scale up to transposition. The number of alterations (sharps and flats) needed to match each scale can then be used as a measure of its diatonic/chromatic character.
Such a pythagorean index will be compared with other measures (e.g. Anatol Vieru's DIA/CHRO measure, or evenness measured by euclidian distances between vertices of heptagones) and will be applied to a recent representation of the collection of heptachords which is used as a geometric model for jazz improvisation. In particular, our approach enables the musician to order the scales from the most diatonic to the most chromatic. The aim is to obtain a graphical representation of the scales and their modes in which the ordering follows a musical rather than a purely combinatorial logic. This research has been started by Pierre Audétat, a musician and composer who discovered this atlas of modes. He will discuss and illustrate some musical questions by showing examples on the keyboard.

Cette journée d'étude est organisée dans le cadre du groupe de travail « Mathématiques/Musique & Cognition », sous l'égide de l'AFIM (Association française d'Informatique Musicale). Pour plus de renseignements sur le groupe de travail "Mathématiques/Musique & Cognition" :
`http://recherche.ircam.fr/equipes/repmus/mamux/Cognition.html`