Appendice I

Les propriétés géométriques de la gamme diatonique dans les groupes cycliques d'ordre n=k(k+1), d'après « Balzano » : Balzano a montré que Z_{12} est isomorphe au produit direct de deux sous-groupes, Z_3 et Z_4. On peut écrire $Z_{12} \cong Z_3 \times Z_4$. Chaque élément dans $Z_3 \times Z_4$ est décrit par deux nombres $(a,b) : a \leftarrow \{0,1,2\} \wedge b \leftarrow \{0,1,2,3\}$, à la place d'un seul nombre $(a) \leftarrow \{0,1,2,...,11\}$. La règle de combinaison est $(a,b) \circ (a',b') = ([a+a'] \bmod 3, [b+b'] \bmod 4)$, une simple l'adition vectorielle. Cela signifie que les éléments $(0,0),(0,1),(0,2),(0,3),(1,0),(1,1),...,(2,2),(2,3)$ avec la règle de combinaison génèrent une structure qui est isomorphe au Z_{12}. L'isomorphisme est très simple : $(a,b) \leftrightarrow (4a+3b)_{12}$.

La représentation graphique est également très simple : un axe représente la tierce majeure (4_{12} génère Z_3), et un autre, la tierce mineure (3_{12} génère Z_4). Chaque point correspond donc aux nombres de tierces majeures et/ou mineures contenues dans cet intervalle. La quinte par exemple qui contient à la fois une tierce majeure et une tierce mineure, sera de ce fait le point (1,1).

Les structures les plus compactes et les plus connectées sont les quatre accords principaux. De plus, la gamme diatonique est construite par les accords voisins sur l'axe diagonal. En fait, cet axe diagonal représente le cycle des quintes et l'autre axe diagonal présente le cycle des semi-tons, ce qui peut se traduire par les voisinages harmonique et mélodique. Voir Figure Ap.I.

Balzano a démontré que ces aspects existent si et seulement si le groupe cyclique est de d'ordre $n = k(k+1)$, et décomposé en Z_k et Z_{k+1}. Le système tempéré aux 240 parties égales peut être décomposé en $Z_{15} \times Z_{16}$ qui comportera le système $Z_2 \times Z_3$, $Z_3 \times Z_4$ et $Z_4 \times Z_5$. Il peut être également décomposé en $Z_5 \times Z_6$ (indépendamment de $Z_{15} \times Z_{16}$). L'on arrive donc à retrouver toutes les possibilités de telles caractéristiques symétriques.

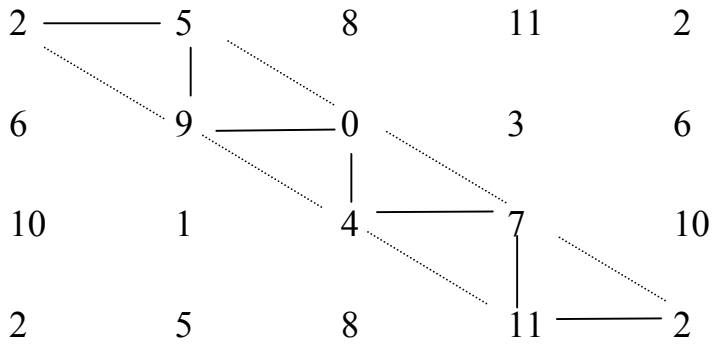

Figure Ap.I : La représentation bidimensionnelle de Z_{12}

Appendice II
La représentation des *Dastgah* par Talai.

Description of the charts:

What follows is a series of charts showing the *dâng*-s (tetrachords) and *mâyeh*-s (modes) and their position in each *dastgâh*.

1. The vertical keyboard at the left of each chart shows the widest range (six *dâng*-s) used for Persian music. The conventional pitches chosen for notation of the *radif* of the *târ* and *setâr* are used in these charts. In practice the instruments can be tuned as much as a fourth lower (but not higher) than the conventional pitches.

2. The numbers along the bottom of the charts show the different componets of the *dastgâh* structure.These numbers will be referred to in the comments.

3. Because the position of the *mayeh*-s and their relation to the open strings is important, the *dastgâh*-s are presented from the position used in the oldest *radif*, that of Mirzâ Abdollâh. The horizontal lines designate the tuning of the open strings of the *târ* or *setâr* for each *dastgâh*. Thus the tuning in Shur, for instance, is f, g, and c.

4. In the vertical keyboard, when the bottom line of a pitch is thicker, this pitch is the stressed pitch in one of the sections of the *dastgâh*.

5. Two *dâng*-s combined to form a *mâyeh* share a common pitch (except for the *mâyeh*-s of Delkesh and Râk).This pitch is designated as a square between the two *dâng*-s in the charts. In Persian music when an instrument does not have the high range of a melody, the pitches that are too high would ordinarily be transposed to the octave below. This means that position of the two *dâng*-s of the *mâyeh* would be reversed (i.e., the higher would be played an octave lower). To better show the relationship of the *mâyeh*-s in the *dastgâh*, in the following chart s the position of the *dâng*-s for all the *mâyeh*-s are shown in the position where they share a pitch. But *dâng*-s designated +8 are played an octave higher in the *dastgâh* of Mâhur in numbers 3 and 6 and in the *dastgâh* of Chahârgâh in number 2.

6.The *mâyeh* of Oj is shown throughout with the higher *dâng* only one tone higher than the one to its left instead of stacked vertically. In shur, for instance, Oj appears in steps 6 and 11.

7. The names of the *gusheh*-s listed below each chart are not all the *gusheh*-s in any given *mâyeh*, but the most important *gusheh*-s, which have a significant and distinguished modal character.

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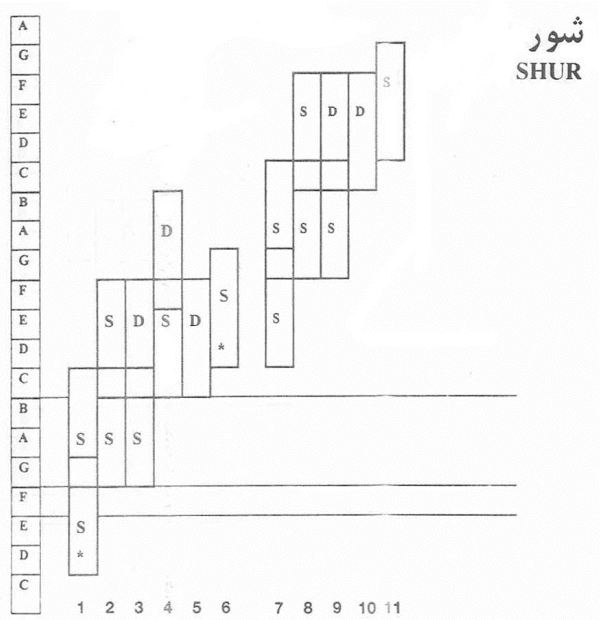
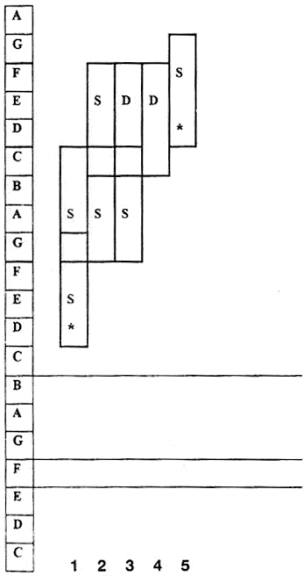
شور
SHUR



1. Darâmad of Shur
2. Oj
3. Salmak
4. Bozorg
5 & 6. 'Ozzal
7. Darâmad of Shur-e Pâyin Dasteh
8 & 9. Shahnâz, Qaracheh & Razavi
10 & 11. Oj in Shahrâshub
(* The asterisks enclose an octave.)

۱) درآمد شور
۲) اوج
۳) سلمك
۴) بزرگ
۵و۶) عزّال
۷) درآمد شور پایین دسته
۸ و ۹) شهناز، قرچه و رضوی
۱۰ و ۱۱) اوج در شهر آشوب
(* ستاره‌ها یك اکتاو را در برمی گیرند.)

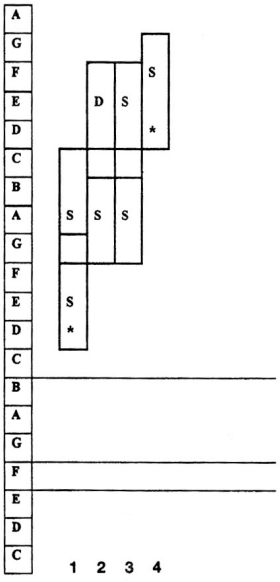
دشتی
DASHTI



1, 2 & 3. Dashti
5 & 6. Oj

۱ و ۲ و ۳) دشتی
۵ و ۶) اوج

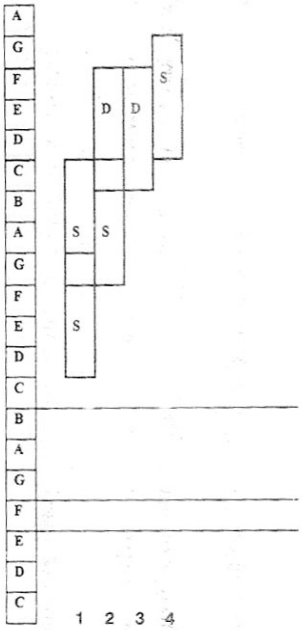
بیات ترك
BAYAT-E TORK



1 & 2. bayât-e tork
1. Ruh ol-arvâh
2. Mehrabâni
3 & 4. Shekasteh

۱ و ۲) بیات ترك
۱) روح الارواح
۲) مهربانی
۳ و ۴) شکسته

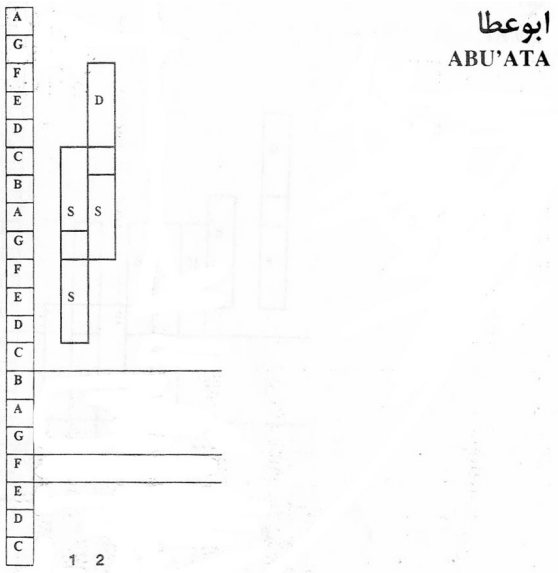
بیات کرد
BAYAT-E KORD



1 & 2. Bayât-e Kord
3 & 4. Oj

۱ و ۲) بیات کرد
۳ و ۴) اوج

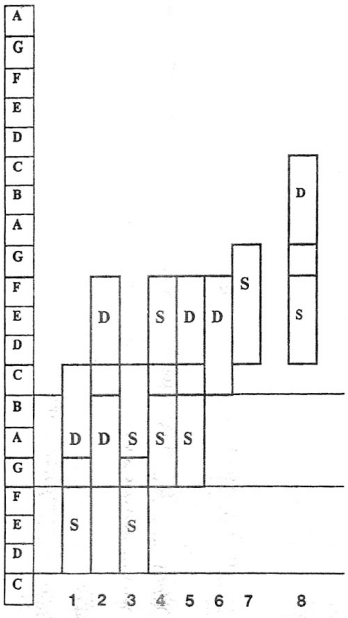
ابوعطا
ABU'ATA



1. Abuatâ
2. Hejâz

۱) ابوعطا
۲) حجاز

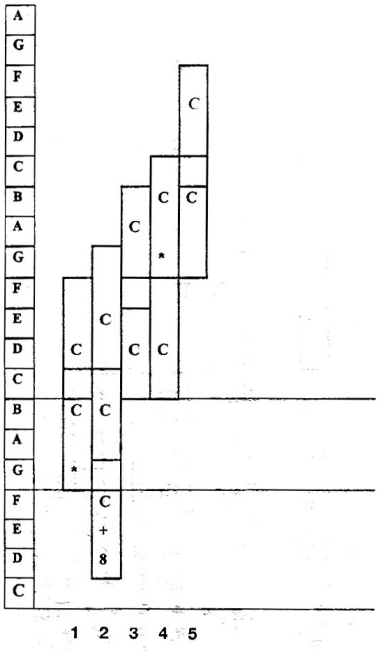
نوا
NAVA



1. Darâmad of Navâ
1. Bayât-e Râjeh (The stressed pitch is A.)
2. Nahoft
3. Ashirân
4. Gavesht
5. Khojasteh
5. & 7. Hosain
8. Busalik

۱) درآمد نوا
۱) بیات راجه (نت شاهد A است.)
۲) نهفت
۳) عشیران
۴) گوشت
۵) خجسته
۵ و ۷) حسین
۸) بوسلیك

چهارگاه
CHAHARGAH



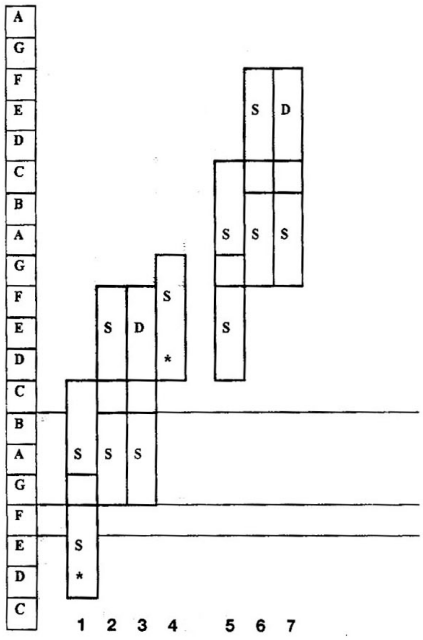
The lower C string serves primarily as a pedal tone (*vakhum*) and the two other strings are the melody strings.

1. Darâmad of Chahârgâh (The stressed pitch is C.)
1. Zâbol (The stressed pitch is E.)
2. Hesâr is played one octave above (+8) and the G shown as stressed pitch is for this *gusheh*.
3. Muyeh 4. Mokhâlef & Maghlub
5. Mansuri
(* The asterisks enclose an octave.)

سیم C پائین در اصل برای واخوان استفاده می‌شود و دو سیم دیگر سیمهای ملودیك هستند.

۱) درآمد چهارگاه (نت شاهد C است)
۱) زابل (نت شاهد E است)
۲) حصار (این گوشه از یك کتاو بالاتر «8+» اجرا می‌شود و نت شاهد G است)
۳) مویه ۴) مخالف و مغلوب
۵) منصوری
(* ستاره‌ها یك اکتاو را در بر می‌گیرند.)

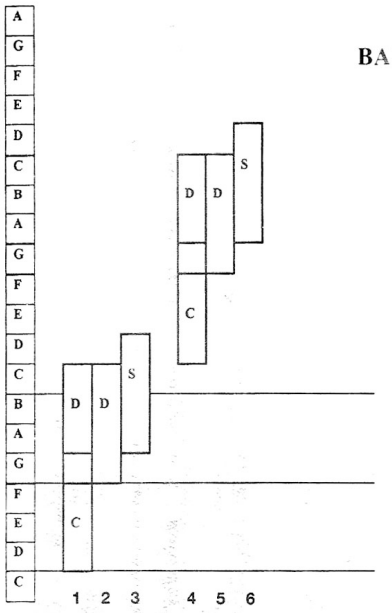
سه‌گاه
SEGAH



1 & 2. Darâmad of Segâh
2. Zâbol (The stressed pitch is C.)
2. Muyeh (The stressed pitches are Dp and Eb.)
3 & 4. Hesâr (C, D from number 3 and Ep, F, G from number 4 The stressed pitch is Ep.)
2 & 4. Mokhâlef (C, Dp from number 2 and Ep, F, G from number 4 The stressed pitch is F.)
5. Maghlub
5 & 6. Rahâb & Masihi
5, 6 & 7. Shâh Khatâyi
(* The asterisks enclose an octave.)

۱ و ۲) درآمد سه‌گاه
۲) زابل (نت شاهد C است)
۲) مویه (انتهای شاهد Dp و Eb هستند)
۳ و ۴) حصار (انتهای C و D از شماره ۳ و F، EP و G از شماره ۴. نت شاهد EP است)
۲ و ۴) مخالف (انتهای C و DP از شماره ۲ و F، EP و G از شماره ۴. نت شاهد F است)
۵) مغلوب
۵ و ۶) رهاب و مسیحی
۵ و ۶ و ۷) شاه ختایی
(* ستاره‌ها یك اکتاو را در برمی‌گیرند.)

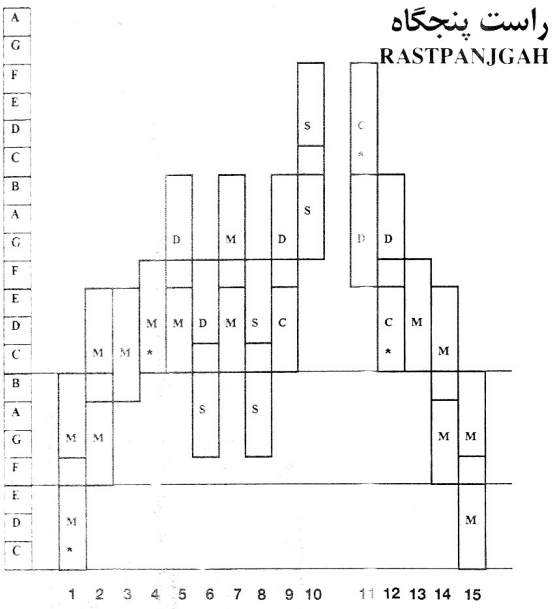
بیات اصفهان
BAYAT-E ESFEHAN



۴) اصفهان
۵ و ۶) اوج
۱ و ۲ و ۳) فرود
(این قسمت یك اکتاو پائین‌تر از ۴ و ۵ و ۶ است که برای فرود اواز استفاده می‌شود)

4. Esfehân
5 & 6. Oj
1, 2 & 3. Forud (This part is the same as 4, 5 & 6 an octave lower, used for the descent of the *âvâz*.)

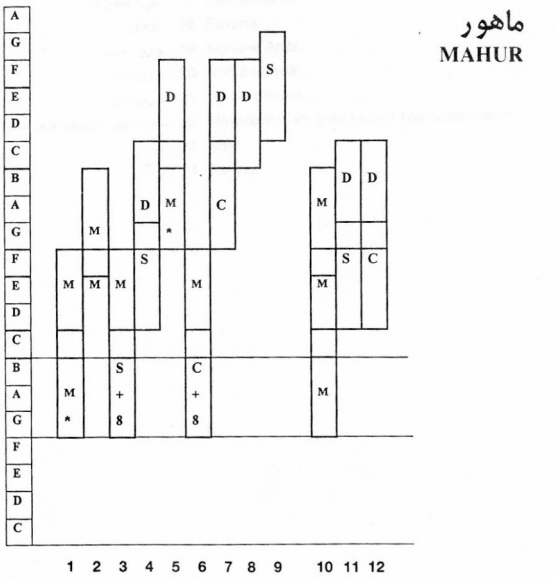
راست پنجگاه
RASTPANJGAH



۱و۲) راست
۳و۴) عراق(همان نقش اوج را دارد)
۵) عراق و نهیب
۶) پنجگاه و روح‌افزا
۶و۴) عشّاق
۷) سپهر
۸) بیات عجم و قرچه
۹) طرز و ابوالچپ
۱۰) نوروز عرب و ماوراءالنهر
۱۱ و ۱۲ و ۱۳ و ۱۴ و ۱۵) فرنگ (۱۵ فرود)
(* ستاره‌ها یك اکتاو را در برمی گیرند)

1 & 2. Râst
3 & 4. 'Arâq (the same function as Oj)
5. 'Arâq & Nahib
6. Panjgâh & Ruhafzâ
6 & 4. Oshshâq
7. Sepehr
8. Bayât-e Ajam & Qaracheh
9. Tarz & Abolchap
10. Noruz-e Arab & Mavarâ ol-Nahr
11, 12, 13, 14 & 15. Farang (15 Forud)
(* The asterisks enclose an octave.)

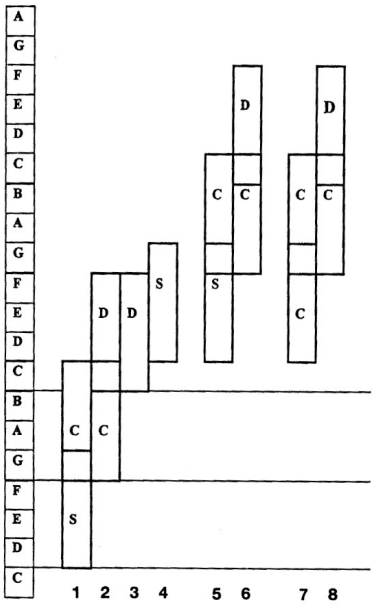
ماهور
MAHUR



سیم C پائین در اصل برای واخوان استفاده می‌شود و دو سیم دیگر سیمهای ملودیك هستند.
۱) درآمد ماهور
۲) حصار ماهور (یا پس ماهور) ۳) دلکش
۴) شکسته ۵) عراق
۶) راك ۷) راك و اصفهانك
۸ و ۹) سفیر راك و اشور
۱۰) ساقی نامه
۱۱) کُشته
۱۲) صوفی نامه

The lower C string serves primarily as a pedal tone (*vakhum*) and the two other strings are the melody strings.
1. Darâmad of Mâhur
2. Hesâr Mâhur (or Pas Mâhur) 3. Delkesh
4. Shekasteh 5. 'Arâq
6. Râk 7. Râk & Esfehânak
8. & 9. Safir-e Râk & Ashur
10. Sâqi Nâmeh
11. Koshteh
12. Sufi Nâmeh

همایون
HOMAYUN



۱) درآمد همایون
۲) چکاوك
۲) بیداد
۳ و ۴) اوج بیداد
۵ و ۲) نوروز عرب
۶) شوشتری
۷ و ۸) موالیان

1. Darâmad of Homâyun
2. Chakâvak
2. Bidâd
3 & 4. Oj of Bidâd
5 & 2. Noruz-e Arab
6. Shushtari
7 & 8. Mavâliân

Appendice III

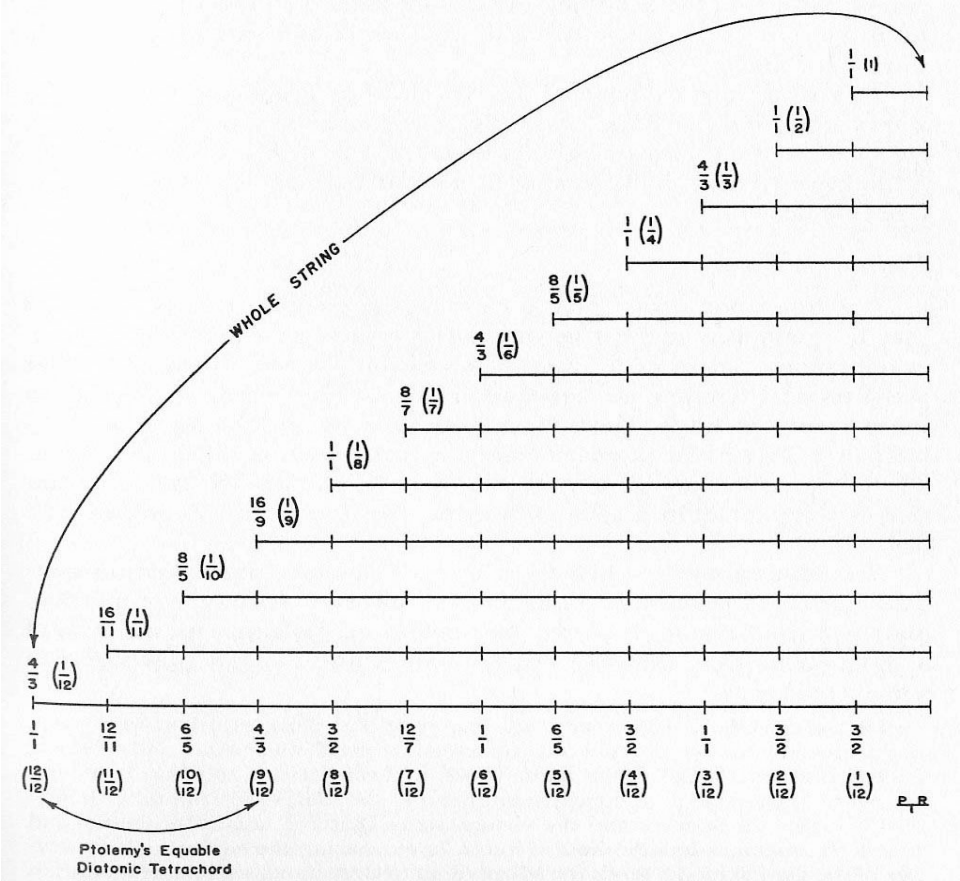


DIAGRAM —THE ARITHMETICAL PROPORTION IN TWELVE PARTS

Calcule de la longueur de la corde correspondante aux rapports fréquentiels.

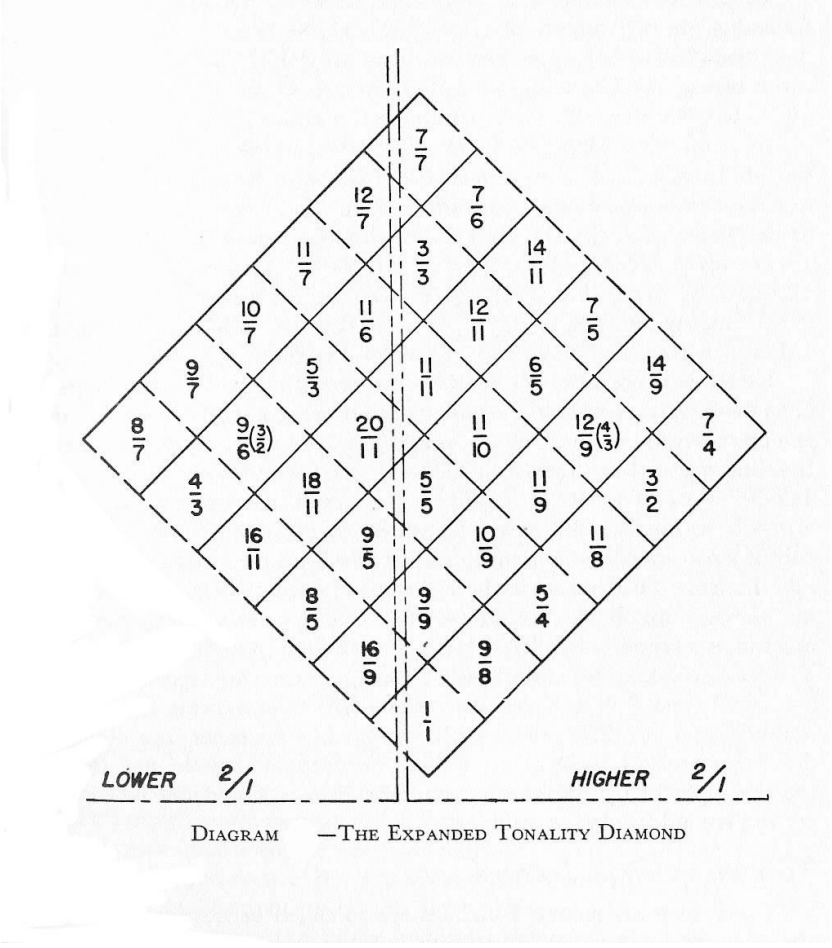


DIAGRAM —THE EXPANDED TONALITY DIAMOND

Appendice IV

Tonality Relationships and Common Tones

In Table 1 on the next page Otonalities are indicated by the letter *O* and Utonalities by *U*. For the benefit of music students it should be explained that the names of degrees for "minor" are the reverse of "major," and all relationships for "minor" are downward; that is, the "dominant" of "minor" is found five scale "steps" below the "tonic," and the "subdominant" is found four scale "steps" below. Also, the "dominants" and "subdominants" of Otonalities are Otonalities, and the "mediants" and "relatives" of Otonalities are Utonalities; the "dominants" and "subdominants" of Utonalities are Utonalities, and the "mediants" and "relatives" of Utonalities are Otonalities. The relationships given in Table 1 are confined to those involving the smallest numbers.

In Table 2 the Monophonic tonalities are listed according to the place of their 1–5–3 triads (upward for Otonality and downward for Utonality) in the 2/1. The lowest tones of these triads are the 1 Odentities of Otonalities and the 3 Udentities of Utonalities. The 3 Udentities, ordinarily considered the "tonic" degree in "minor," are shown in parentheses, and the 1 Udentities, which Monophony considers the true unities in Utonality, are shown on the same lines, without parentheses.

In the chart on pages 338 and 339 the unities of the Monophonic tonalities are given at right and left, and the forty-three Monophonic ratios horizontally at top and bottom. Converging points show the identity of each ratio in a tonality, and the number of senses in which a ratio may be taken is simply the total number of identities in its vertical column.

TABLE I

Tonalities	Their 3/2 Tonalities ("Dominant")	Their 4/3 Tonalities ("Subdominant")	Their 5/4 Tonalities ("Relative")	Their 15/8 Tonalities ("Mediant")
1. 1/1–O	3/2–O	4/3–O	5/4–U	15/8–U
2. 1/1–U	4/3–U	3/2–U	8/5–O	16/15–O
3. 16/15–O	8/5–O	.....	4/3–U	1/1–U
4. 10/9–U	40/27–U	5/3–U	16/9–O	32/27–O
5. 9/8–U	3/2–U	27/16–U	9/5–O	6/5–O
6. 8/7–O	.....	32/21–O	10/7–U	.....
7. 32/27–O	16/9–O	.....	40/27–U	10/9–U
8. 6/5–O	9/5–O	8/5–O	3/2–U	9/8–U
9. 5/4–U	5/3–U	15/8–U	1/1–O	4/3–O
10. 21/16–U	7/4–U	.....	.....	7/5–O
11. 4/3–O	1/1–O	16/9–O	5/3–U	5/4–U
12. 4/3–U	.....	1/1–U	16/15–O	.....
13. 27/20–O	.....	9/5–O	.....	.....
14. 11/8–U	.....	.....	.....	.....
15. 7/5–O	.....	.....	7/4–U	21/16–U
16. 10/7–U	.....	.....	8/7–O	32/21–O
17. 16/11–O	.....	.....	.....	.....
18. 40/27–U	.....	10/9–U	.....	.....
19. 3/2–O	.....	1/1–O	15/8–U	.....
20. 3/2–U	1/1–U	9/8–U	6/5–O	8/5–O
21. 32/21–O	8/7–O	.....	.....	10/7–U
22. 8/5–O	6/5–O	16/15–O	1/1–U	3/2–U
23. 5/3–U	10/9–U	5/4–U	4/3–O	16/9–O
24. 27/16–U	9/8–U	.....	27/20–O	9/5–O
25. 7/4–U	.....	21/16–U	7/5–O	.....
26. 16/9–O	4/3–O	32/27–O	10/9–U	5/3–U
27. 9/5–O	27/20–O	6/5–O	9/8–U	27/16–U
28. 15/8–U	5/4–U	.....	3/2–O	1/1–O

TABLE 2

1. 1/1–O	8. 32/27–O	15. 16/11–O	22. (7/4)21/16–U
2. (1/1)3/2–U	9. 6/5–O	16. (40/27)10/9–U	23. 16/9–O
3. 16/15–O	10. (5/4)15/8–U	17. 3/2–O	24. (16/9)4/3–U
4. (10/9)5/3–U	11. 4/3–O	18. (3/2)9/8–U	25. 9/5–O
5. (9/8)27/16–U	12. (4/3)1/1–U	19. 32/21–O	26. (11/6)11/8–U
6. 8/7–O	13. 27/20–O	20. 8/5–O	27. (40/21)10/7–U
7. (7/6)7/4–U	14. 7/5–O	21. (5/3)5/4–U	28. (160/81)40/27–U

Chart of

	10/7	16/11	40/27	3/2	32/21	14/9	11/7	8/5	18/11	5/3	27/16	12/7	7/4	16/9	9/5	20/11	11/6	15/8	40/21	64/33	160/81	1/1
1/1-O				3									7									1
1/1-U		11						5						9								1
16/15-O								3														
10/9-U			3											5							9	
9/8-U				3					11						5							9
8/7-O	5						11					3										7
32/27-O			5											3								
6/5-O				5											3							
5/4-U	7									3						11						5
21/16-U				7									3									
4/3-O				9						5							11					3
4/3-U					7									3						11		
27/20-O											5											
11/8-U							7										3					11
7/5-O													5									
10/7-U	1																		3			
16/11-O		1							9							5						11
40/27-U			1																		3	
3/2-O				1							9							5				
3/2-U				1								7										3
32/21-O					1							9							5			
8/5-O								1							9							5
5/3-U			9							1									7			
27/16-U				9							1											
7/4-U						9							1									7
16/9-O						7								1								9
9/5-O															1							
15/8-U				5						9								1				
	10	16	40	3	32	14	11	8	18	5	27	12	7	16	9	20	11	15	40	64	160	1

Common Tones

81/80	33/32	21/20	16/15	12/11	11/10	10/9	9/8	8/7	7/6	32/27	6/5	11/9	5/4	14/11	9/7	21/16	4/3	27/20	11/8	7/5	
							9						5						11		1/1-O
								7									3				1/1-U
			1								9						5				16/15-O
						1															10/9-U
							1								7						9/8-U
								1							9						8/7-O
										1							9				32/27-O
		7									1							9			6/5-O
						9							1								5/4-U
		5							9							1					21/16-U
									7								1				4/3-O
			5							9							1				4/3-U
3																		1			27/20-O
					5							9							1		11/8-U
		3																		1	7/5-O
								5													10/7-U
				3										7							16/11-O
										5											40/27-U
	11						3									7					3/2-O
				11							5						9				3/2-U
								3									7				32/21-O
					11						3									7	8/5-O
						3											5				5/3-U
							3											5			27/16-U
									3					11						5	7/4-U
						5						11					3				16/9-O
9							5											3			9/5-O
													3								15/8-U
81/80	33/32	21/20	16/15	12/11	11/10	10/9	9/8	8/7	7/6	32/27	6/5	11/9	5/4	14/11	9/7	21/16	4/3	27/20	11/8	7/5	