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Mazzola, Guerino

The topos of music. Geometric logic of concepts, theory, and performance. In collaboration with Stefan Göller and Stefan Müller. Contributions by Carlos Agon, Moreno Andreatta, Gérard Assayag, Jan Beran, Chantal Buteau, Roberto Ferretti, Anja Fleischer, Harald Fripertinger, Jörg Garbers, Werner Hemmert, Michael Leyton, Emilio Lluis Puebla, Mariana Montiel Hernandez, Thomas Noll, Joachim Stange-Elbe, Hans Straub, Oliver Zahorka. With 1 CD-ROM. (English)

Basel: Birkhäuser. xxx, 1338 p. EUR 91.58/net; sFr. 144.00 (2002).

Topos of Music (abbreviated as ToM in the sequel) is an extensive and elaborate body of mathematical investigations into music and involves several and ontologically different levels of musical description. Albeit the author Guerino Mazzola lists 17 contributors and 2 collaborators, the book should be characterized as a monograph. Large portions of the content represent original research of Mazzola himself, and the material from other work is exposed from Mazzola's point of view and is well referenced. The preface preintimates an intended double meaning of the term *topos* in the title. On the one hand, it provides a mathematical anchor, which is programmatic for the entire approach: the concept of a cartesian closed category with a subobject classifier. Mazzola's motivations to exploit especially categories of set-valued functors for the description and investigation of musical structures are discussed further below. On the other hand, the choice of the title alludes to the general ontological and epistemological problems of music research, especially to the difficulties to communicate musical and meta-musical knowledge by means of a clear conceptual universe. Therefore the title encapsulates a scientific program, namely to follow the pathbreaking ideas on inner-mathematical concept formation in the intellectual tradition of Alexander Grothendieck, William Lawvere and others and thereby to dispel the widespread epistemological doubts about the eligibility of mathematical investigations into music in general.

The entire book spans 1335 pages and consists of a main exposition (ca. 1000 pages, subdivided into 14 parts and covering 52 chapters) and a series of clearly ordered appendices. Parts I–IV (covering almost one half of the main exposition) are devoted to the development of a general theoretical setup and include the presentation of a meta-language (Chapter 6: denotators and Chapter 18: predicates) and several prolegomena about the constitution and accessibility of musical and music-theoretical objects. The general orientation of this approach may be characterized as a consequential continuation of ideas, which already shaped two earlier monographs [*G. Mazzola*, Gruppen und Kategorien in der Musik. Berlin: Heldermann (1985; Zbl 0574.00016); *G. Mazzola*, Geometrie der Töne. Basel: Birkhäuser (1990; Zbl 0729.00008)]. The exposition of concrete music-theoretical levels of description is distributed over several parts of the book. In order to read ToM along such pathways (like metric analysis) it is highly recommendable to take advantage of the navigation tools on the CD-ROM, which contains the full text with all necessary hyperlinks.

This present review dispenses from giving a summary of ToM, which on its own provides a summary of many strands of research including music theory, music analysis, musical performance, computer-aided composition, musical semiotics, visualisation, and classification of mathematical objects (with potential musical meaning). Instead the review concentrates on two selected aspects which exemplify central lines of thought in ToM.

Affine Transformations and Denotators: One of the central ideas of the entire approach is that musical objects are inhabitants of ambient spaces whose transformations contribute to the constitution of their musical meaning. A more radical formulation of that idea – upon which the denotator language is built – considers transformations themselves as basic constituents of musical objects. The main type of transformations in this approach are affine ones, defined over modules. Thus, to understand and to estimate this central idea in its radical formulation one needs first of all to understand and estimate the musical relevance of affine transformations. Furthermore one needs to grasp the mathematical shift from modules – or generally from objects of a category – to representable functors and needs to understand and estimate the musical implication of this shift. For giving a thorough judgement of this theoretical position further research seems necessary. But it is possible to give some hints, crosslinks and arguments which may serve as points of departure. The musical meaning of pure translations is broadly acknowledged throughout music theory. Likewise pitch inversions are prominent in music theory and are manifestations of affine isometries with respect to the absolute value of pitch height. The full 48-elemented affine group of Z_{12} appears aside of [Mazzola (1985)] also in *Robert Morris*' book [Composition with pitch classes. New Haven: Yale University Press (1987)] under the name 'Twelve-Tone-Operations' (TTOs). The multiplication by the units 5 and 7 modulo 12 exchanges the chromatic circle with the circle of fifths. This exchange plays also a crucial role in the theory of well-formed scales by *Norman Carey* and *David Clampitt* ["Aspects of well-formed scales", Music Theory Spectrum 11, No. 2, 187–206 (1989); "Self-similar pitch structures, their duals, and their rhythmic analogues", Perspectives of New Music 34, No. 2, 62–87 (1996)]. A generated scale is said to be well-formed if the conversion between the circle of steps and the circle of generator intervals is a linear transformation.

Aside from these and other pitch-related transformations there are examples from the domain of musical time and also transformations which non-trivially connect pitch and time. In Chapter 6 Mazzola cites an interesting example from *David Lewin*'s book [Generalized musical intervals and transformations. New Haven: Yale University Press (1987)], where Lewin studies a non-commutative group of 'generalized' intervals between musical times spans, which is (anti)-isomorphic to the one-dimensional case of affine transformations on Q (or R respectively). A time-span s can be seen as a pair (a, x) consisting of a temporal anchor a and its temporal extension x. The generalized interval from time-span (a, x) to time span (b, y) consists of the scaled difference $(b-a)/x$ of the two anchors and the ratio y/x of the two extensions. The concatenation of these intervals is best understood in terms of affine transformations: rescalings followed by translations. Mazzola therefore characterizes Lewinian time-spans as instances of self-adressed denotators. This means that these time-spans are interpreted as elements of the set $Q@Q$ of affine endomorphisms of Q, which is the evaluation of the representable (contravariant) functor $@Q$ at the concrete module Q. Time spans are thereby identified with the acts of their creation from a unit time span $(0, 1)$. The same argument connects the 144 pitch intervals mod 12 with the 144 affine endomorphisms of Z_{12} [see

T. Noll, Morphologische Grundlagen der abendländischen Harmonik. Bochum: Brockmeyer (1997)] and thereby connects aspects of counterpoint with aspects of harmonic morphology. Chapters 24 and 25 of ToM contain a thorough review of these results and embed them into the framework of ToM. The aspect of non-invertibility leads to instructive applications of topos theory [see also *T. Noll*, “The topos of triads”, in: H. Fripertinger and L. Reich (eds.), Colloquium on mathematical music theory. Proceedings of the colloquium MaMuTh, Graz, Austria, May 6–9, 2004. Grazer Math. Ber. 347 (2005)].

Denotators are (associated with) elements of some set $A@F$, where F is a contravariant functor from Modules to Sets and A is a concrete Address-Module, where this functor is evaluated. This functor F serves as the ambient space for the given denotator, which carries this functor (together with information for its construction) as a ‘snake house’ which is called its form. Representable functors correspond with simple forms – such as in the case of the time spans – and the universal constructions of limits, co-limits and power-objects correspond to compound forms. The denotator setup is therefore a rather natural consequence of two basic requirements: (1) the need for affine transformations in music theory and (2) the need for universal constructions in order to build complex denotators from simple ones. The book lists convincing examples for these constructions. A more recent contribution interprets Klumpenhouwer networks as instances of denotators of the limit-type, i.e. elements of the limit of a diagram of representable functors. Aside from this clear motivation and useful applications the setup has two technical peculiarities, which – to put it mildly – need getting used to. (1) All denotators which are simply meant as points in a module M are to be understood as Zero-addressed, i.e. as transformations from the Zero-module into that module M. This is fine from a mathematical point of view, but creates an unpleasant notational overhead. (2) Non-affine maps – such as permutations – need to appear in a blown-up linearized form. This is an unproblematic mathematical overhead, but its musical interpretation is often not self-evident. These drawbacks do not put the approach as such into question. But they suggest a more flexible attitude of plurality in the choice of the basis category (or categories) to replace the monolithic case of **Mod**. The denotator theory, which is built upon the category of modules **Mod** is not essentially dependent on the choice of this basis category (see also [*G. Mazzola*, “Towards a Galois theory of concepts”, in: G. Mazzola, T. Noll and E. Lluis-Puebla (eds.), Perspectives in mathematical and computational music theory. Osnabrück: EposMusic (2004)], where Mazzola himself relativizes the position taken in ToM).

Local/Global Structures and Paradigmatic Analysis: The book offers a broad palette of methods and strategies for musical analysis, including contributions to the traditional levels of description Rhythm/Meter, Melody and Harmony. Most of these approaches share a common attitude which can be suitably explained with reference to the semiological distinction between syntagmatics (combinatorial relations between signs in terms of their positioning within a given sign complex) and paradigmatics (relations between signs like kinship, similarity, contrast within an a-priorily given sign system). The approaches discussed in ToM typically (1) depart from a previously chosen paradigmatic aspect, (2) investigate a given piece of music with respect to (possibly all) manifestations of this aspect and (3) investigate the combinatoriality of (all or selected) carriers of this analytical aspect as an instance of an associated syntagmatic structure. This can be easily explained on the basis of metrical analysis. The following is a simplified paraphrase of content of Section 13.4.3 in ToM. Suppose, the onsets of the notes of musical piece are represented by a set $X \subset R$. The paradigmatic interest in musical meter leads to the consideration of local meters, i.e. arithmetic sequences $\{x_0 + kd \mid k = 0, \ldots, n\}$ within X. For example, one may cover X by all its maximal local meters and their finite intersections. The nerve of this covering – a simplicial complex – represents the combinatoriality of this metric structure. Each onset in X can be characterized as the incidence of those meters in which it is contained. By simply counting them one obtains a quantitative result: a metric weight. Section 21 provides a topological refinement of this characterization. The consideration of the local meters as an atlas of charts which cover X is an example for a 'musical manifold' or global composition. *Manifolds* are showpieces for mathematical objects which are accessible through several local and "simple" or "elementary" perspectives, but which nevertheless inhere a non-trivial global structure. Mazzola's concept of *global composition* is a highly geometric one and the techniques for global classification (Chapter 15) are anchored in algebraic geometry. Parts III and IV reflect a music-theoretical division of labor which is driven by the local/global dichotomy. The local/global dichotomy is also applied to the modeling of musical performances in Part VIII, where global performance scores are covered by atlases of local performances scores. This part spends special attention to modalities of analytic performance, where analytical facts – being encoded in numerical weights – are expressed through the shaping of a performance. The shaping operations take particular advantage of differential geometry.

Mazzola's music-analytical strategies have parallels in the methods of transformational analysis according to *David Lewin* [cf. Musical form and transformation: 4 analytical essays. New Haven: Yale University Press (1993)]. Some differences in the analytical attitude illuminate a potential for a fruitful application of the topos-theoretic machinery to Lewin-style analyses [see also *G. Mazzola* and *M. Andreatta*, "From a categorical point of view: K-nets as limit denotators", Perspectives of New Music 44, No. 2 (2006)]. Within the analytical methods for motivic/melodic analysis in ToM a fixed paradigm (Symmetry group and Gestalt paradigm) is chosen and all manifestations of this paradigm in a given piece are established in the analysis. In his network analyses Lewin instead tends to play around with certain transformations until he finds a salient network that grasps and interconnects relevant elements of the score and makes their syntagmatic position plausible in terms of an analysis of the piece (or selected aspects) as a trajectory through the network. In semiological terms, Lewin cuts back the full paradigmatic control over the piece in favor of a salient poetic function. Lewin's analytical networks are motivated by a phenomenological line of thought rather than a semiological one, but the mathematical approach is closely related. Mazzola adopts the concept of *paradigmatic theme* from structuralist semiology (originally introduced by Jean Jacques Nattiez) and makes a mathematically plausible proposal for its formalization. The totality of classification criteria in a given analysis is associated with a special subcategory of local compositions or more specifically with a subcategory of the category **Mod** of modules. Particularly it involves a selection of a paradigmatic subgroup for each module M. Lewinian networks are no sub-categories, but they can be suitably related to diagrams in a category. This crosslink suggests a more general exploration of the concept of *paradigmatic theme*.

Outlook: ToM is a highly valuable pioneering contribution to the ongoing mathematisation process in music research and it is mainly addressed to a professional readership in this newly growing field. The tight connections between mathematical meta-language and data structures is particularly interesting for composers of computer music or musicians with ambitions in computer-aided music creation, for music theorists with interest in mathematical and/or computational analysis, as well as for computer scientists working on the modeling of musical structures in the growing domain of music information retrieval.

The above mentioned anchors in structuralist semiology also allow one to read large parts of the book as a contribution to music semiology. And – last but not least – it offers concrete ideas for academic music theory, such as in the investigations into modulation and counterpoint, which have been inherited from [Mazzola (1985)] and [Mazzola (1990)]. Many considerations, such as in the introductory sections to each chapter, are also of interest to a more general readership. But to read the whole book – even with help of the appendices – essentially requires general education in mathematics. Mazzola somewhat provocatively takes this requirement of as a matter of course and thereby creates a source of irritation for unprepared readers. But the more it becomes accepted that insights into music can be convincingly communicated through mathematics and that the underlying mathematical facts cannot effectively paraphrased in a non-mathematical language, the easier it becomes for the community to digest a book like ToM.

The critical play with levels of generality under which certain music-theoretical propositions may be formulated is a valuable source of knowledge in its own right. In a seminar on applications of topos theory to music and arts at IRCAM (Paris) [MaMuX-Seminar, March 20, 2004, http://recherche.ircam.fr/equipes/repmus/mamux] the leading topos theorist Peter Johnstone gave the advice to use geometric morphisms in order to play with the possibility to change the topoi where musical objects are studied. This reinforces the desire for the plural form "Topoi of Music", which Mazzola uses as a title for Chapter 19 in order sketch some future work along with Grothendieck topologies. But it may likewise include the general topos-theoretic exploration of explicit and implicit challenges that eventually emerge from the reception of the singular "Topos of Music".

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Classification :

∗00A06 Mathematics for non-mathematicians
00A69 General applied mathematics
00-02 Research monographs (general mathematics)
14F20 Grothendieck cohomology and topology
14J15 Analytic moduli, classification (surfaces)
18-02 Research monographs (category theory)
18B25 Topoi
18D15 Closed categories