CANONI RITMICI

Come, quanto ?

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New algorithms

* 3D (Kolountzakis)
* Using completion & Coven-Meyerowitz ideas (Matolcsi, with examples of mine)

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* Flatten all this as a tiling of $Z_a \times Z_b$: you get a Vuza !

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  - There are many, many, Vuza canons (K & M, to be published): for arbitrary large N, there are > k $e^{c\sqrt{N}}$
  - We musicians can produce canons for values of n as little as 120 with that method (6 voices of 20 notes).

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  * Do not fill in B linearly: look for non covered elements with the smallest number of choices (Matolcsi)

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- Choose a number with minimal choice — it will reduce the combinatorial explosion: Ex. 20 can only come from A+20={20,6}, not A+10={10,20} (as 10 is already covered).

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  - Complete these A's (one for each value of R_A) in (non periodic) B's.
    Il colpo è fatto!

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(select a representative for each R_A before second completion)

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gComplete[{1, 3, 3, 6, 11, 4, 9, 6, 5, 1, 3, 20}]
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    - Ex: $A_0 = \{a \in A \ / \ 15 | a\} \ / \ 15$
  - But here it means A_i=complete$(1\ 1\ 1\ 3) = (0\ 4)$
    => A is 60-periodic...

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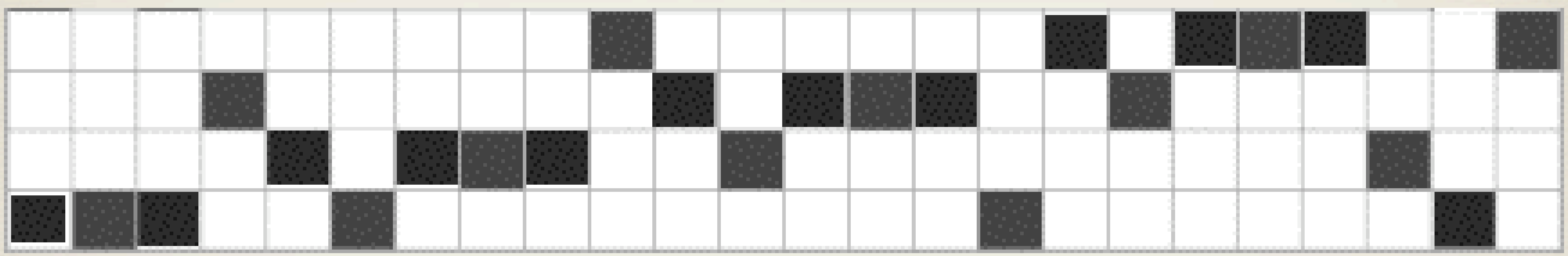

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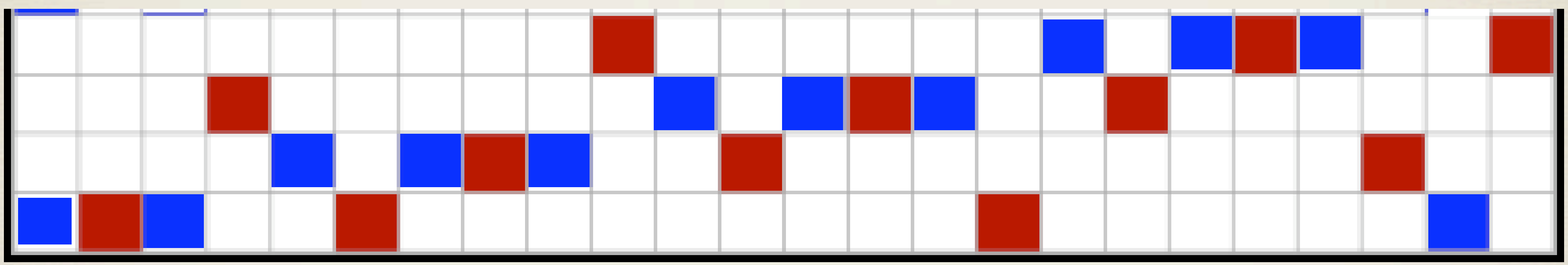

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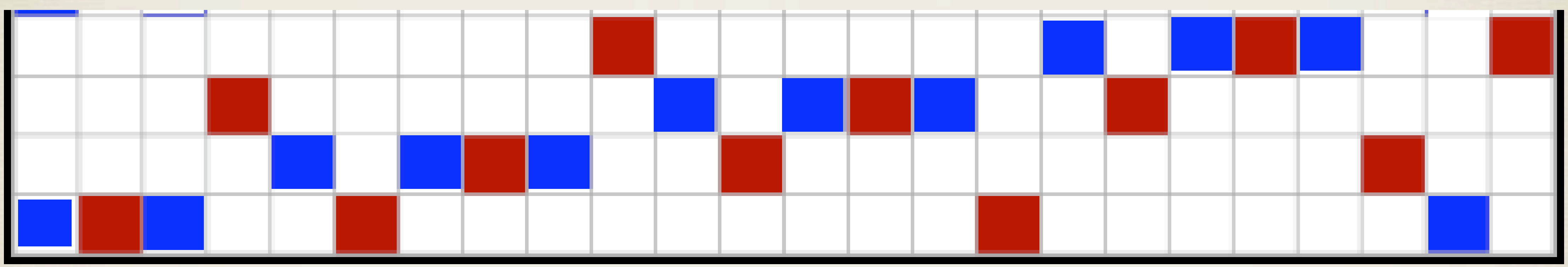

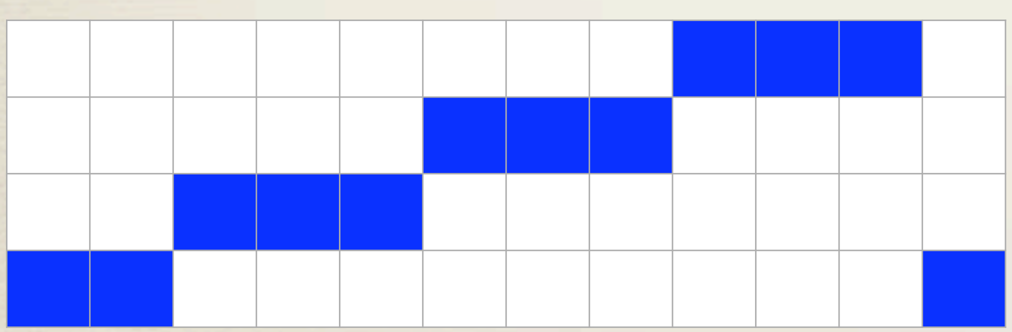

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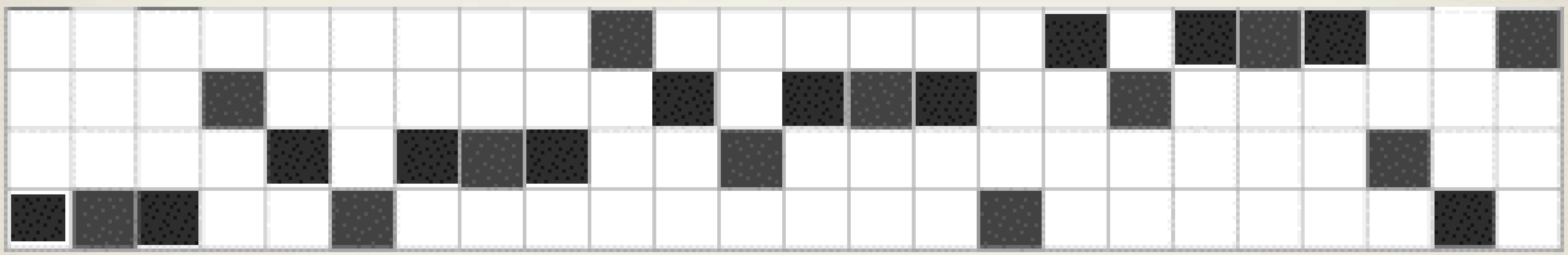

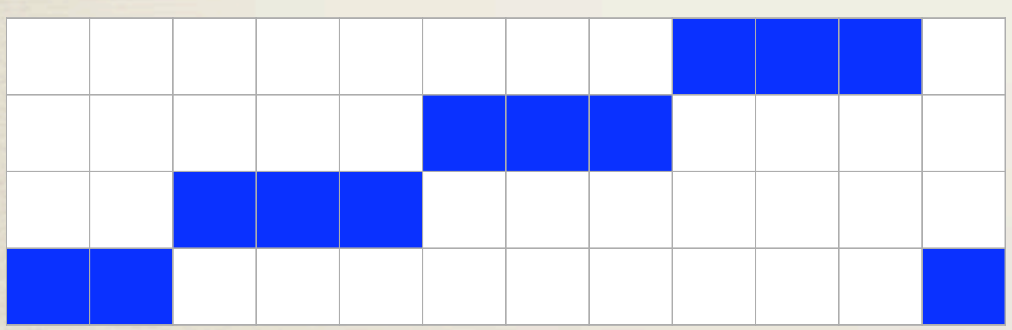

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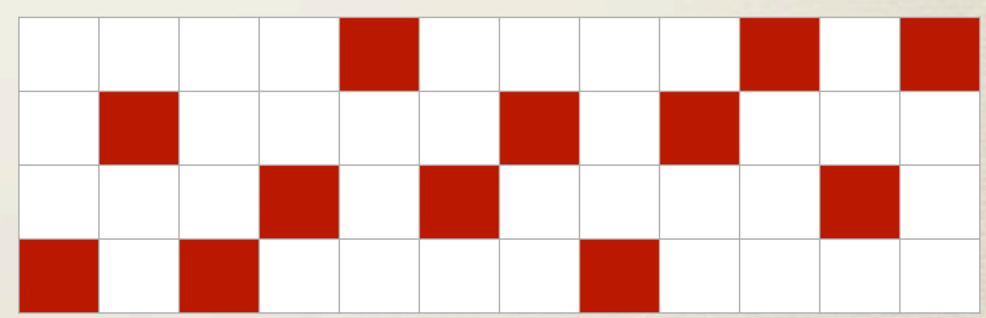

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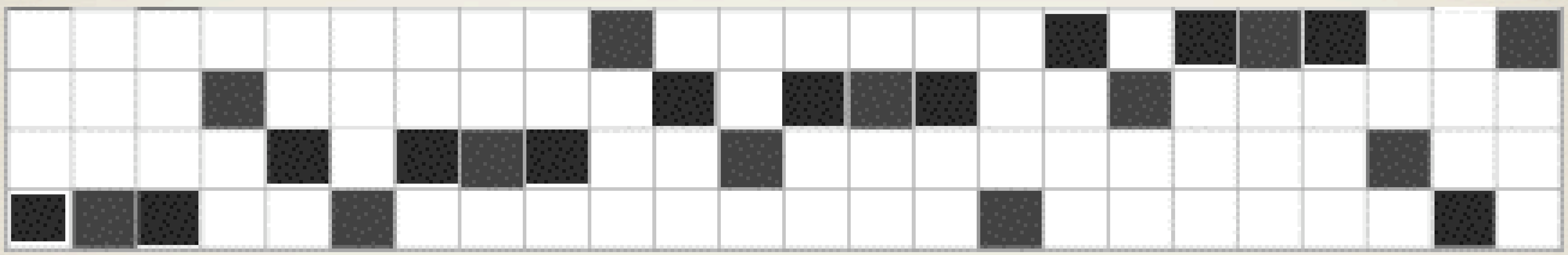

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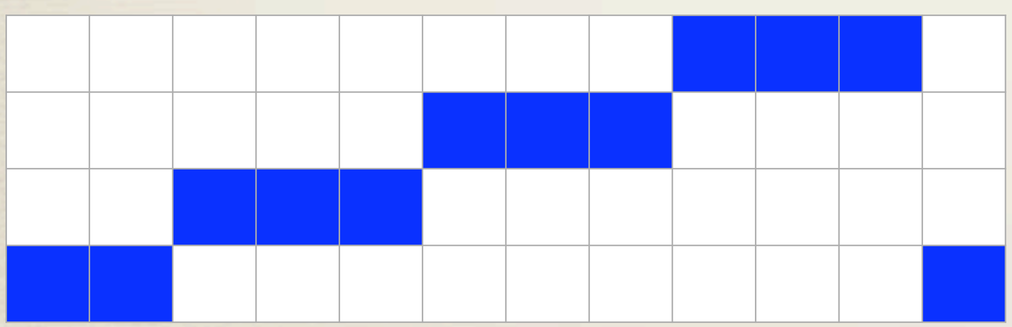

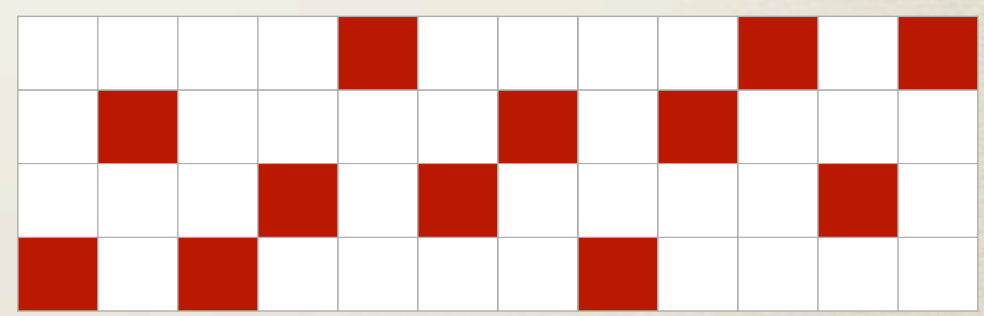

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- Hang on, it is not proved for n=120 that *(T2) holds for all tilings* (so C is not necessarily a complement for A)
- Take it from the other side, B: $S_B = \{2, 4\}$
- => B mod 4= Z_4, necessarily 0,1,2,3 mod 4. Lifting this mod 120 (adding multiples of 4) yields 18000 tiles, 225 (in basic form) only have S_B, only 16≠ R_B, all complements must be periodic (from consideration of R_B, known as R_A contains the complement of R_B).

  (Ex: $R_B=\{3,5,6,8,10,12,15,20,24,40,60,120\}$)

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  - Complete mod n, finding all B's

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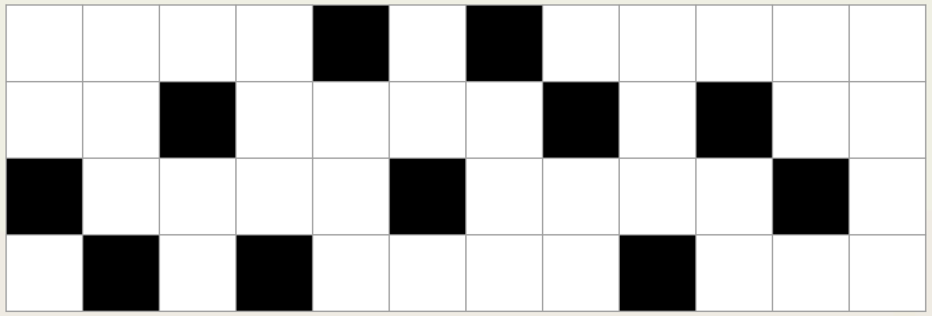

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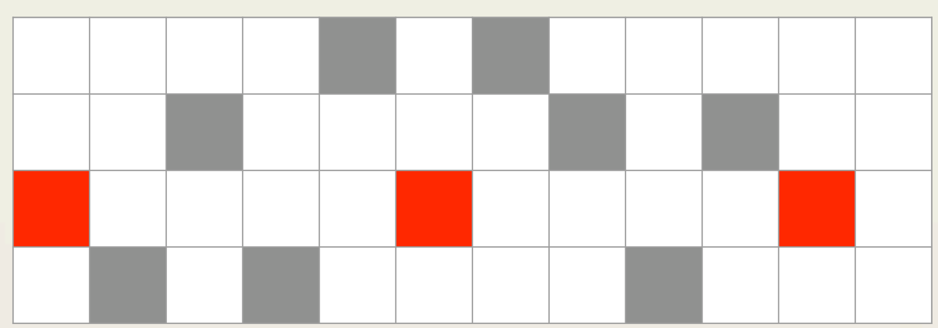

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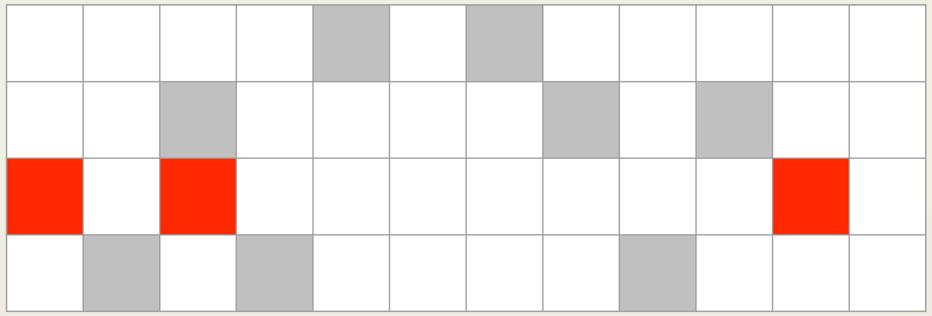

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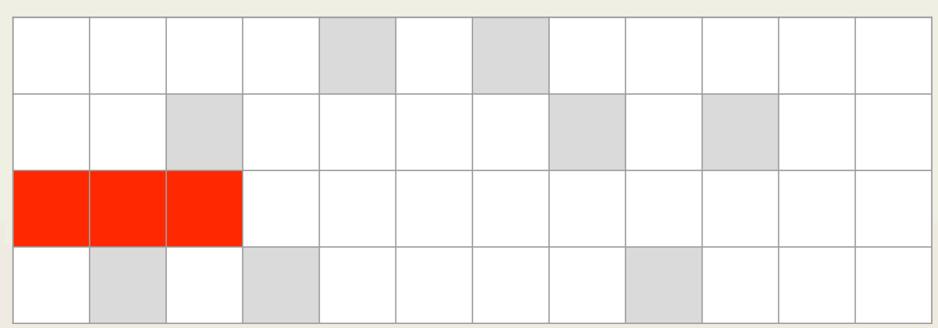

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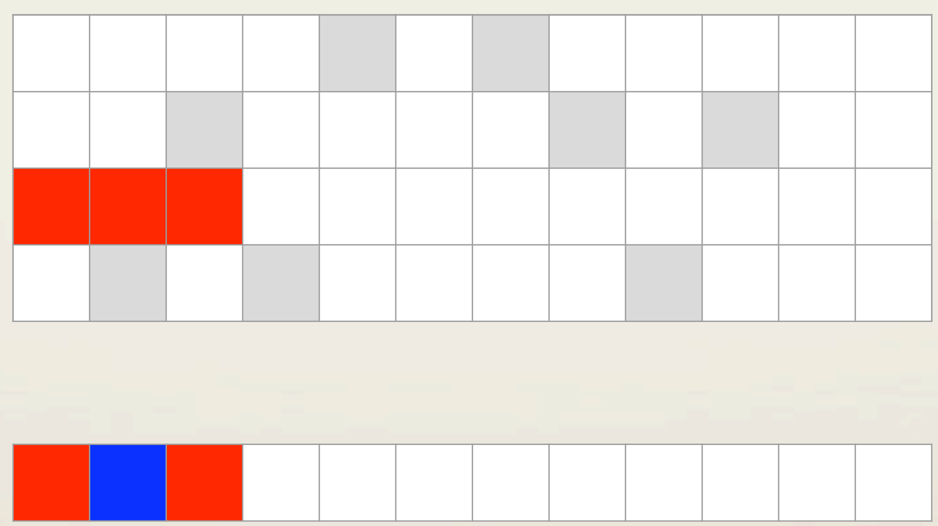

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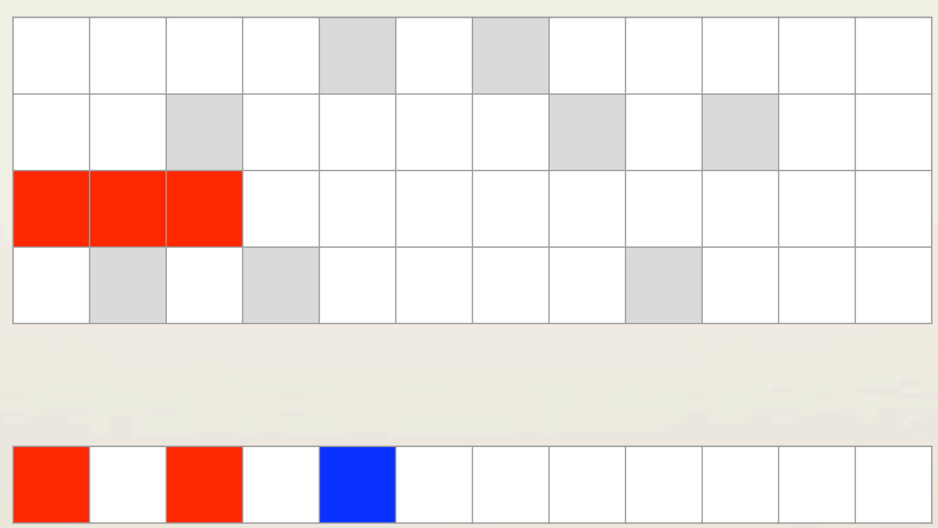

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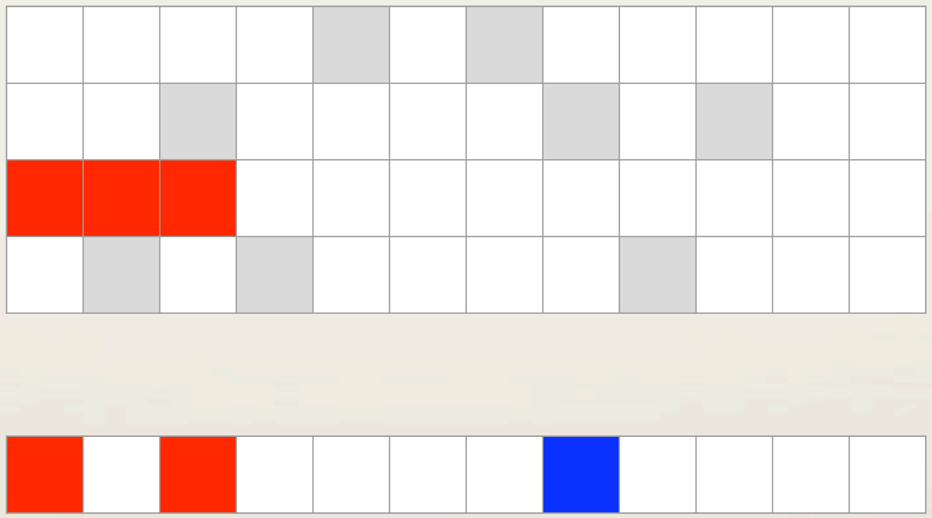

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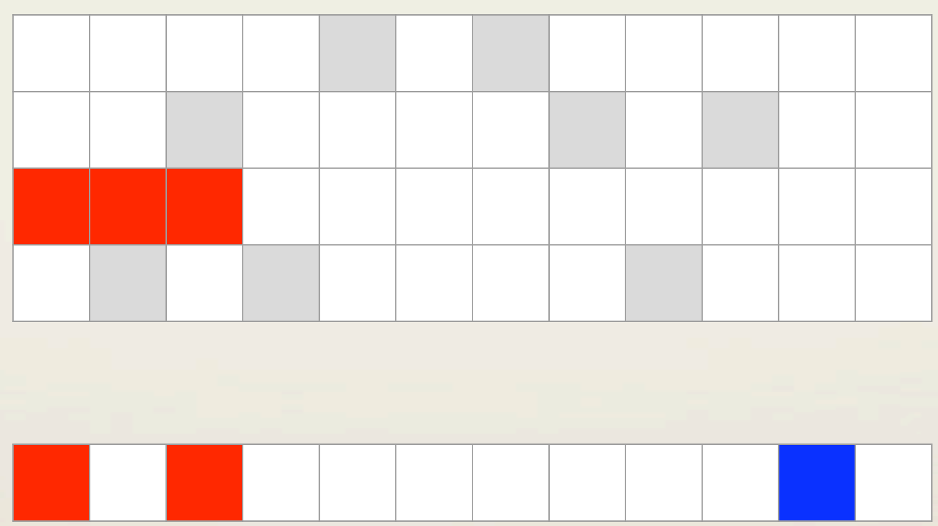

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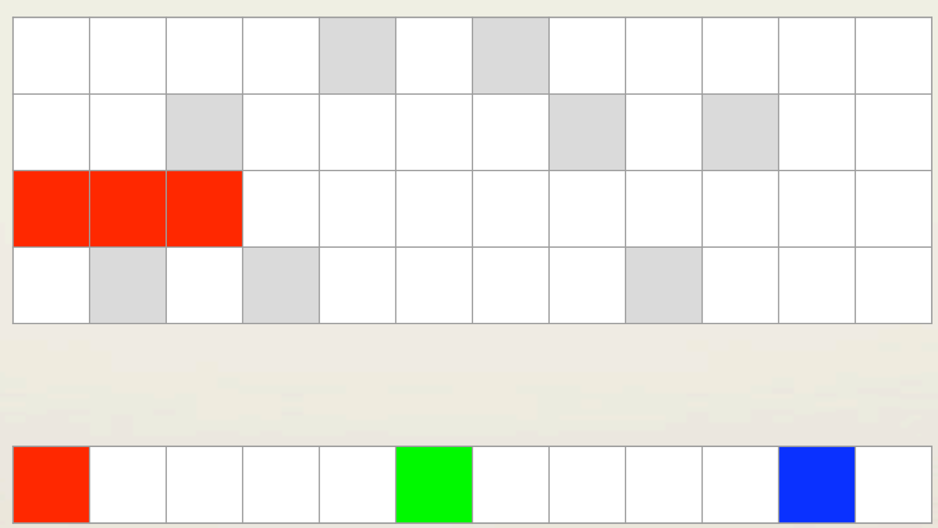

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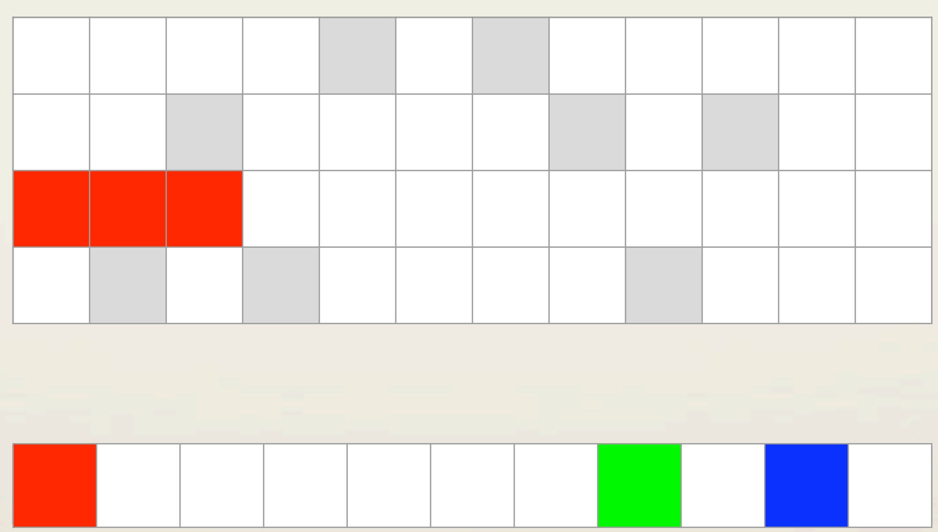

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The theory was initiated in Coven-Meyerowitz:

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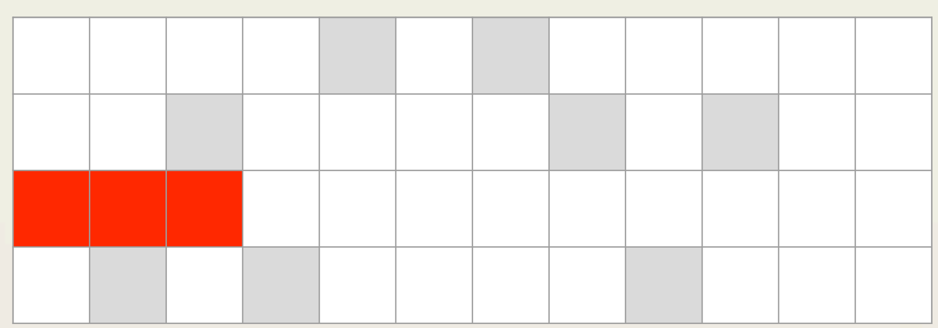

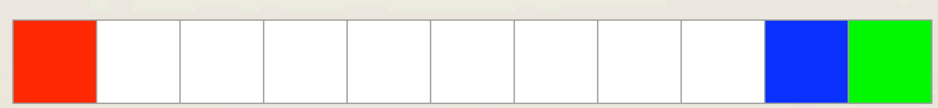

Ex: {3,8} / {2,4,5}

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```
complementCM[{3, 8}, 24]

{3, 3, 3, 15}
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```
tiles24 = fromBasicForm /@ gComplete[{3, 3, 3, 15}]

{{0, 1, 11, 12, 13, 23}, {0, 4, 11, 12, 16, 23}, {0, 2, 10, 12, 14, 22},
 {0, 5, 10, 12, 17, 22}, {0, 1, 8, 12, 13, 20}, {0, 4, 8, 12, 16, 20}}
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```

```
lift[i_] :=
 Select[ basicForm[#, 120] & /@
    Partition[
     Flatten[Table[tiles24[[i]] + {0, 24 a, 24 b, 24 c, 24 d, 24 e},
       {a, 0, 4}, {b, 0, 4} , {c, 0, 4} , {d, 0, 4} , {e, 0, 4} ]], 6],
   aperQ] // Union
```

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```

A very large output was generated. Here is a sample of it:

```
{{{1, 1, 10, 1, 1, 106}, {1, 1, 10, 1, 25, 82}, {1, 1, 10, 1, 49, 58},
  {1, 1, 10, 2, 23, 83}, {1, 1, 10, 2, 47, 59}, <<1540>>,
  {46, 1, 11, 1, 1, 60}, {46, 12, 1, 1, 11, 49}, {47, 2, 10, 1, 1, 59},
  {47, 11, 1, 1, 10, 50}, {49, 1, 10, 1, 1, 58}}, <<4>>, {<<1>>}}
```

Show Less | Show More | Show Full Output | Set Size Limit...

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```

```
élague[Partition[Flatten[%], 6]]

{{1, 1, 10, 1, 1, 106}, {2, 2, 8, 2, 2, 104}, {1, 19, 16, 20, 29, 35},
 {1, 11, 8, 12, 29, 59}, {4, 4, 4, 4, 4, 100}, {2, 12, 8, 12, 26, 60},
 {4, 8, 8, 12, 32, 56}, {4, 10, 26, 10, 34, 36}, {12, 8, 12, 8, 12, 68}}
```

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```

```
Select[%, aperiodicDiv[ Complement[div, factCyclo[#]] , 120] &]

{{1, 19, 16, 20, 29, 35}, {1, 11, 8, 12, 29, 59}, {2, 12, 8, 12, 26, 60},
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```

```
Map[Select[gComplete[#], aperQ] &,
 {{1, 19, 16, 20, 29, 35}, {1, 11, 8, 12, 29, 59}, {2, 12, 8, 12, 26, 60},
  {4, 8, 8, 12, 32, 56}, {4, 10, 26, 10, 34, 36}, {12, 8, 12, 8, 12, 68}}]

{{}, {}, {}, {}, {}, {}}
```

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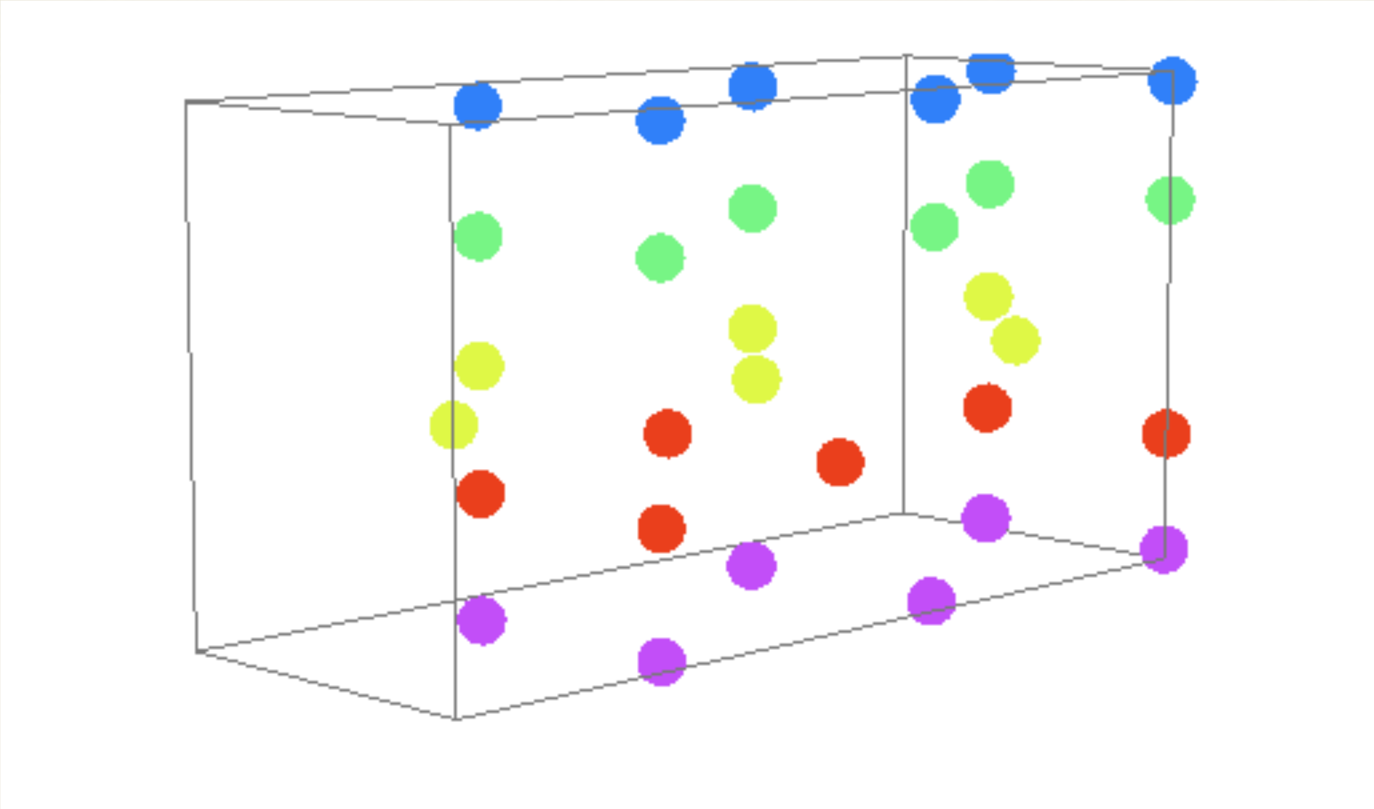
* Mathematical interest: some conditions related to tiling are still hypothetical:

  Tiling => (T_2), Tiling => Spectral
* If counter examples exist, then Vuza counter examples exist (Amiot 2003, Gilbert, 2006). Hence musical questions can help solve mathematical conjectures!



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