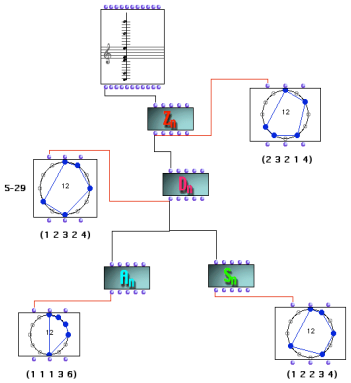
PAVIA FESTIVAL DEI SAPERI

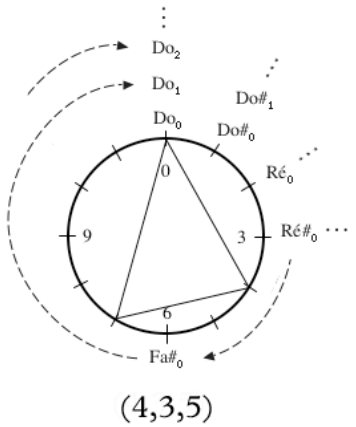
3-7 settembre 2008

Linguaggi della creatività:
matematica e musica

Musica e simmetria: trasformazioni geometriche e strutture matematiche nell'analisi musicale



Moreno Andreatta & Carlos Agon
Equipe Représentations Musicales
IRCAM/CNRS

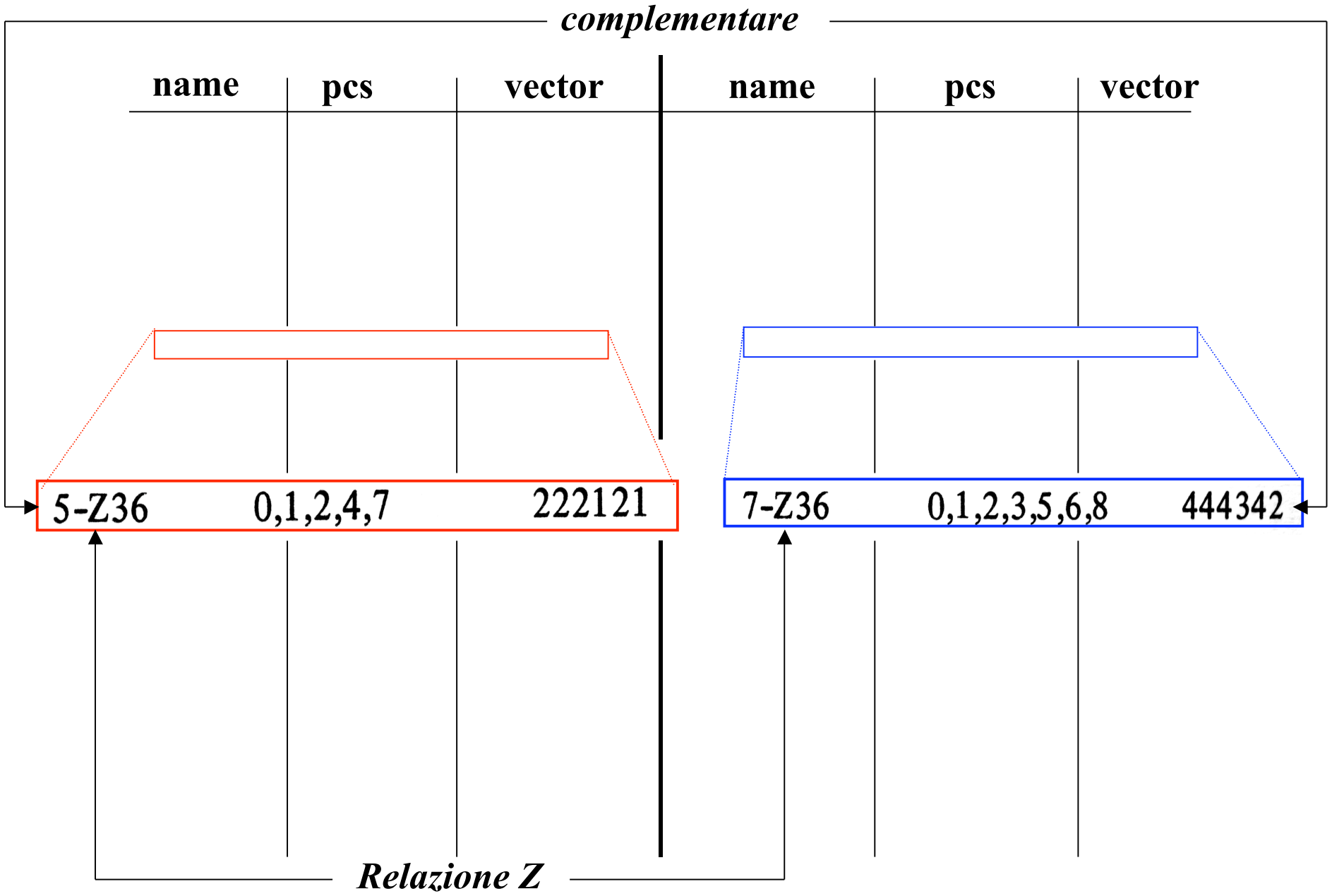


(4,3,5)

ircam Centre Pompidou

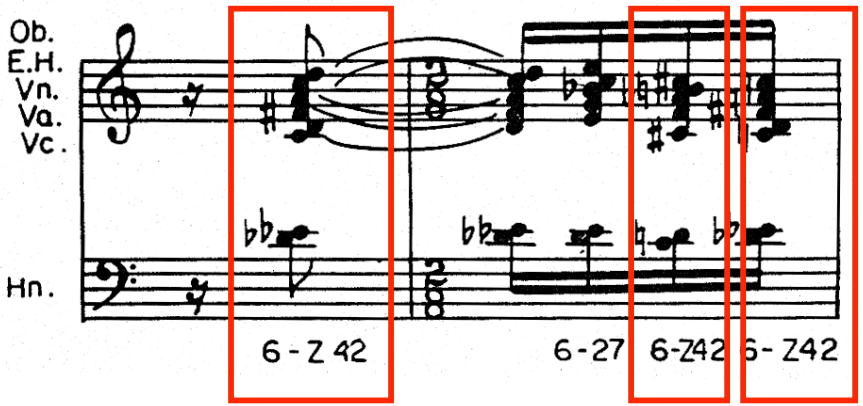
CNRS CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

La *Set Theory* d'Allen Forte: catalogo dei *pitch-class sets*

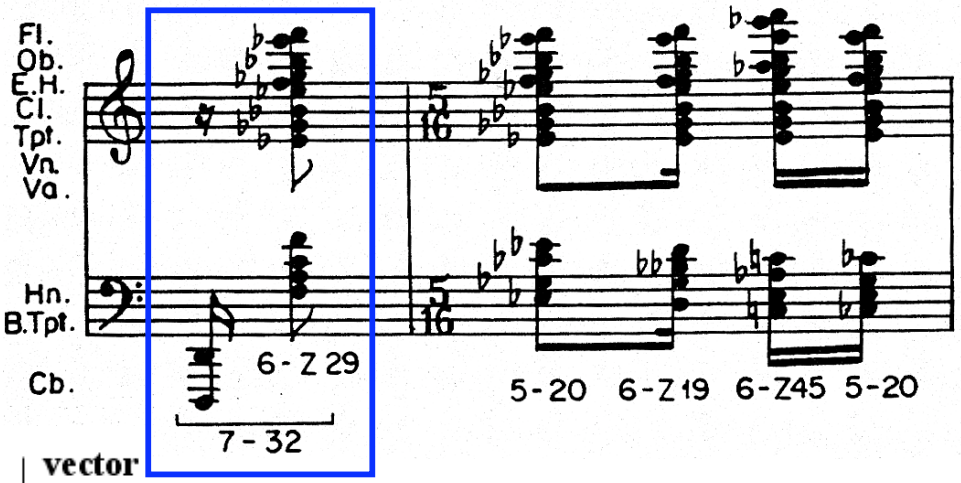


La *Set Theory* d'Allen Forte: esempio d'analisi musicale

Ex. 89. *Sacrificial Dance:* R142+3



Ex. 90. *Sacrificial Dance:* R144



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5-32	0,1,4,6,9	113221
5-33(12)	0,2,4,6,8	040402
5-34(12)	0,2,4,6,9	032221
5-35(12)	0,2,4,7,9	032140
5-Z36	0,1,2,4,7	222121
5-Z37(12)	0,3,4,5,8	212320
5-Z38	0,1,2,5,8	212221
6-1(12)	0,1,2,3,4,5	543210
6-2	0,1,2,3,4,6	443211
6-Z3	0,1,2,3,5,6	433221
6-Z4(12)	0,1,2,4,5,6	432321
6-5	0,1,2,3,6,7	422232
6-Z6(12)	0,1,2,5,6,7	421242
6-7(6)	0,1,2,6,7,8	420243
6-8(12)	0,2,3,4,5,7	343230
6-9	0,1,2,3,5,7	342231
6-Z10	0,1,3,4,5,7	333321
6-Z11	0,1,2,4,5,7	333231
6-Z12	0,1,2,4,6,7	332232
6-Z13(12)	0,1,3,4,6,7	324222

name	pcs	vector
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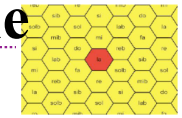
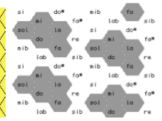
7-31	0,1,3,4,6,7,9	336333
7-32	0,1,3,4,6,8,9	335442
7-33	0,1,2,4,6,8,10	262623
7-34	0,1,3,4,6,8,10	254442
7-35	0,1,3,5,6,8,10	254361
7-Z36	0,1,2,3,5,6,8	444342
7-Z37	0,1,3,4,5,7,8	434541
7-Z38	0,1,2,4,5,7,8	434442
6-Z36	0,1,2,3,4,7	
6-Z37(12)	0,1,2,3,4,8	
6-Z38(12)	0,1,2,3,7,8	
6-Z39	0,2,3,4,5,8	
6-Z40	0,1,2,3,5,8	
6-Z41	0,1,2,3,6,8	
6-Z42(12)	0,1,2,3,6,9	

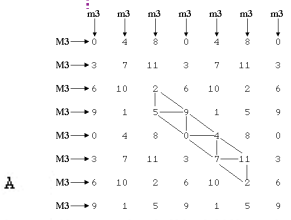
5-16
5-31
5-32
⊂ 6-27 ⊂
7-16
7-31
7-32

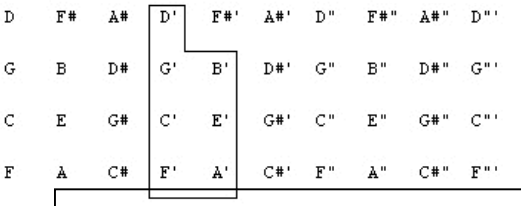
5-31 ⊂
6-z42
6-27
6-z28

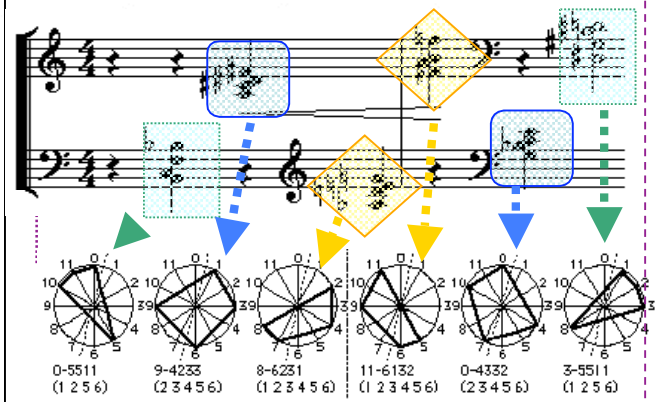
Teorie diatoniche

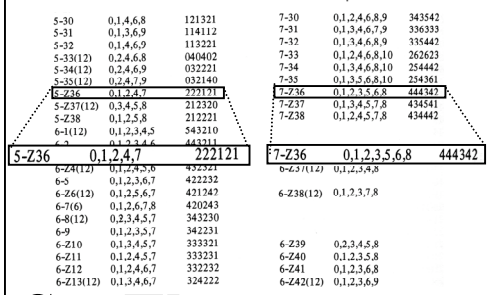






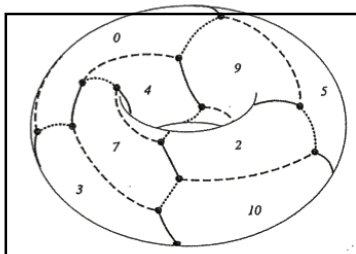


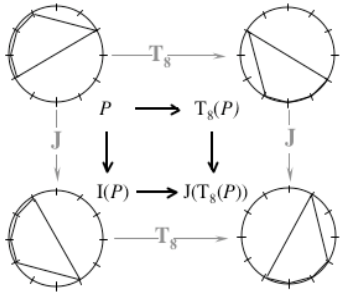


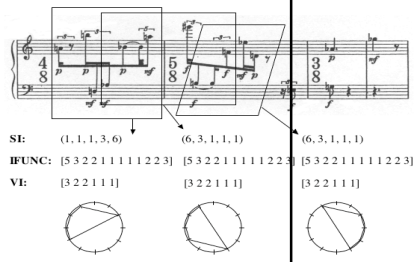


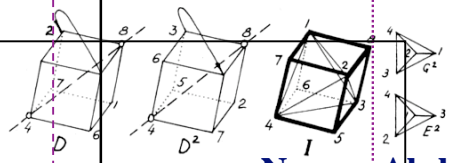
Set Theory

Teorie trasformazionali

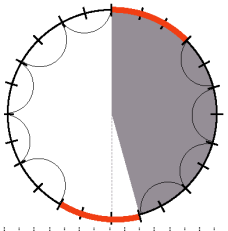


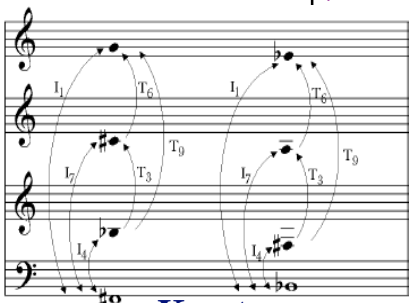




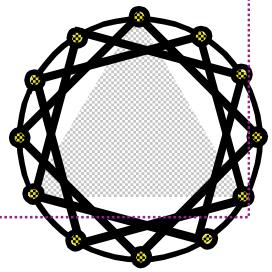


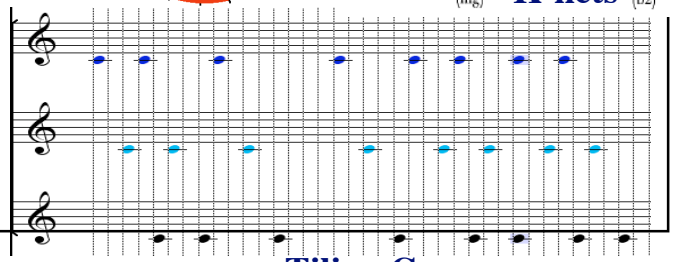
Nomos Alpha



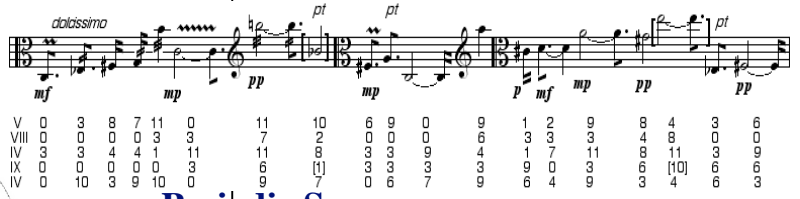


K-nets (b2)

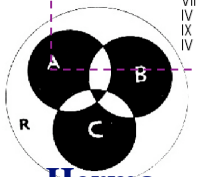




Tiling Canons

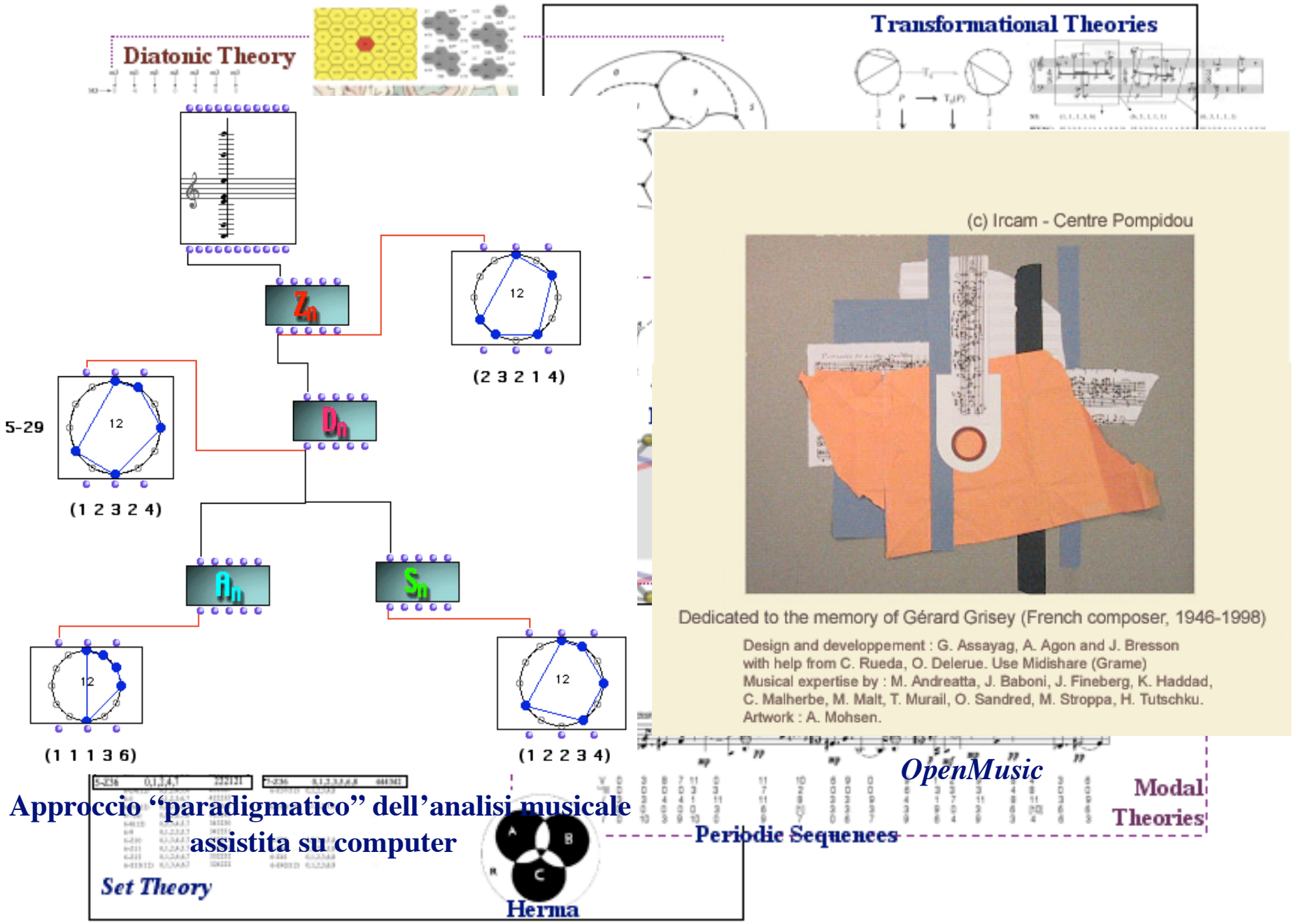


Periodic Sequences



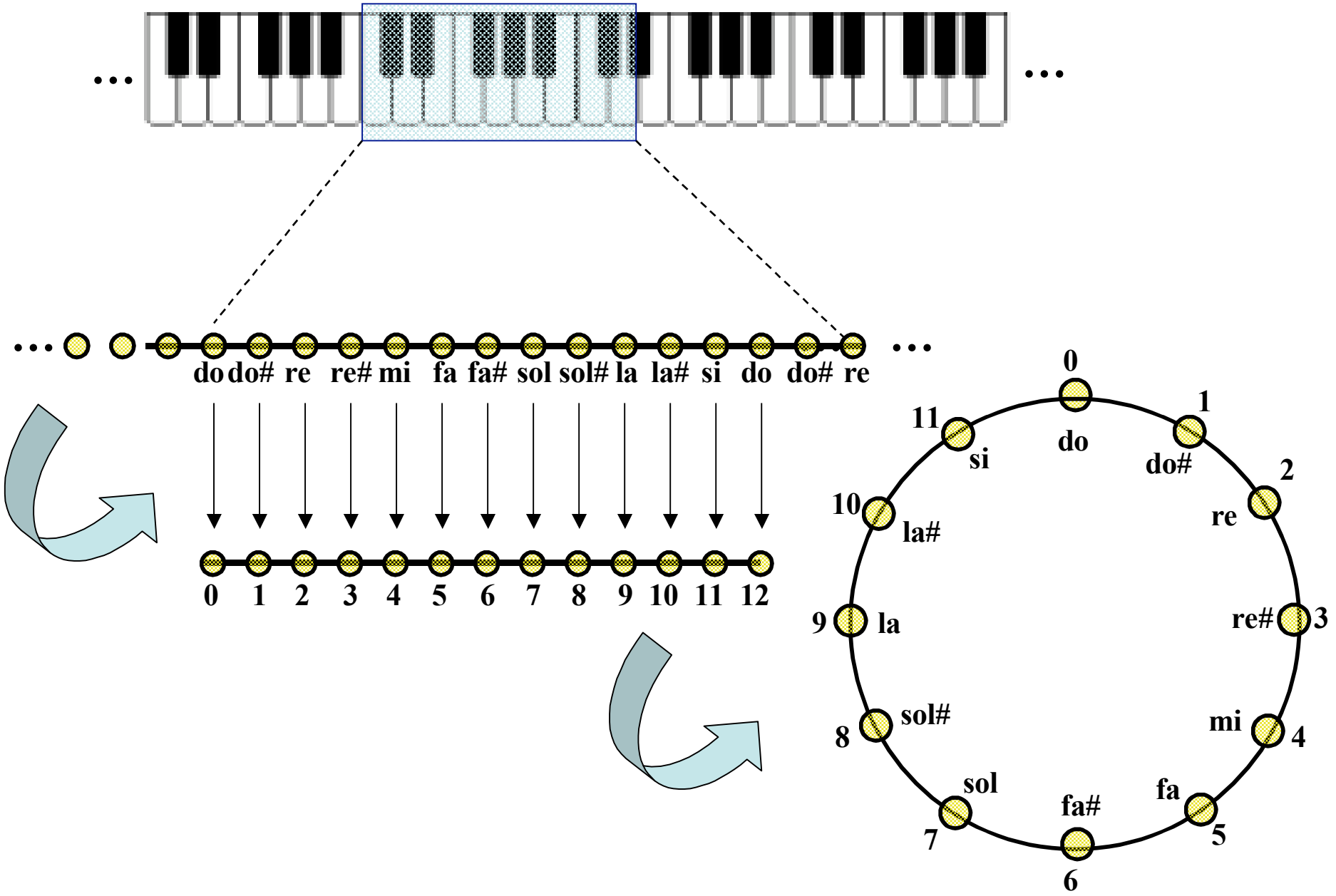
Herma

Teorie ritmiche

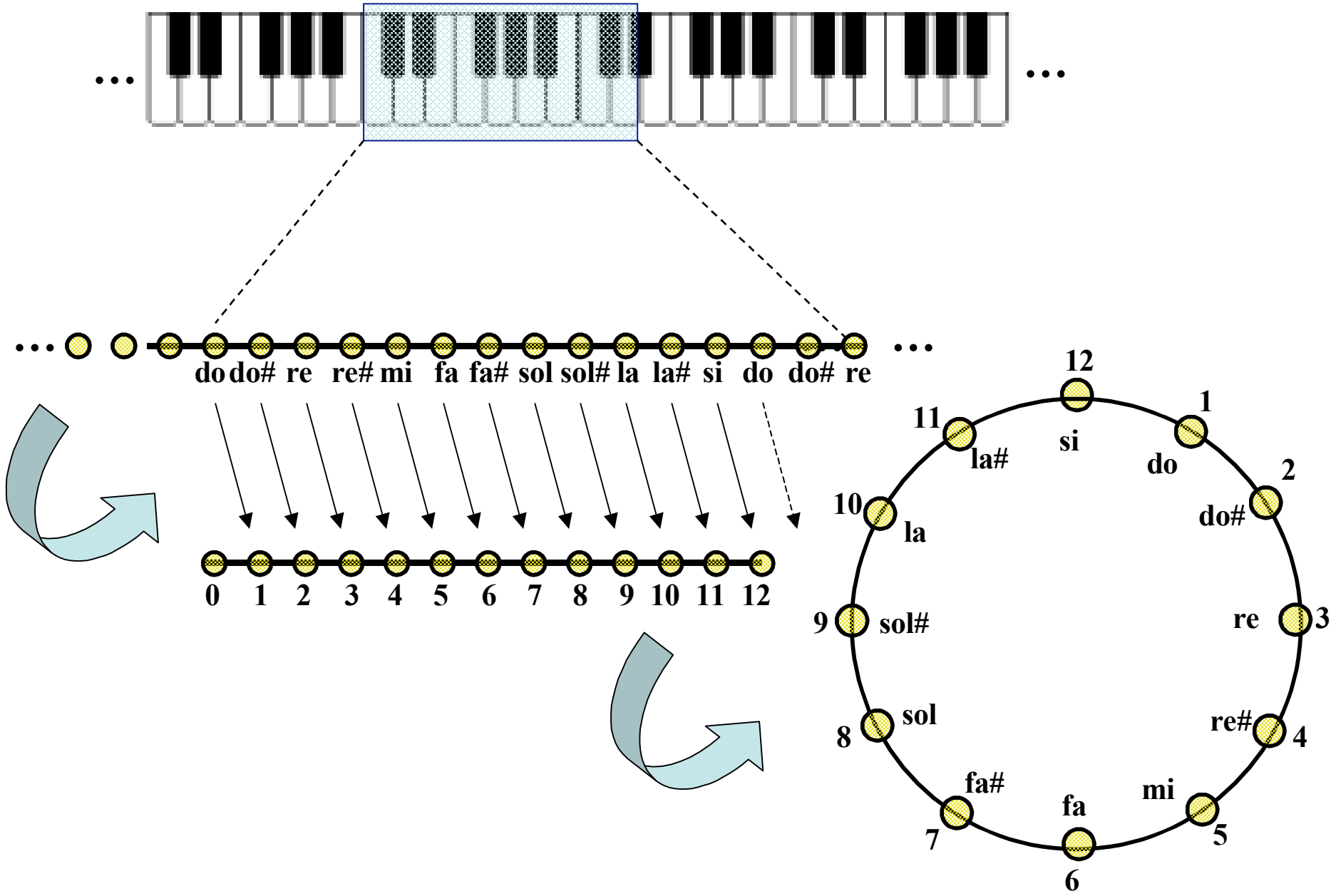


Approccio "paradigmatico" dell'analisi musicale assistita su computer

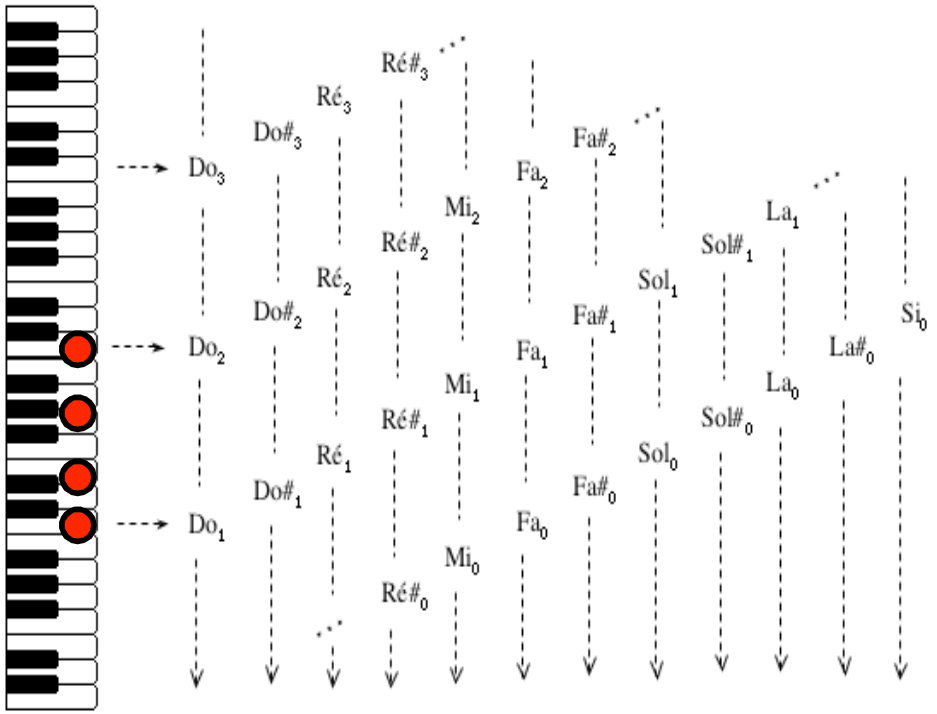
Riduzione all'ottava e congruenza modulo 12

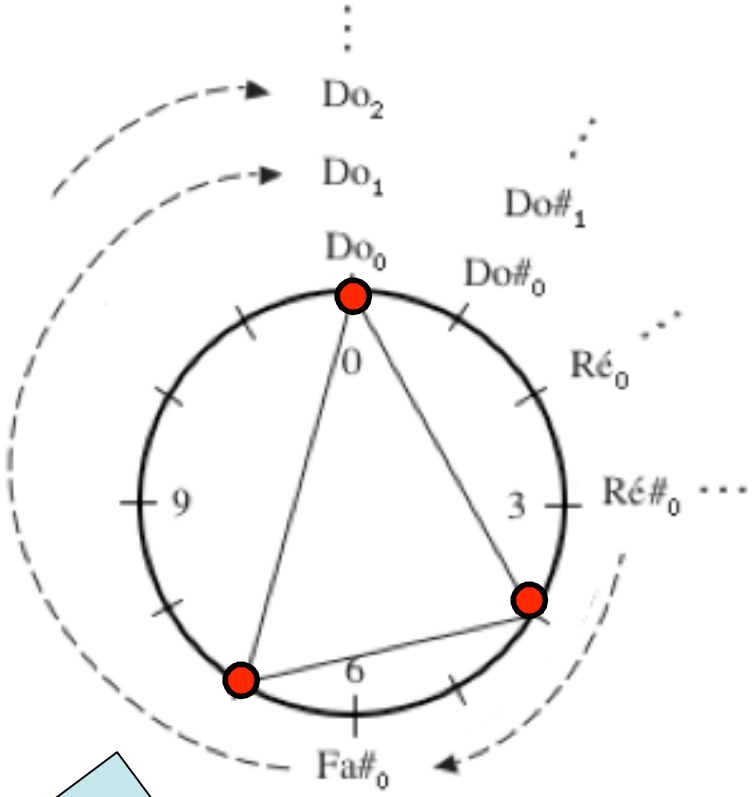


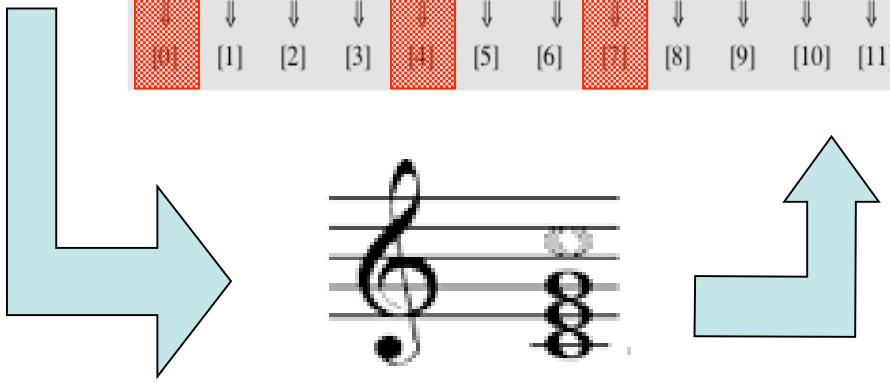
Riduzione all'ottava e congruenza modulo 12



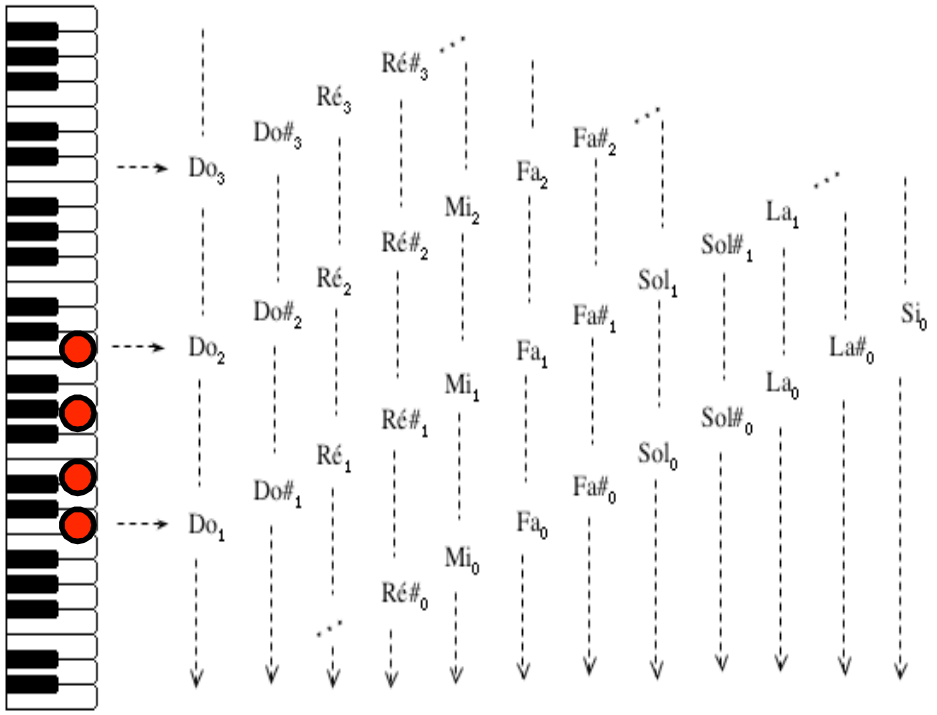
Riduzione all'ottava e congruenza modulo 12



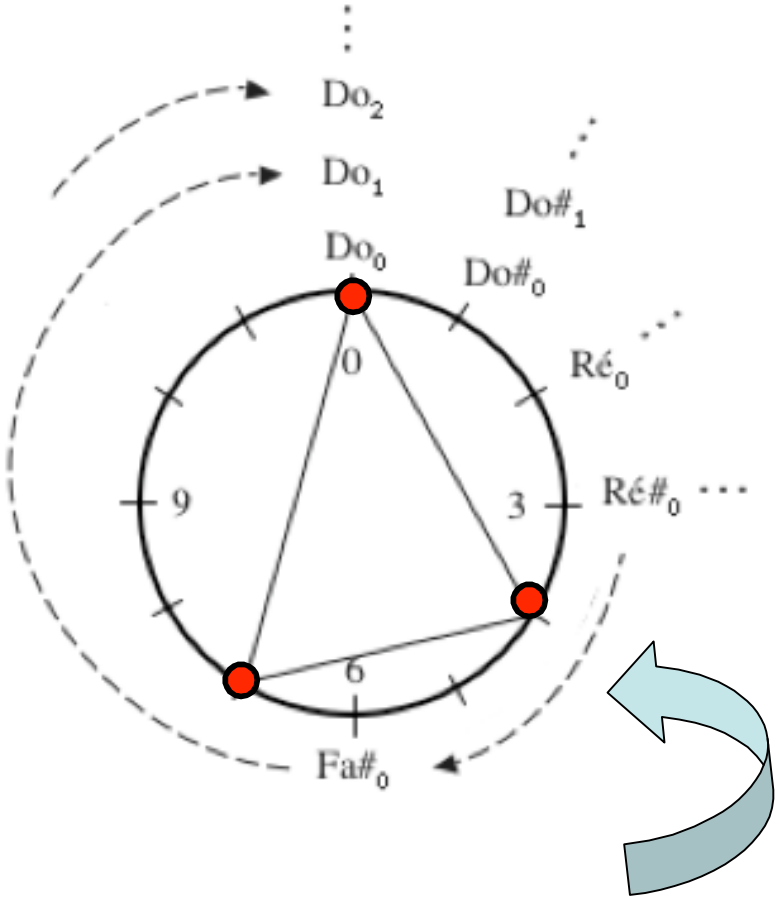


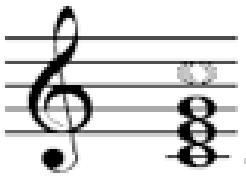


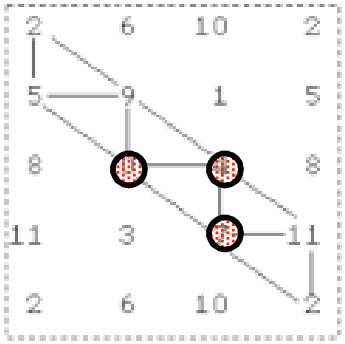
Riduzione all'ottava e congruenza modulo 12

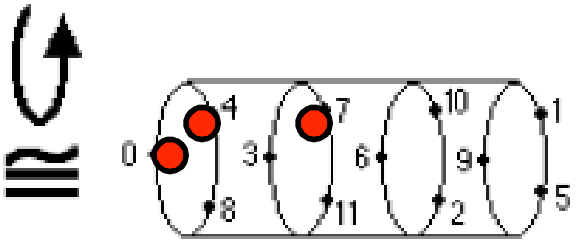


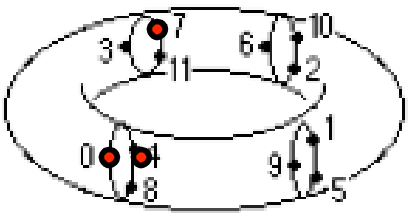
Do	Do#	Ré	Ré#	Mi	Fa	Fa#	Sol	Sol#	La	La#	Si
⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]



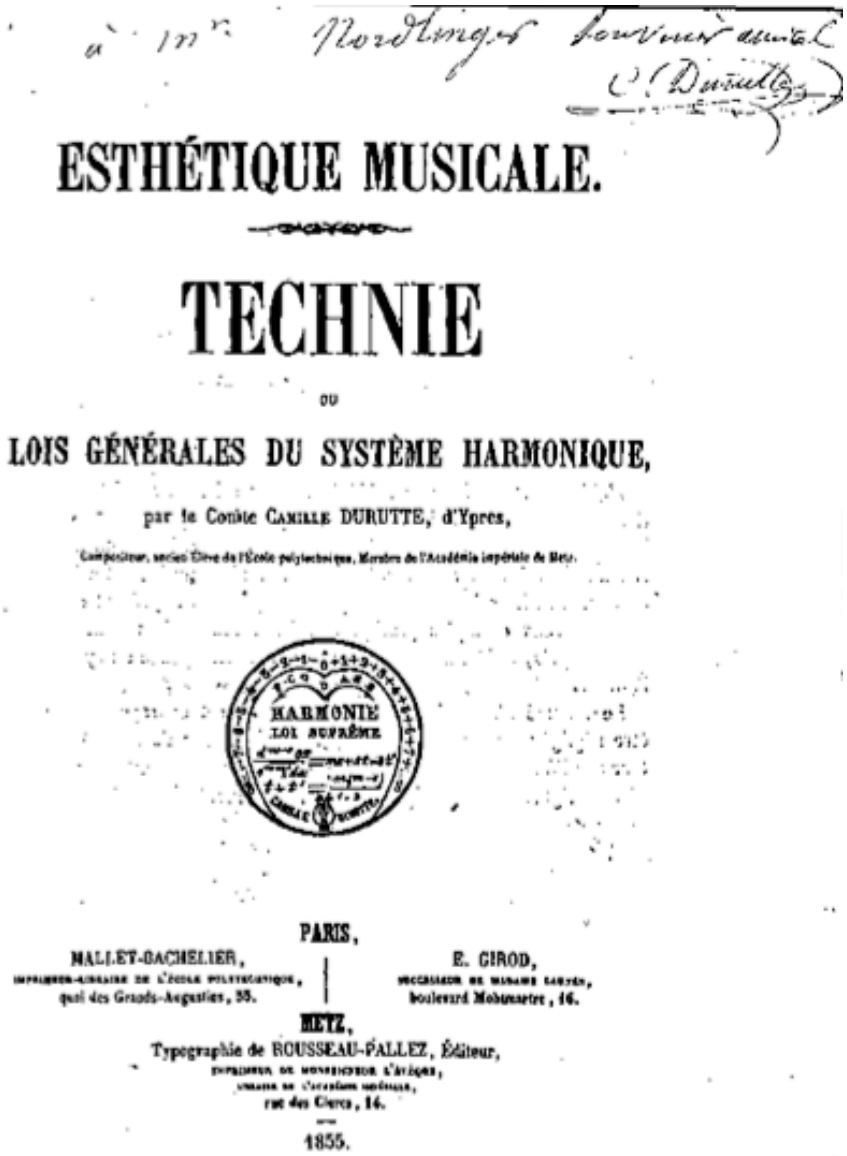








La relazione di congruenza modulo 12 in musica



ESTHÉTIQUE MUSICALE.

TECHNIE

OU

LOIS GÉNÉRALES DU SYSTÈME HARMONIQUE,

par le Comte CAMILLE DURUTTE, d'Ypres,

Compositeur, ancien Élève de l'École polytechnique, Membre de l'Académie impériale de Metz.

HARMONIE
LOI SUPRÊME

PARIS,

MALLET-BACHELIER, | E. CIROD,

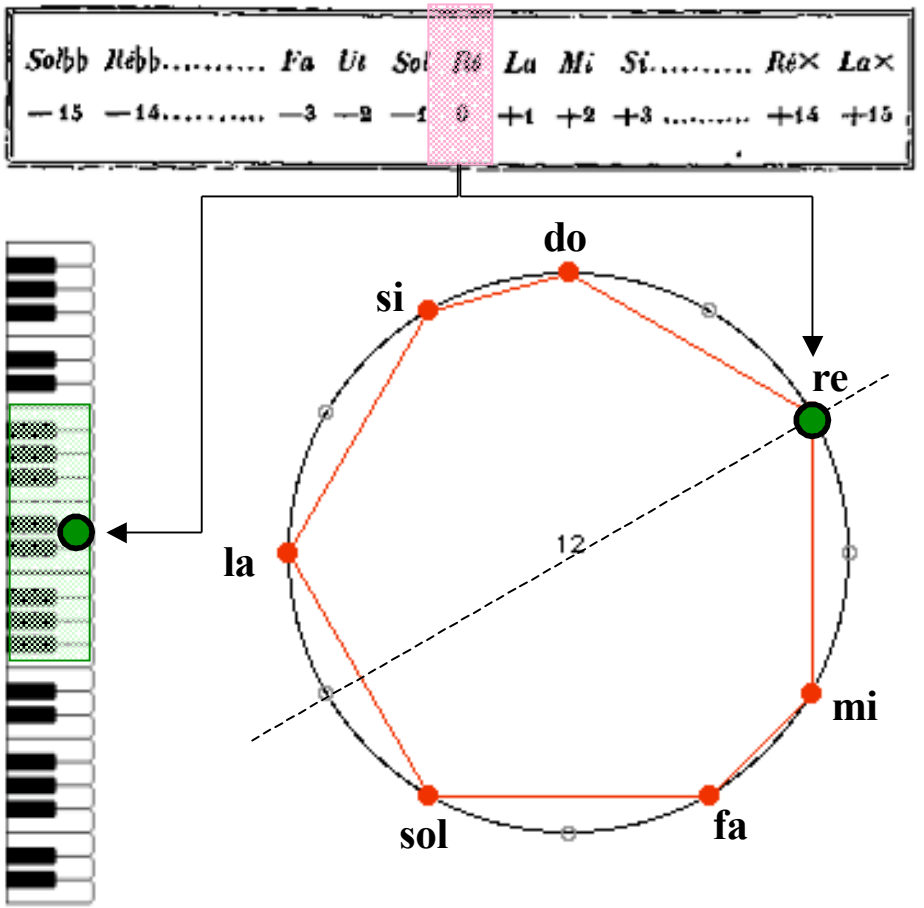
METZ,

Typographie de ROUSSEAU-PALLEZ, Éditeur,

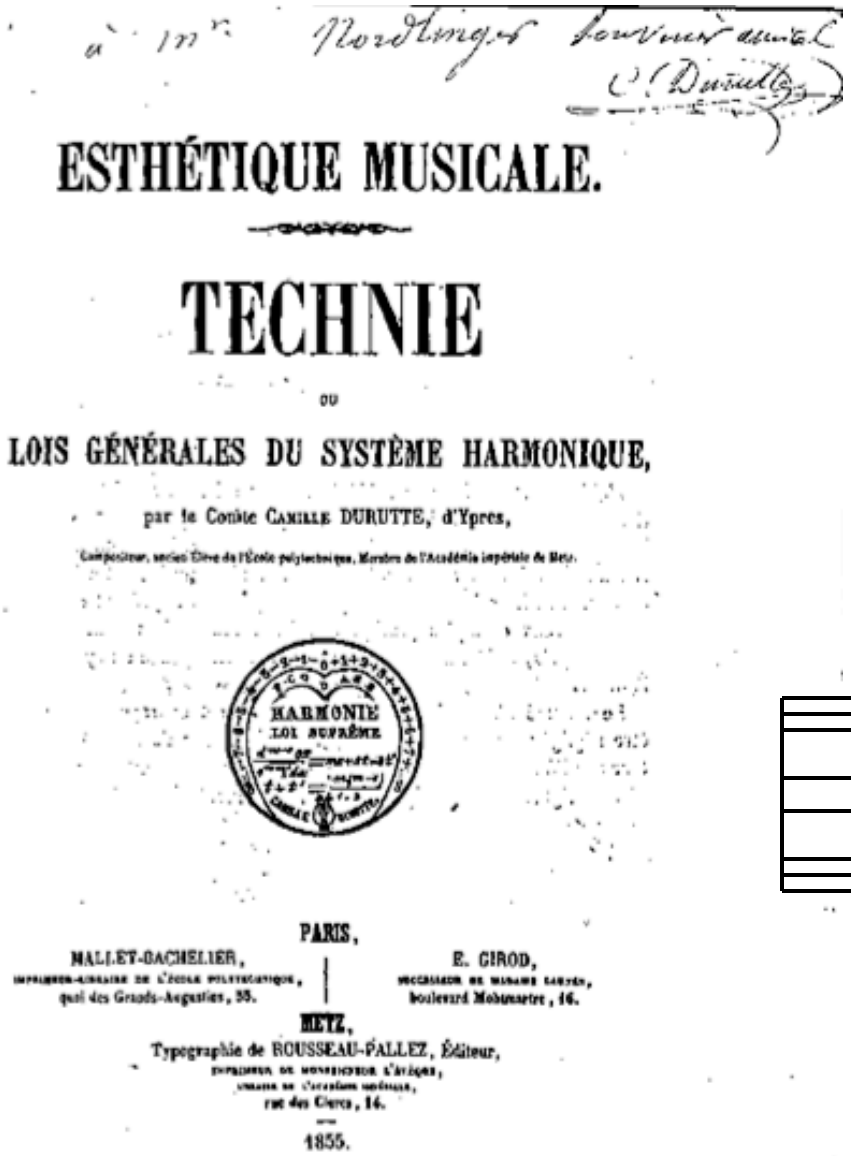
1855.

Camille Durutte:

- *Technie, ou lois générales du système harmonique* (1855)
- *Résumé élémentaire de la Technie harmonique, et complément de cette Technie* (1876)



La relazione di congruenza modulo 12 in musica



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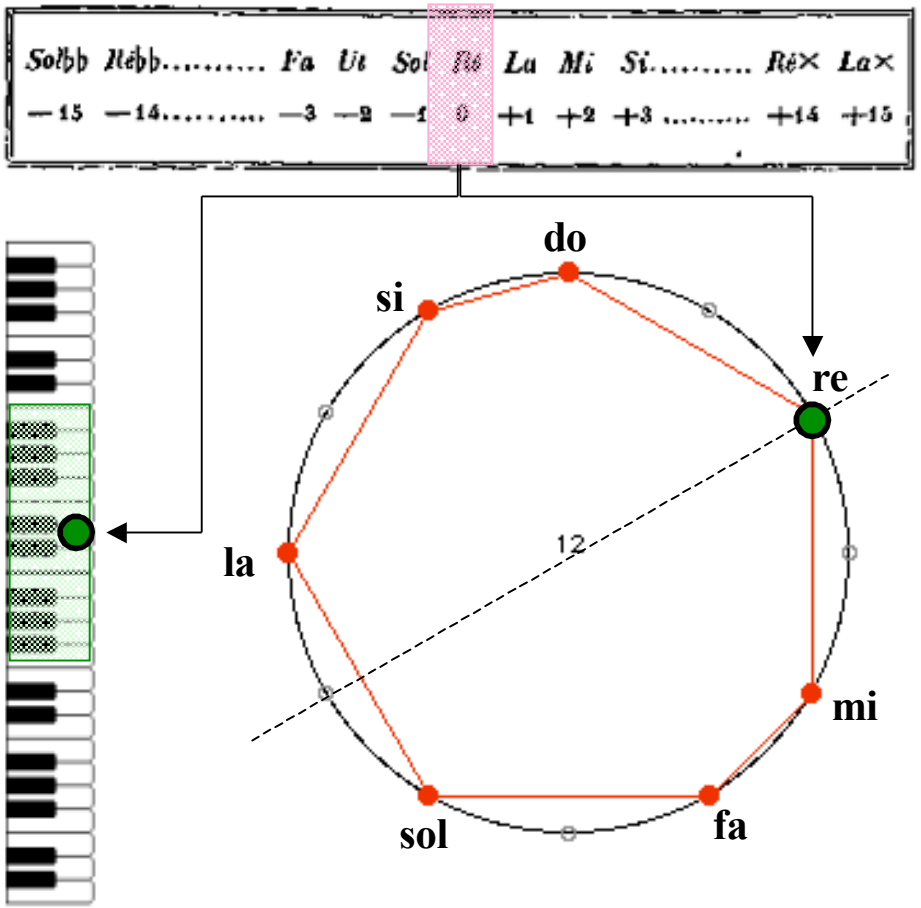
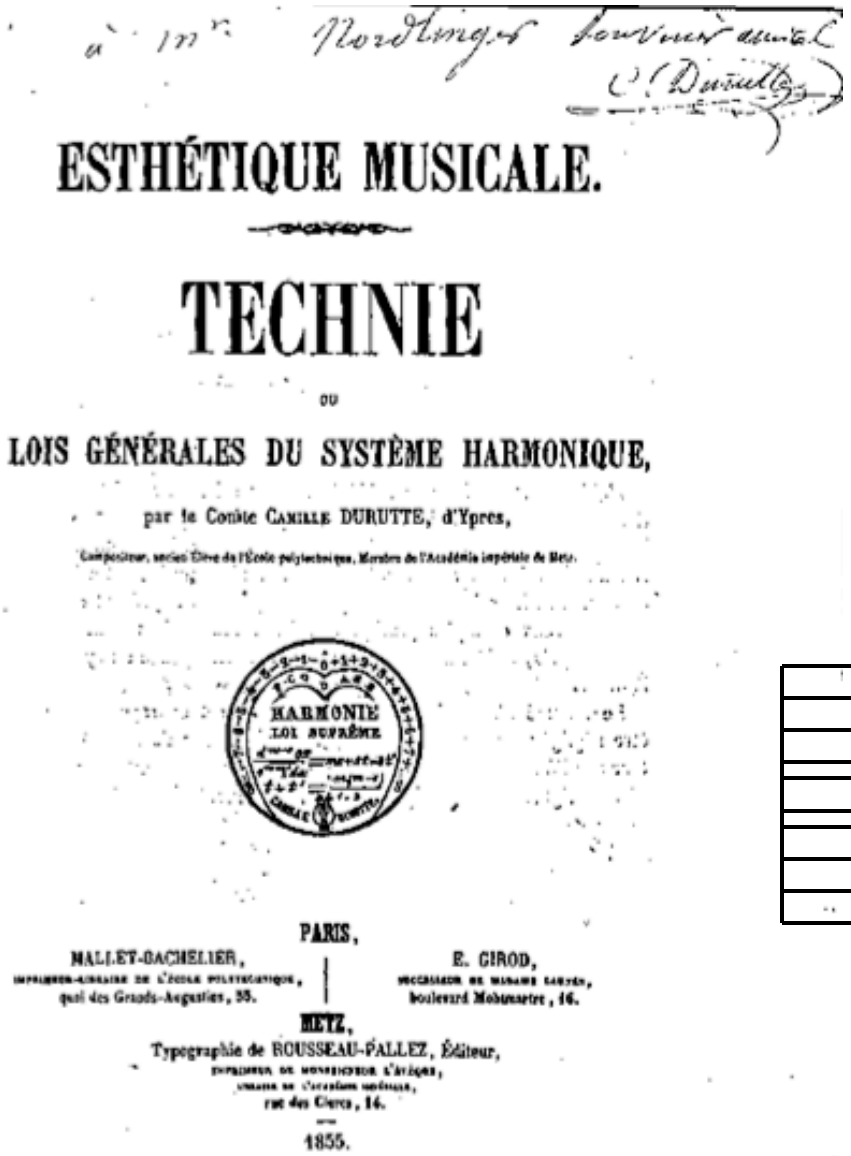


La relazione di congruenza modulo 12 in musica



ESTHÉTIQUE MUSICALE.

TECHNIE

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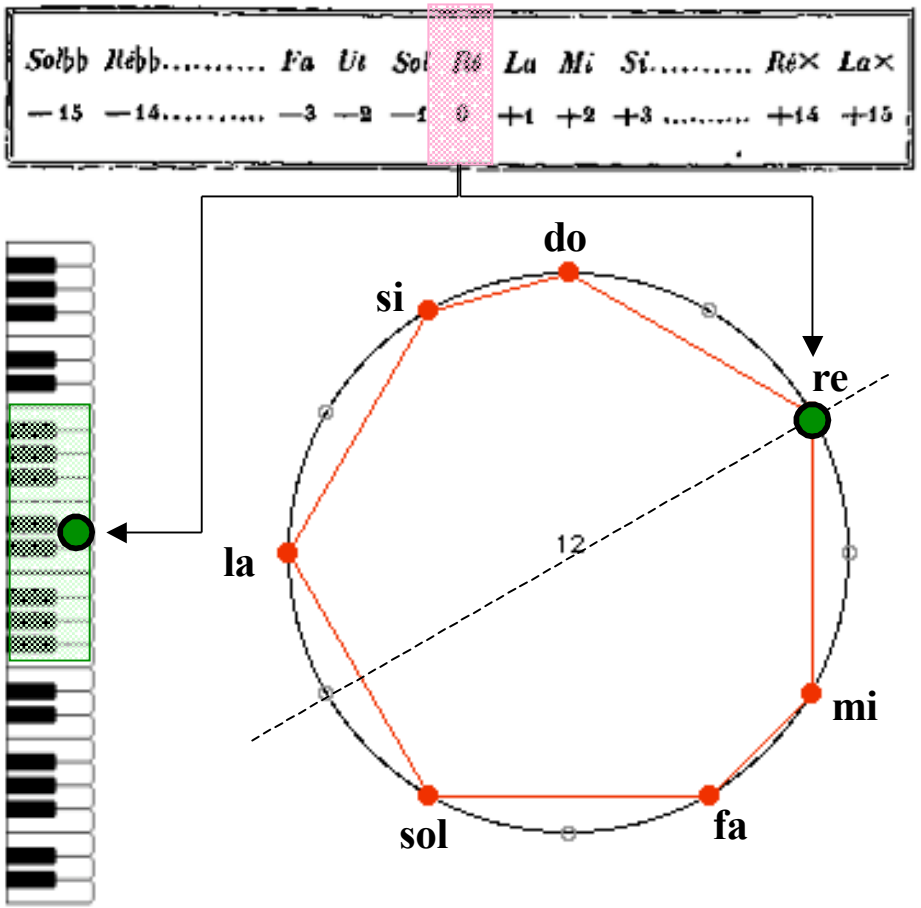
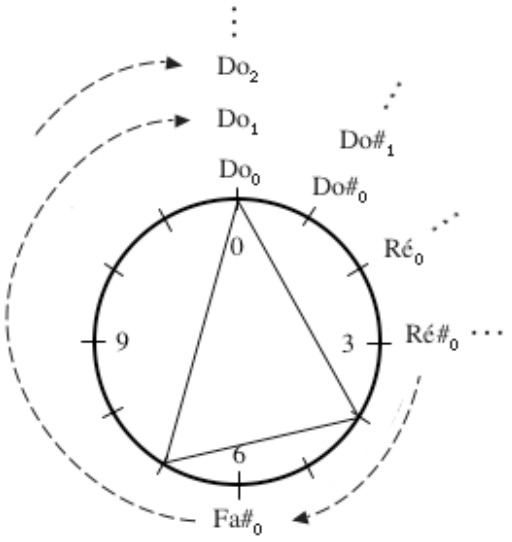


Rappresentazione circolare e struttura intervallare



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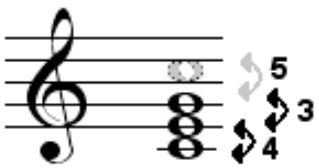
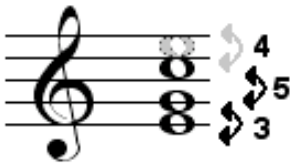
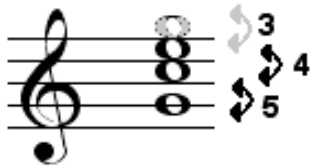


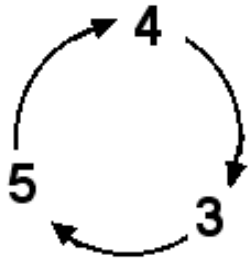
(4 3 5)

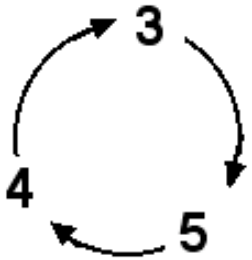


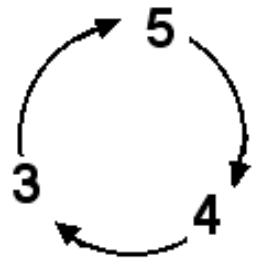
(3 5 4)



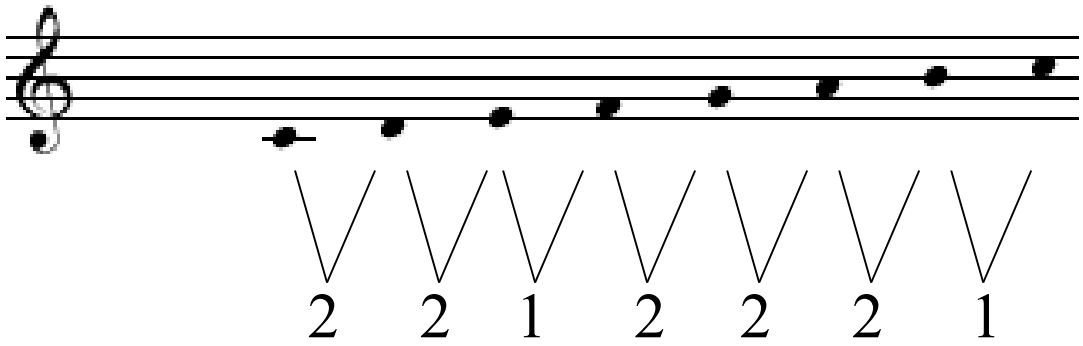
(5 4 3)

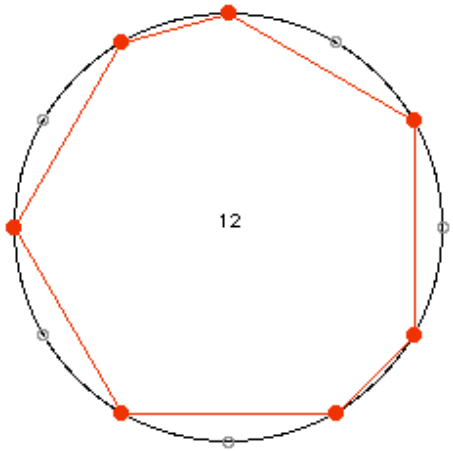




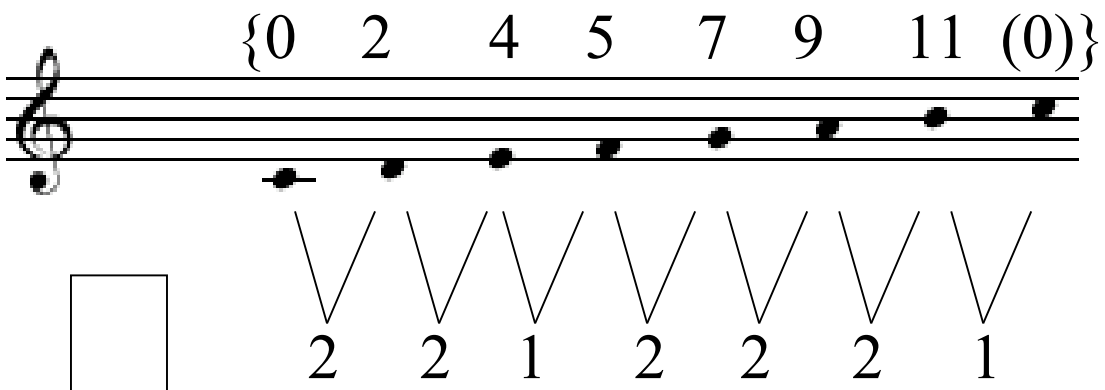


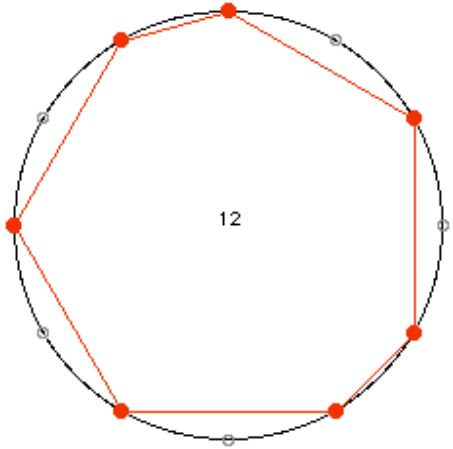
I rivolti di un accordo corrispondono alle permutazioni circolari della struttura intervallare





Trasformazioni geometriche: la trasposizione

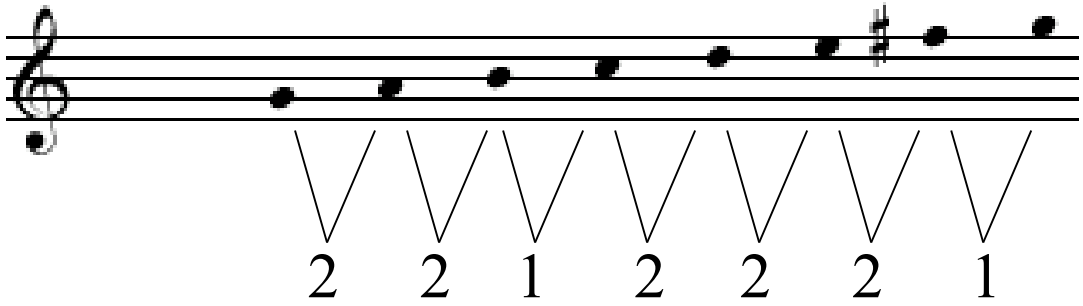


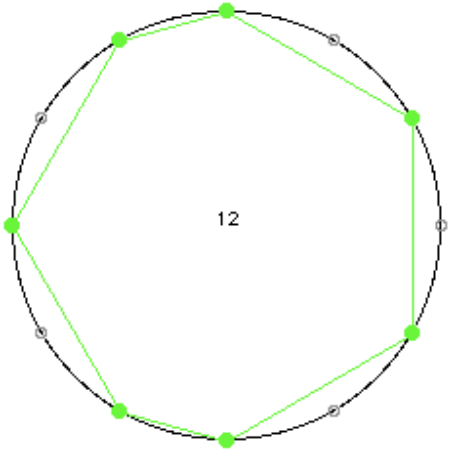


T_7

$T_7(x)=7+x \pmod{12}$

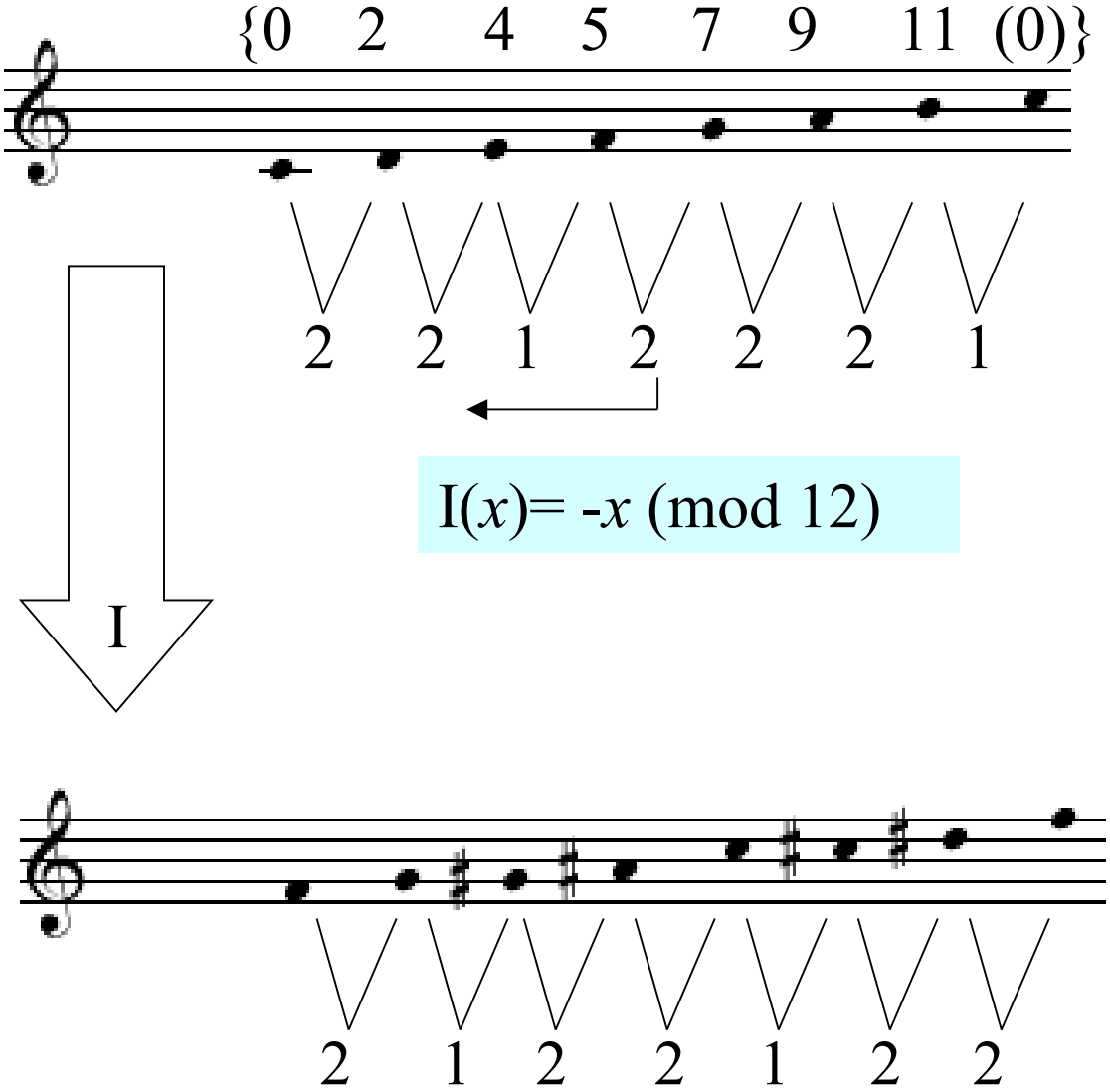
$\alpha=210°$



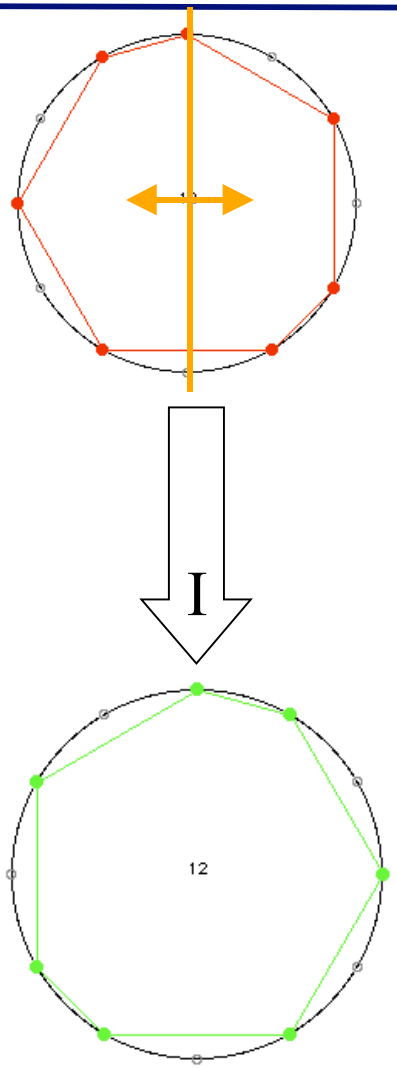


Equivalenza modulo la trasposizione

Trasformazioni geometriche: l'inversione

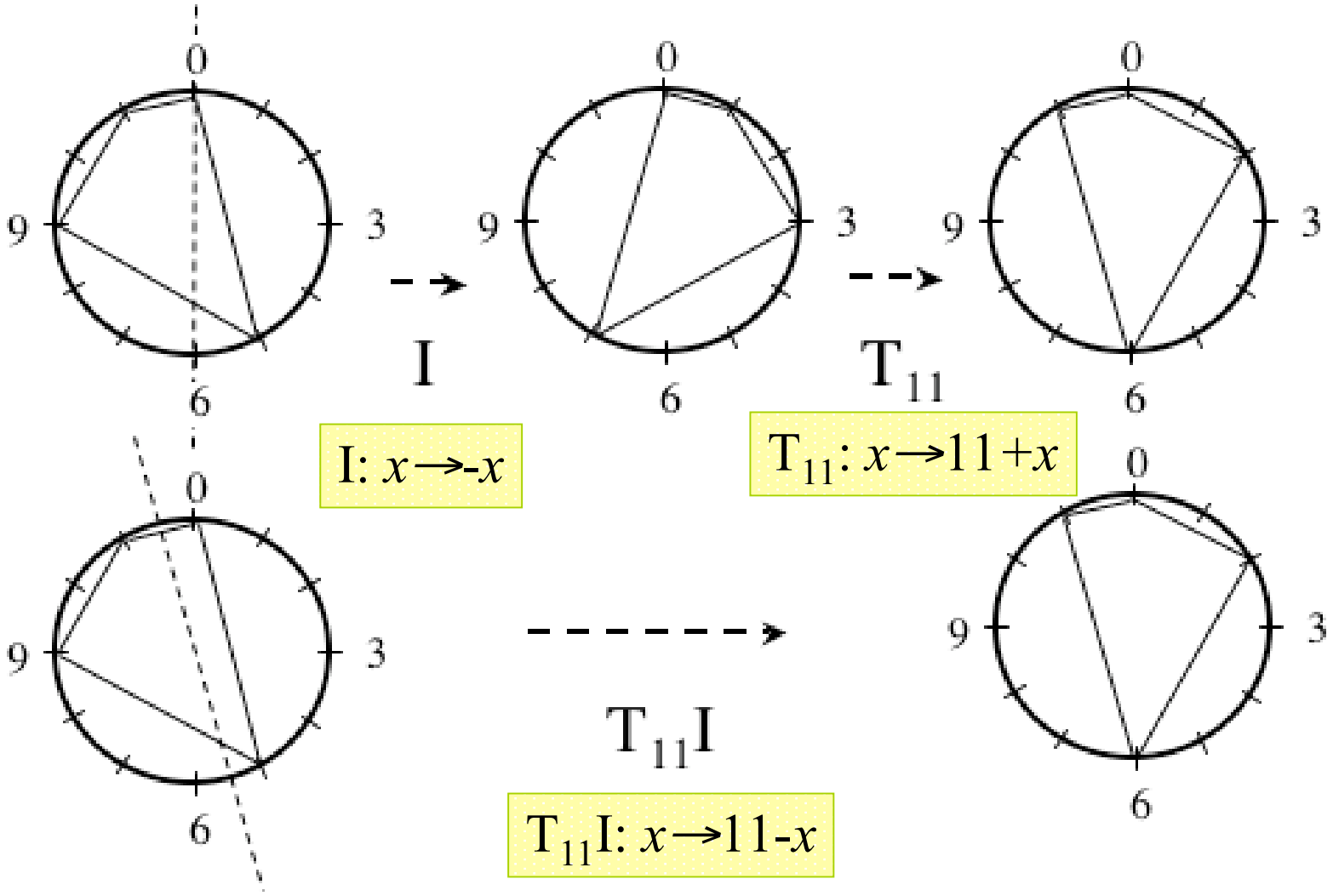


$I(x) = -x \pmod{12}$



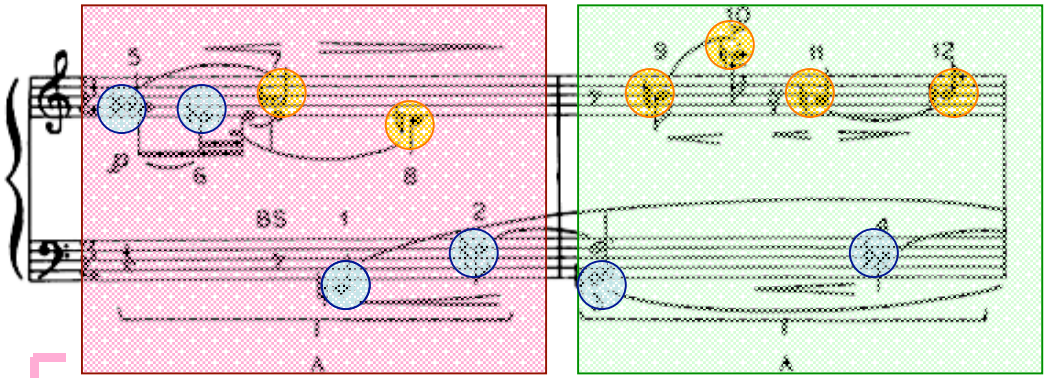
Equivalenza modulo l'inversione

La *Set Theory*: equivalenza per trasposizione/inversione

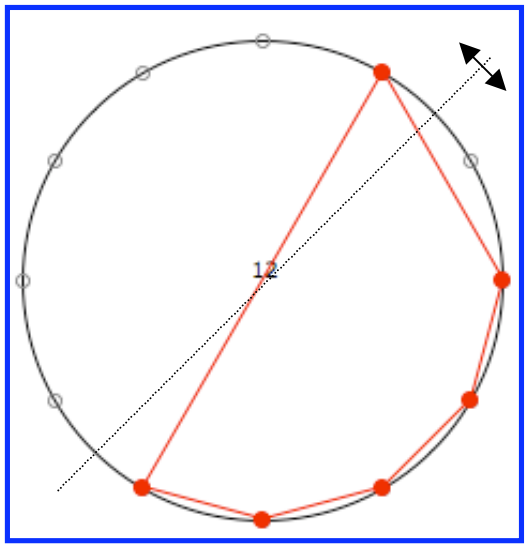


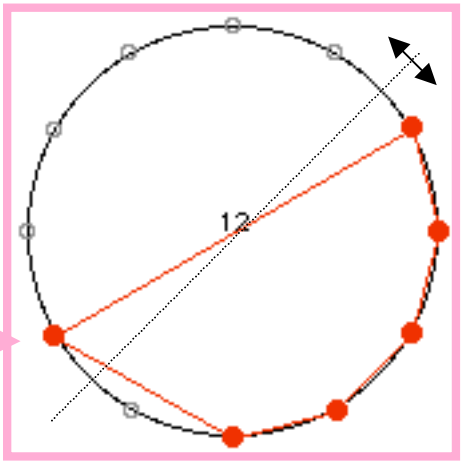
Set Theory, serialismo e equivalenza modulo trasposizione/inversione

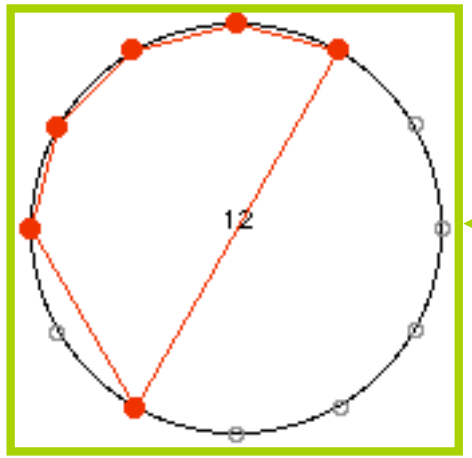
Schoenberg: Suite Op.25, Minuetto

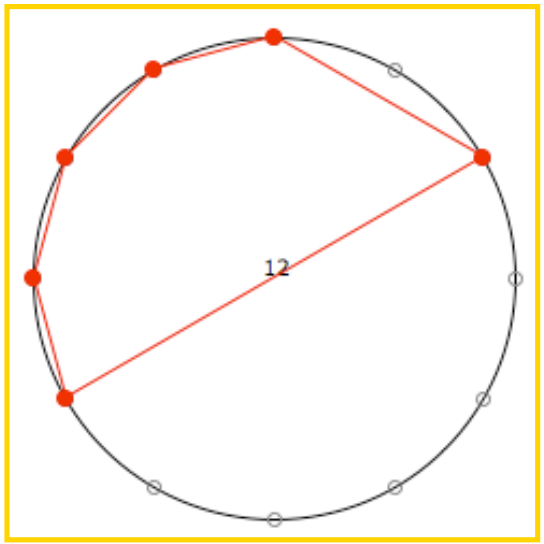


“Combinatorialità” esacordale

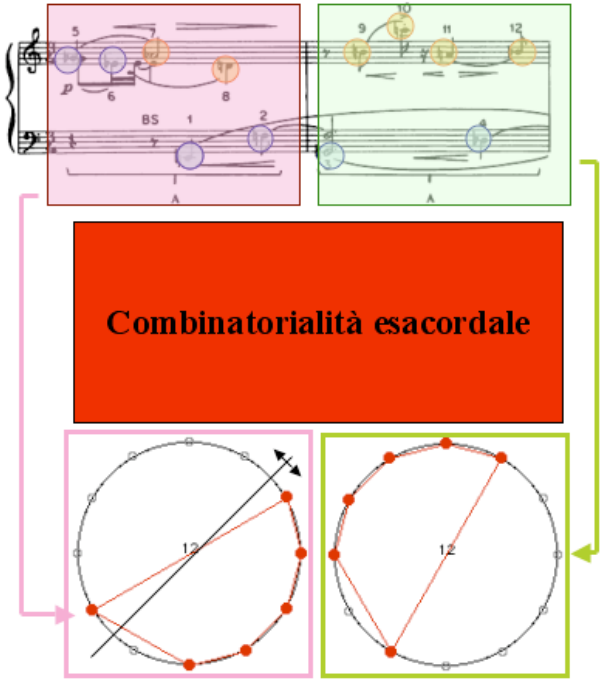


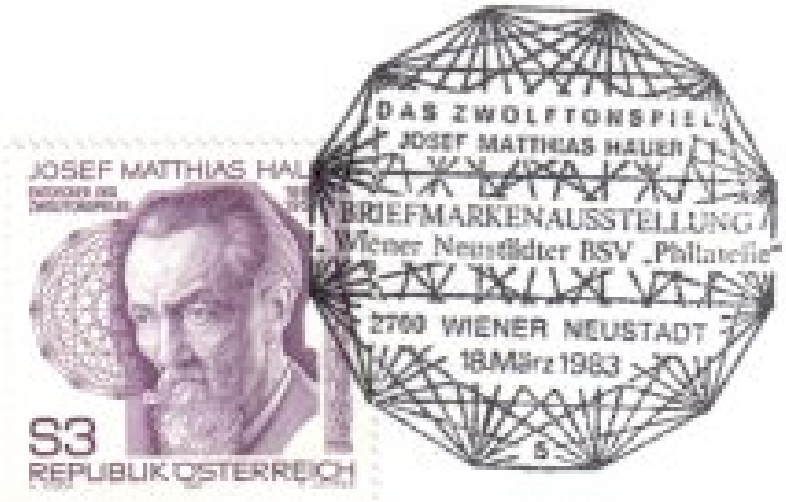


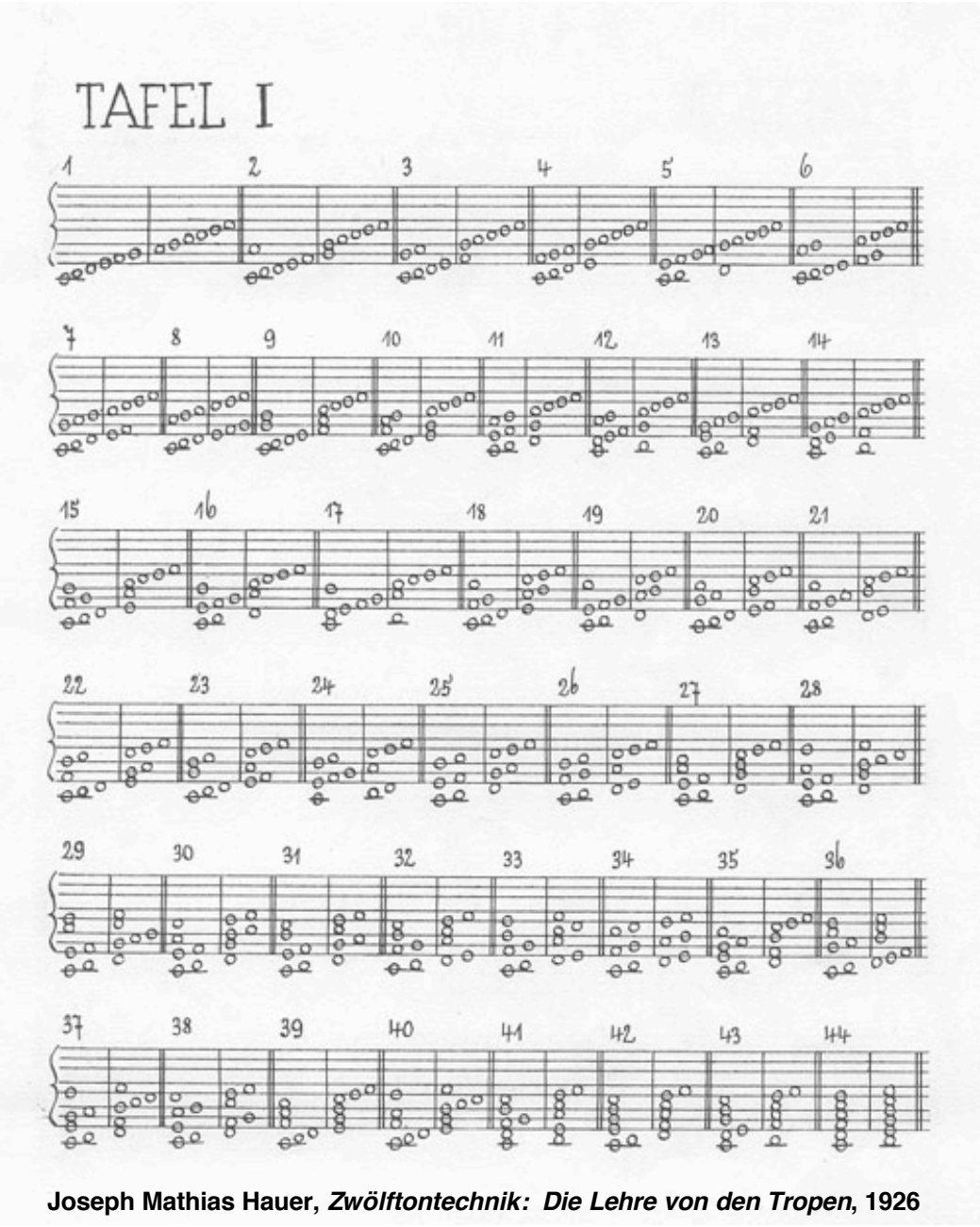




“Combinatorialità” esacordale e teoria dei tropi (Joseph Mathias Hauer)

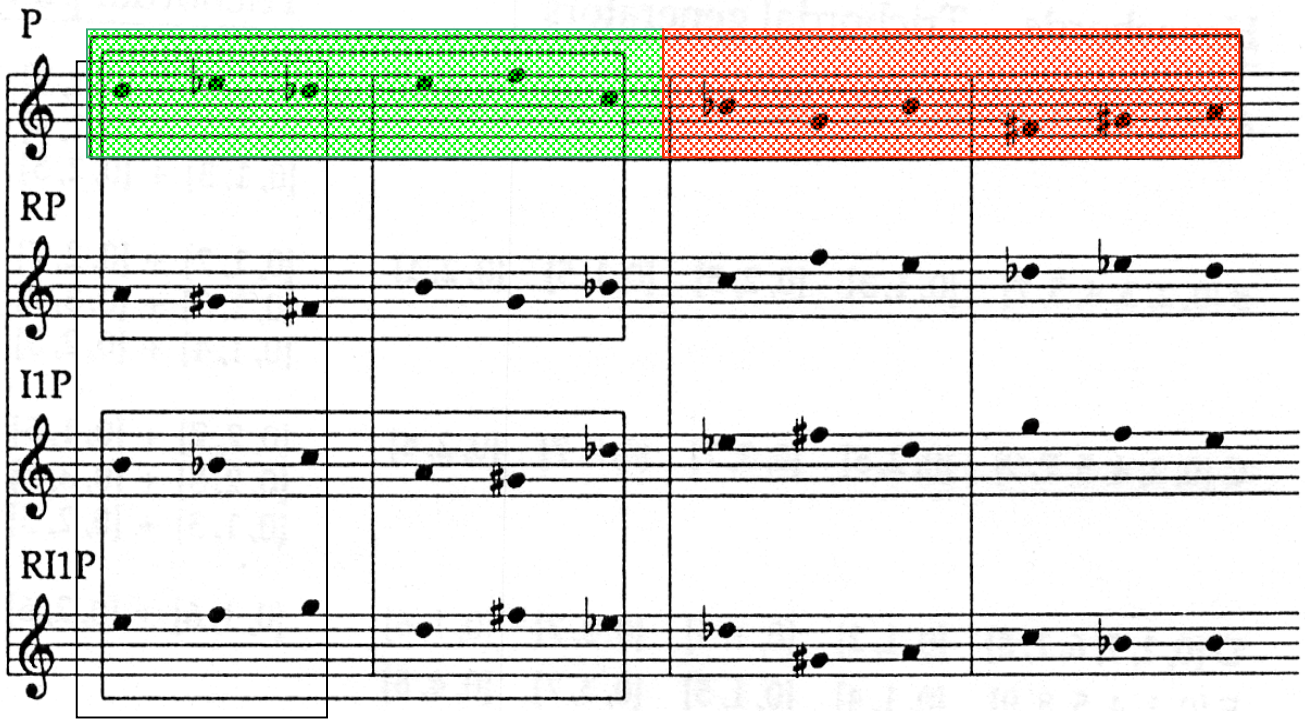


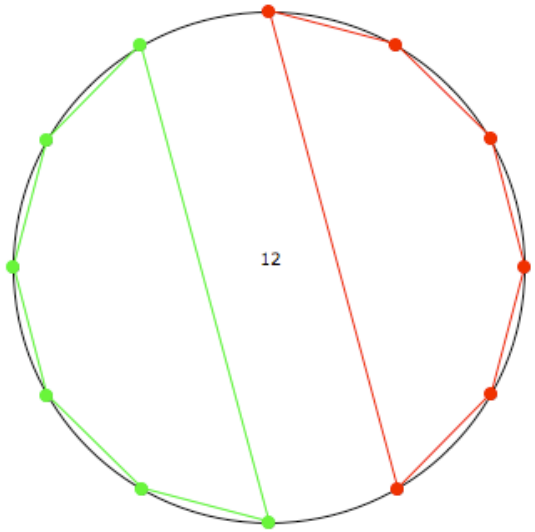




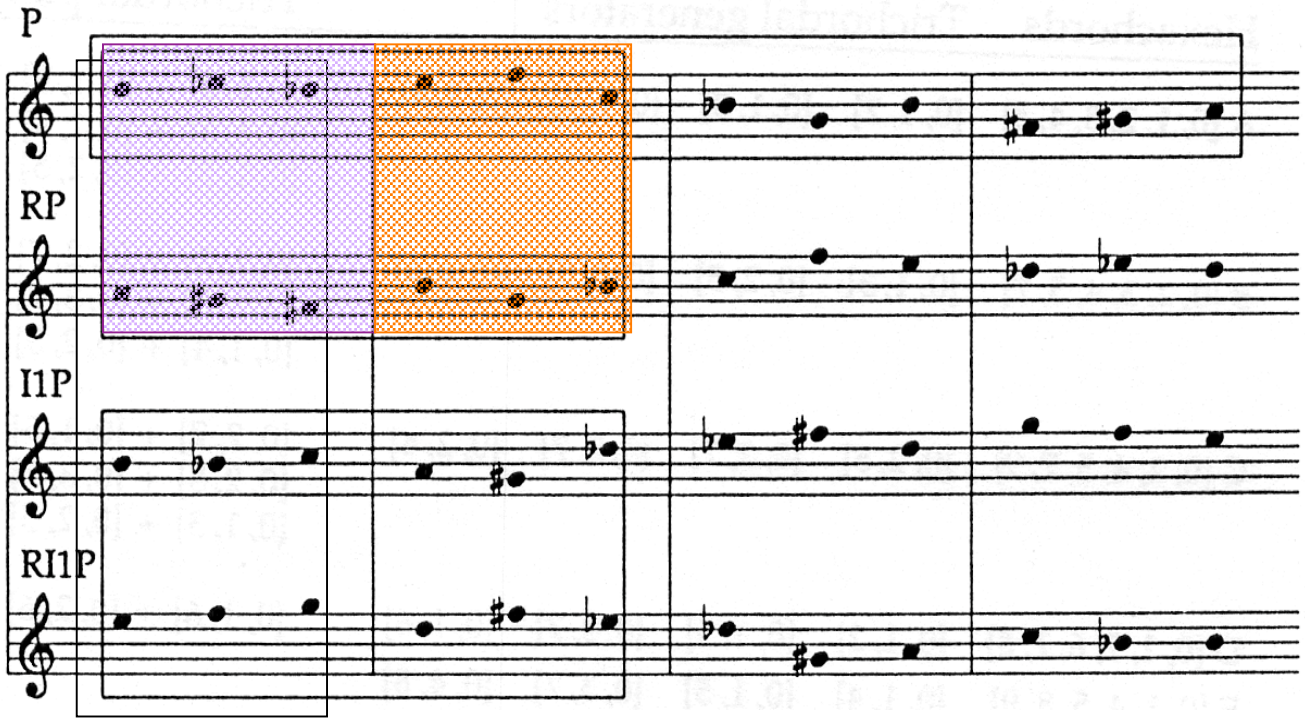
Joseph Mathias Hauer, ***Zwölftontechnik: Die Lehre von den Tropen***, 1926

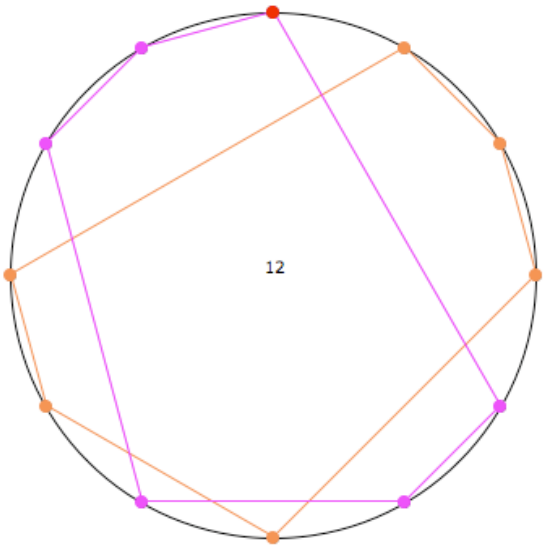
Combinatorialità esacordali e tricordali (Milton Babbitt)

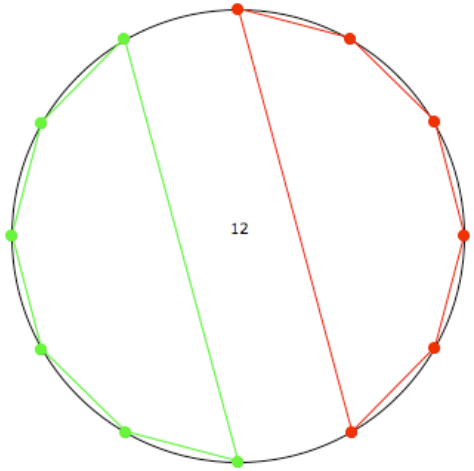




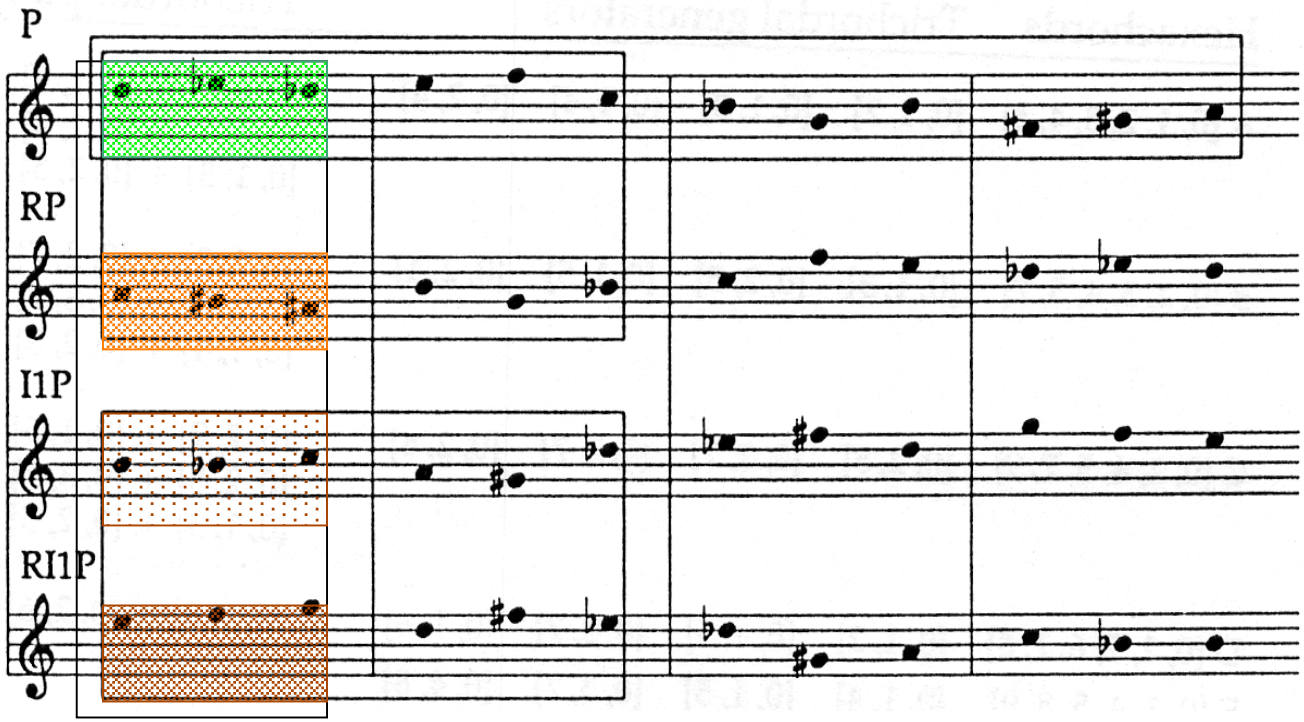
Combinatorialità esacordali e tricordali (Milton Babbitt)

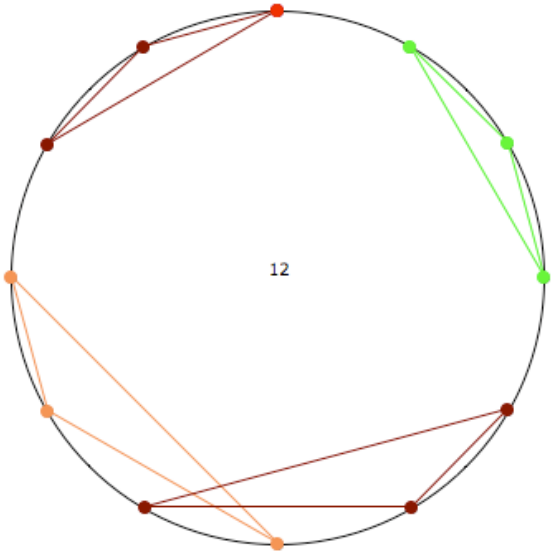


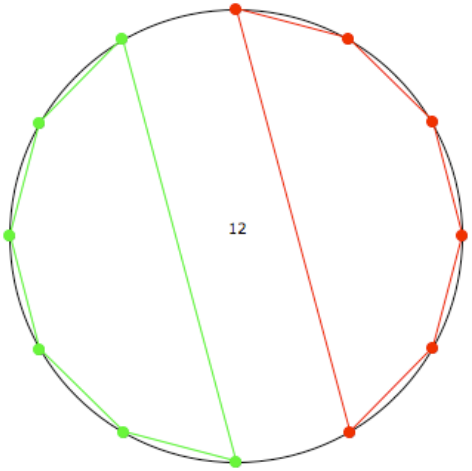


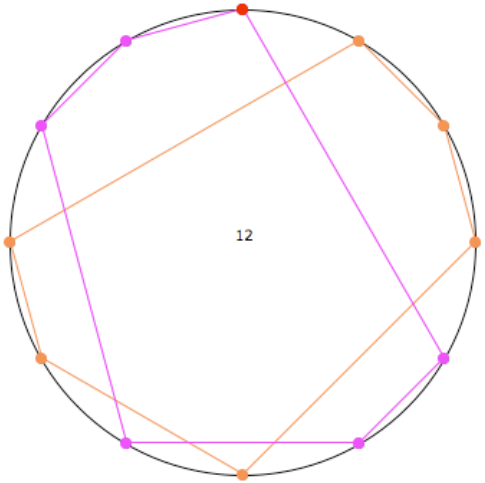


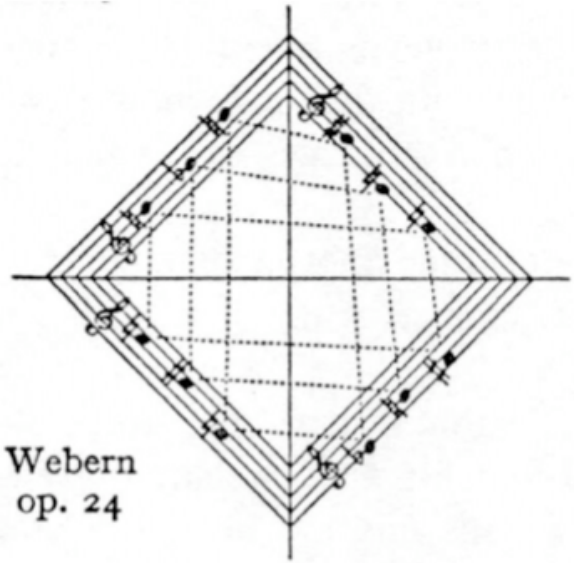
Combinatorialità esacordali e tricordali (Milton Babbitt)



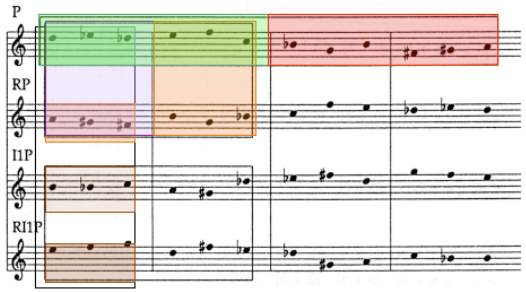


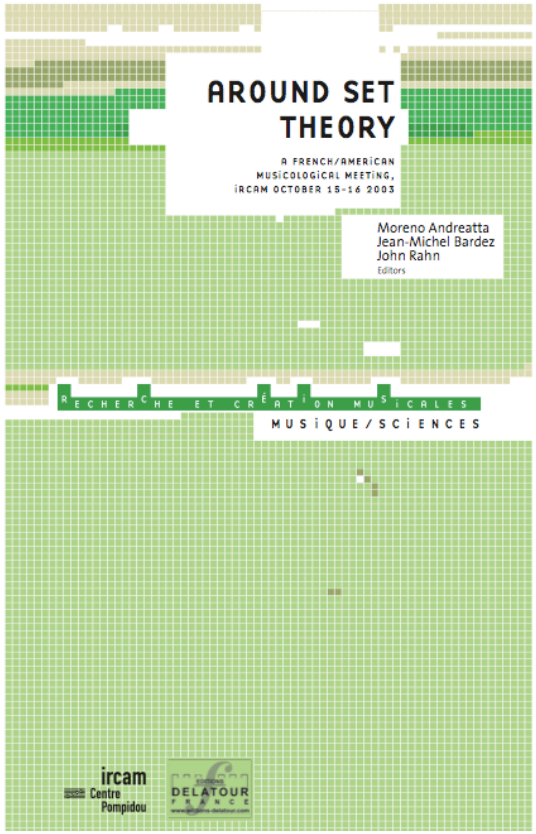




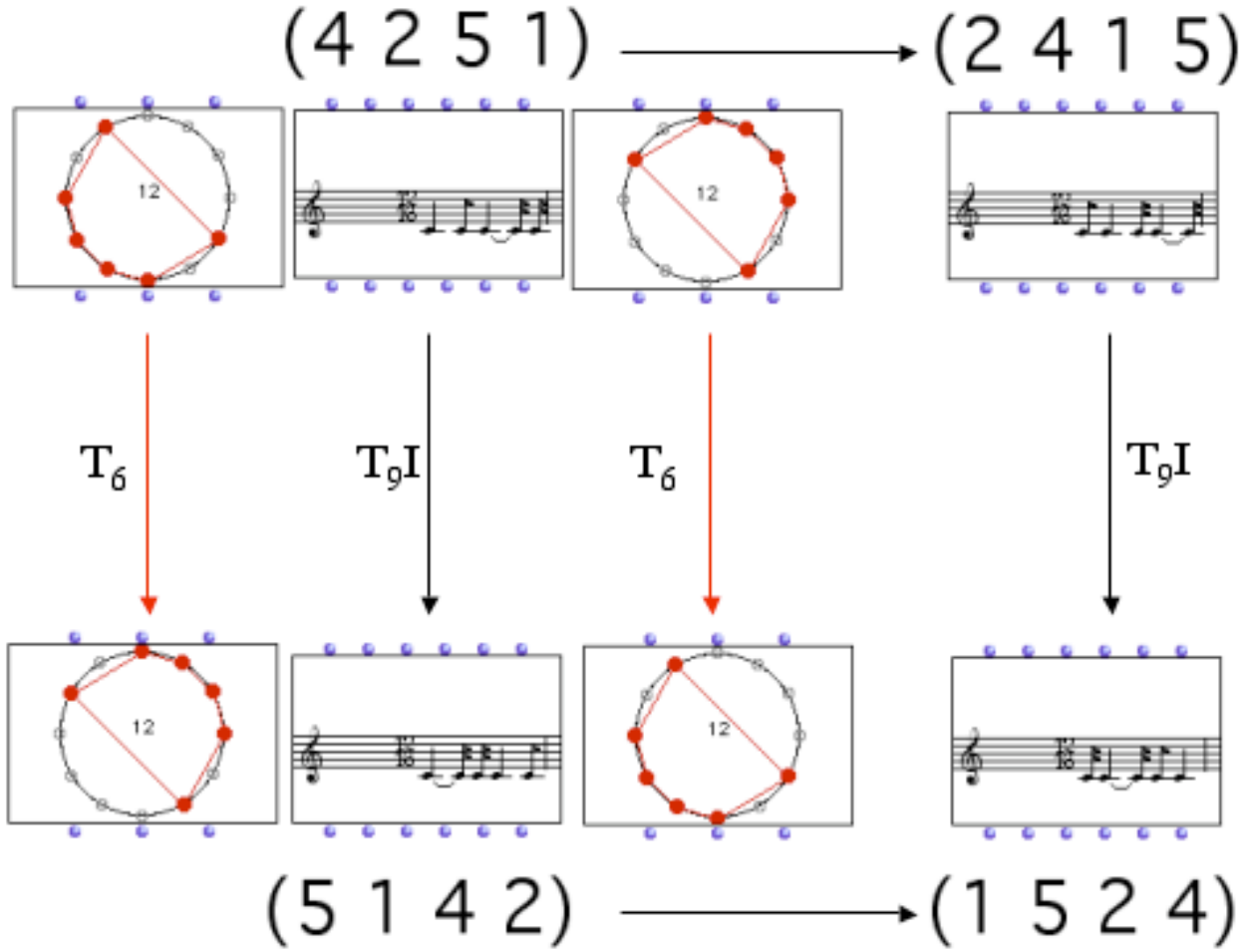


Combinatorialità esacordali e tricordali (Milton Babbitt)



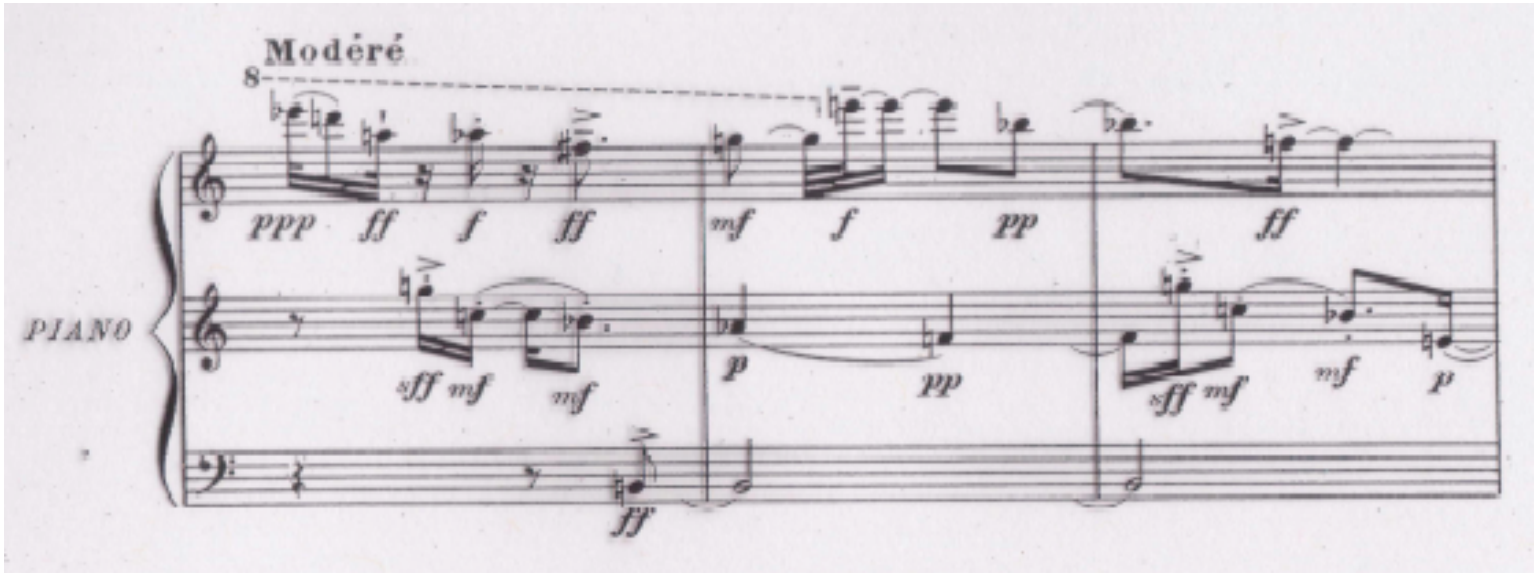


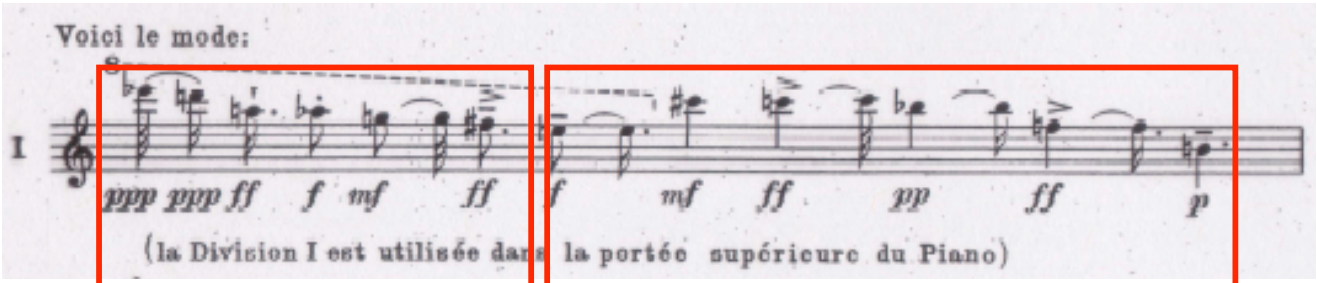
- Milton Babbitt: *Three compositions for piano* (1948)

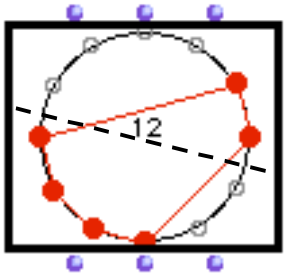


Combinatorialità esacordale in Messiaen

- Mode de valeurs et d'intensités (1950)

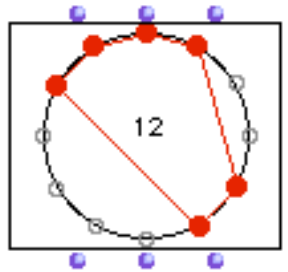






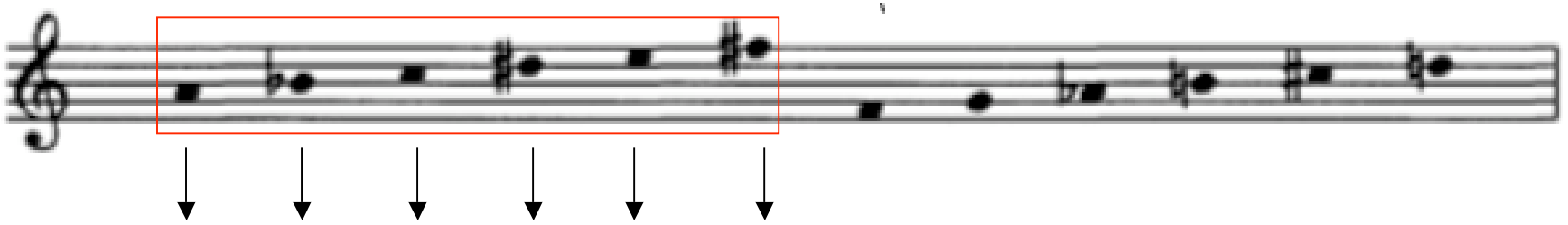
$\{3,2,9,8,7,6\} \longrightarrow \{4,5,10,11,0,1\}$

$T_7I : x \rightarrow 7\text{-}x$

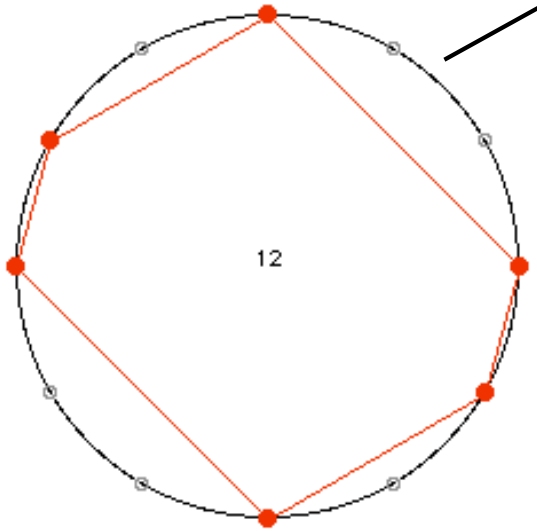


“Combinatorialità” esacordale e simmetria trasposizionale

Schoenberg: Serenade Op.24, Mouvement 5

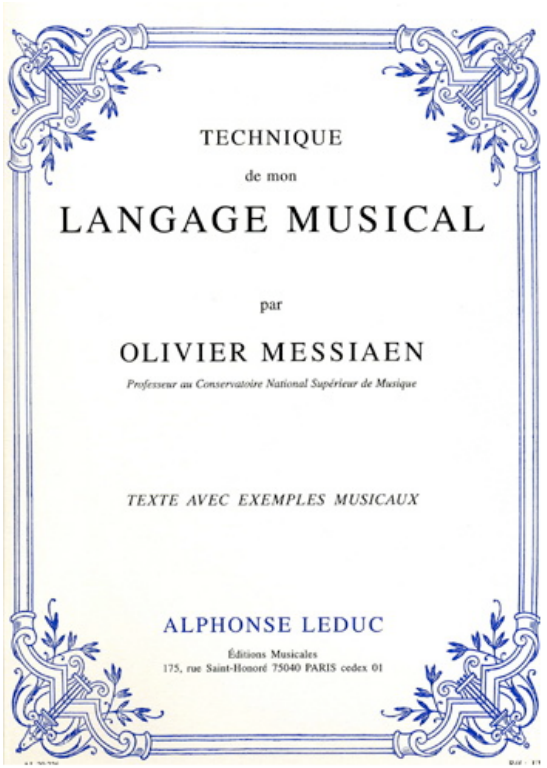


A = {9 , 10 , 0 , 3, 4 , 6}



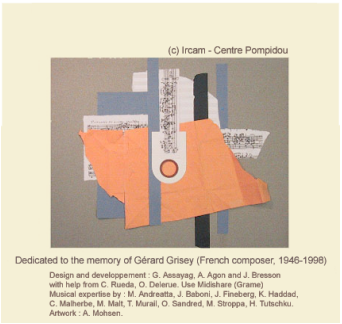
$T_6\{9,10,0,3,4,6\}=$
$=\{6+9,6+10,6,6+3,6+4,6+6\}=$
$=\{3,4,6,9,10,0\}$

$T_6(A)=A$



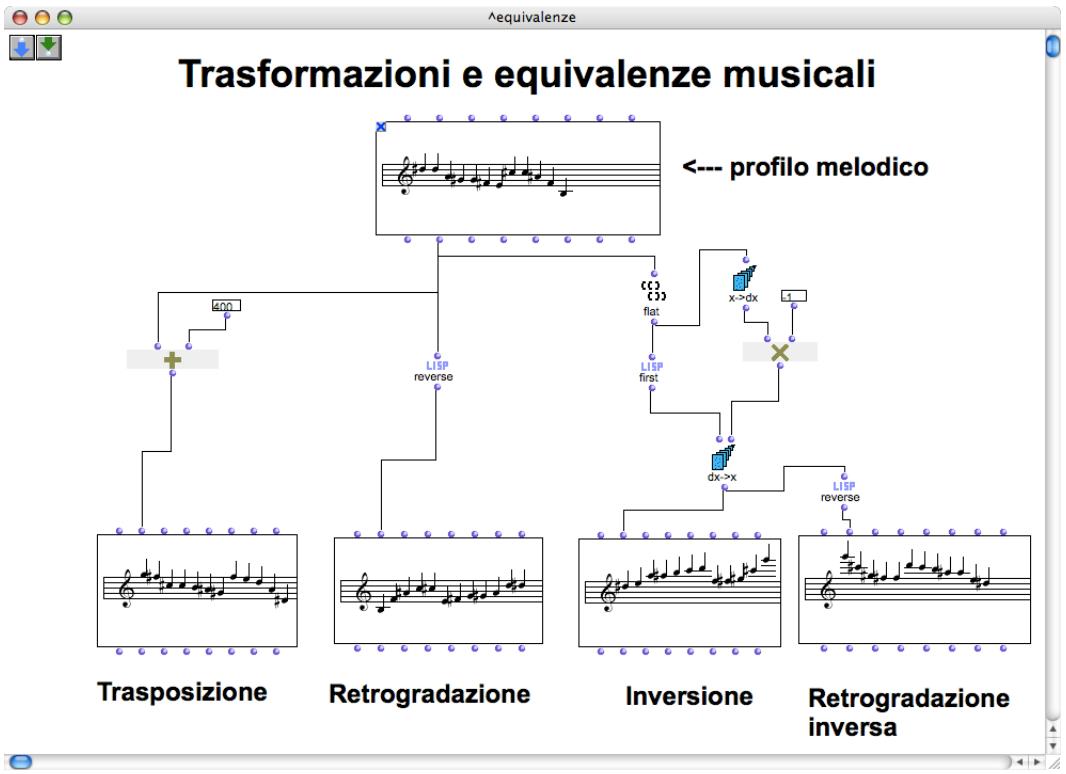
(3, 1, 2, *3, 1, 2*)

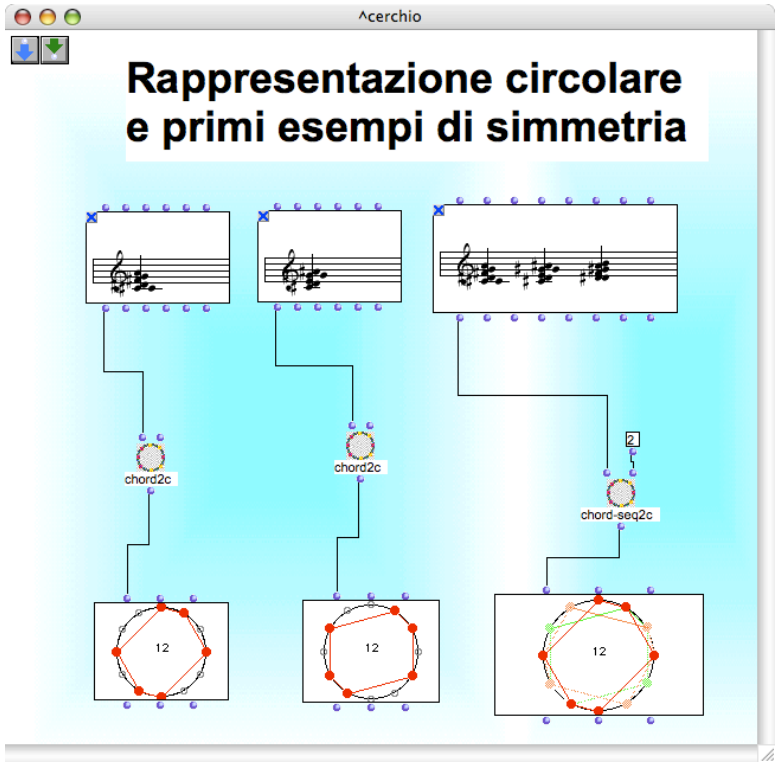
=> *OpenMusic*

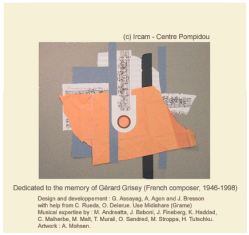

(c) Ircam - Centre Pompidou
Dedicated to the memory of Gérard Grisey (French composer, 1946-1998)
Design and developpement : G. Assayag, A. Agon and J. Bresson
with help from C. Rueda, O. Delerue. Use Midishare (Grame)
Musical expertise by : M. Andreatta, J. Baboni, J. Fineberg, K. Haddad,
C. Malherbe, M. Malt, T. Murail, O. Sandred, M. Stroppa, H. Tutschku.
Artwork : A. Mohsen.

OpenMusic


codificazione-MIDI
Codificazione tradizionale delle altezze in MIDI
6000
(6000 6400 6700)
((6000 6400 6700) (6000 6500 6900) (6500 6200 6900) (6200 6500 6700) (6000 6400 6700))


^equivalenze
Trasformazioni e equivalenze musicali
<--- profilo melodico
400
LISP reverse
flat
LISP first
x->dx
1
dx->x
LISP reverse
Trasposizione
Retrogradazione
Inversione
Retrogradazione inversa

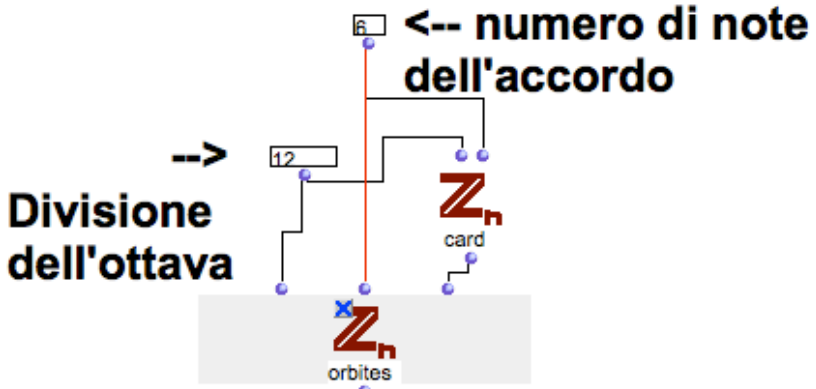

^cerchio
Rappresentazione circolare e primi esempi di simmetria
chord2c
chord2c
2
chord-seq2c
12
12
12



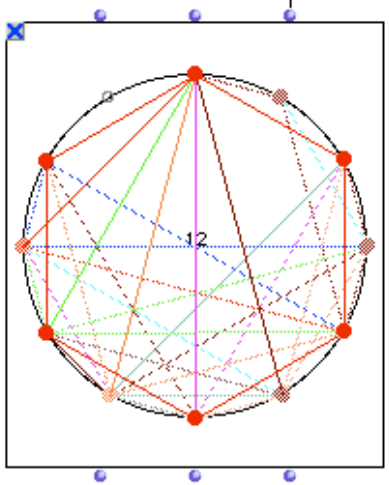
OpenMusic

^Catalogo-Tn

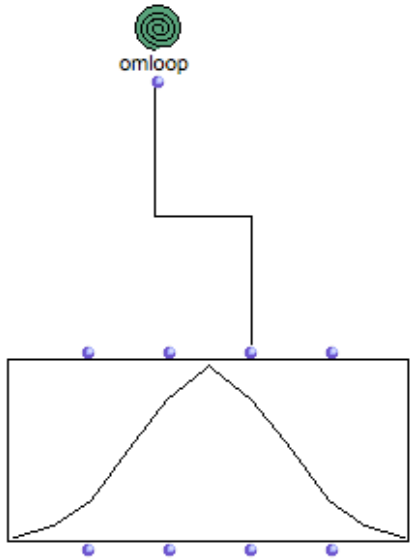
Catalogo d'accordi (modulo la trasposizione)



Paradigma del gruppo ciclico

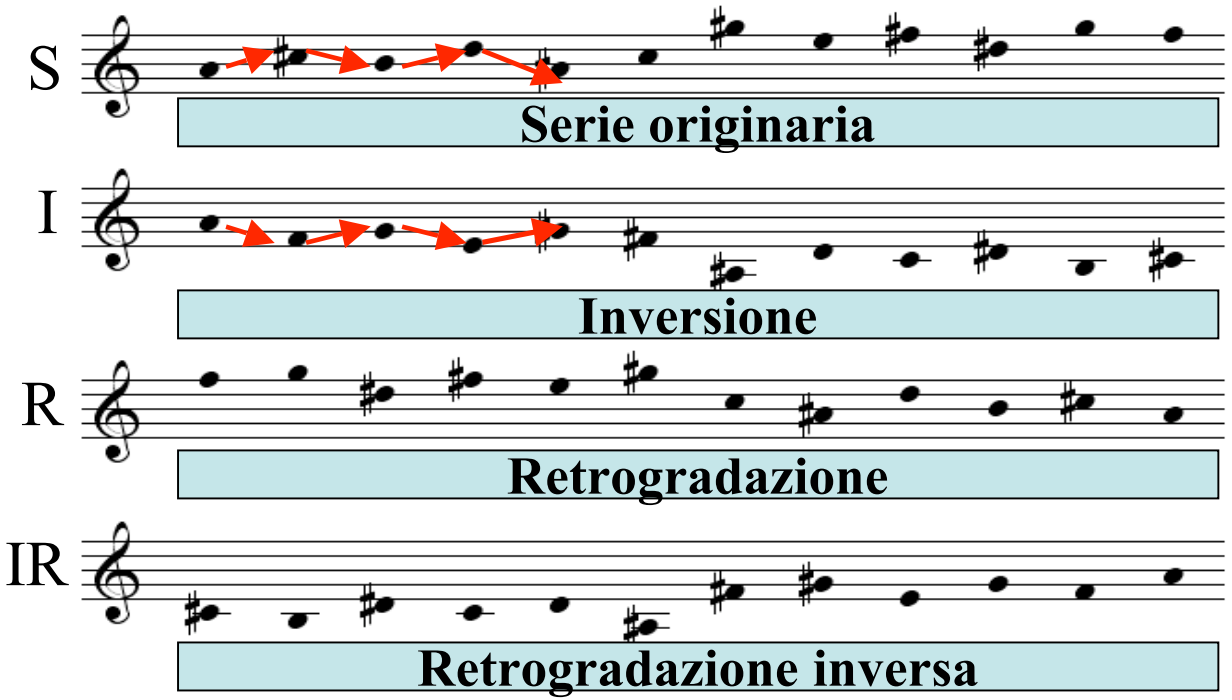


Gli 80 esacordi



Distribuzione degli accordi

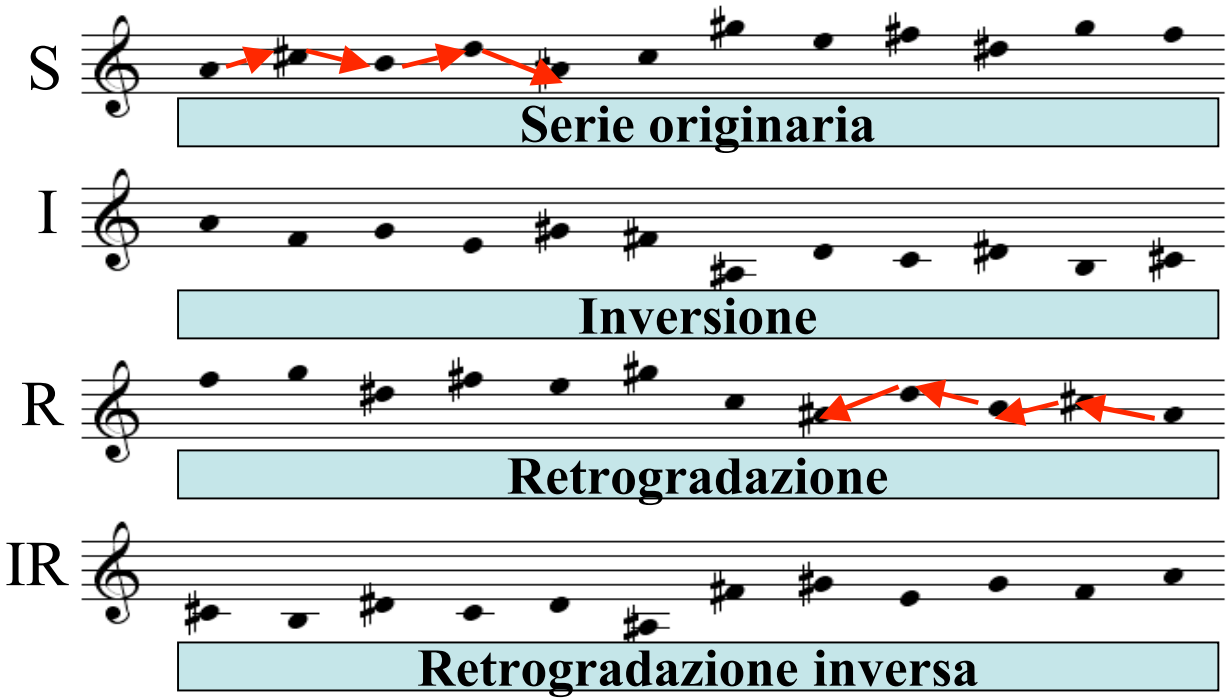
Operazioni dodecafoniche e strutture matematiche



Il sistema dodecafonico è "*un insieme d'elementi,* ***relazioni*** *fra gli elementi e* ***operazioni*** *sugli elementi*. […] *Un'effettiva matematizzazione avrebbe bisogno di una formulazione e di una presentazione dettata dal fatto che il sistema dodecafonico è un* ***gruppo di permutazioni*** *determinato* [shaped] *dalla struttura di questo modello matematico*"

M. Babbitt: *The function of Set Structure in the Twelve-Tone System*, PhD (1946/1992)

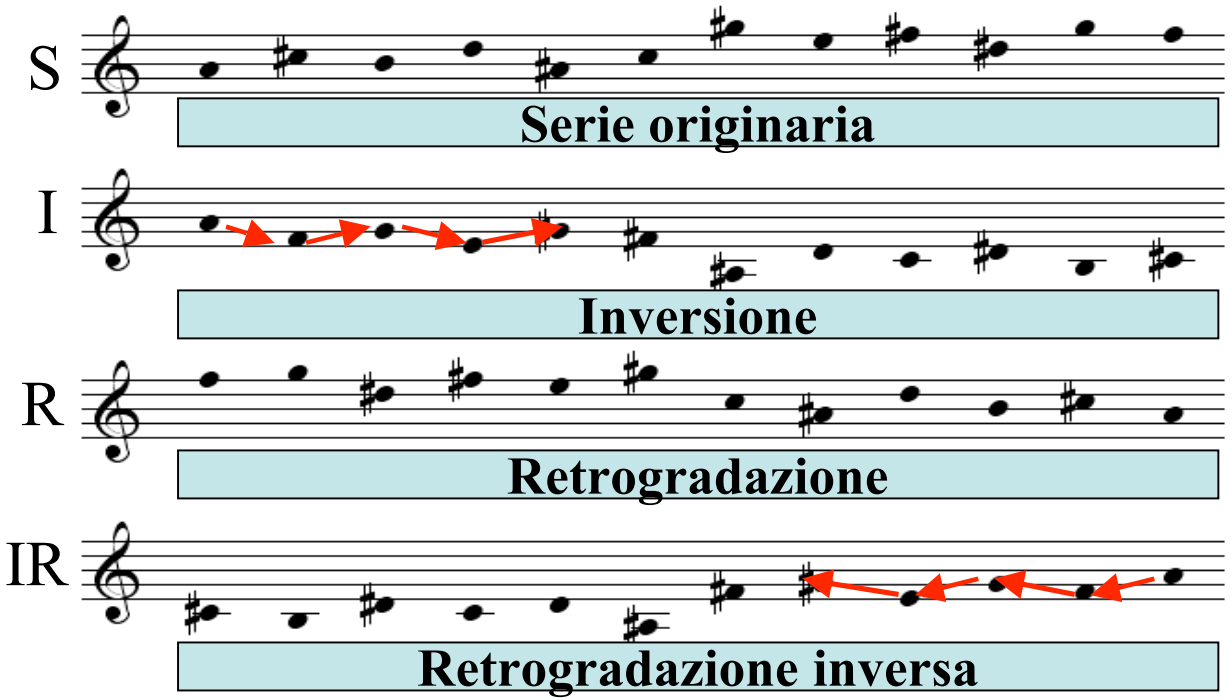
Operazioni dodecafoniche e strutture matematiche



Il sistema dodecafonico è "*un insieme d'elementi,* ***relazioni*** *fra gli elementi e* ***operazioni*** *sugli elementi*. […] *Un'effettiva matematizzazione avrebbe bisogno di una formulazione e di una presentazione dettata dal fatto che il sistema dodecafonico è un* ***gruppo di permutazioni*** *determinato* [shaped] *dalla struttura di questo modello matematico*"

M. Babbitt: *The function of Set Structure in the Twelve-Tone System*, PhD (1946/1992)

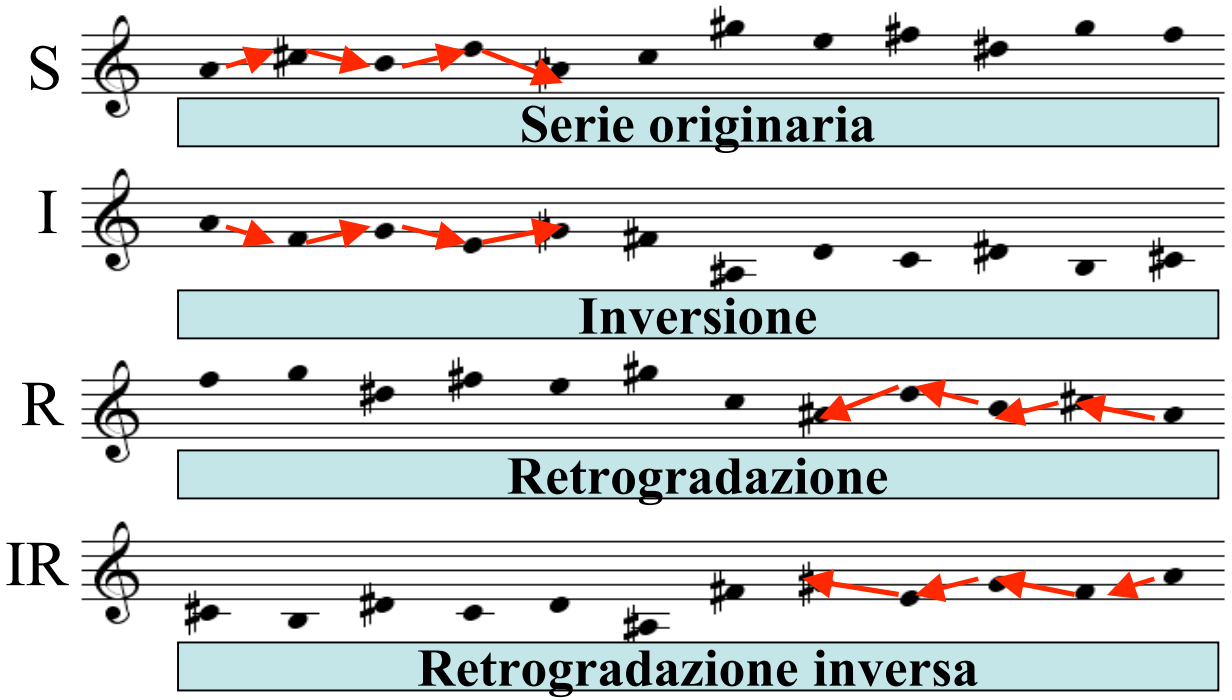
Operazioni dodecafoniche e strutture matematiche



Il sistema dodecafonico è "*un insieme d'elementi,* ***relazioni*** *fra gli elementi e* ***operazioni*** *sugli elementi*. […] *Un'effettiva matematizzazione avrebbe bisogno di una formulazione e di una presentazione dettata dal fatto che il sistema dodecafonico è un* ***gruppo di permutazioni*** *determinato* [shaped] *dalla struttura di questo modello matematico*"

M. Babbitt: *The function of Set Structure in the Twelve-Tone System*, PhD (1946/1992)

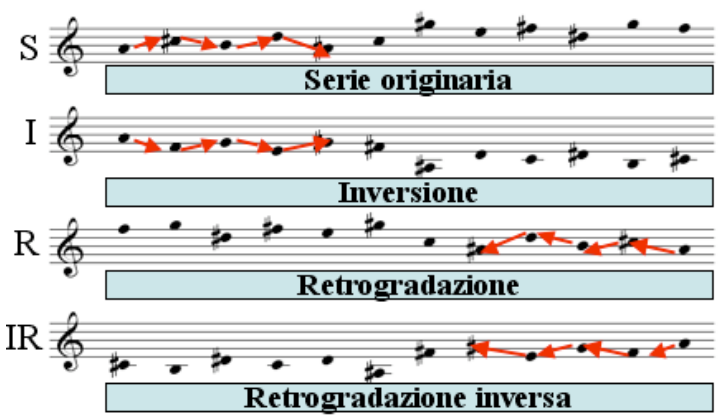
Operazioni dodecafoniche e strutture algebriche



+	S	I	R	RI
S	S	I	R	RI
I	I	S	RI	R
R	R	RI	S	I
RI	RI	R	I	S

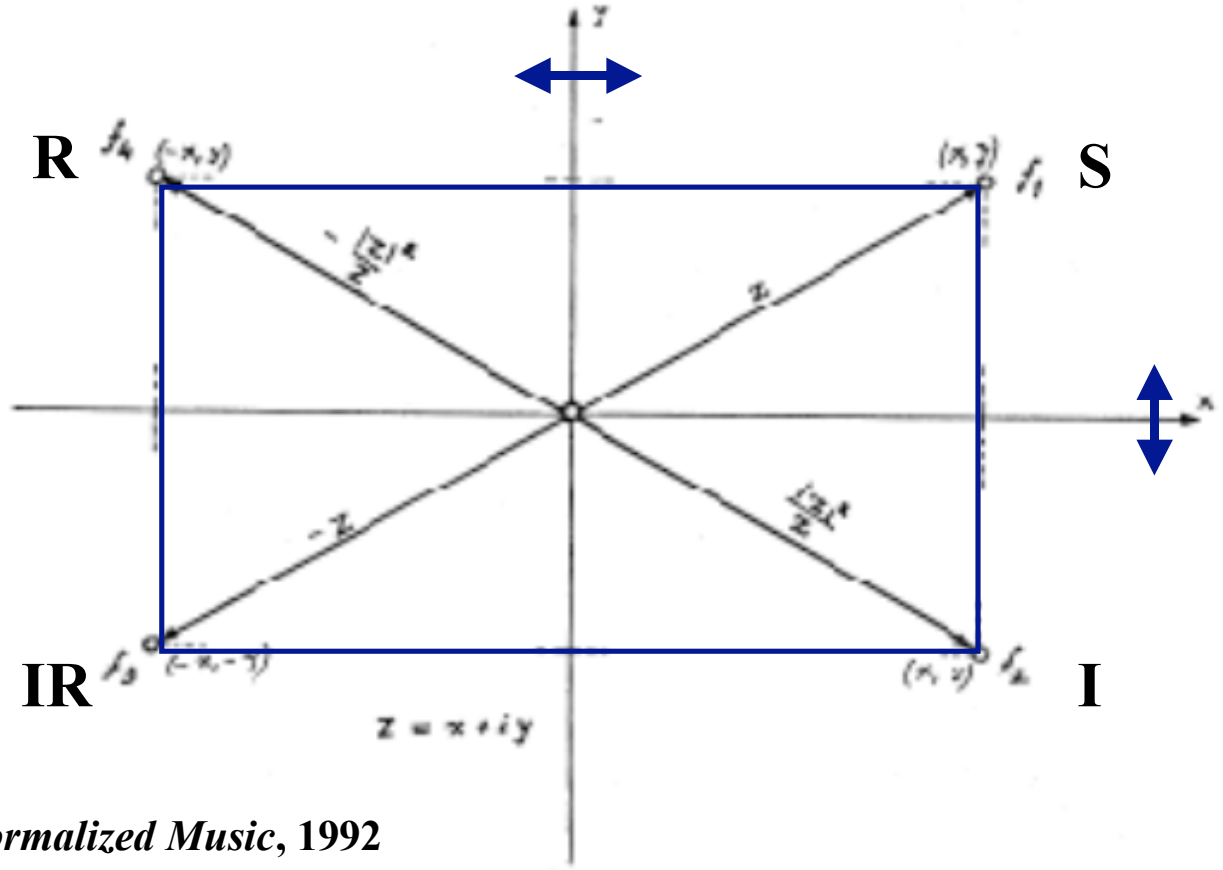
Struttura di *gruppo:*
- **Chiusura**
- **Esistenza dell'elemento neutro**
- **Esistenza dell'inverso**
- **Associatività: $(a+b)+c=a+(b+c)$**

Operazioni dodecafoniche e strutture algebriche



	S	I	R	RI
S	S	I	R	RI
I	I	S	RI	R
R	R	RI	S	I
RI	RI	R	I	S

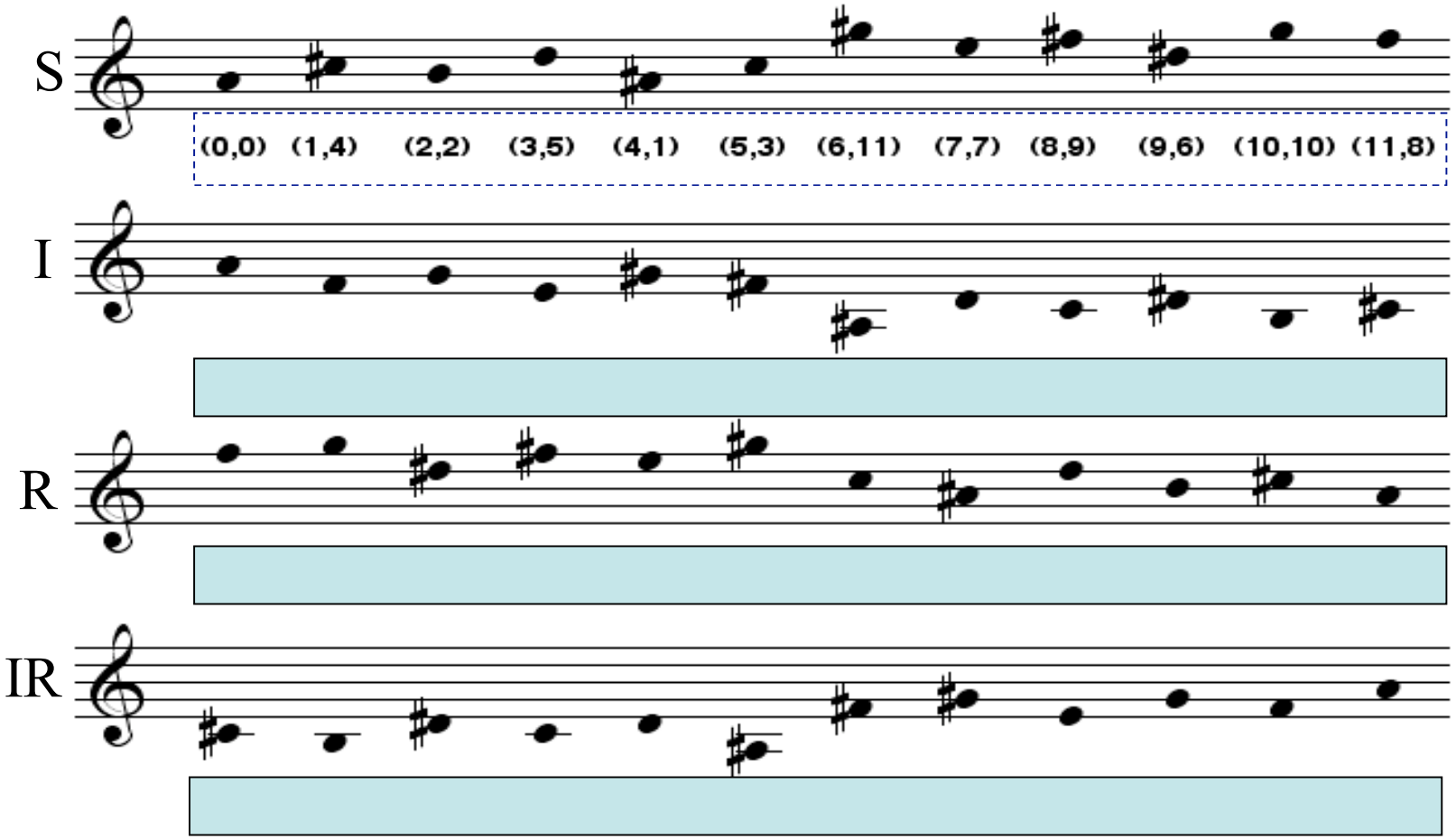
Struttura di *gruppo*
- **Chiusura**
- **Esistenza dell'elemento neutro**
- **Esistenza dell'inverso**
- **Associatività**



Iannis Xenakis, *Formalized Music*, 1992

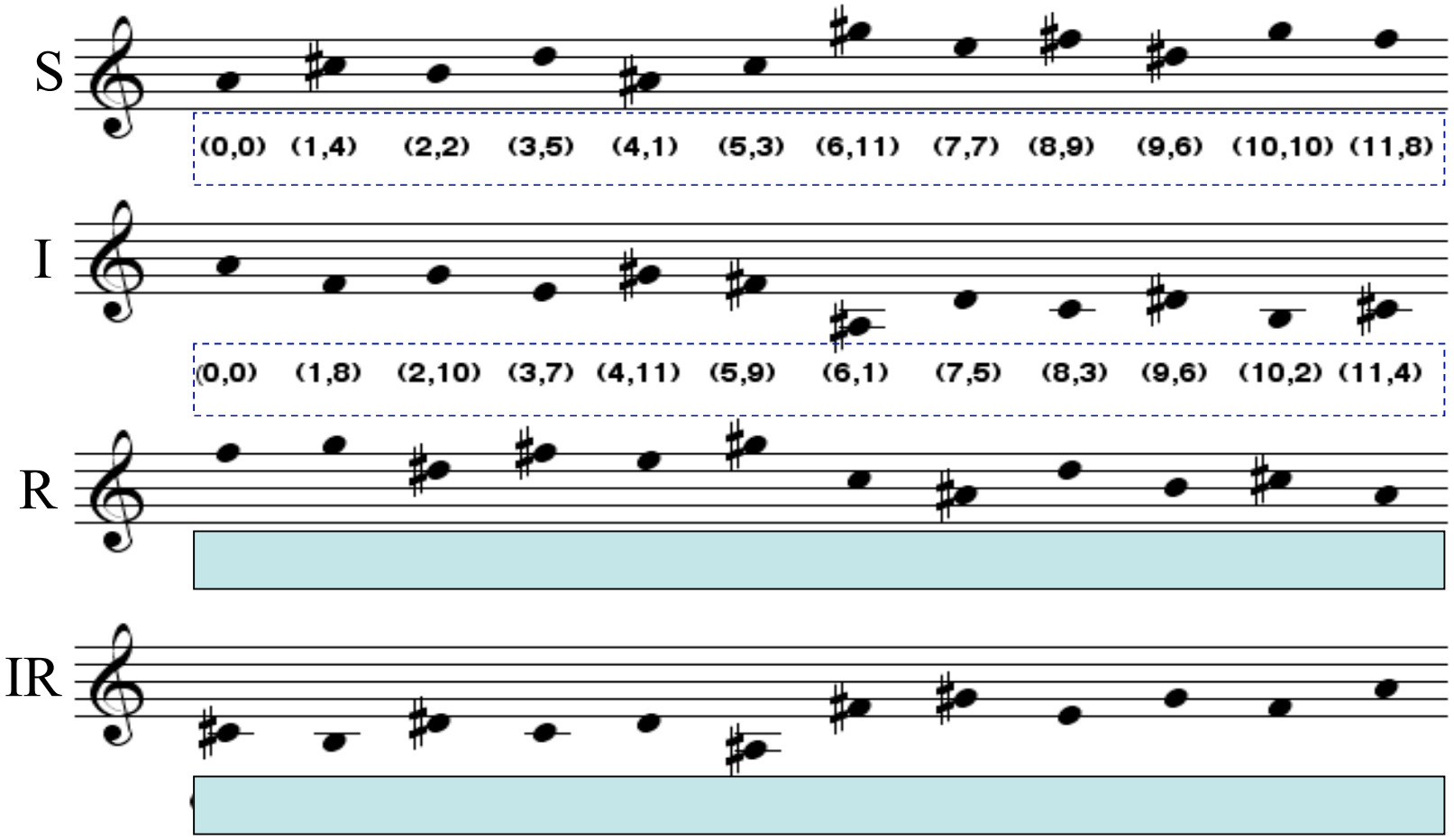
Felix Klein

Operazioni dodecafoniche e strutture algebriche



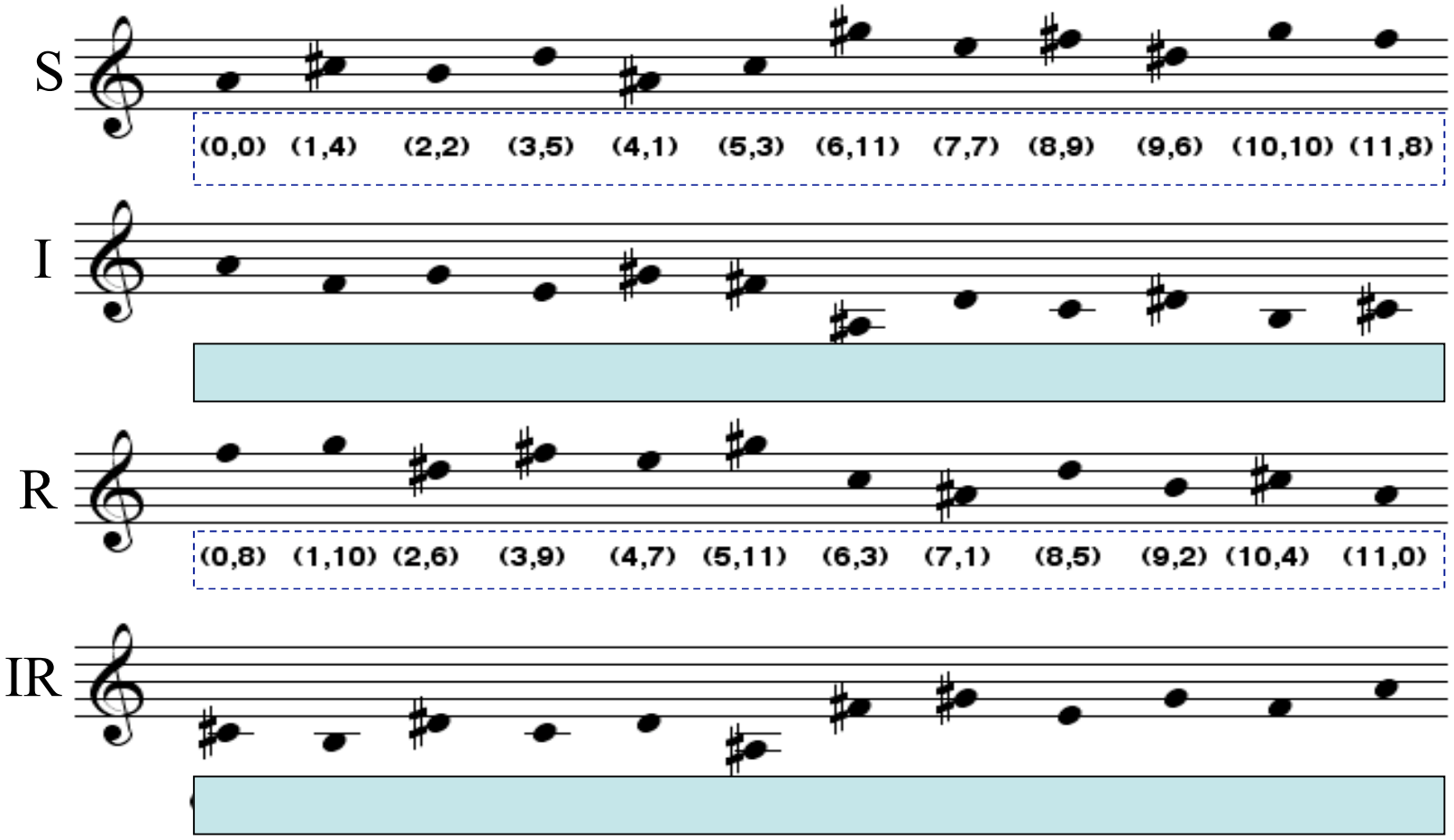
S: $(a,b) \rightarrow (a,b)$

Operazioni dodecafoniche e strutture algebriche



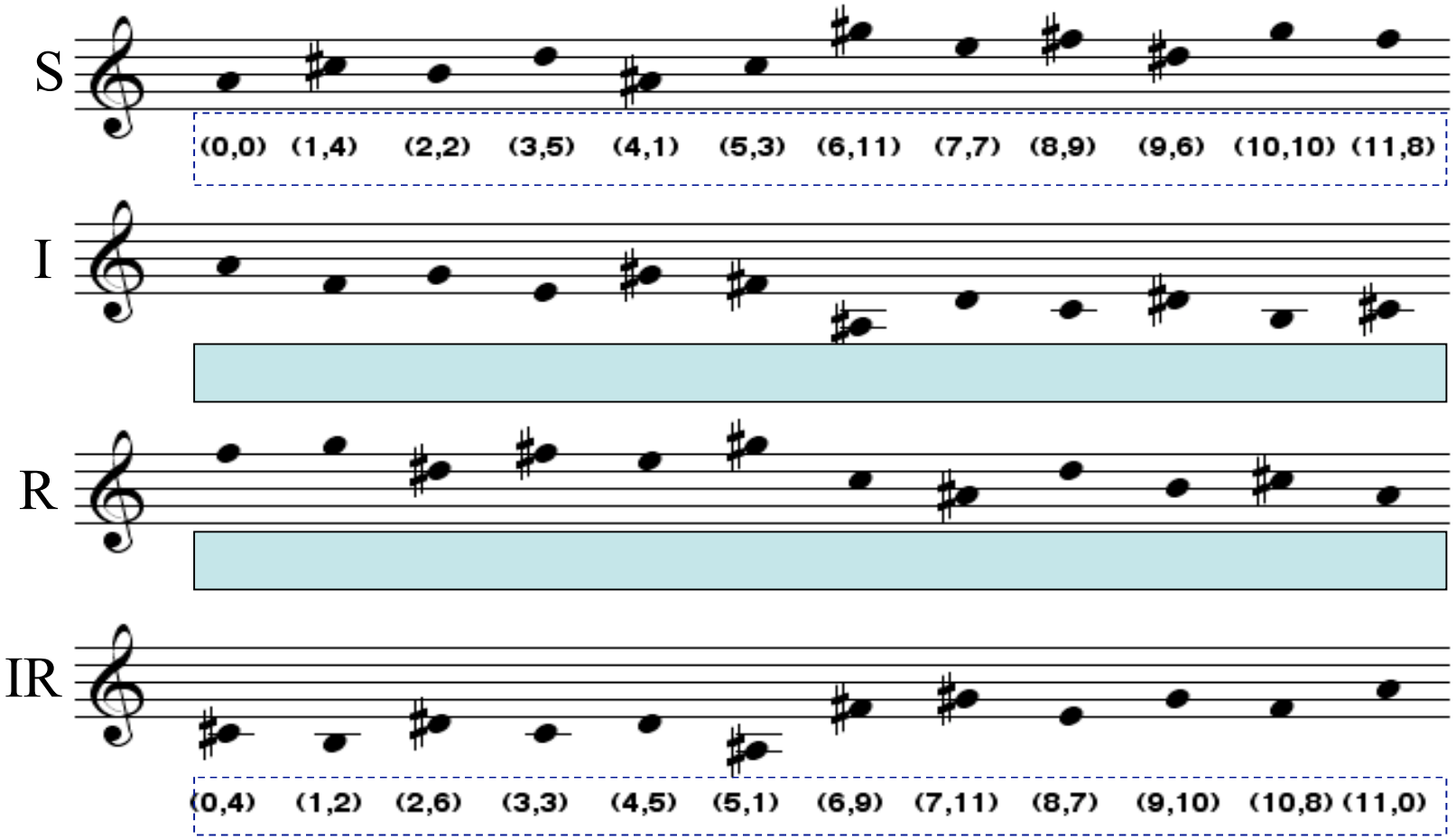
I: $(a,b) \rightarrow (a,$ **12-*b* mod.12**$)$

Operazioni dodecafoniche e strutture algebriche



R: $(a,b) \rightarrow (11\text{-}a,b)$.

Operazioni dodecafoniche e strutture algebriche



IR:$(a,b) \rightarrow (11\text{-}a,\ b \bmod 12)$
$\downarrow$
$(11\text{-}a,\ 12\text{-}b \bmod 12)$

$=$

RI:$(a,b) \rightarrow (a,\ 12\text{-}b \bmod 12)$
$\downarrow$
$(11\text{-}a,\ 12\text{-}b \bmod 12)$

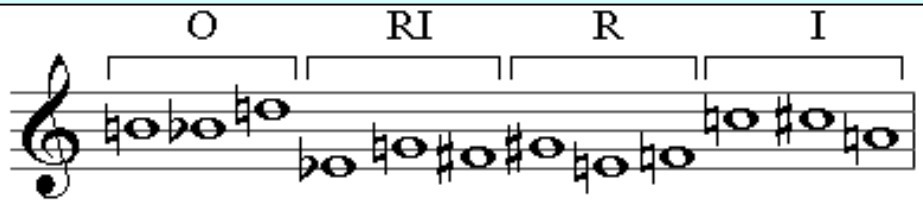
Simmetrie musicali e strutture matematiche

Ernst Krenek e l'approccio assiomatico in musica

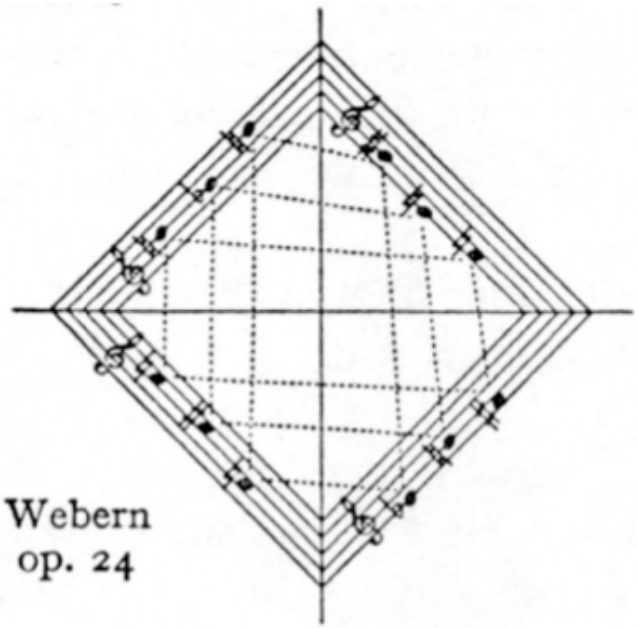
- *The Relativity of Scientific Systems*
- *The Significance of Axioms*
- *Axioms in music*
- *Musical Theory and Musical Practice*

Ernst Krenek : *Über Neue Musik*, 1937 (Engl. Transl. *Music here and now,* 1939).

Physicists and mathematicians are far in advance of musicians in realizing that their respective sciences do not serve to establish a concept of the universe conforming to an objectively existent nature



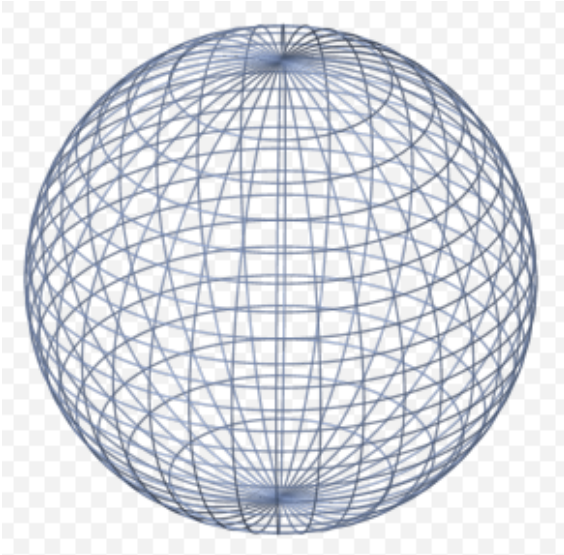
As the study of axioms eliminates the idea that axioms are something absolute, conceiving them instead as ***free propositions of the human mind****, just so would this* ***musical theory*** *free us from the concept of major/minor tonality [...] as an irrevocable law of nature.*



L'approccio assiomatico in matematica

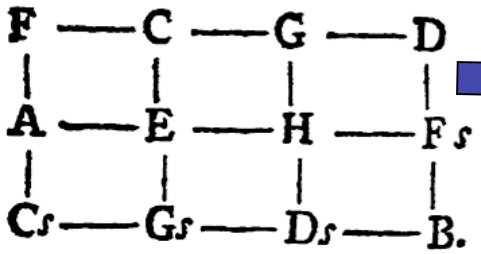
David Hilbert: i fondamenti assiomatici della geometria e il ruolo dell'intuizione

In order to be constructed in a right way, geometry [...] only needs few simple principles. These principles are called the ***axioms*** *of the geometry. [...] This study (of the axioms) goes back to the* ***logical analysis of our spatial intuition***
(Grundlage der Geometrie, 1899).

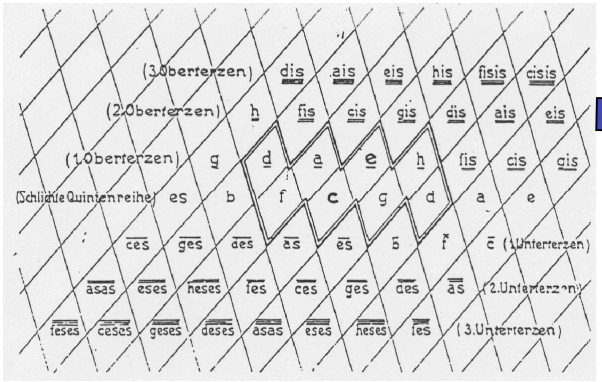


At the moment there are two tendencies in mathematics. From one side, the tendency toward abstraction aims at 'cristallizing' the logical relations inside of a study object and at organizing this material in a systematic way. But there is also a tendency towards the ***intuitive understanding*** *which aims at understanding the* ***concret meaning of their relations***
(Anschauliche Geometrie, 1932)

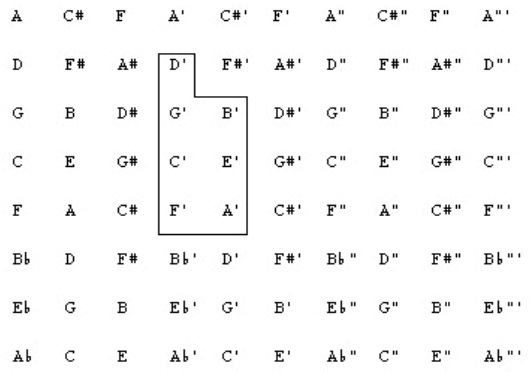
Rappresentazioni geometriche delle strutture musicali



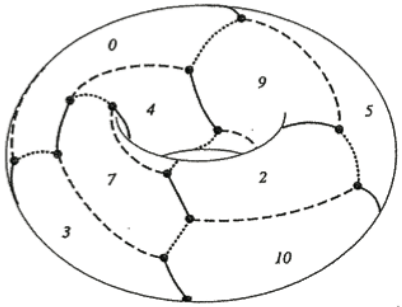
Euler : *Speculum musicum*, 1773



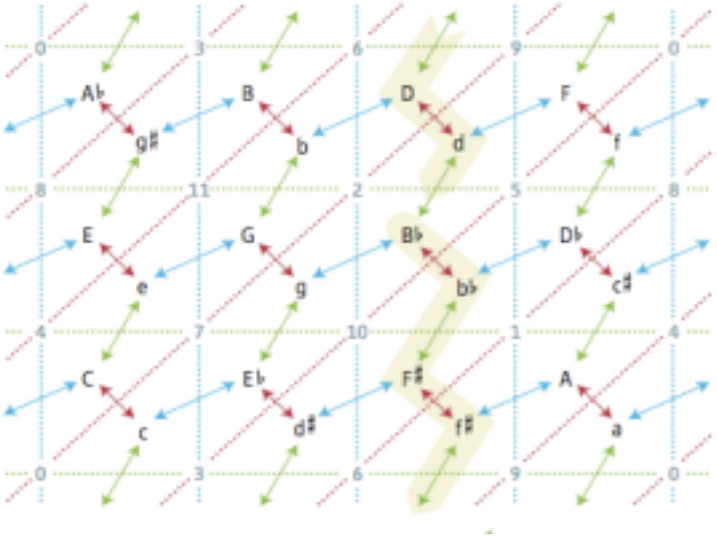
Hugo Riemann : « Ideen zu einer Lehre von den Tonvorstellung », 1914



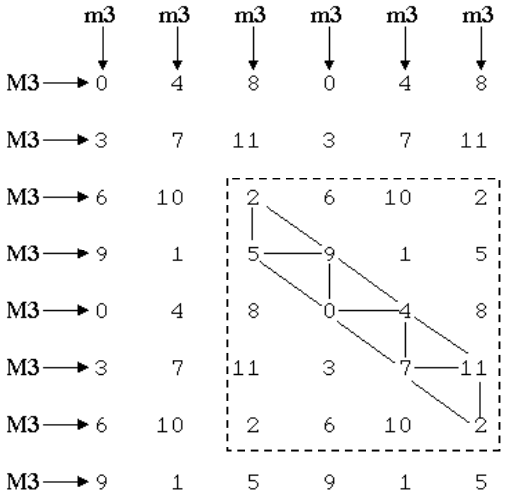
Longuet-Higgins (1962)



Douthett & Steinbach, *JMT*, 1998

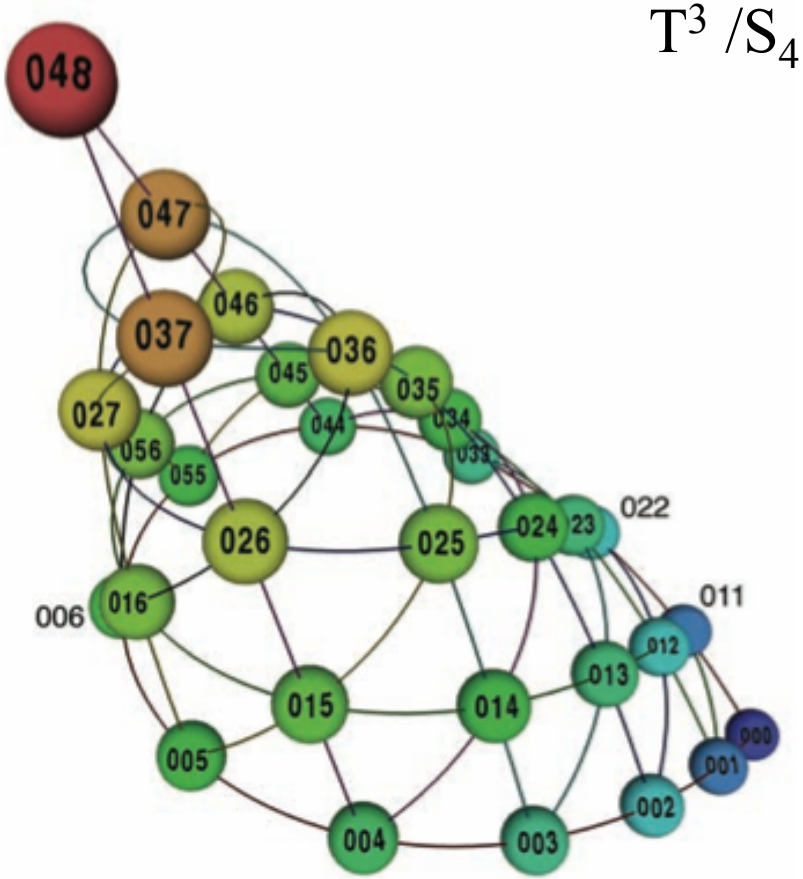


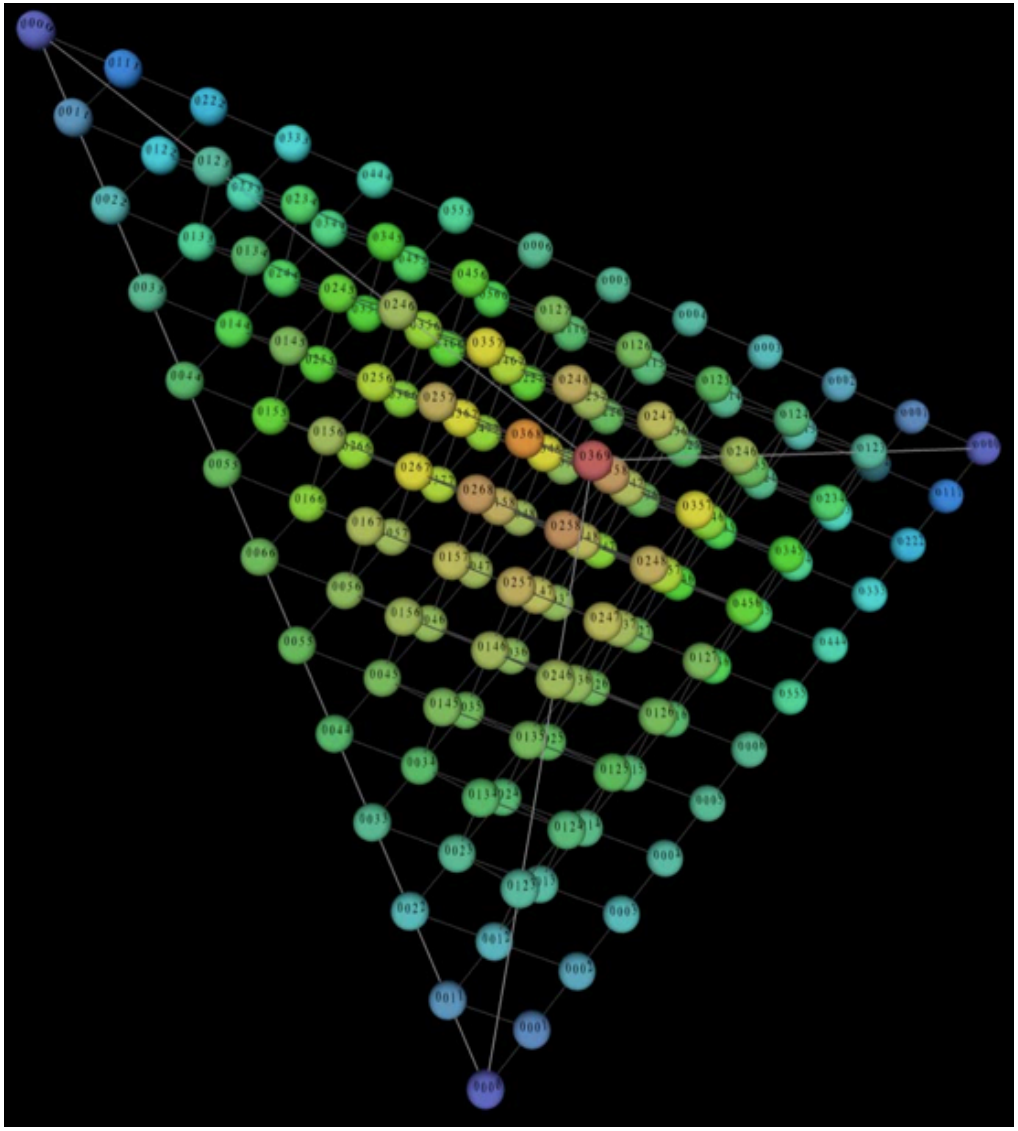
J. Hook, « Exploring Musical Space », *Science, 2006*



Balzano (1980)

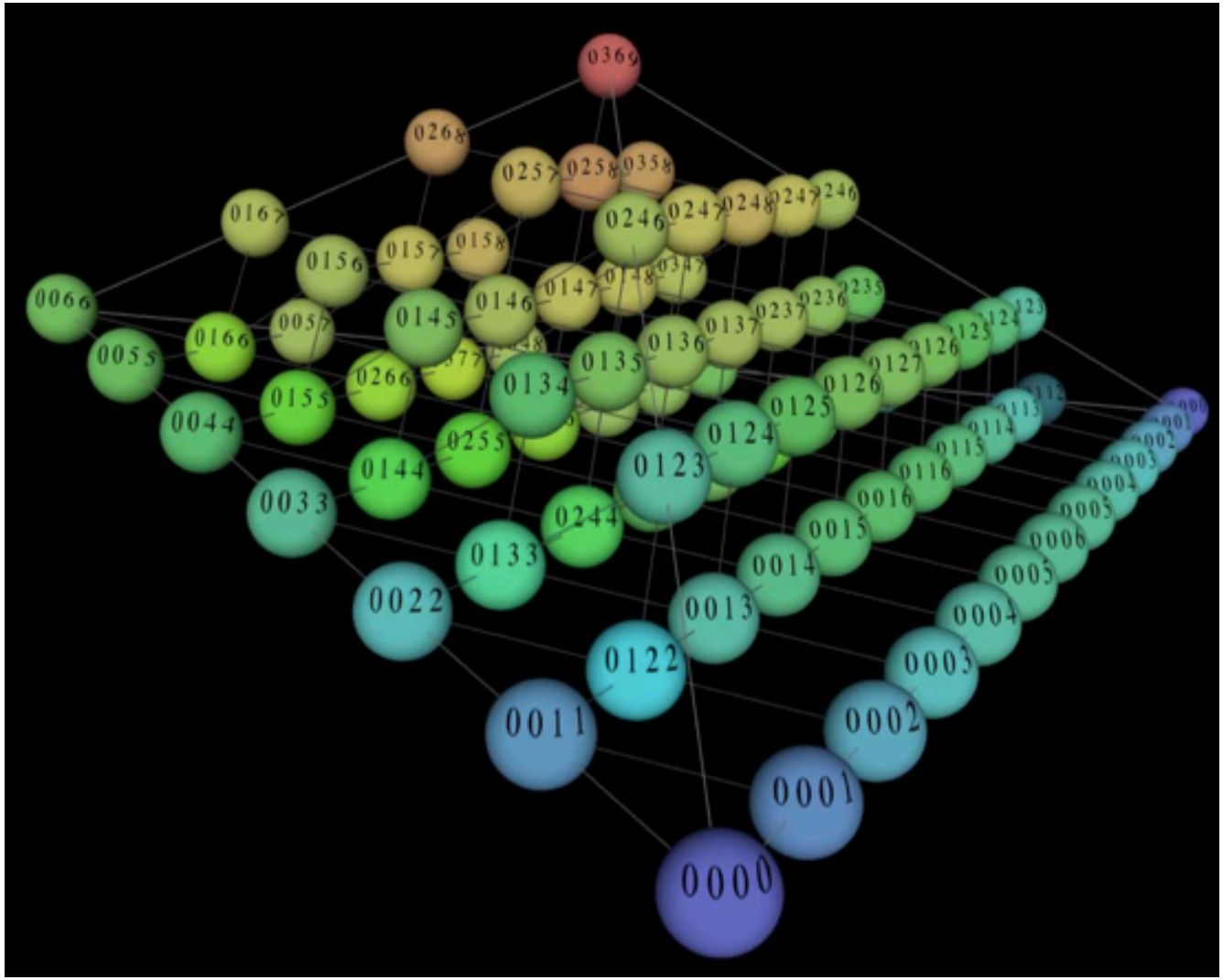
T^3 / S_4





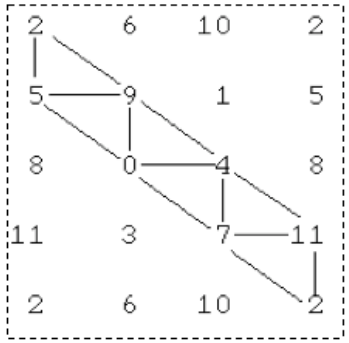
C. Callender, I. Quinn & D. Tymoczko, « Generalized Voice-Leading Spaces », *Science*, 320, 2008

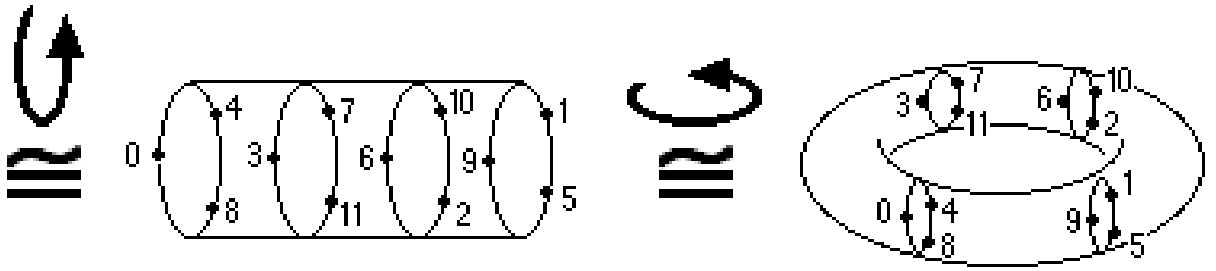
$T^3/(S_4 \times Z_2)$



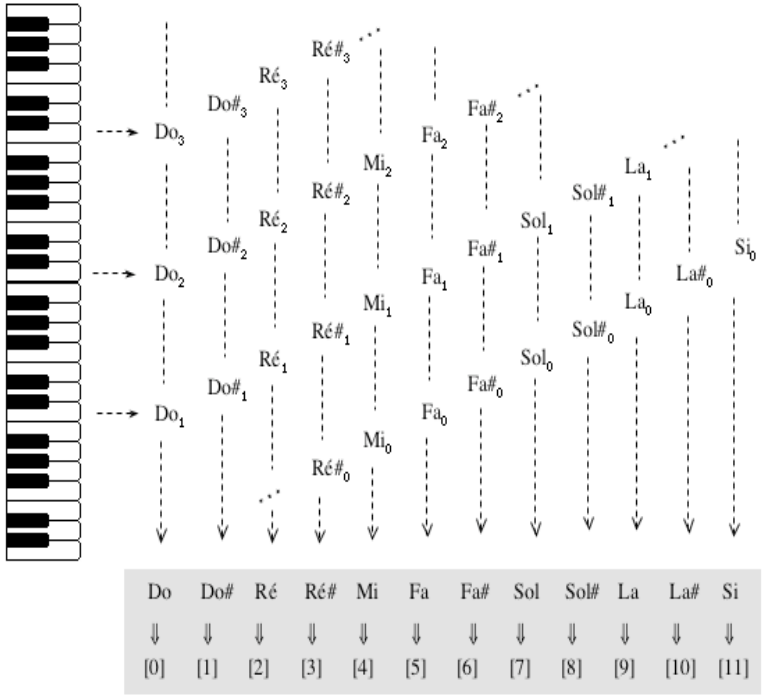
C. Callender, I. Quinn & D. Tymoczko, « Generalized Voice-Leading Spaces », *Science*, 320, 2008

Equivalenza algebrica fra rappresentazioni geometriche

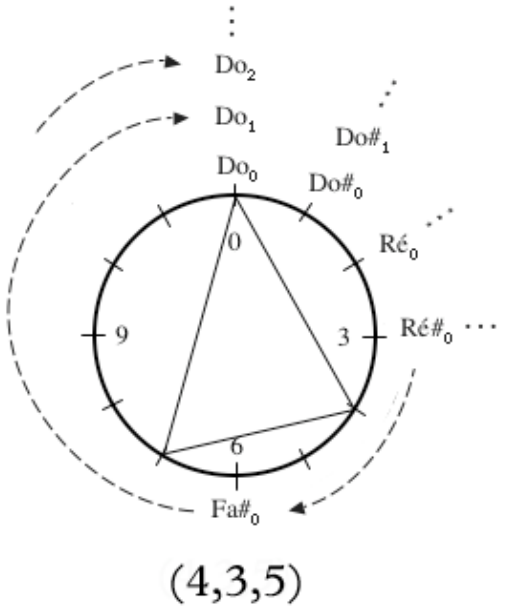




Rappresentazione toroidale

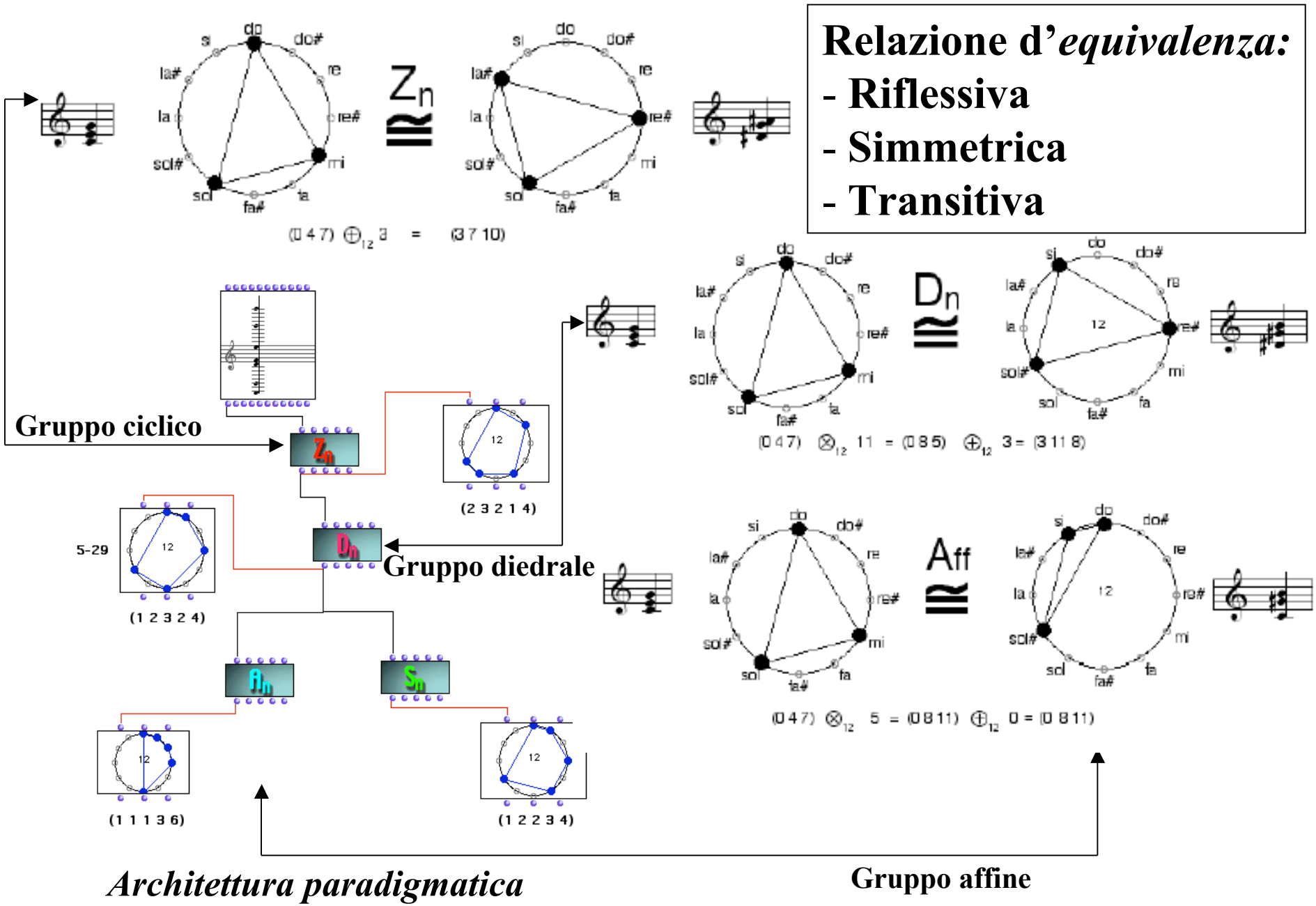


$$Z_{12} = Z_3 \times Z_4$$



Rappresentazione circolare

I gruppi come “paradigmi” per l’equivalenza fra accordi



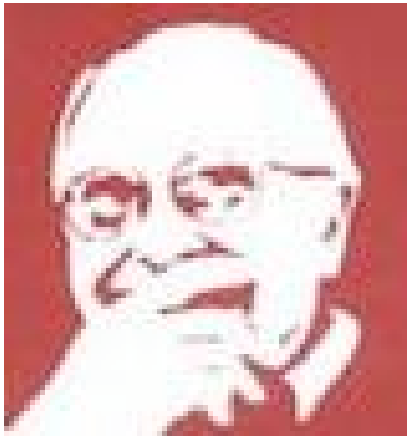
Architettura paradigmatica e strutture algebriche in musica

« [C'est la notion de groupe qui] donne un sens précis à l'idée de structure d'un ensemble [et] permet de déterminer les éléments efficaces des transformations en réduisant en quelque sorte à son schéma opératoire le domaine envisagé. [...]L'objet véritable de la science est le ***système des relations*** *et non pas les termes supposés qu'il relie. [...] Intégrer les résultats - symbolisés - d'une* ***expérience*** *nouvelle revient [...] à créer un canevas nouveau, un* ***groupe de transformations*** *plus complexe et plus compréhensif »*

G.-G. Granger : « Pygmalion. Réflexions sur la pensée formelle », 1947

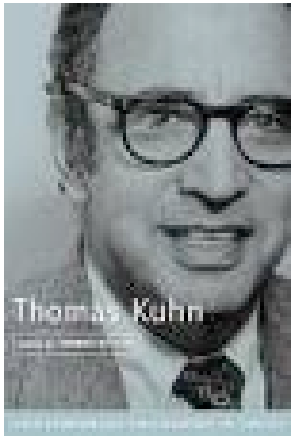
Felix Klein

Ernst Cassirer



Gilles-Gaston Granger

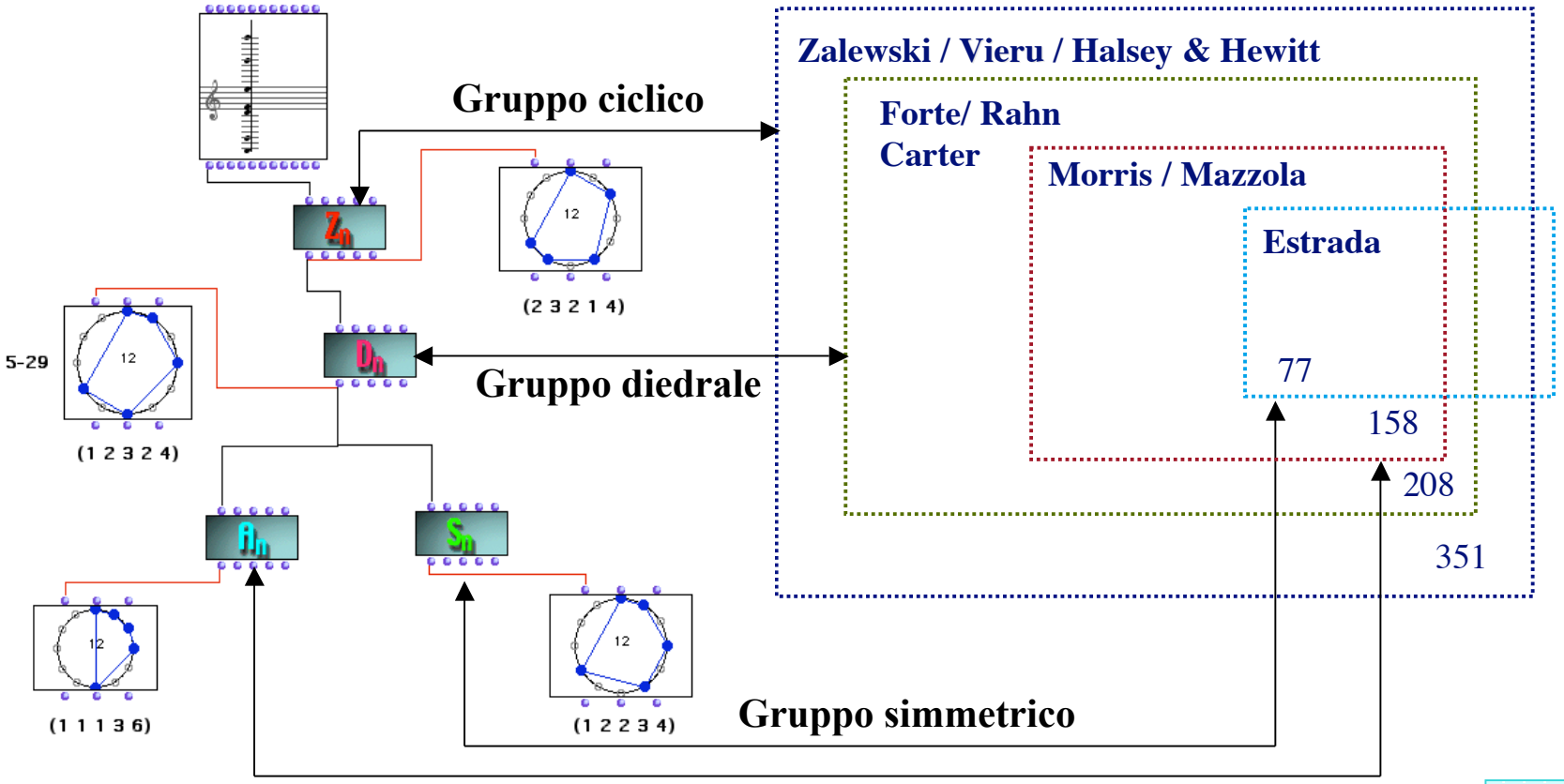
Jean Piaget



Thomas Kuhn

Classificazione paradigmatica delle strutture musicali

$G\backslash k$	1	2	3	4	5	6	7	8	9	10	11	12	
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1	
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1	**Set Theory**
$\mathrm{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1	



Architettura paradigmatica

=> *OpenMusic*

OpenMusic

^Catalogo-Dn

Catalogo d'accordi (modulo la trasposizione e l'inversione)

Paradigma del gruppo diedrale (Set Theory)

6 <-- numero di note dell'accordo

dn-orbites

chord2c

12

pc-set

12

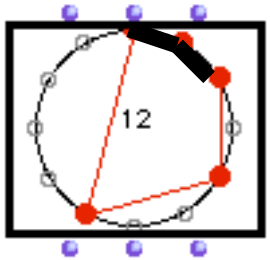
omloop

Distribuzione dei pitch-class sets

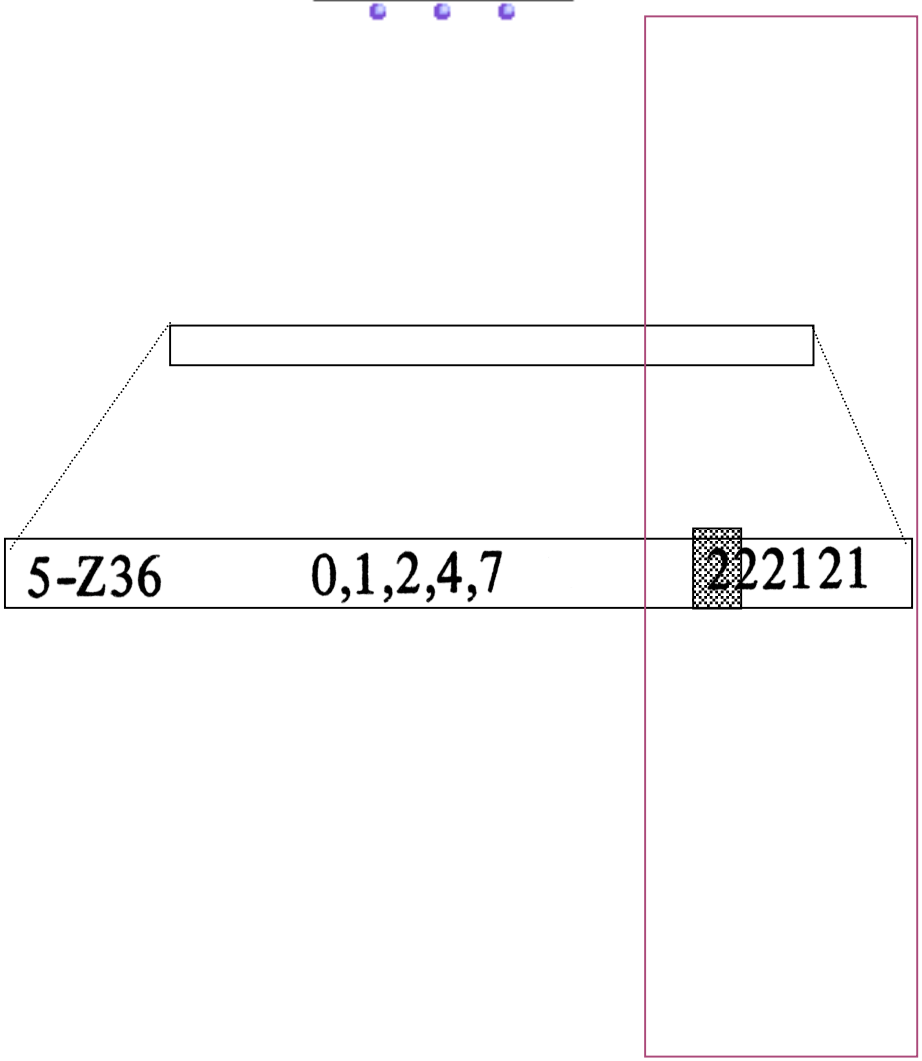
c2chord

I 50 esacordi della Set Theory

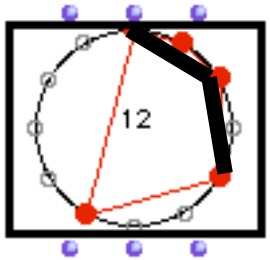
Vettore intervallare e relazione Z



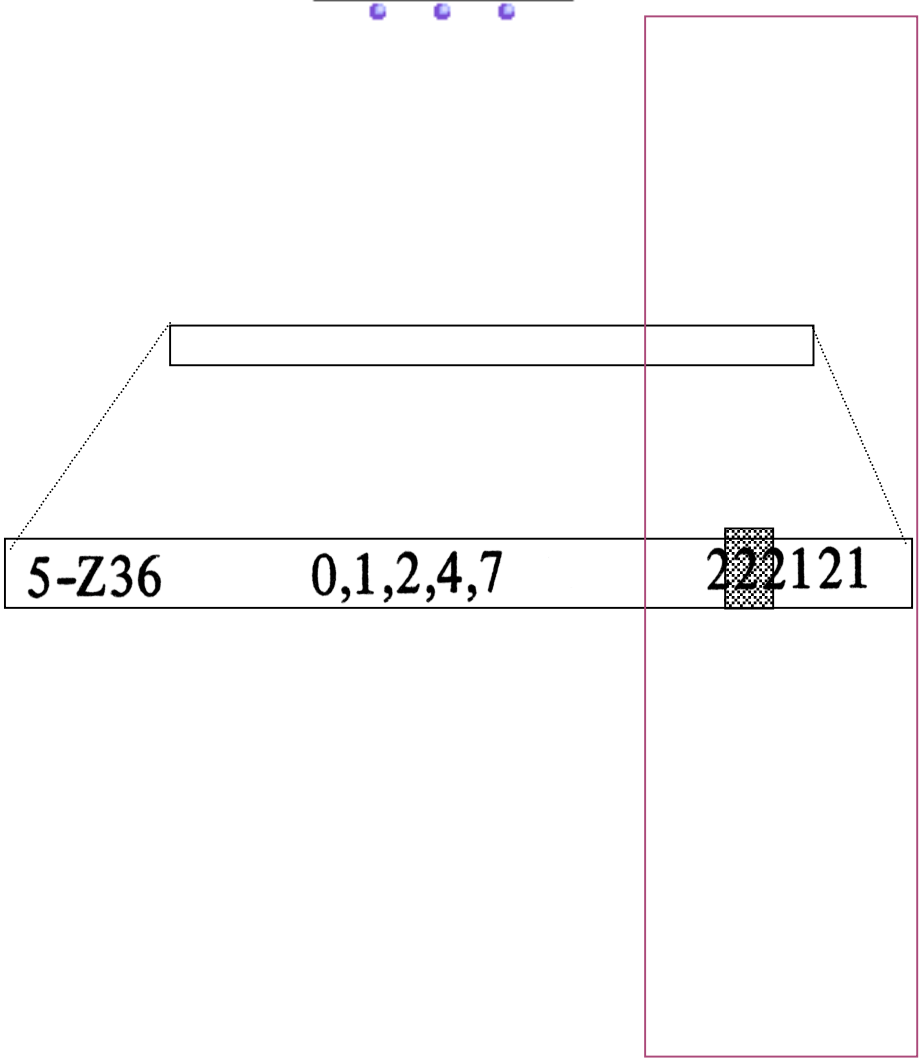
Il vettore intervallare (Forte) esprime la frequenza di apparizione di ogni intervallo (modulo il suo complementare)



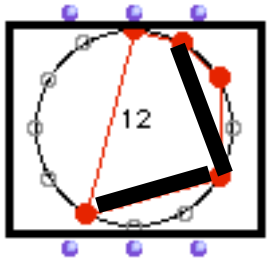
Vettore intervallare e relazione Z



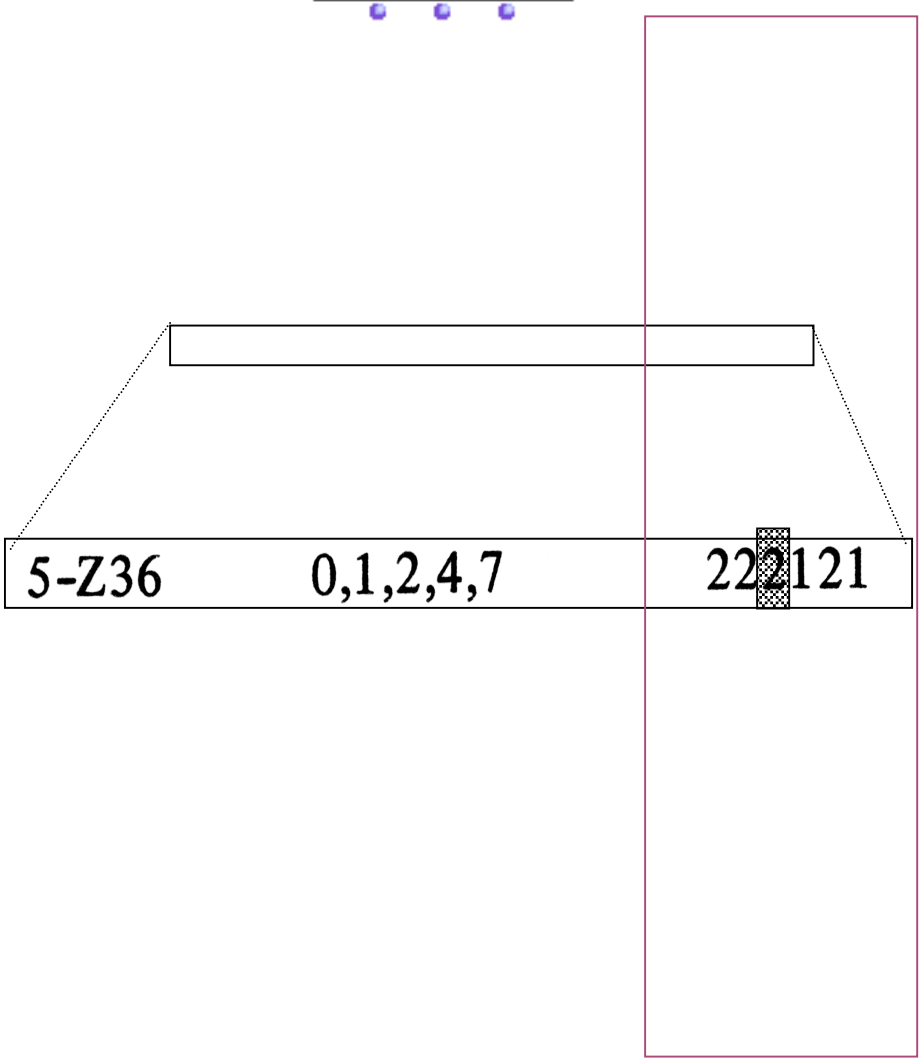
Il vettore intervallare (Forte) esprime la frequenza di apparizione di ogni intervallo (modulo il suo complementare)



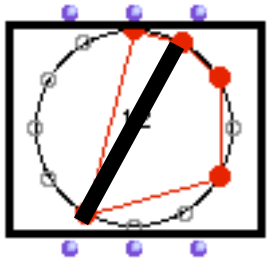
Vettore intervallare e relazione Z



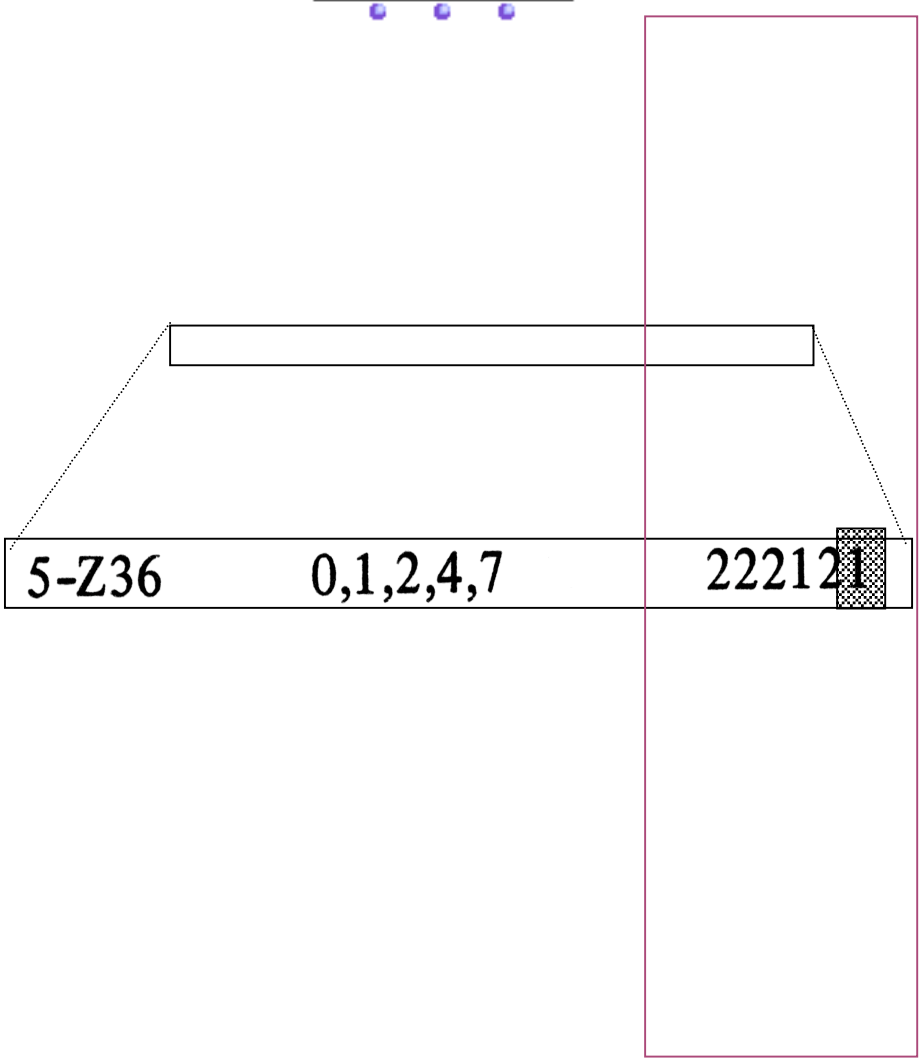
Il vettore intervallare (Forte) esprime la frequenza di apparizione di ogni intervallo (modulo il suo complementare)



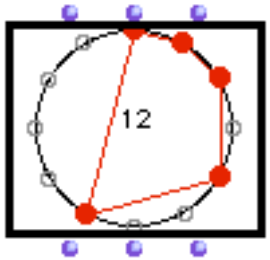
Vettore intervallare e relazione Z



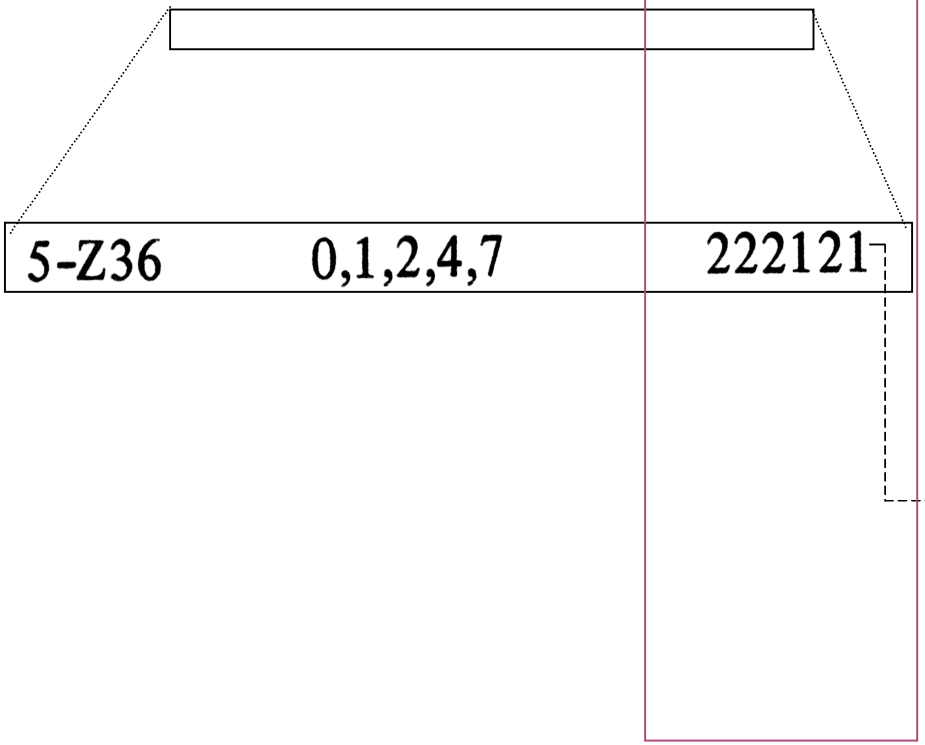
Il vettore intervallare (Forte) esprime la frequenza di apparizione di ogni intervallo (modulo il suo complementare)

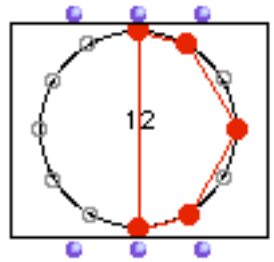


Vettore intervallare e relazione Z



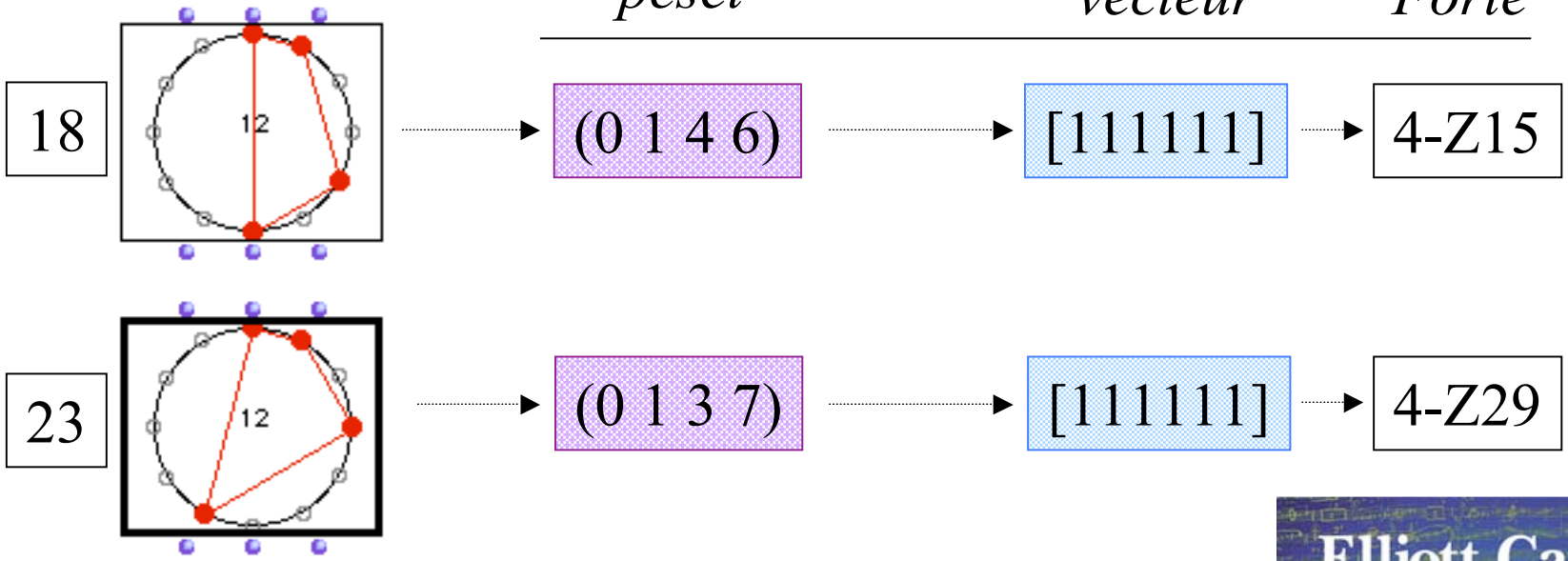
Il vettore intervallare (Forte) esprime la frequenza di apparizione di ogni intervallo (modulo il suo complementare)





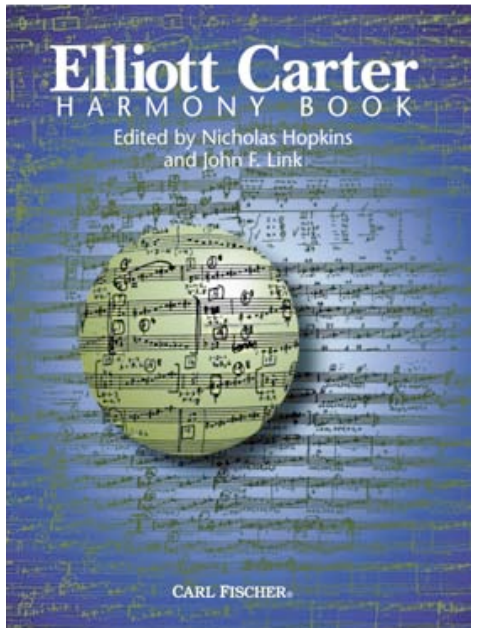
5-Z12

Elliott Carter's *Harmony Book* (2002)



Utilizzazione (implicita) della *Z-relation*

- *Quartetto n°1* (1951)
- *Night Fantasies* (1980)
- *90+* (1994)
- …

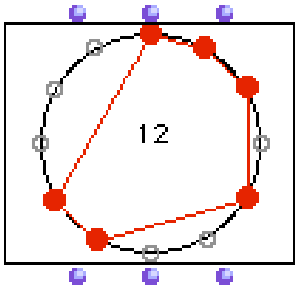


D. Ghisi, *Vettori intervallari : non degenerazione e Z-relation*, tesi di laurea, Milano, ottobre 2006

Elliott Carter: 90+ (1994)

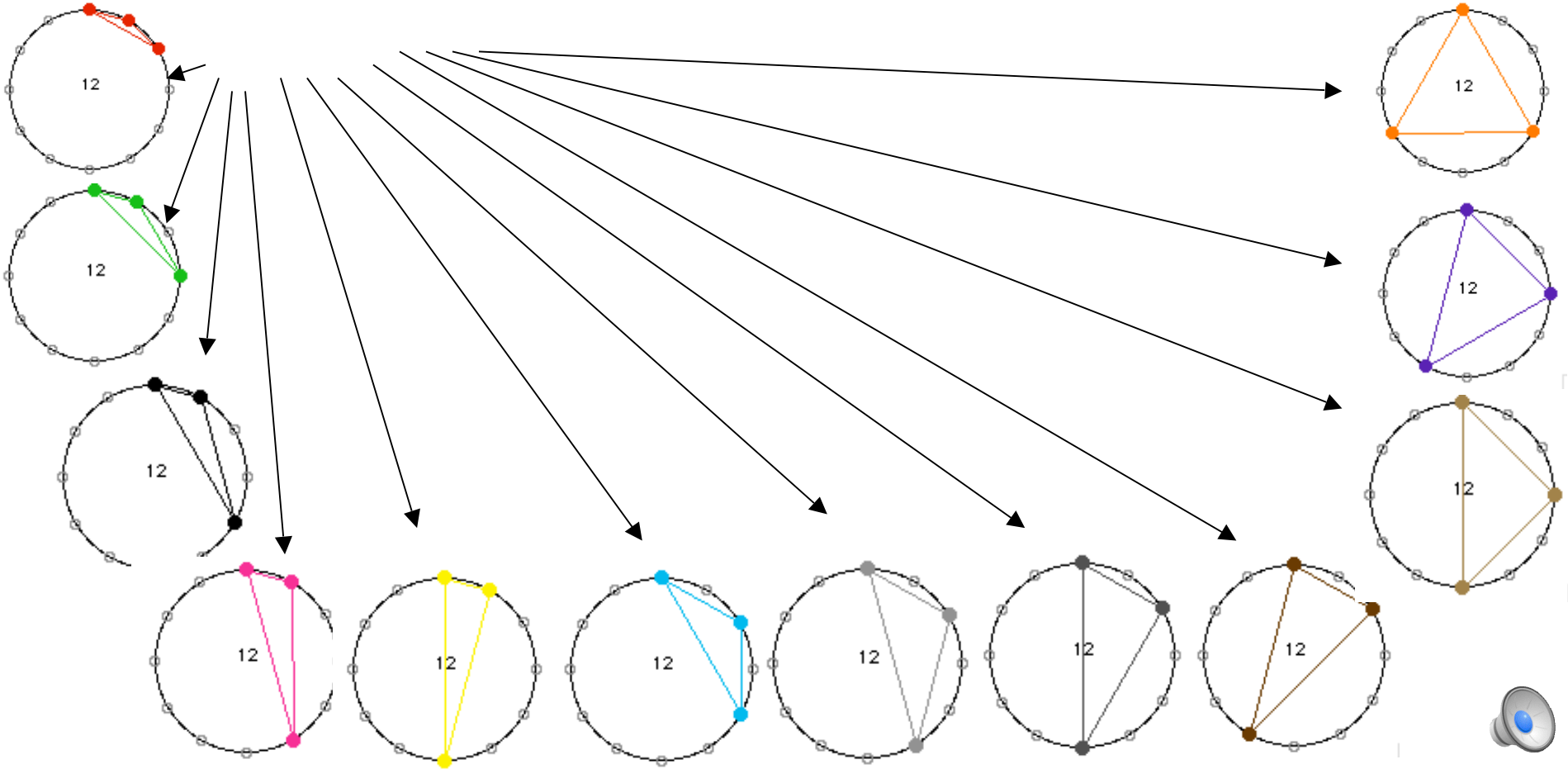
$G\backslash k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1
$\mathrm{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1

35



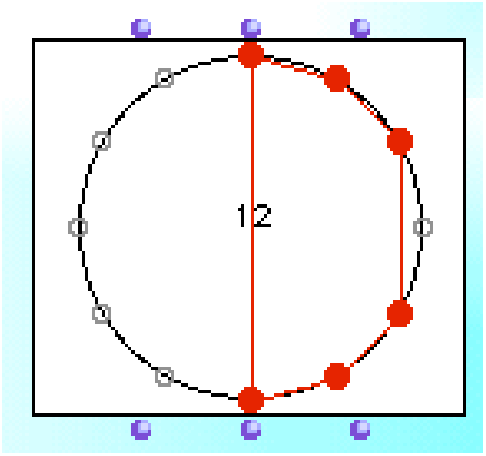
pcset	*vector*	*name*
(0 1 2 4 7 8)	[322332]	6-Z17

All-triad hexachord

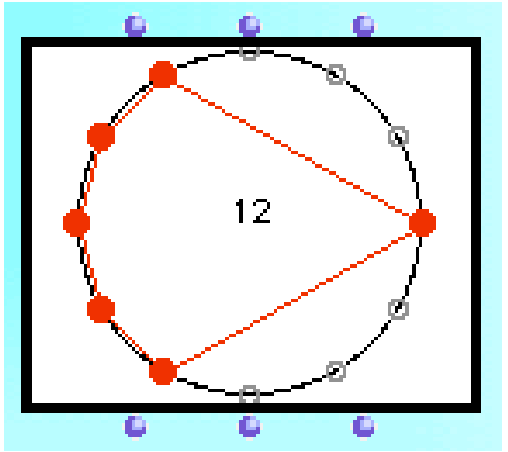


Teorema dell'esacordo (o teorema di Babbitt)

(Wilcox, Ralph Fox (?), Chemillier, Lewin, Mazzola, Schaub, …, Amiot [2006])



A



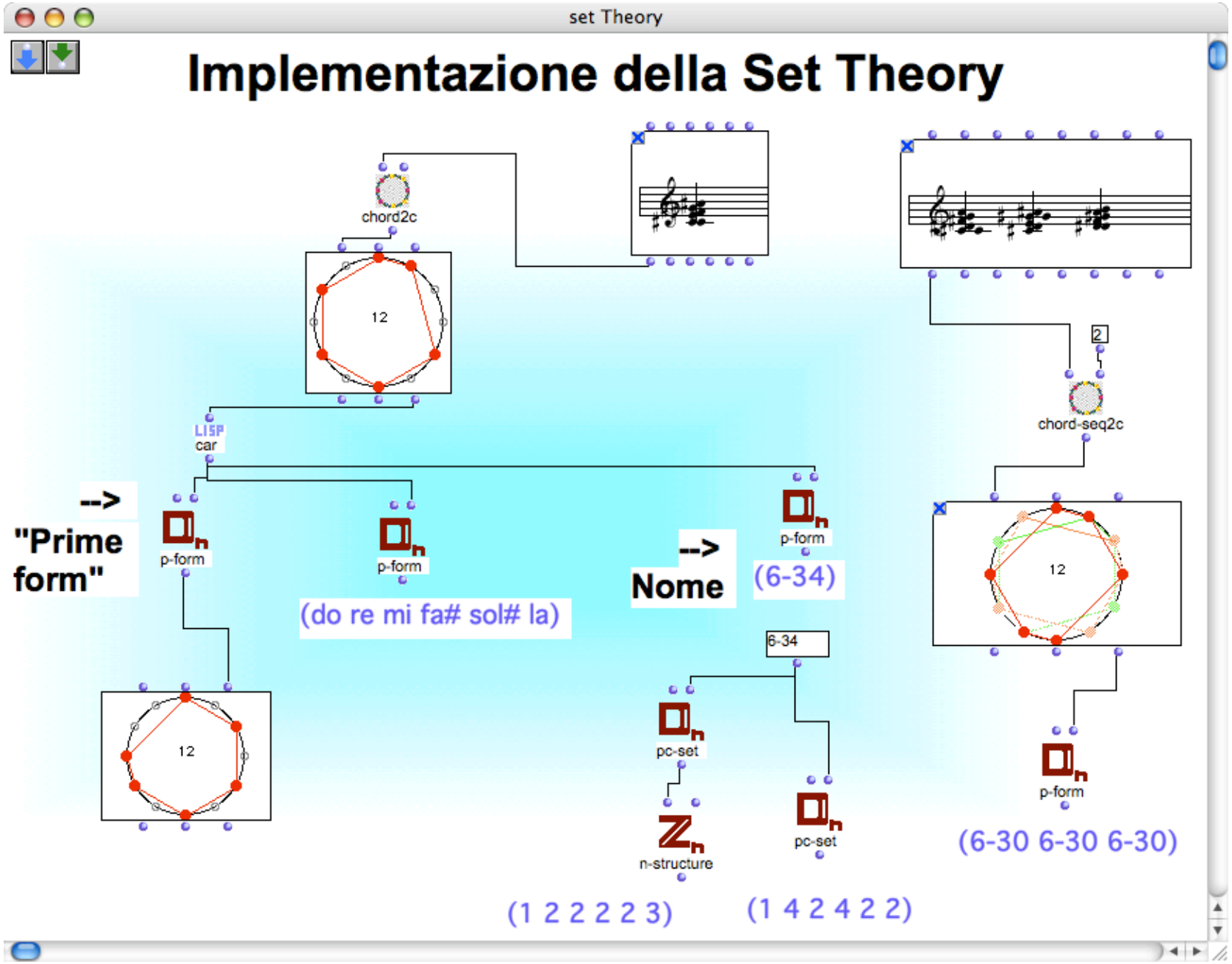
A'

IV(*A*) = [4, 3, 2, 3, 2, 1] = [4, 3, 2, 3, 2, 1] = IV(*A'*)

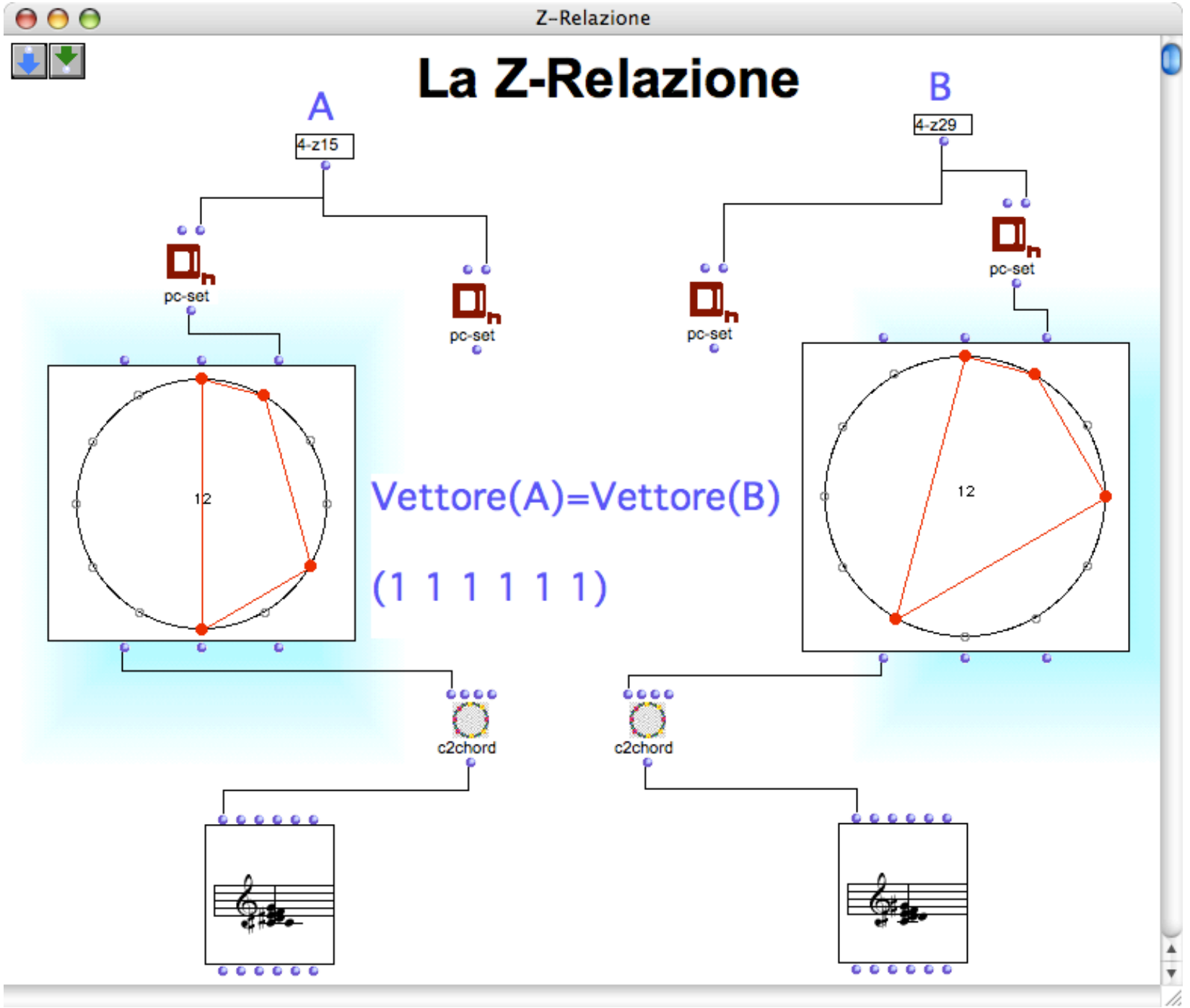
Un esacordo e il suo complementare hanno lo stesso vettore intervallare

=> *OpenMusic*

OpenMusic


set Theory
Implementazione della Set Theory
chord2c
12
LISP
car
-->
"Prime
form"
p-form
p-form
(do re mi fa# sol# la)
12
-->
Nome
p-form
(6-34)
6-34
pc-set
n-structure
pc-set
(1 2 2 2 2 3)
(1 4 2 4 2 2)
2
chord-seq2c
12
p-form
(6-30 6-30 6-30)

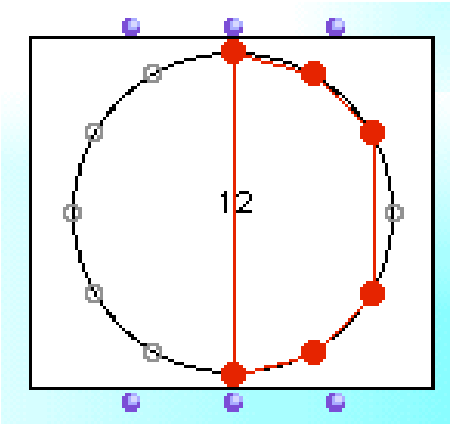
OpenMusic


Z-Relazione
La Z-Relazione
A
4-z15
pc-set
pc-set
12
B
4-z29
pc-set
pc-set
12
Vettore(A)=Vettore(B)
(1 1 1 1 1 1)
c2chord
c2chord

Contenuto intervallare e trasformata di Fourier discreta

David Lewin, *Journal of Music Theory,* 1958

- Il **contenuto intervallare** (*IC*) di un accordo esprime la frequenza di apparizione di ogni intervallo (dall'unisono all'intervallo di settima maggiore)



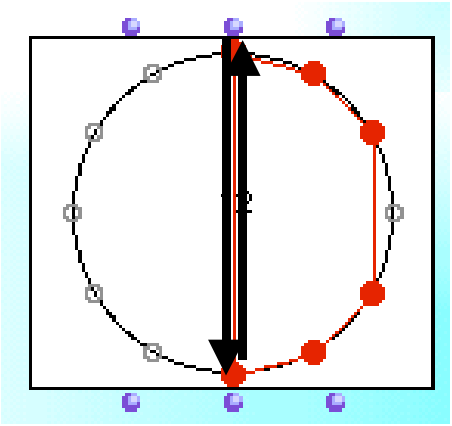
$IV_A =$ [4, 3, 2, 3, 2, 1]

$IC_A =$ [6, 4, 3, 2, 3, 2, 2, 2, 3, 2, 3, 4]

Contenuto intervallare e trasformata di Fourier discreta

David Lewin, *Journal of Music Theory*, 1958

- Il **contenuto intervallare** (*IC*) di un accordo esprime la frequenza di apparizione di ogni intervallo (dall'unisono all'intervallo di settima maggiore)



$IV_A =$ [4, 3, 2, 3, 2, 1]

$IC_A =$ [6, 4, 3, 2, 3, 2, 2, 2, 3, 2, 3, 4]

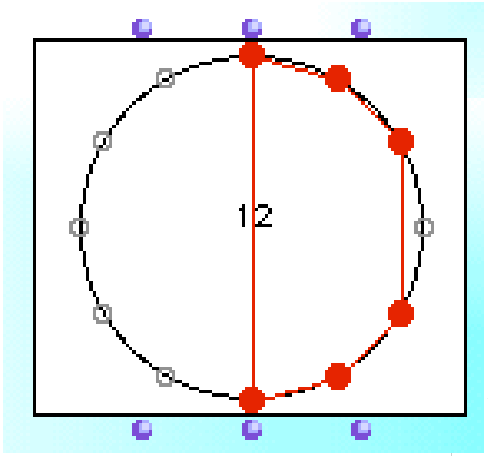
$$IC_A(k) = \mathrm{Card}\{(x, y) \in A \times A \mid x + k = y\}$$

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

$$1_A * \tilde{1}_B(k) = \sum_i 1_A(i) \times 1_B(i-k) = \sum_{\substack{i \in A \\ i-k \in B}} 1$$

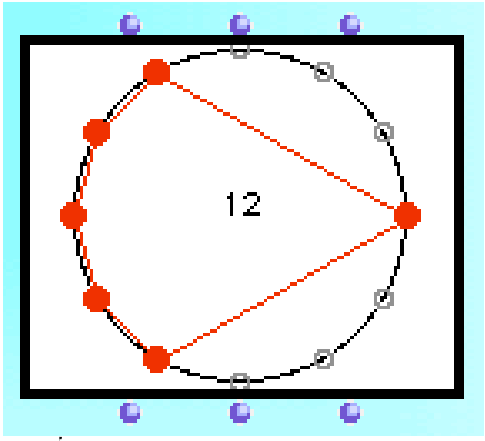
Teorema di Babbitt e trasformata di Fourier discreta

Emmanuel Amiot, *Quadrature*, 2006



A

Un esacordo e il suo complementare hanno lo stesso contenuto intervallare



A'

$$\text{fourier}(\text{set}, \text{t}) := \sum_{k \in set} e^{2i\pi kt/12}$$

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

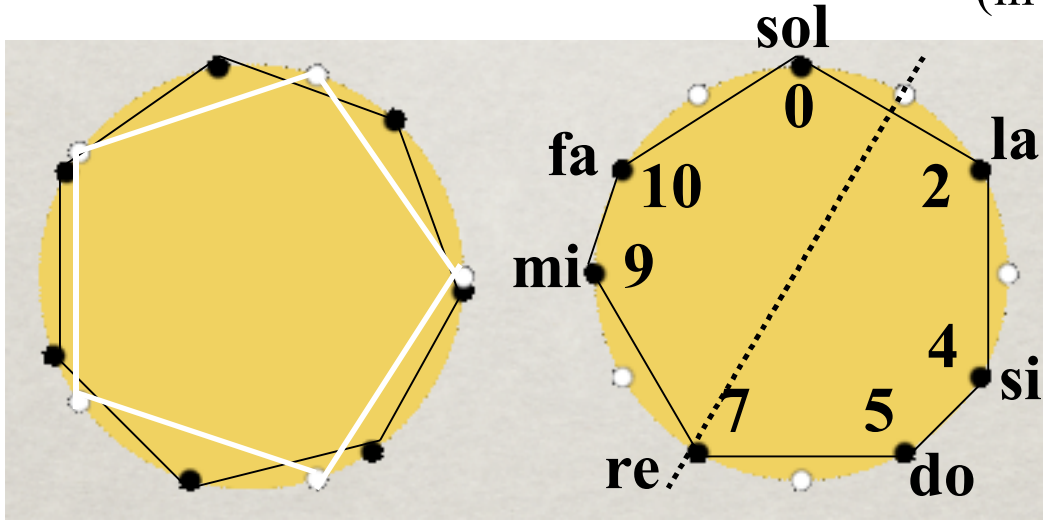
$$\mathcal{F}(1_A * \tilde{1}_B) = \mathcal{F}(1_A) \times \mathcal{F}(\tilde{1}_B)$$

$$\forall k \; \mathcal{F}(\mathrm{IC}_{\mathbb{Z}_c \setminus A})(k) = \mathcal{F}(\mathrm{IC}_A)(k)$$

$$\Rightarrow \quad IC_{A'} = IC_A$$

Transformata di Fourier e *Maximally-Even Sets*

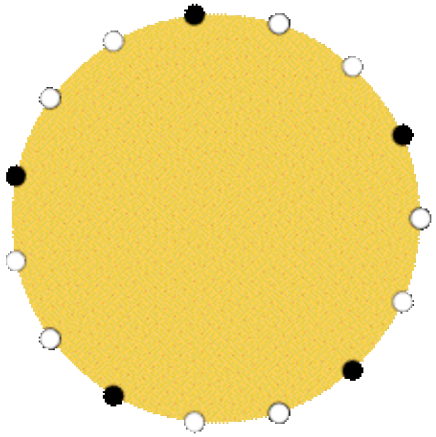
(in collaborazione con Emmanuel Amiot)



Scala diatonica:
{0, 2, 4, 5, 7, 9, 10}

Scala pentatonica:
{1, 3, 6, 8, 11}

$$\text{fourier}(\text{set}, \text{t}) := \sum_{k \in set} e^{2i\pi kt/12}$$

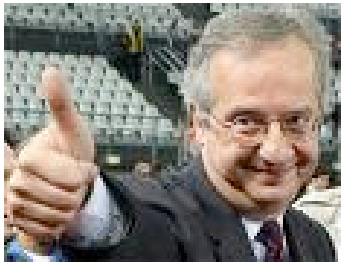


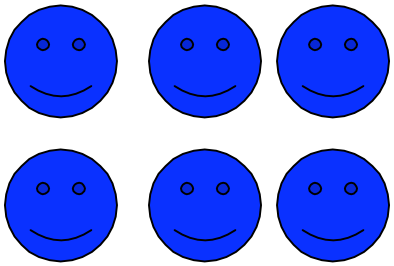
$|F_A(5)| = 1+1+1+1+1 = 5$

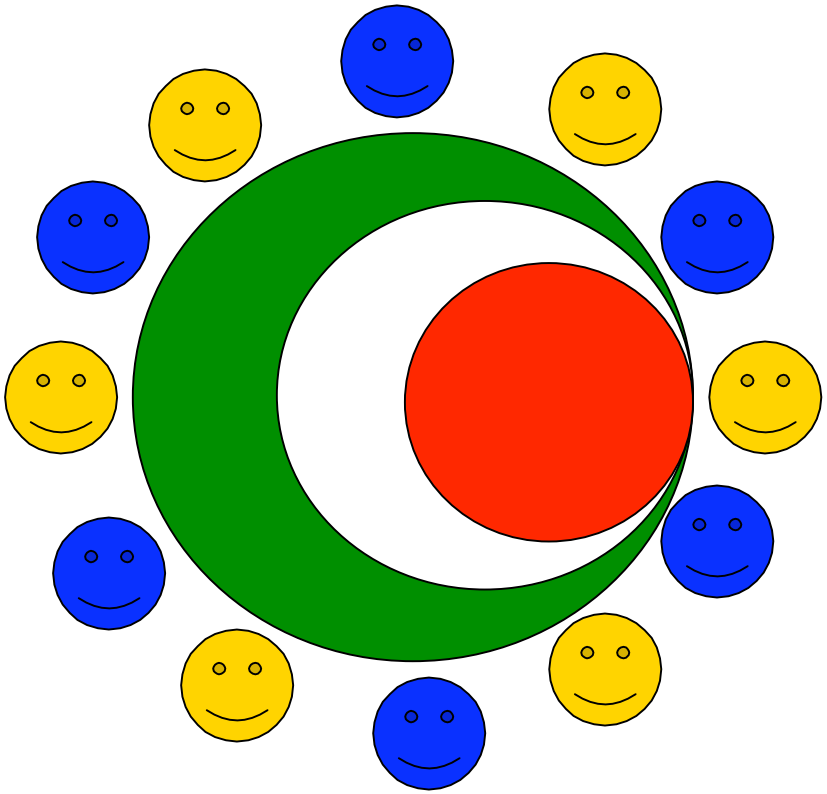
In generale, $|F_A(t)| \leq \#A$

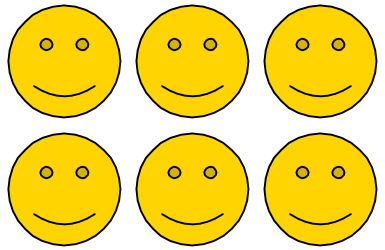
La scala diatonica come *ME-set*

The dinner table problem (Italian version)



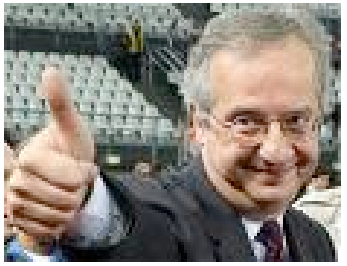


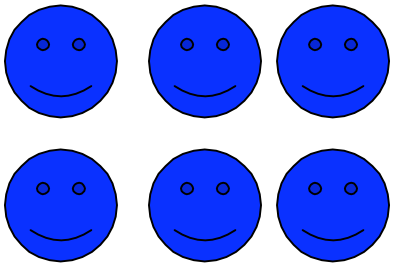


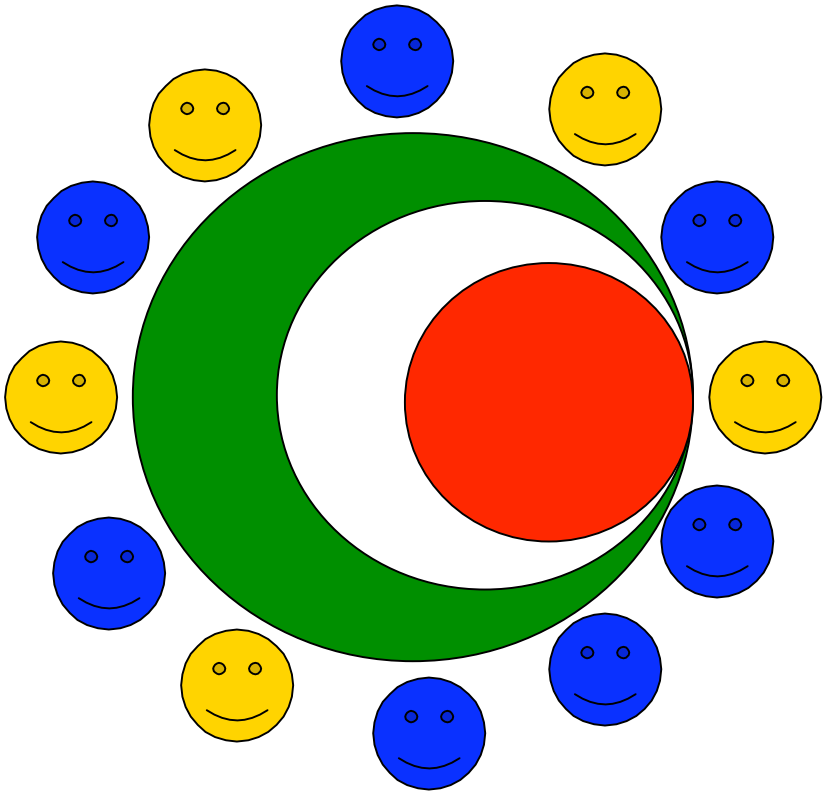


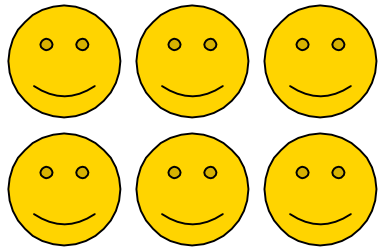
La scala diatonica come *ME-set*

The dinner table problem (Italian version)



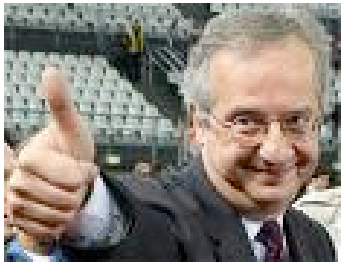


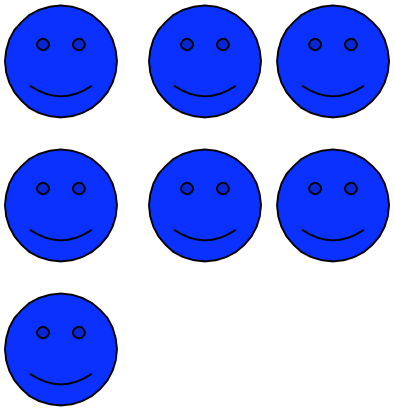


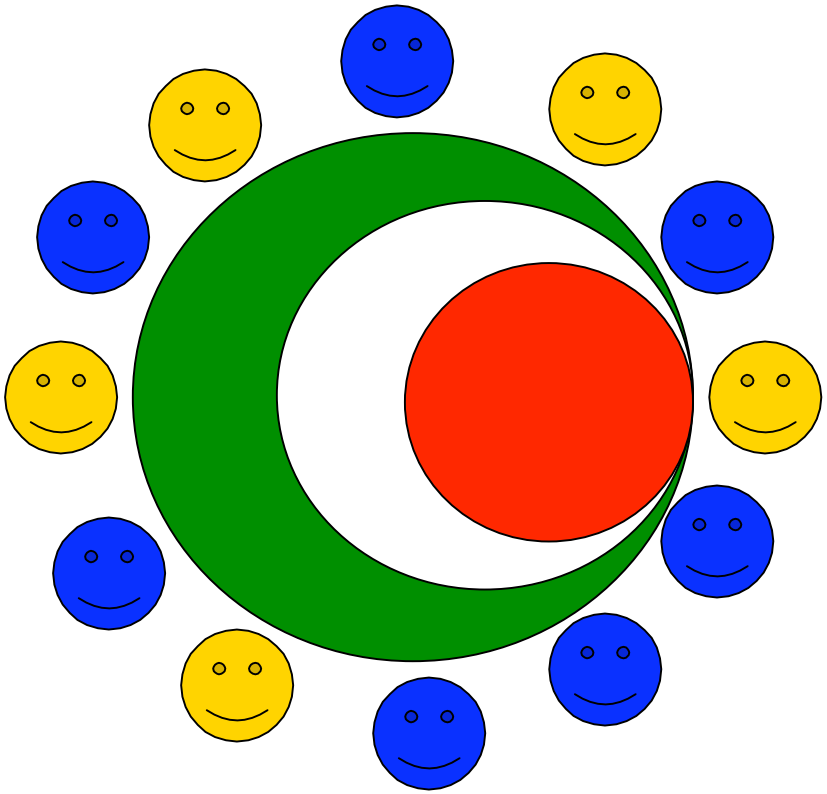


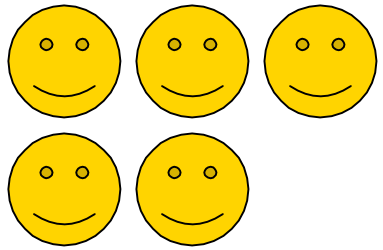
La scala diatonica come *ME-set*

The dinner table problem (Italian version)



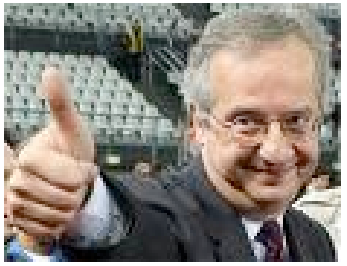


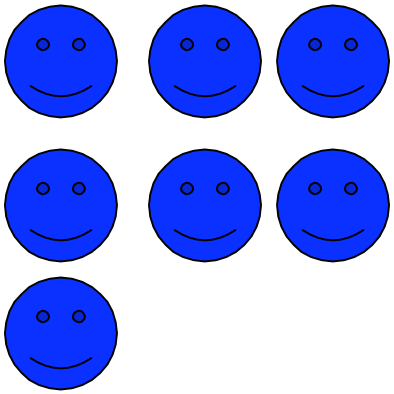


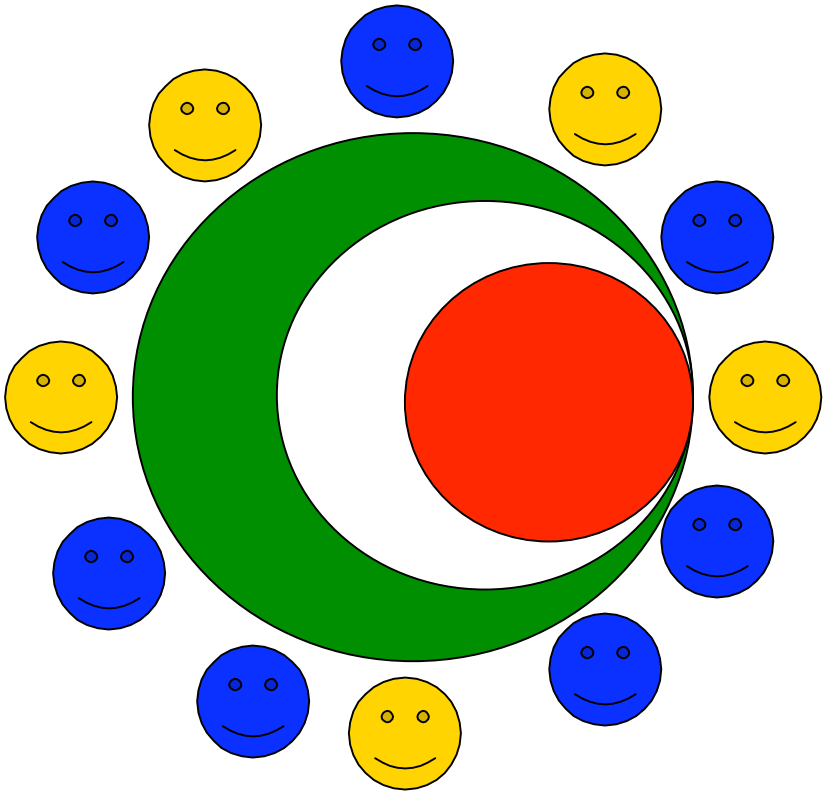


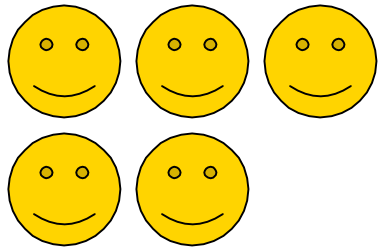
La scala diatonica come *ME-set*

The dinner table problem (Italian version)



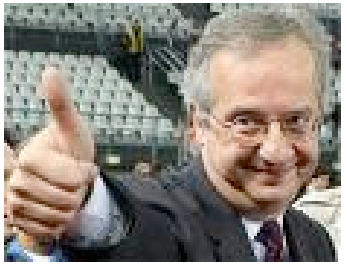


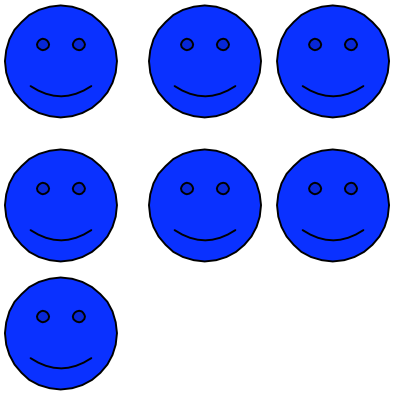


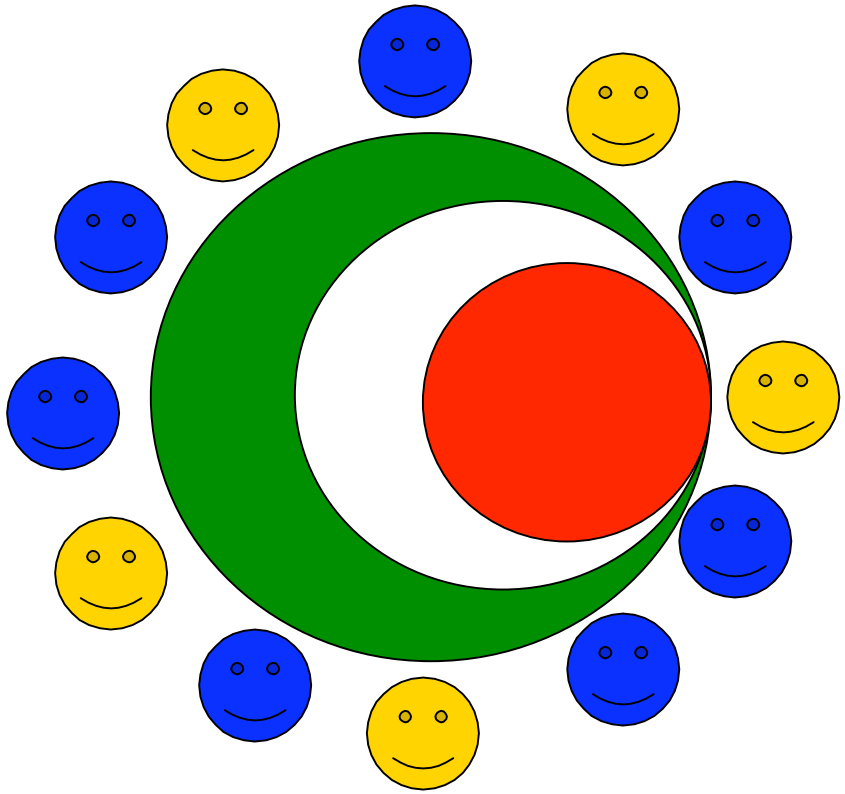


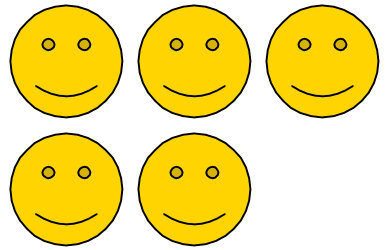
La scala diatonica come *ME-set*

The dinner table problem (Italian version)



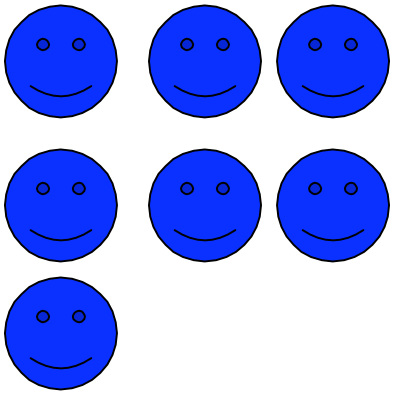


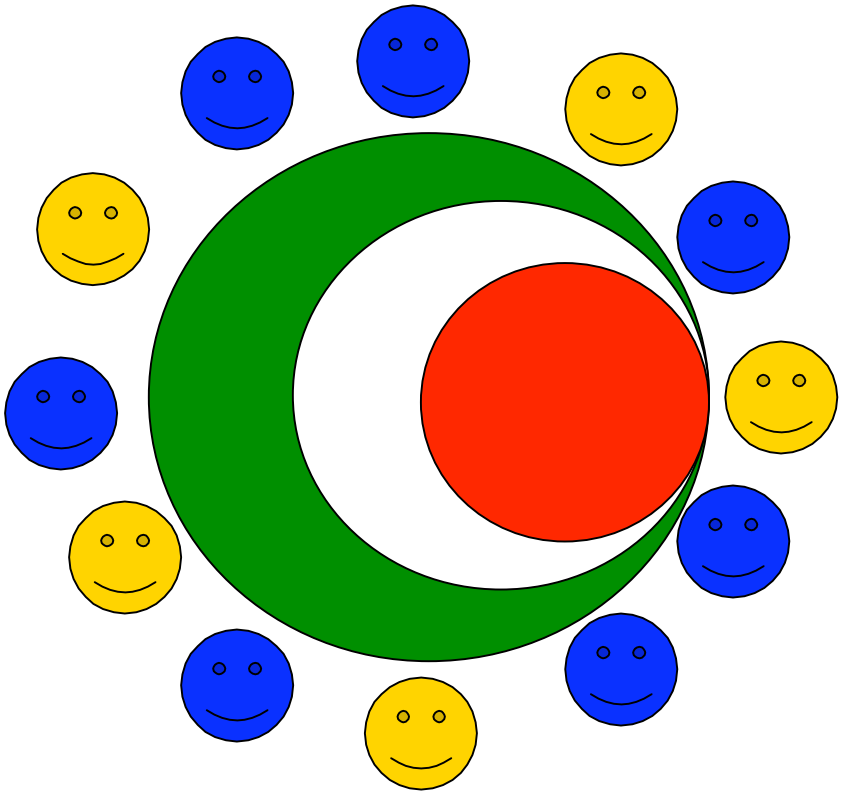


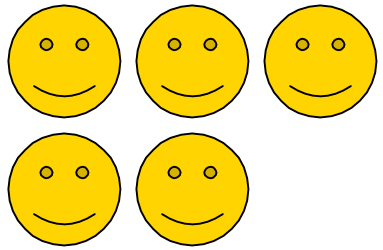


La scala diatonica come *ME-set*

The dinner table problem (Italian version)

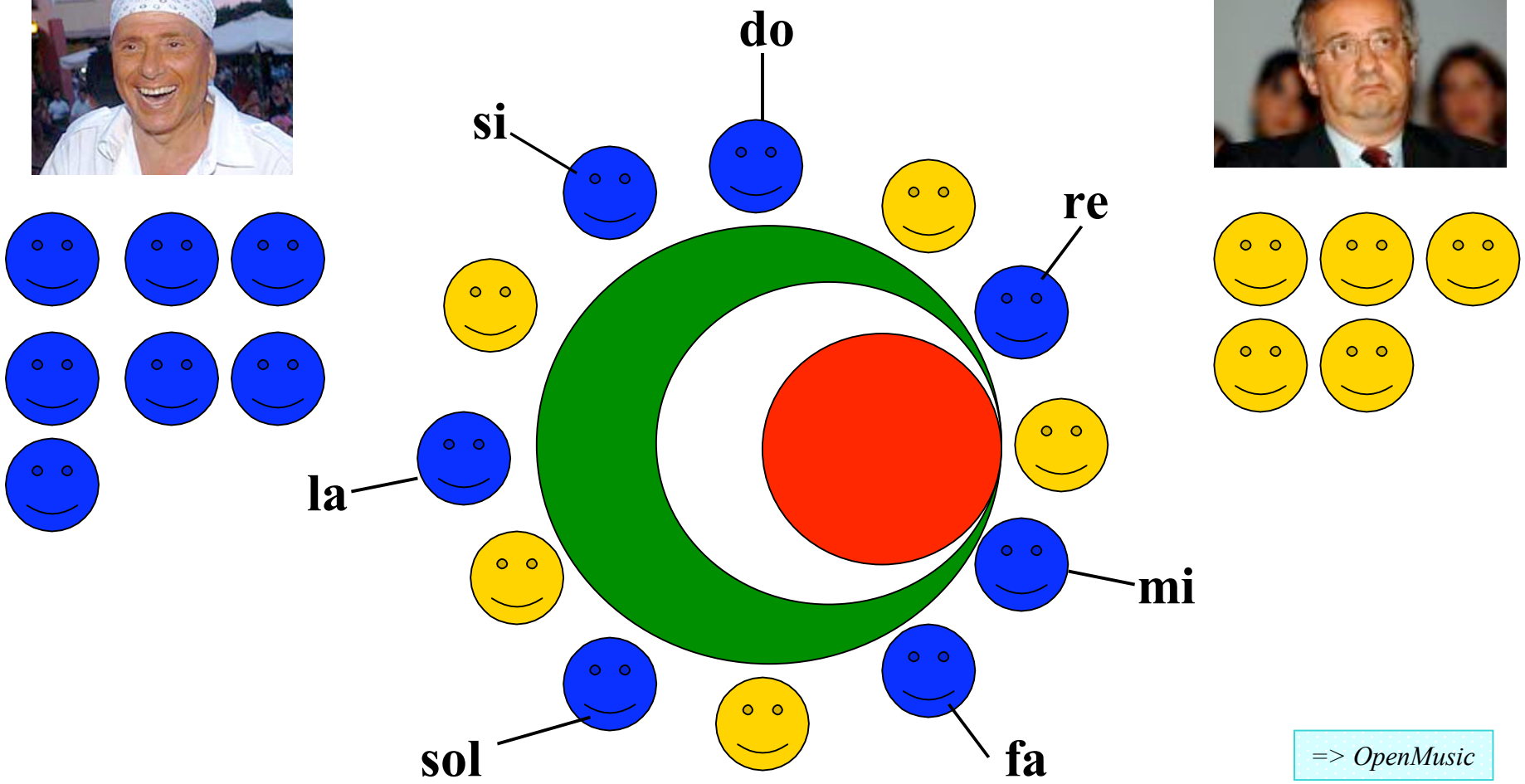






La scala diatonica come *ME-set*

The dinner table problem (Italian version)

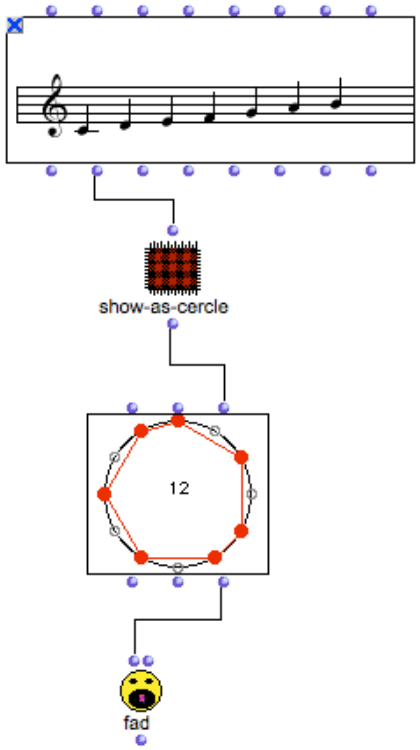


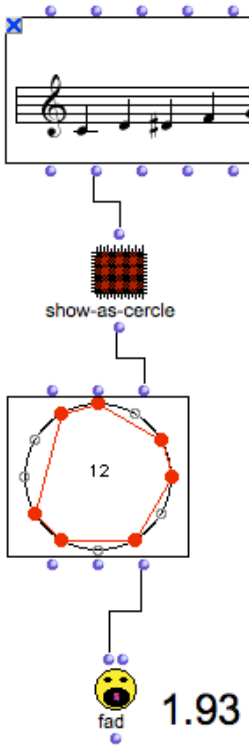
=> *OpenMusic*

Jack Douthett &Richard Krantz, “Energy extremes and spin configurations for the one-dimensional antiferromagnetic Ising model with arbitrary-range interaction”, *J. Math. Phys.* 37 (7), July 1996

^Fourier-1

Trasformata di Fourier discreta e scale musicali





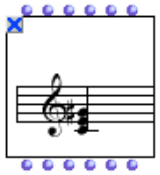
1.93

3.73 <--

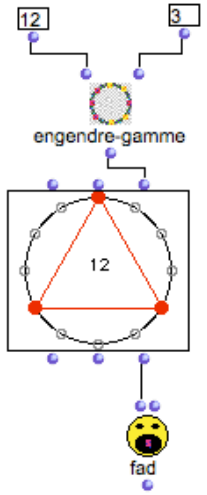
Valore del modulo della trasformata di Fourier

La miglior ripartizione delle altezze

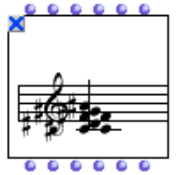
-->



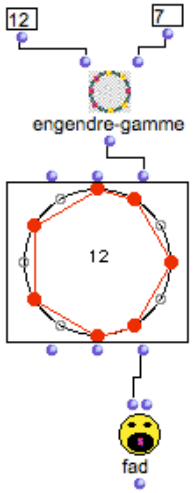
1°) Caso: la cardinalità dell'accordo è un divisore di 12



3.0



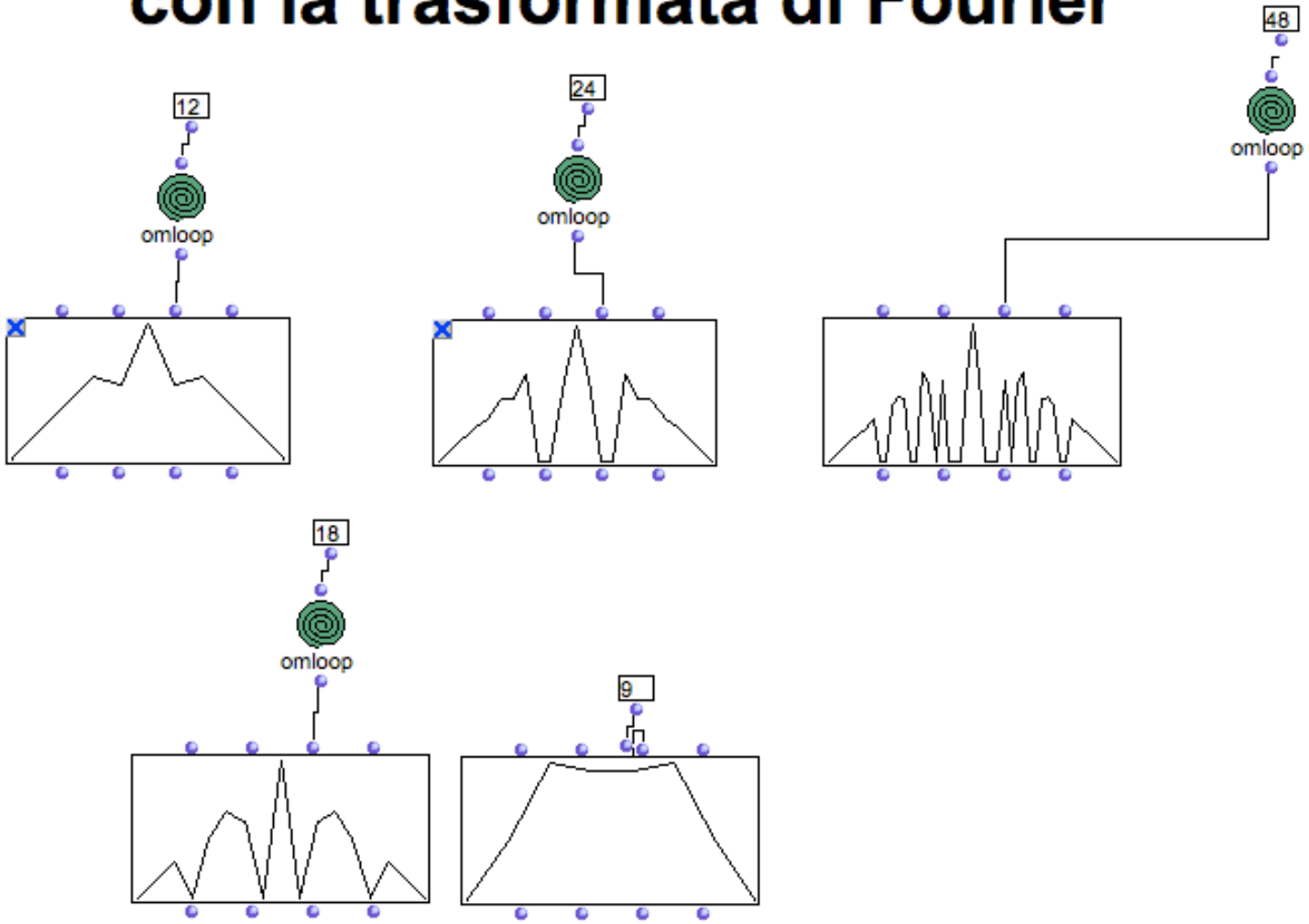
2°) Caso: la cardinalità dell'accordo non è un divisore di 12



3.73

^Fourier-2

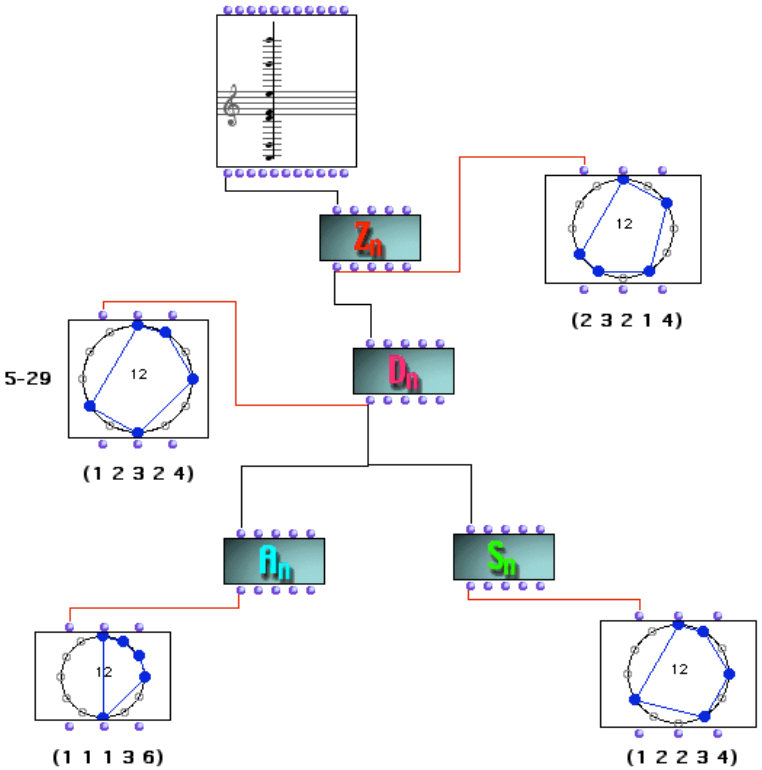
Classificazione delle strutture musicali con la trasformata di Fourier

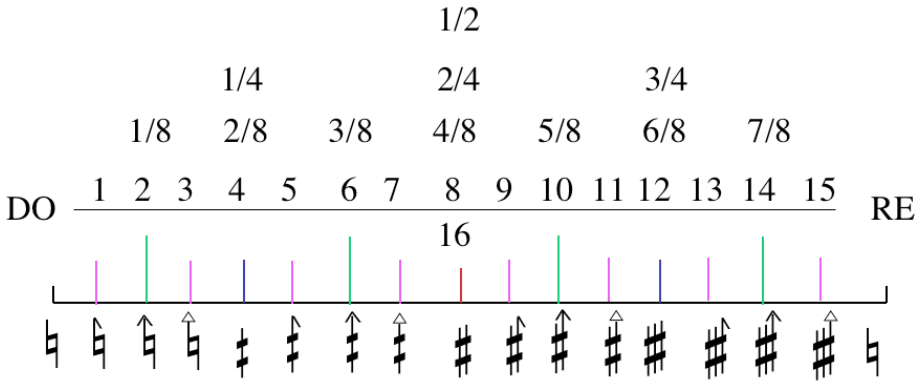


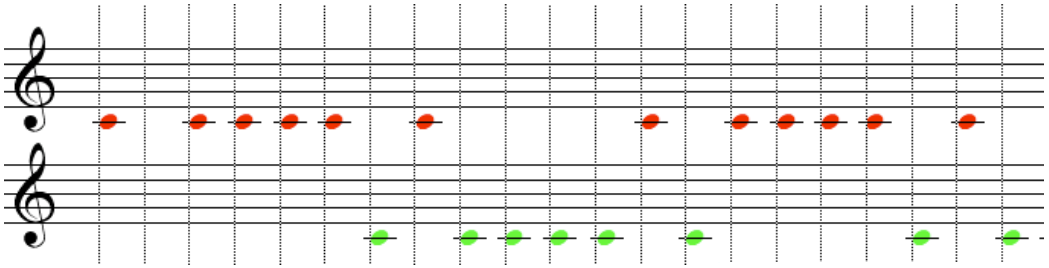

(c) Ircam - Centre Pompidou
Dedicated to the memory of Gérard Grisey (French composer, 1946-1998)
Design and developpement : G. Assayag, A. Agon and J. Bresson
with help from C. Rueda, O. Delerue, Ute McMahan (Grame)
Musical expertise by : M. Andreatta, J. Bobot, J. Pineberg, K. Haddad,
C. Malherbe, M. Mett, T. Murail, O. Sandred, M. Stroppa, H. Tutschku.
Artwork : A. Mohaer.

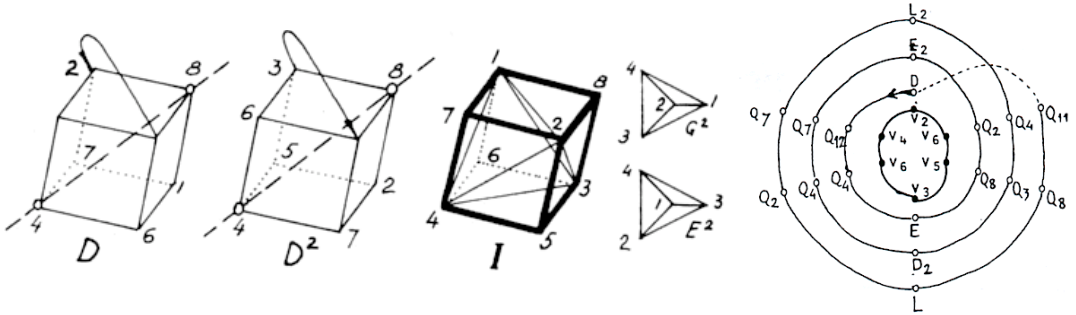
^Fourier-3
Classificazione delle strutture musicali
con la trasformata di Fourier
6
12
card
orbites
Gli 80 esacordi
(modulo la
trasposizione)
12
group-gammes
12
fad
LISP
mapcar
LISP
first
LISP
second
LISP
third
LISP
fourth
4 tipi di accordi
12
12
12
12
c2chord
c2chord
c2chord
c2chord
LISP
length

Trasformazioni geometriche e strutture matematiche



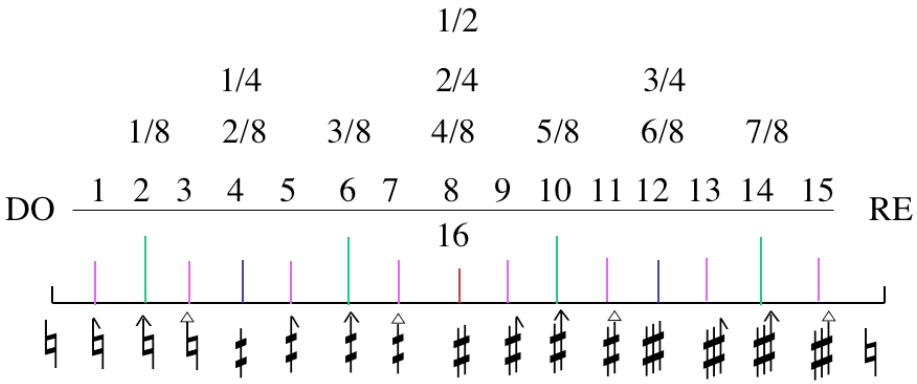




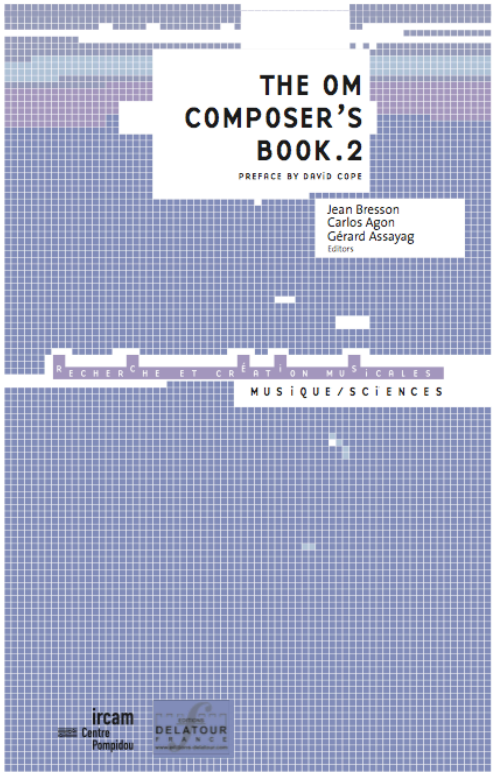


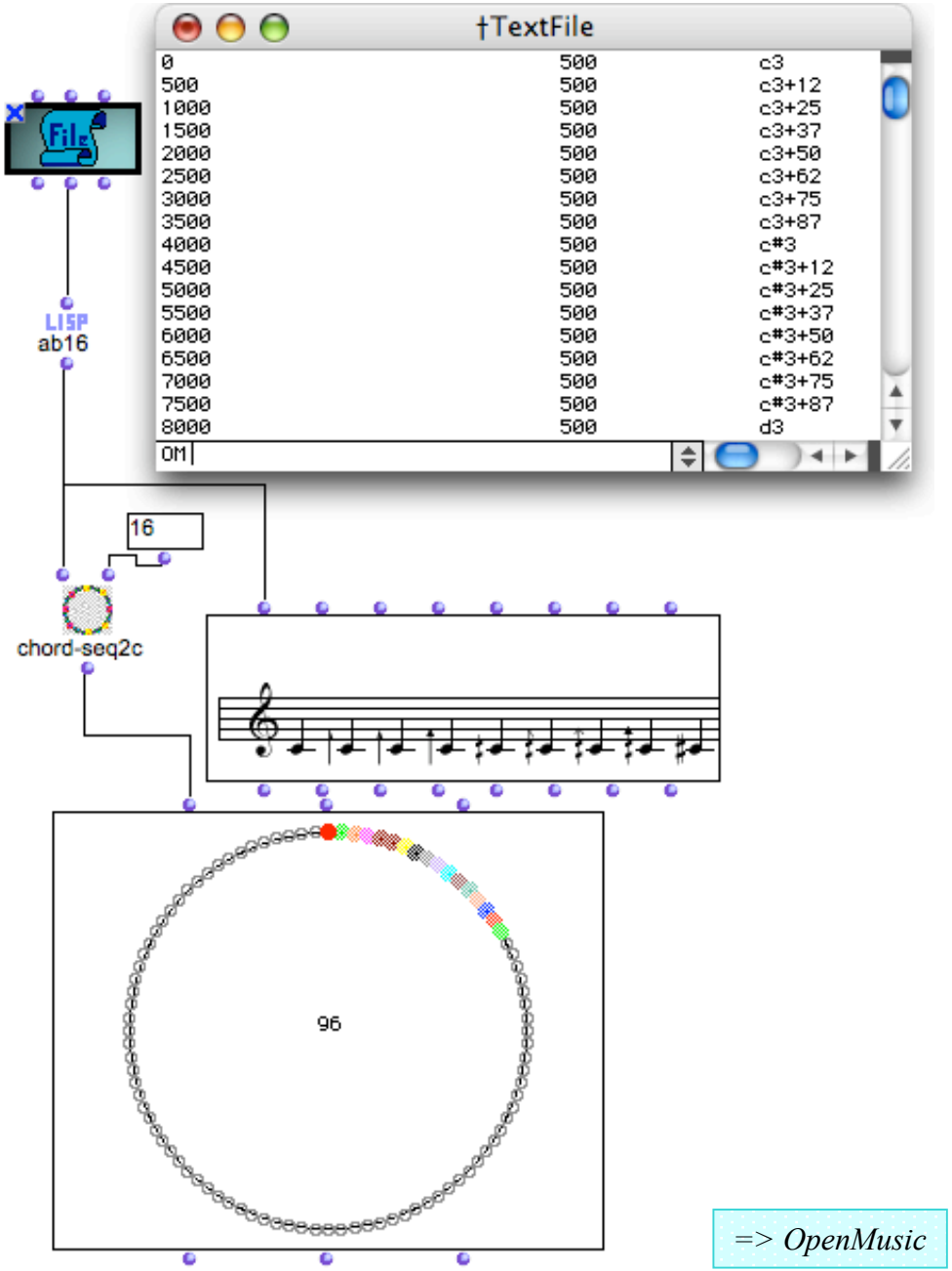
- ***Microtonalità e sistemi non-ottavianti***
- ***Dominio ritmico: canoni ritmici a mosaico***
- ***Analisi musicale computazionale:*** **Nomos Alpha** ***di Iannis Xenakis***

Microtonalità e sistemi non-ottavianti

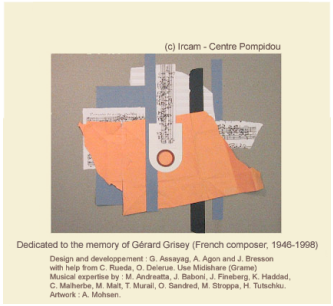


Alain Bancquart





=> *OpenMusic*


(c) Ircam - Centre Pompidou
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Design and developpement : G. Assayag, A. Agon and J. Bresson
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Artwork : A. Mohsen.

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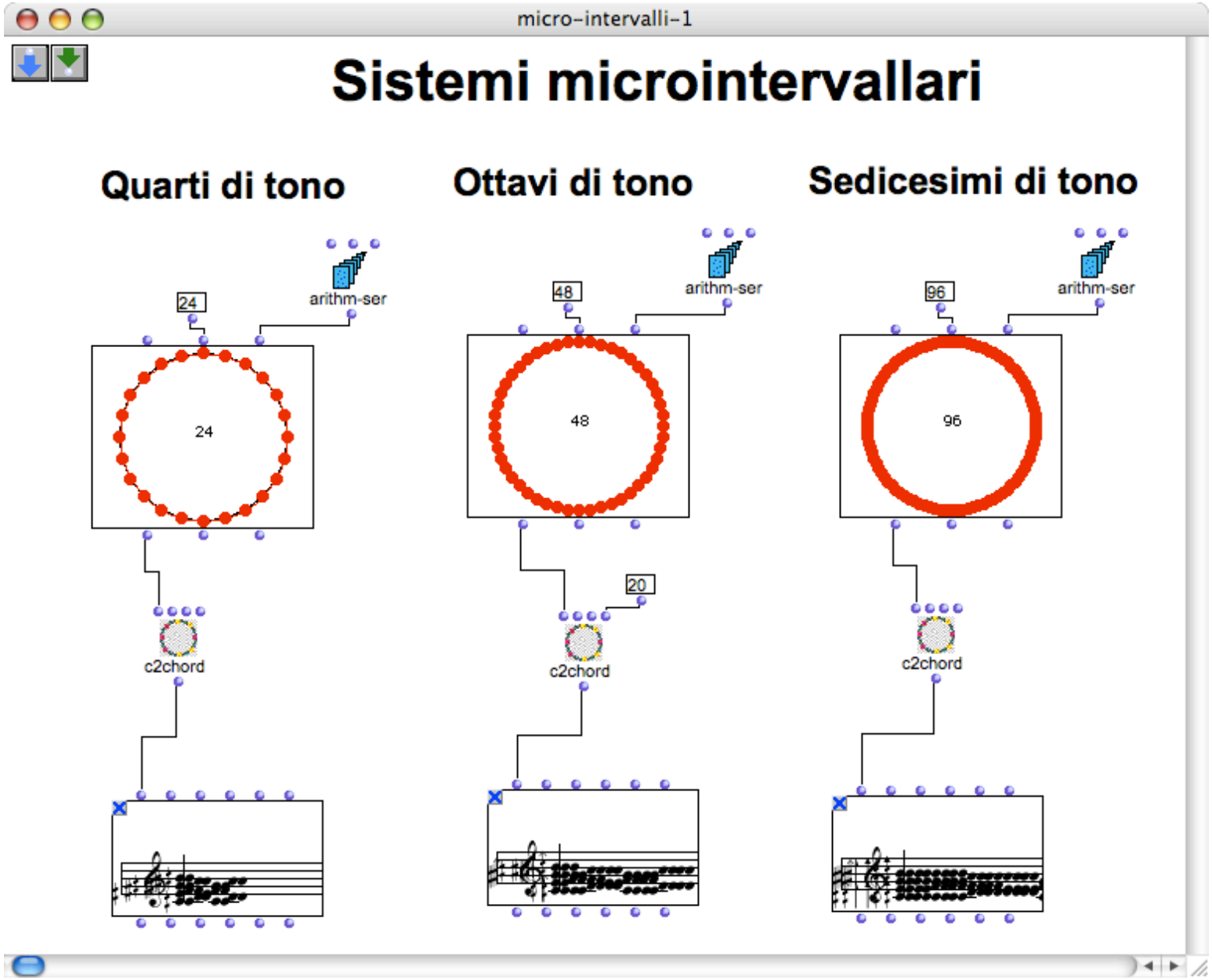
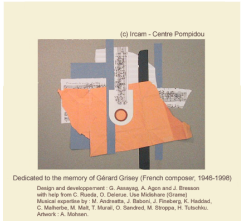
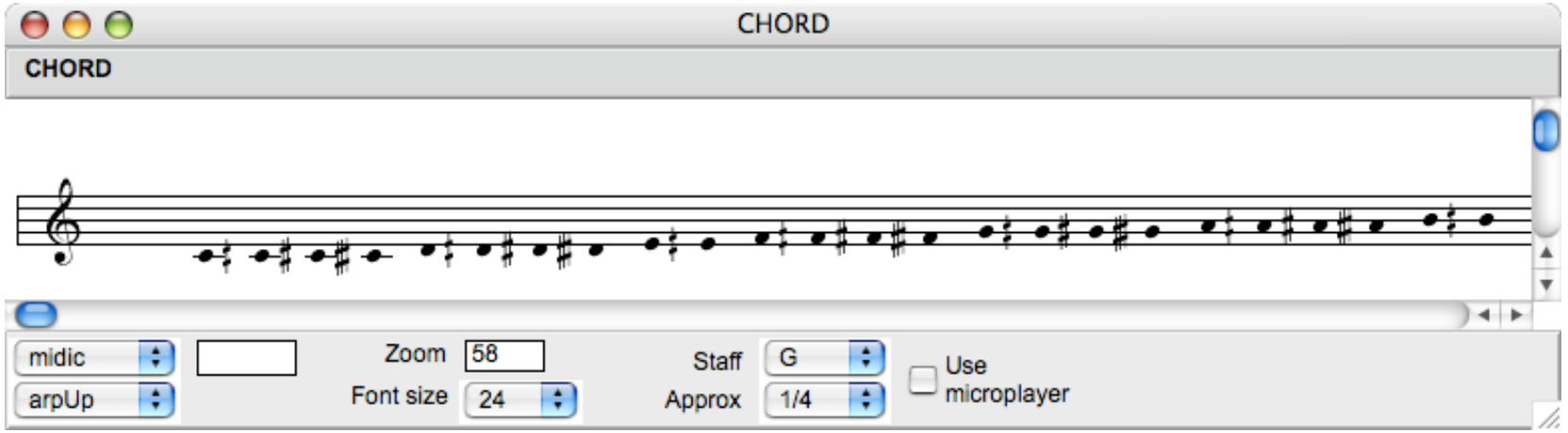
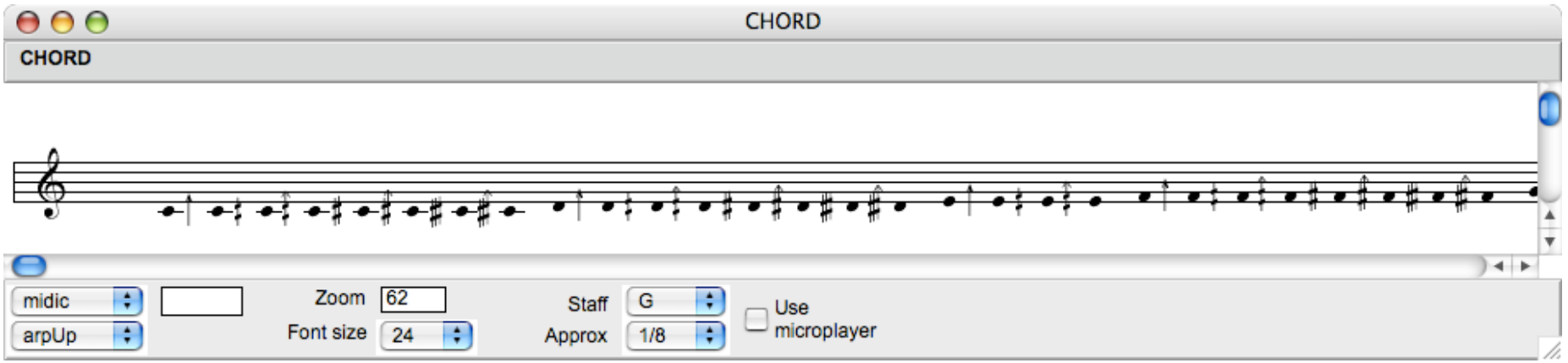
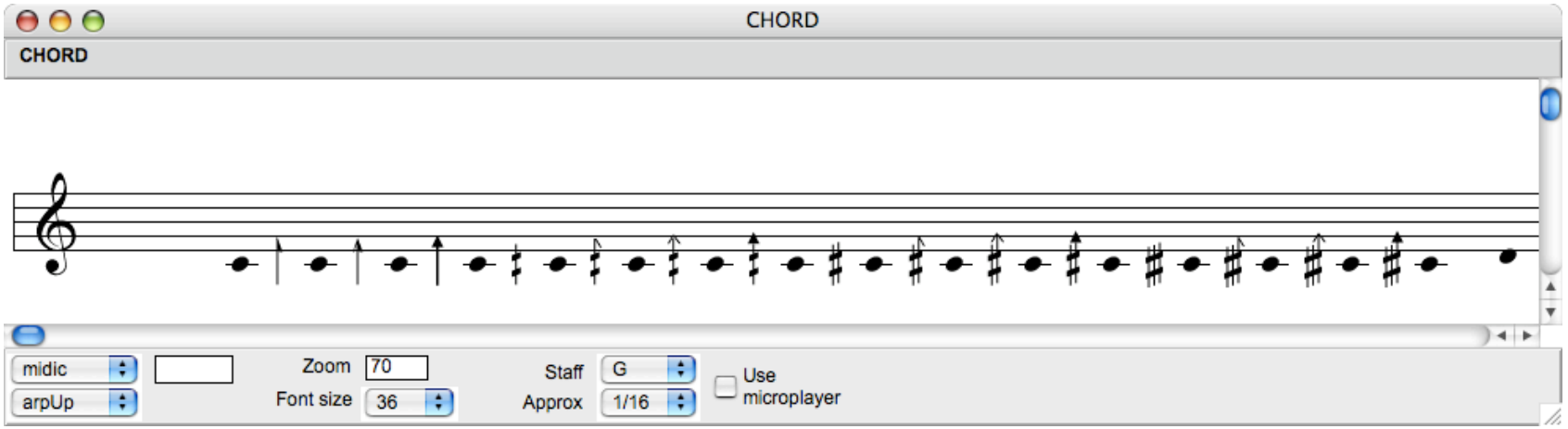

micro-intervalli-1
Sistemi microintervallari
Quarti di tono
Ottavi di tono
Sedicesimi di tono
arithm-ser
24
48
96
20
c2chord


(c) Ircam - Centre Pompidou
Dedicated to the memory of Gérard Grisey (French composer, 1946-1998)

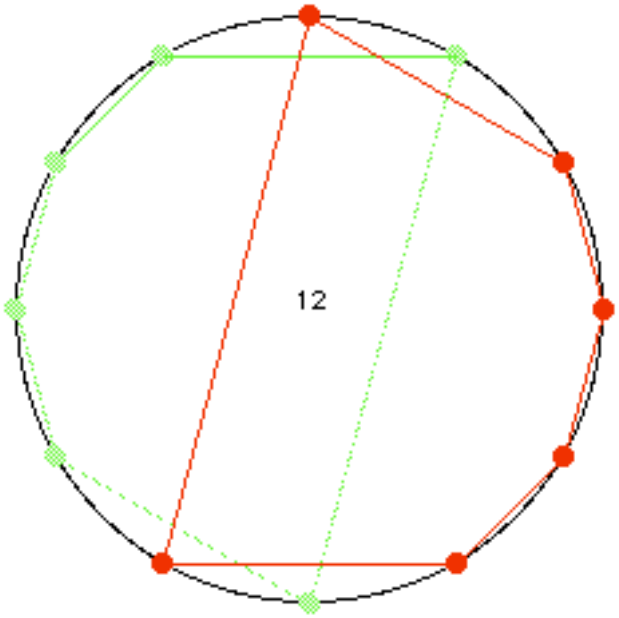
OpenMusic


CHORD
CHORD
midic
arpUp
Zoom 58
Font size 24
Staff G
Approx 1/4
Use microplayer


CHORD
CHORD
midic
arpUp
Zoom 62
Font size 24
Staff G
Approx 1/8
Use microplayer


CHORD
CHORD
midic
arpUp
Zoom 70
Font size 36
Staff G
Approx 1/16
Use microplayer

Interpretazione ritmica: canoni a mosaico *(tiling canons)*



$\mathbf{Z}_{12}$=H1∪H2

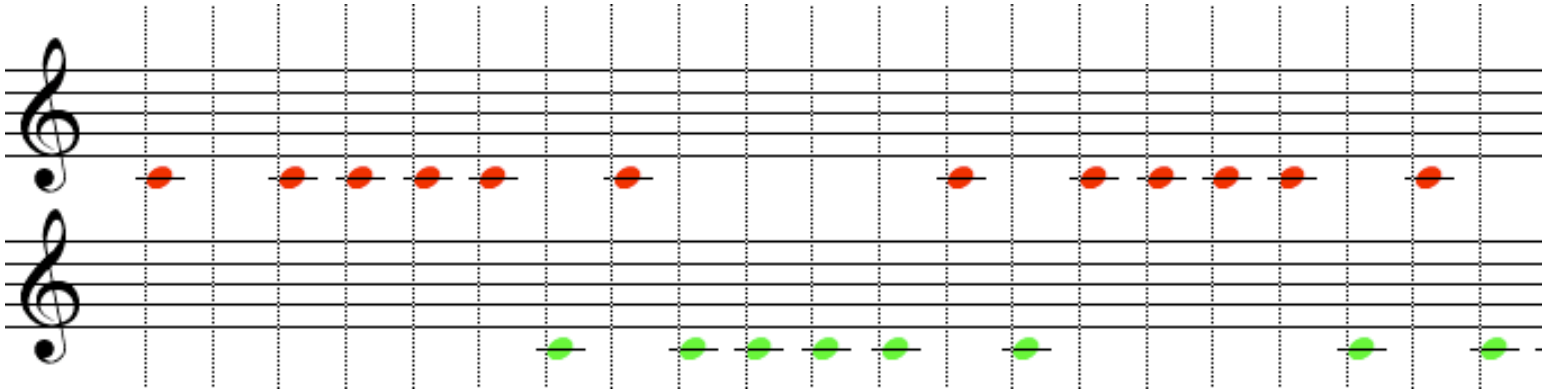
H1= {0,2,3,4,5,7}

H2= {1,6,8,9,10,11}

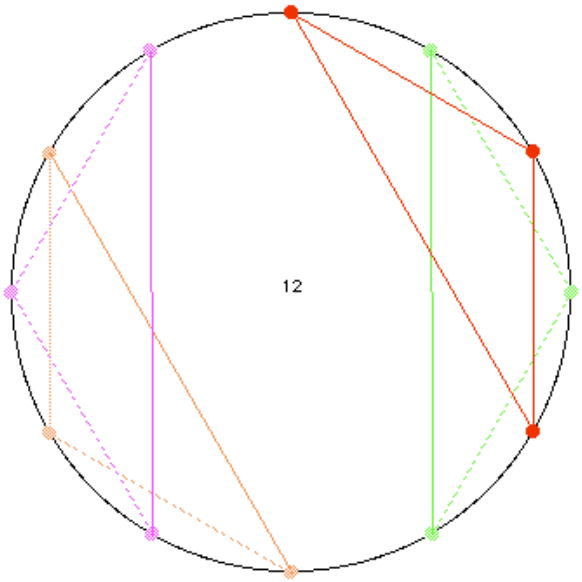
$\mathbf{Z}_{12}$=A⊕B

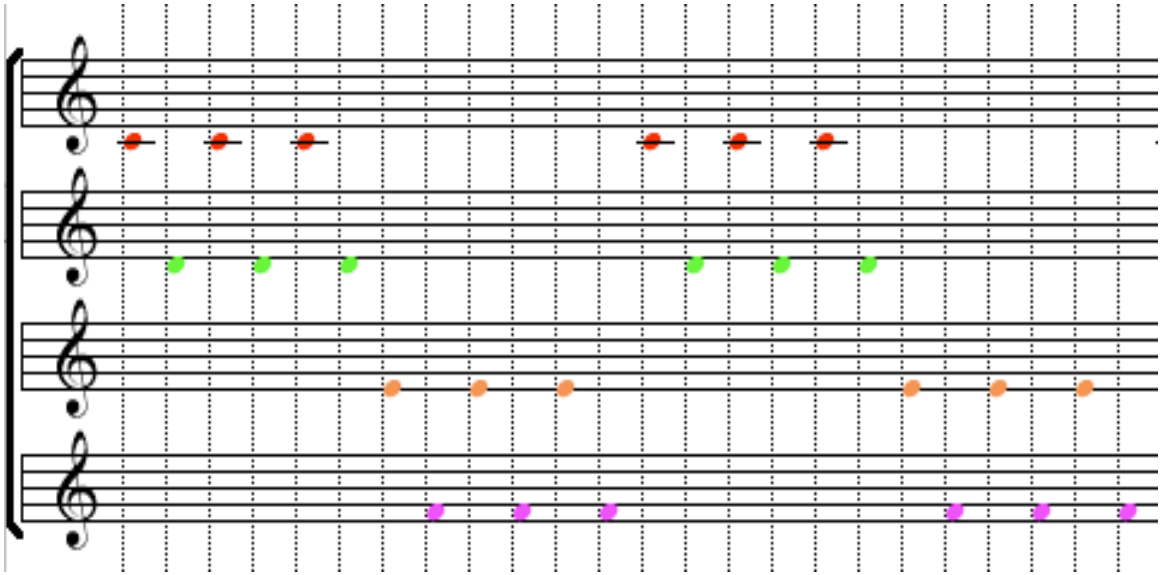
A= {0,2,3,4,5,7}

B= {0,6}



Canoni a mosaico a simmetria trasposizionale

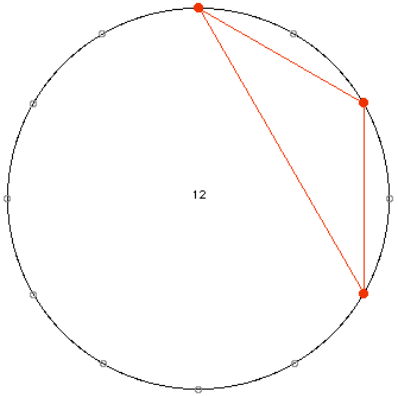


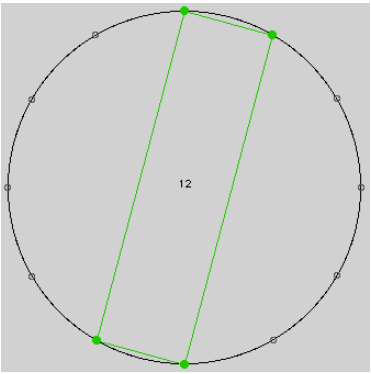


$\mathbf{Z}_{12}$=A⊕B

A= {0,2,4}

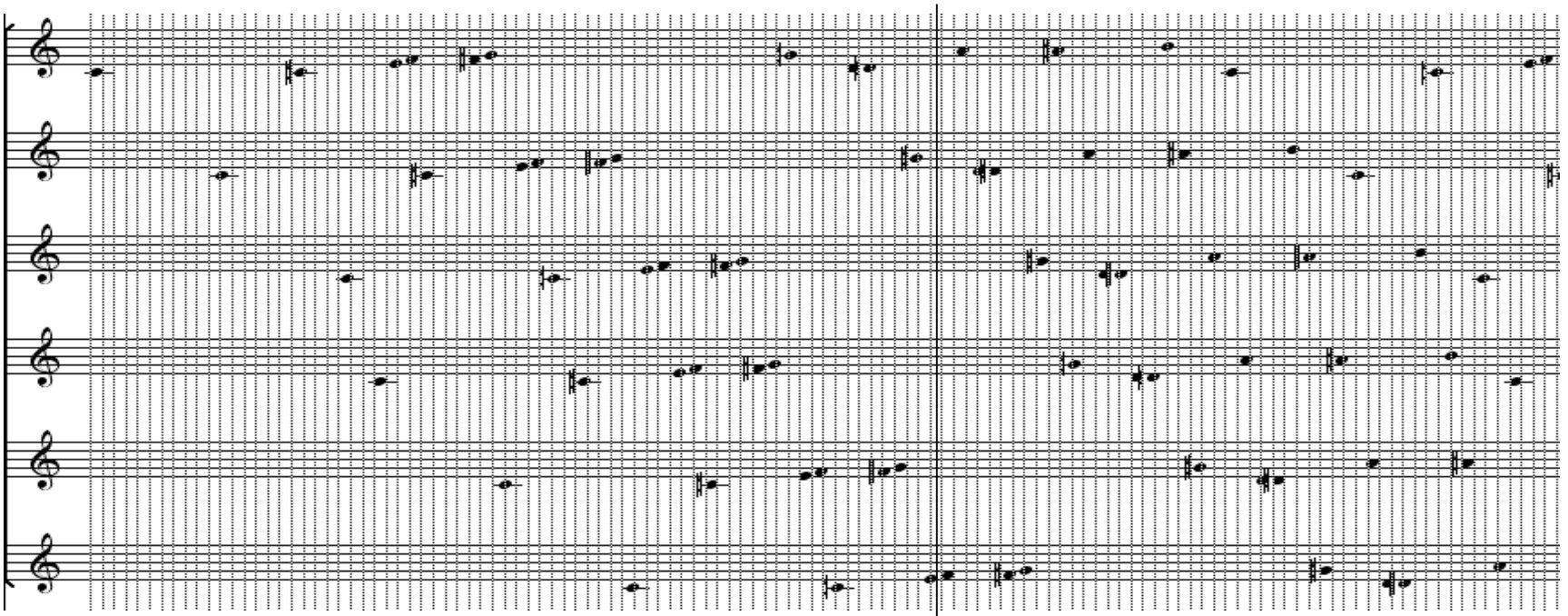
B= {0,1,6,7}

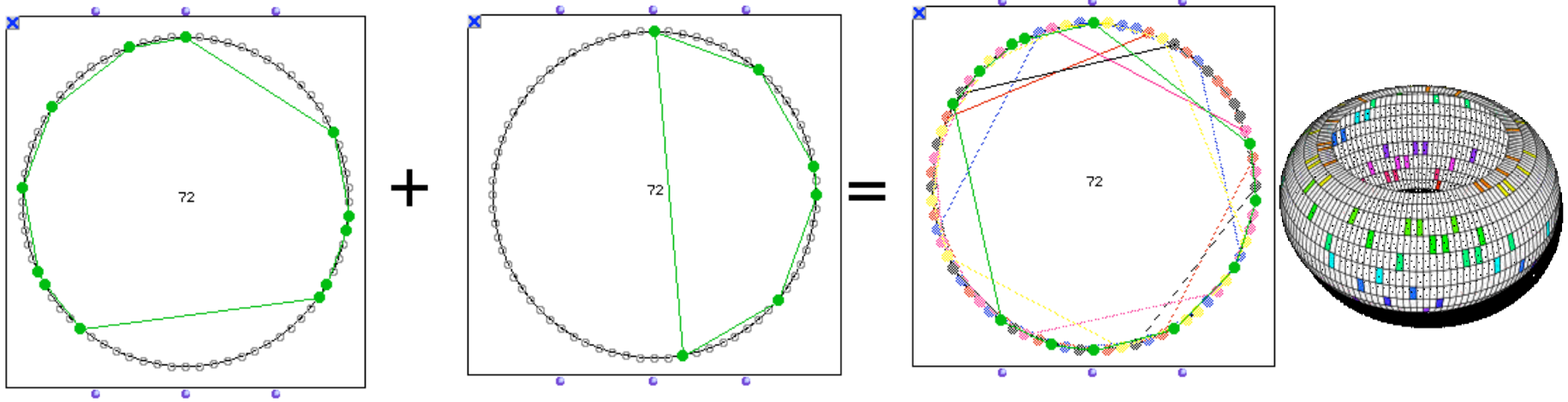




Canoni ritmici di Vuza (canoni senza simmetria trasposizionale)

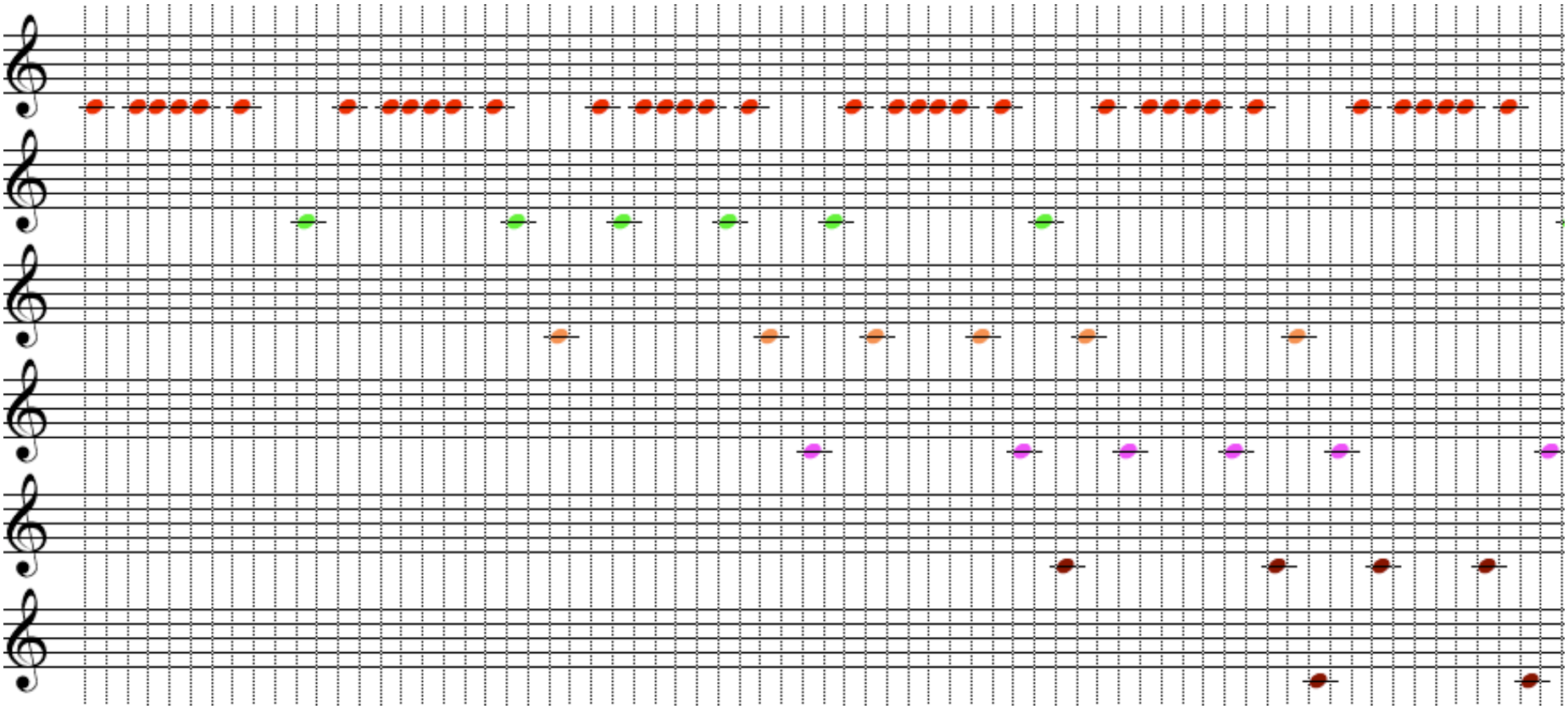
periodo



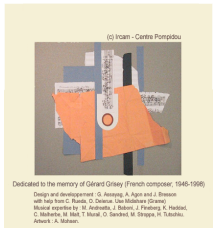


Augmented Tiling Canons o l'azione del gruppo affine

(in collaborazione con Thomas Noll)



=> *OpenMusic*



OpenMusic

canone

Canoni ritmici a mosaico: primi esempi

pattern ritmico di base

24

Entrata delle quattro voci

24

LISP car

c2rhythm

n-structure

n-structure

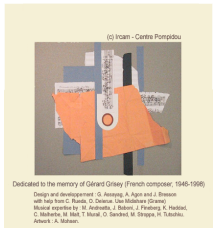
LISP car

canons

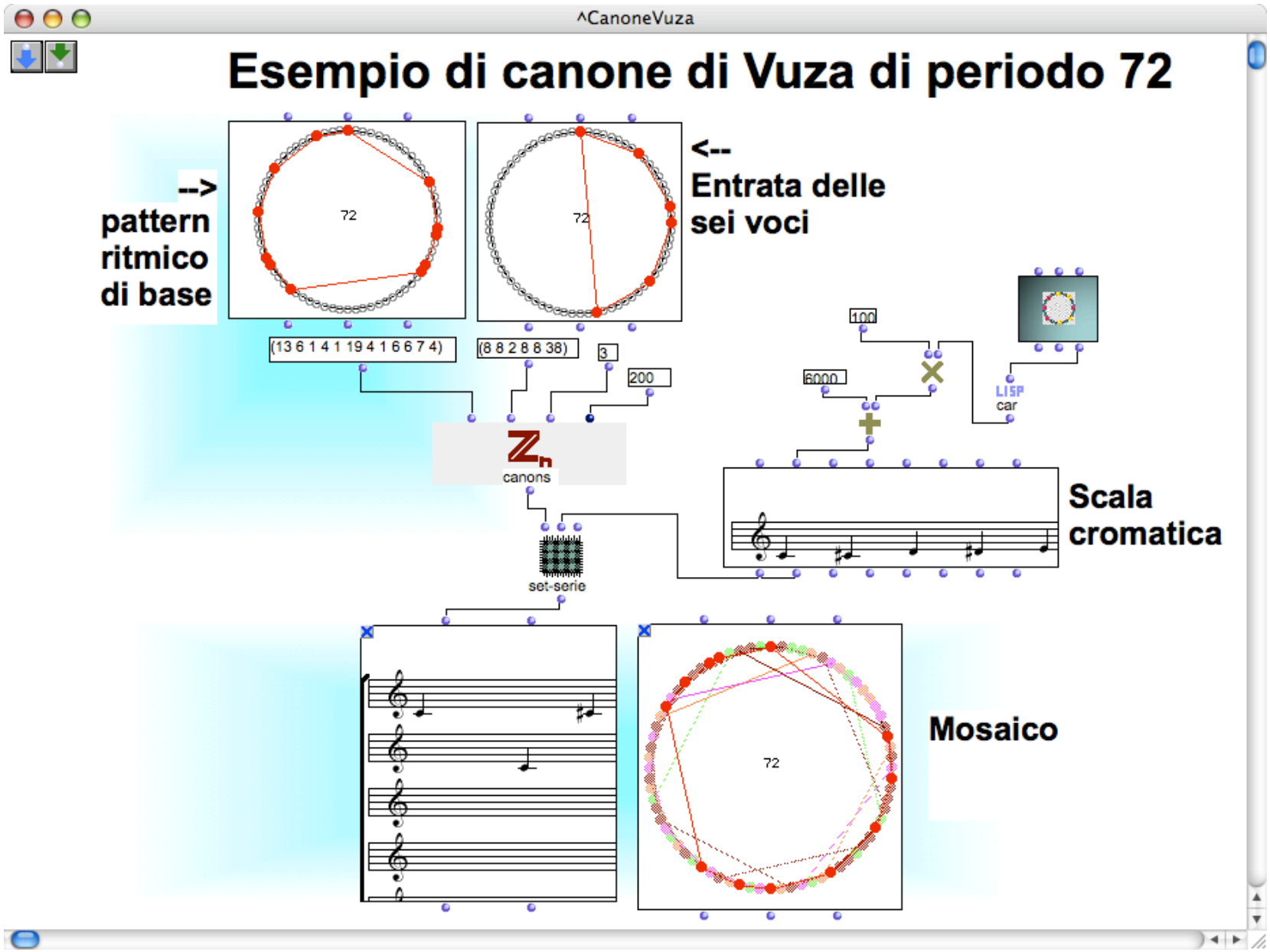
Mosaico

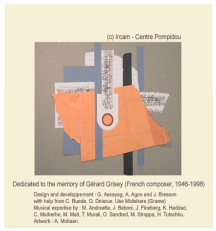
pattern ritmico di base

24



OpenMusic


^CanoneVuza
Esempio di canone di Vuza di periodo 72
--> pattern ritmico di base
72
72
<-- Entrata delle sei voci
(13 6 1 4 1 19 4 1 6 6 7 4)
(8 8 2 8 8 38)
3
200
100
6000
LISP
car
canons
Scala cromatica
set-serie
72
Mosaico



OpenMusic

aug-canon

Augmentazioni ritmiche e mosaici canonici

Pattern ritmico

R

(0 2 3 5)

S

((1 0) (5 10) (5 6))

3

Aff$_n$

augmented-canon

12

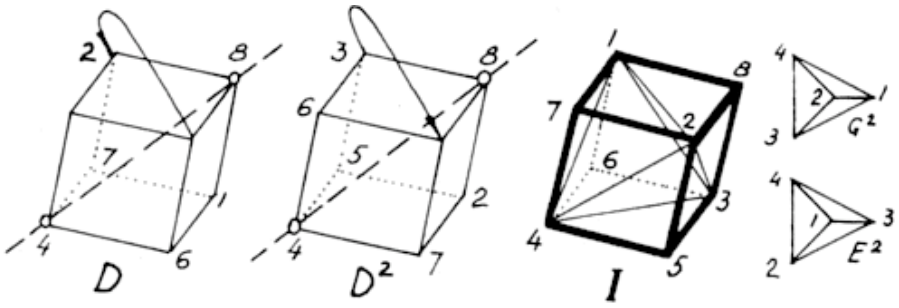
Pattern ritmico e sua trasformazione affine (augmentazione)

Canone a mosaico augmentato

(11 voci)

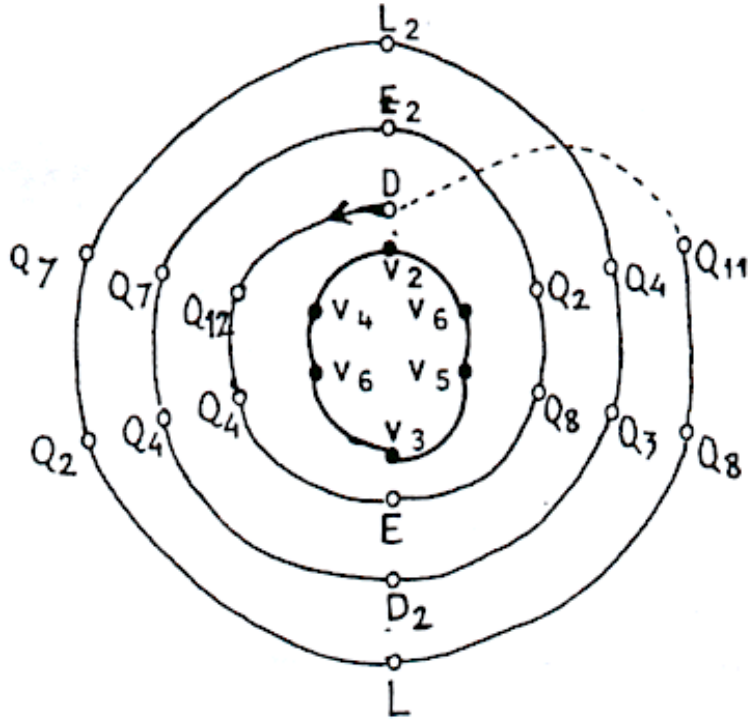
Analisi musicale computazionale: *Nomos Alpha* di I. Xenakis

La questione delle simmetrie (identità spaziali) e delle periodicità (identità nel tempo) ha un ruolo fondamentale nella musica, a tutti i livelli, da quello dei campioni sonori della sintesi del suono mediante computer, fino all'architettura di un intero brano musicale

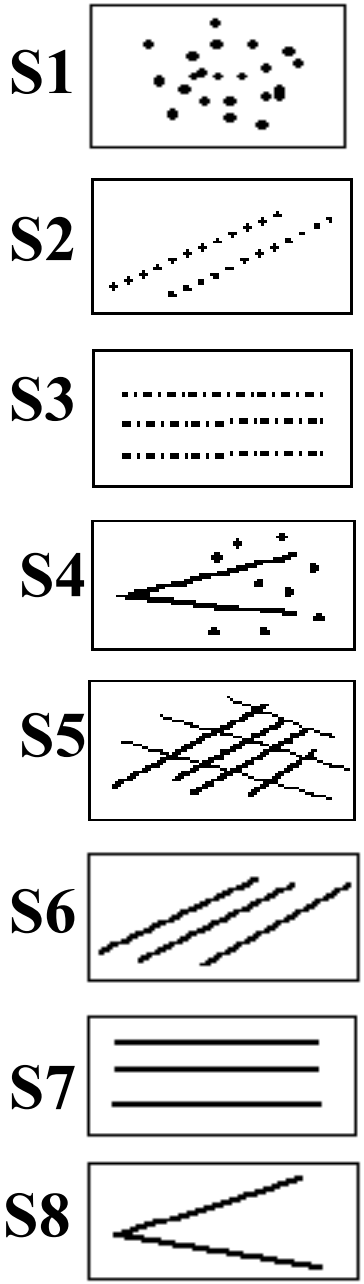


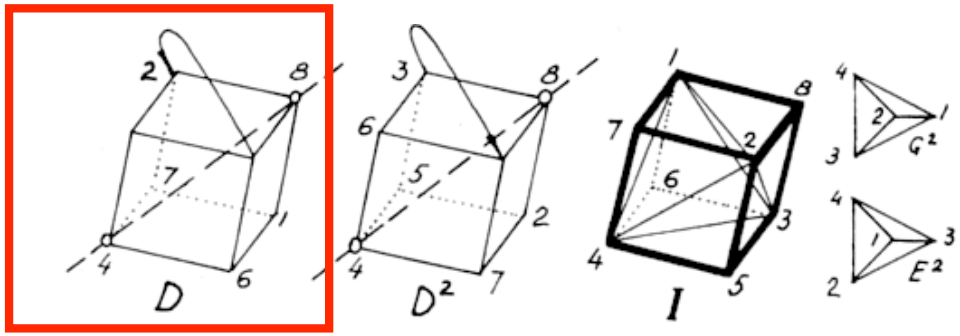
***Nomos Alpha* (1966)**

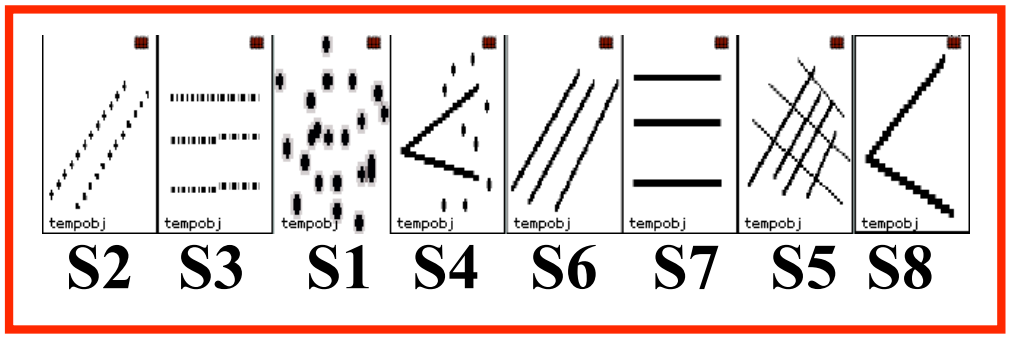
Musique symbolique pour violoncelle seul, possède une architecture "hors-temps" fondée sur la théorie des groupes de transformations.



Analisi musicale computazionale: *Nomos Alpha* di I. Xenakis







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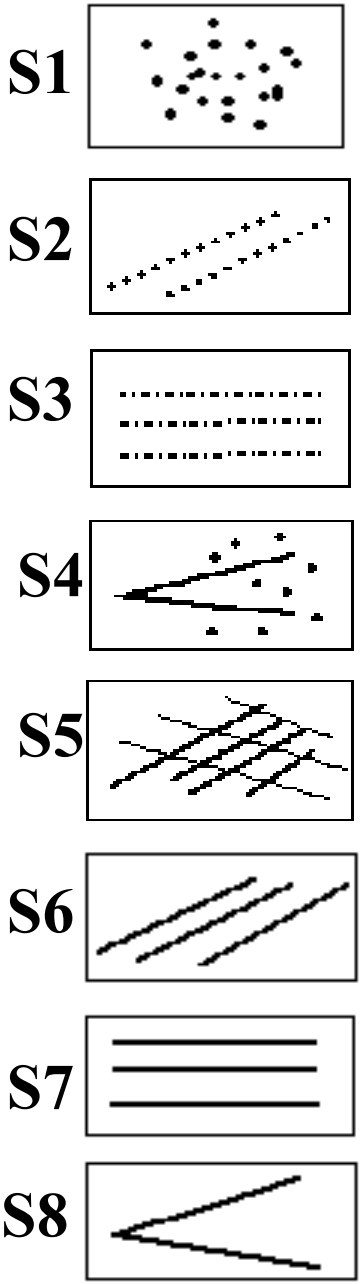
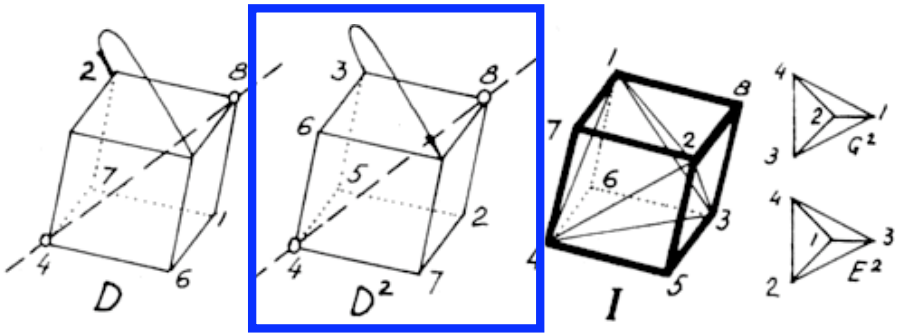
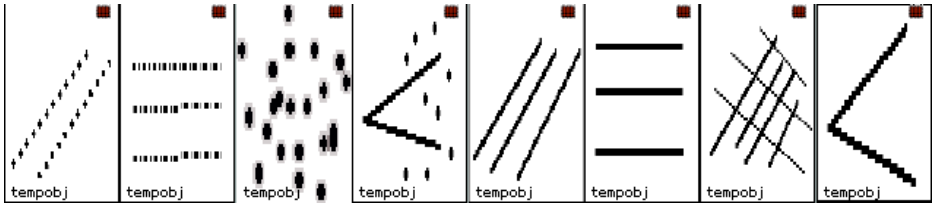
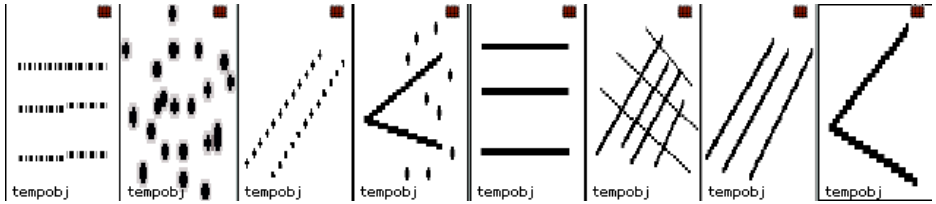
A	21436587
B	34127856
C	43218765
D	23146758
D^2	31247568
E	24316875
E^2	41328576
G	32417685
G^2	42138657
L	13425786
L^2	14235867
Q_1	78653421
Q_2	76583214
Q_3	86754231
Q_4	67852341
Q_5	68572413
Q_6	65782134
Q_7	87564312
Q_8	75863142
Q_9	58761432
Q_{10}	57681324
Q_{11}	85674123
Q_{12}	56871243

Analisi musicale computazionale: *Nomos Alpha* di I. Xenakis









S3 S1 S2 S4 S7 S5 S6 S8

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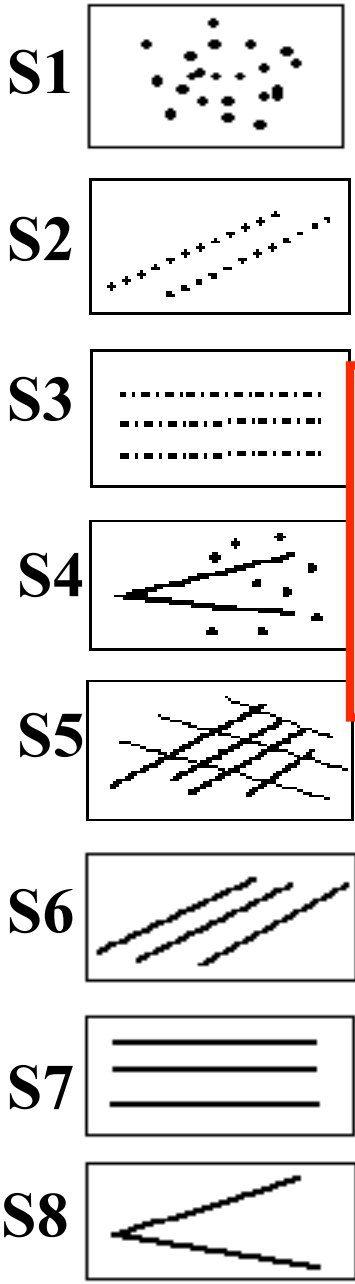
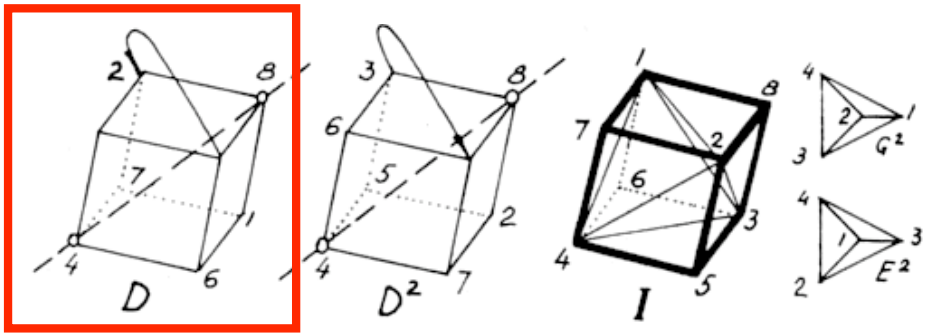
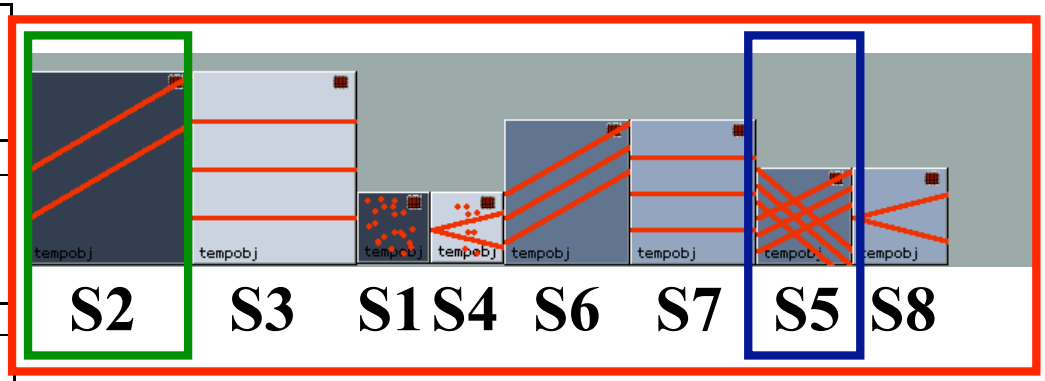
Analisi musicale computazionale: *Nomos Alpha* di I. Xenakis







$\kappa^{\alpha}_{1} = 1 \cdot mf \cdot 2 \rightarrow = 2.mf \rightarrow$

$\kappa^{\alpha}_{2} = 1 \cdot fff \cdot 4.5 = 4.5.fff \rightarrow$

$\kappa^{\alpha}_{3} = 2.5 \cdot fff \cdot 4.5 = 11.25.fff \rightarrow$

$\kappa^{\alpha}_{4} = 2.5 \cdot mf \cdot 2 = 5.mf \rightarrow \rightarrow$

$\kappa^{\alpha}_{5} = 1.5 \cdot f \cdot 2.62 = 3.93.f \rightarrow$

$\kappa^{\alpha}_{6} = 1.5 \cdot ff \cdot 3.44 = 5.15.ff \rightarrow$

$\kappa^{\alpha}_{7} = 2.0 \cdot ff \cdot 3.44 = 6.88.ff \rightarrow$

$\kappa^{\alpha}_{8} = 2.0 \cdot f \cdot 2.62 = 5.24.f \rightarrow$

¶

$\kappa^{\beta}_{1} = 0.5 \cdot mf \cdot 2 = 1.mf \rightarrow \rightarrow$

$\kappa^{\beta}_{2} = 0.5 \cdot fff \cdot 4.5 = 2.25.fff$

$\kappa^{\beta}_{3} = 5 \cdot fff \cdot 4.5 = 22.5.fff \rightarrow$

$\kappa^{\beta}_{4} = 5.0 \cdot mf \cdot 2 = 10.0.mf \rightarrow$

$\kappa^{\beta}_{5} = 1.08 \cdot f \cdot 2.62 = 2.83.f \rightarrow$

$\kappa^{\beta}_{6} = 1.08 \cdot ff \cdot 3.44 = 3.72.ff \rightarrow$

$\kappa^{\beta}_{7} = 2.32 \cdot ff \cdot 3.44 = 7.98.ff \rightarrow$

$\kappa^{\beta}_{8} = 2.32 \cdot f \cdot 2.62 = 6.08.f \rightarrow$

¶

$\kappa^{\gamma}_{1} = 1 \cdot mf \cdot 2 = 2.mf$ ¶

$\kappa^{\gamma}_{2} = 1 \cdot fff \cdot 2 \rightarrow = 2.fff$ ¶

$\kappa^{\gamma}_{3} = 4.0 \cdot fff \cdot 4.5 = 18.0.fff$ ¶

$\kappa^{\gamma}_{4} = 4.0 \cdot mf \cdot 2.0 = 8.0.mf$ ¶

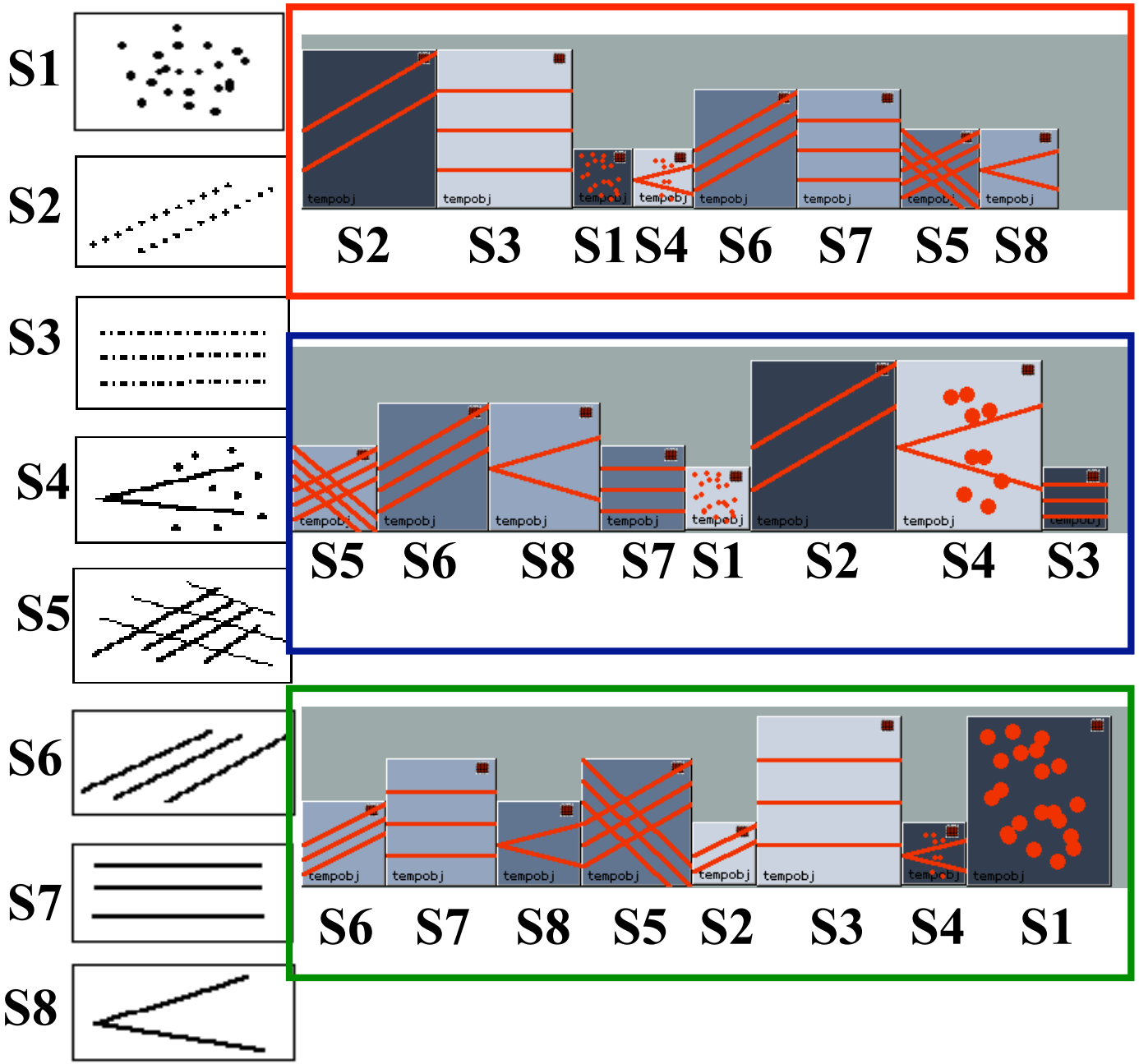
$\kappa^{\gamma}_{5} = 2.0 \cdot f \cdot 2.62 = 5.24.f$ ¶

$\kappa^{\gamma}_{6} = 2.0 \cdot ff \cdot 3.44 = 6.88.ff$ ¶

$\kappa^{\gamma}_{7} = 3.0 \cdot ff \cdot 3.44 = 10.32.ff$ ¶

$\kappa^{\gamma}_{8} = 3.0 \cdot f \cdot 2.62 = 7.86.f$ ¶

Analisi musicale computazionale: *Nomos Alpha* di I. Xenakis



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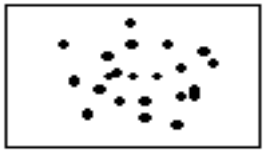
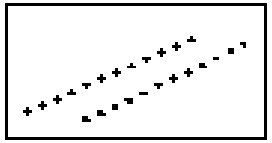
Q_{10}	57681324
Q_{11}	85674123
Q_{12}	56871243

Analisi musicale computazionale: *Nomos Alpha* di I. Xenakis

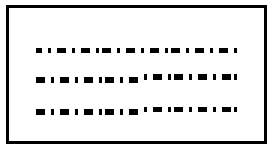
S1



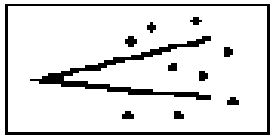
S2



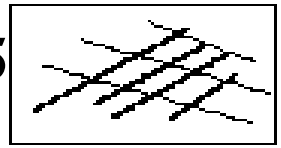
S3



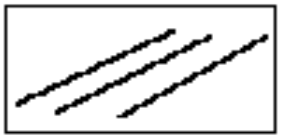
S4



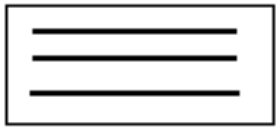
S5



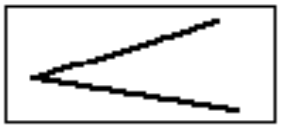
S6

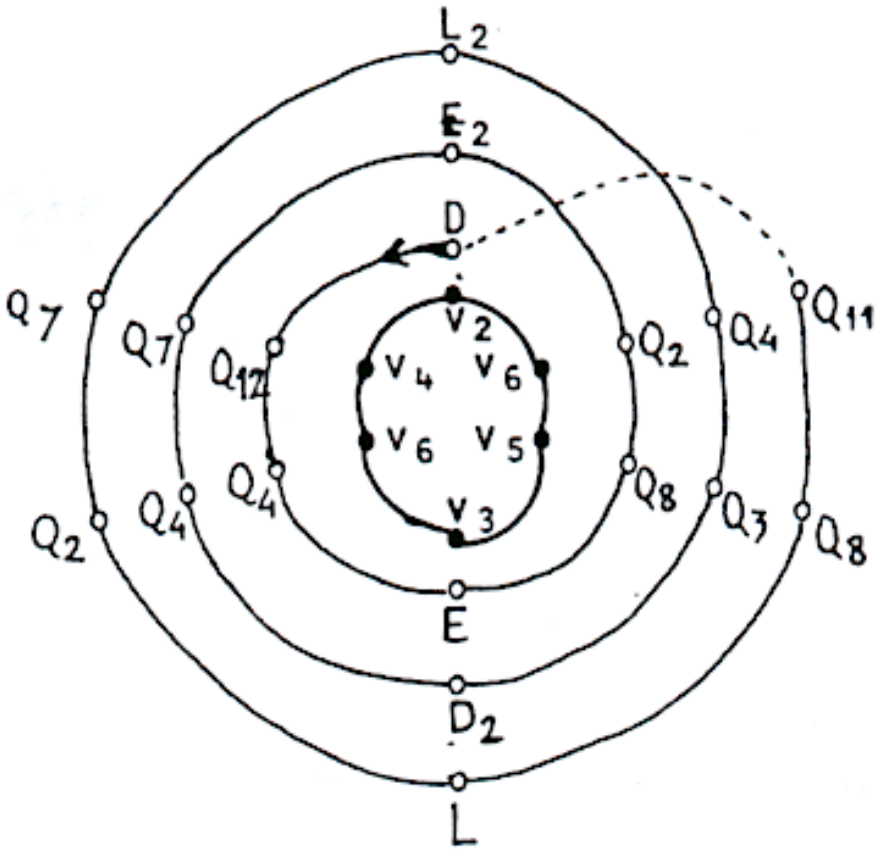


S7



S8





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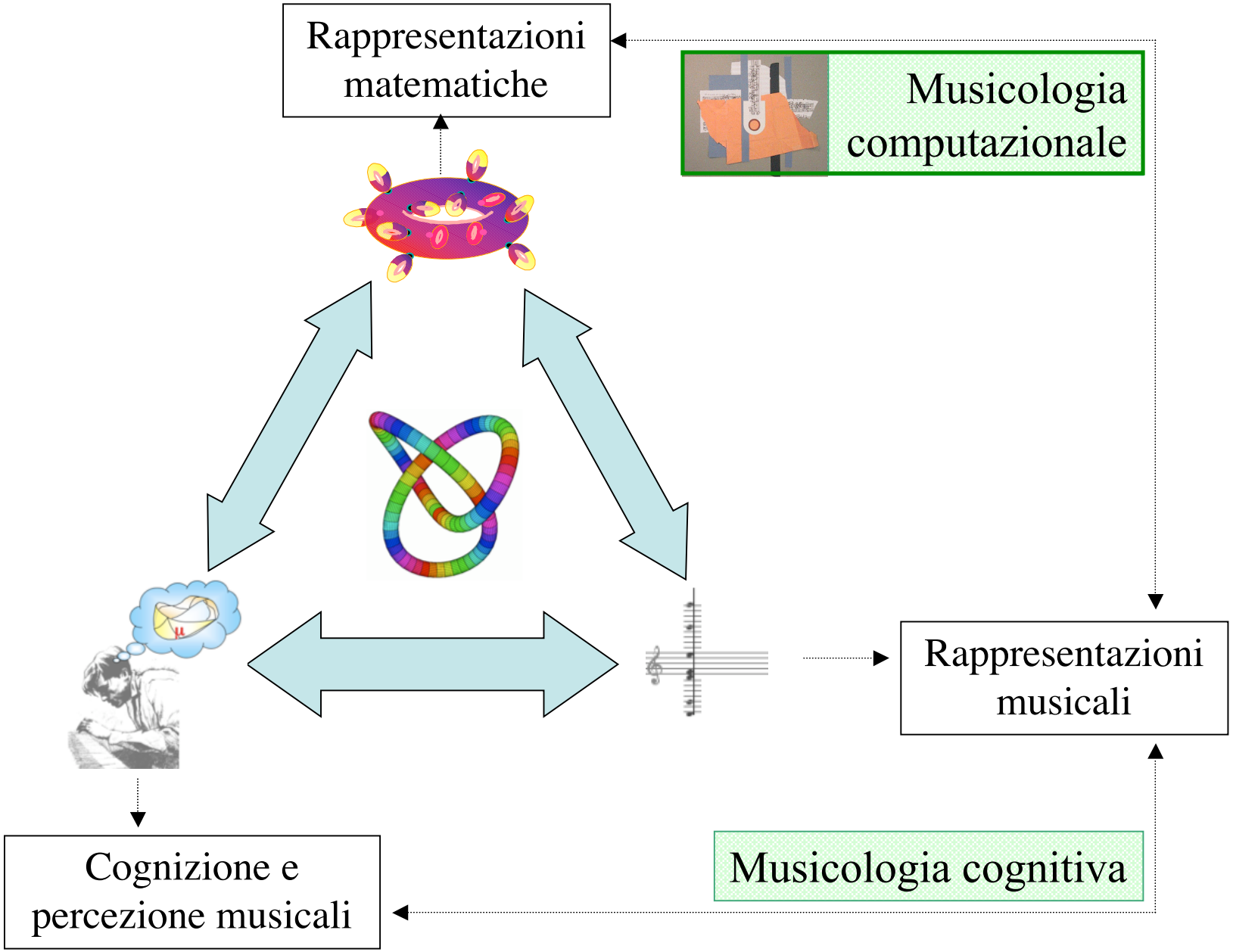
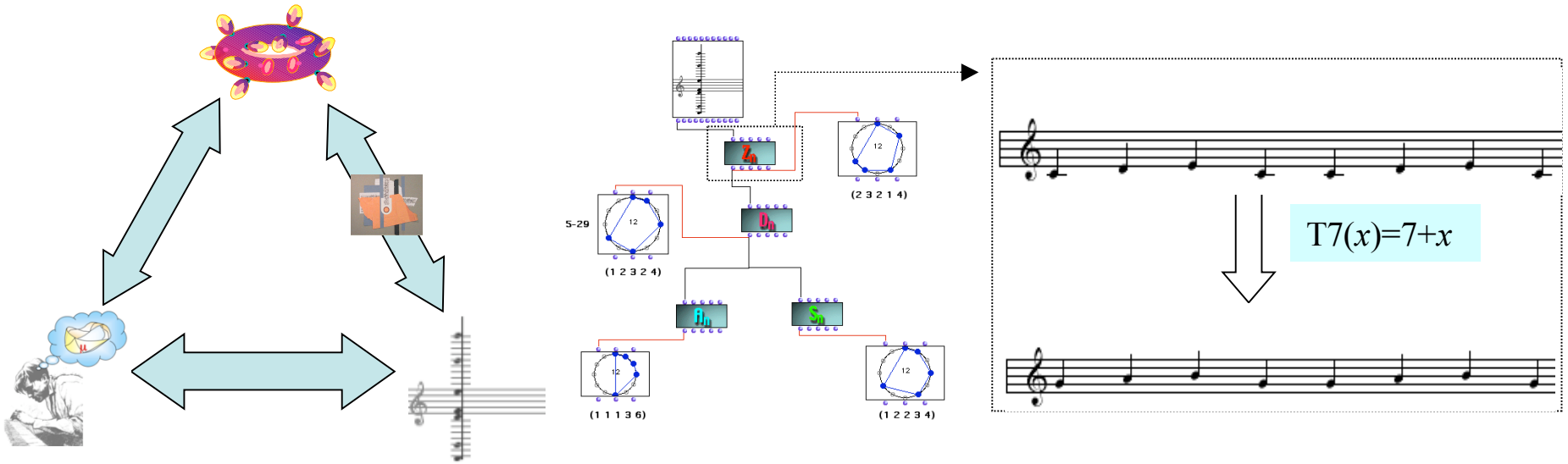
Matematica/Musica & Cognizione/Percezione



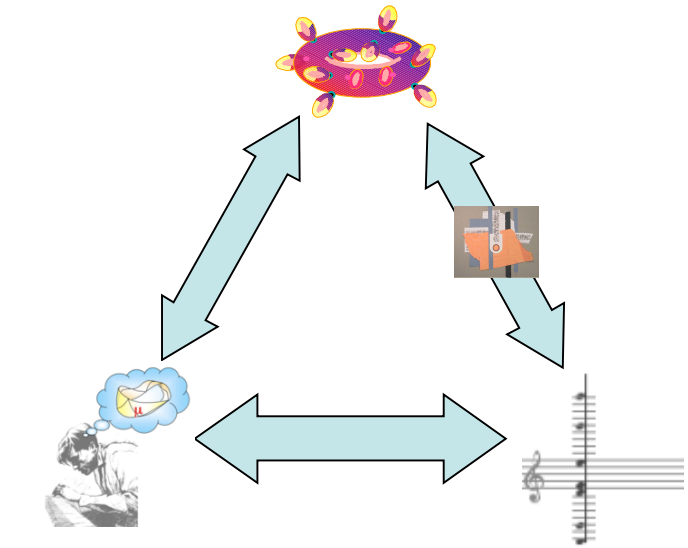
Ramificazioni percettive e cognitive dei metodi algebrici in musica

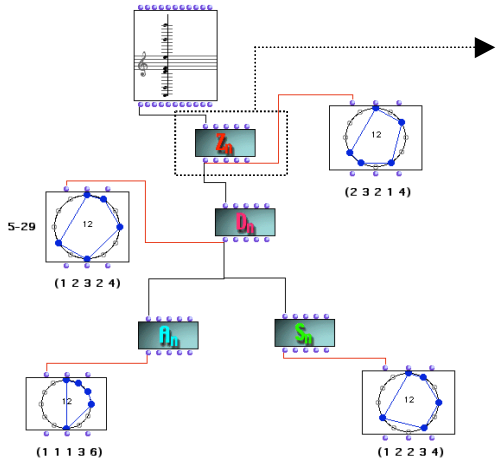


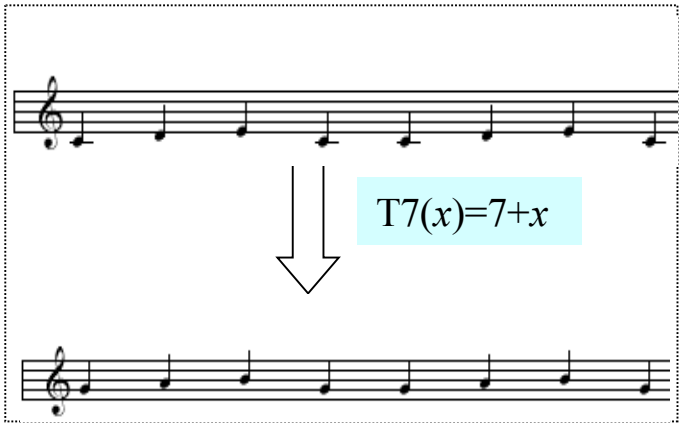
The nature of a given geometry is […] defined by the *reference* to a determinate **group** and the way in which spatial forms are related within that type of geometry. [*Cf. Felix Klein Erlangen Program - 1872*][…] We may raise the question whether there are any concepts and principles that are, although in different ways and different degrees of distinctness, necessary conditions for both the *constitution* of the **perceptual world** and the construction of the universe of geometrical thought. It seems to me that the concept of **group** and the concept of **invariance** are such principles.

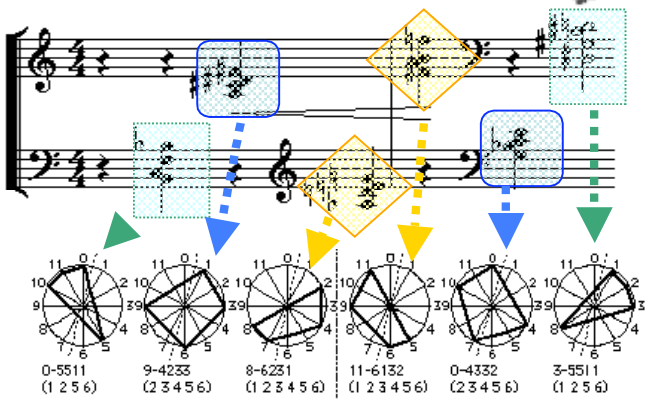
E. Cassirer, "The concept of group and the theory of perception", 1944

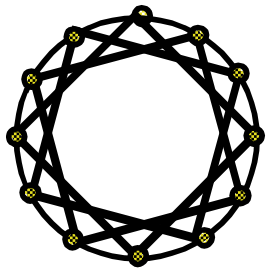
Ramificazioni percettive e cognitive dei metodi algebrici in musica

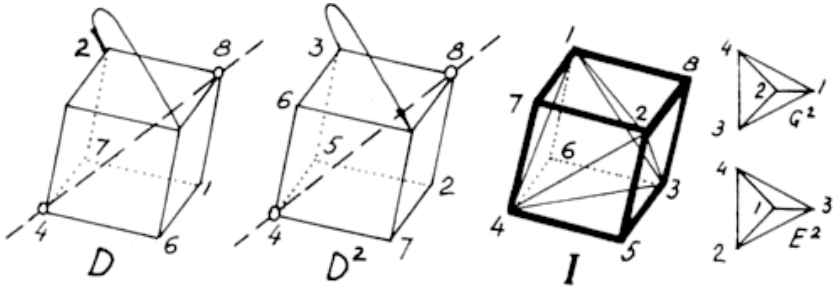






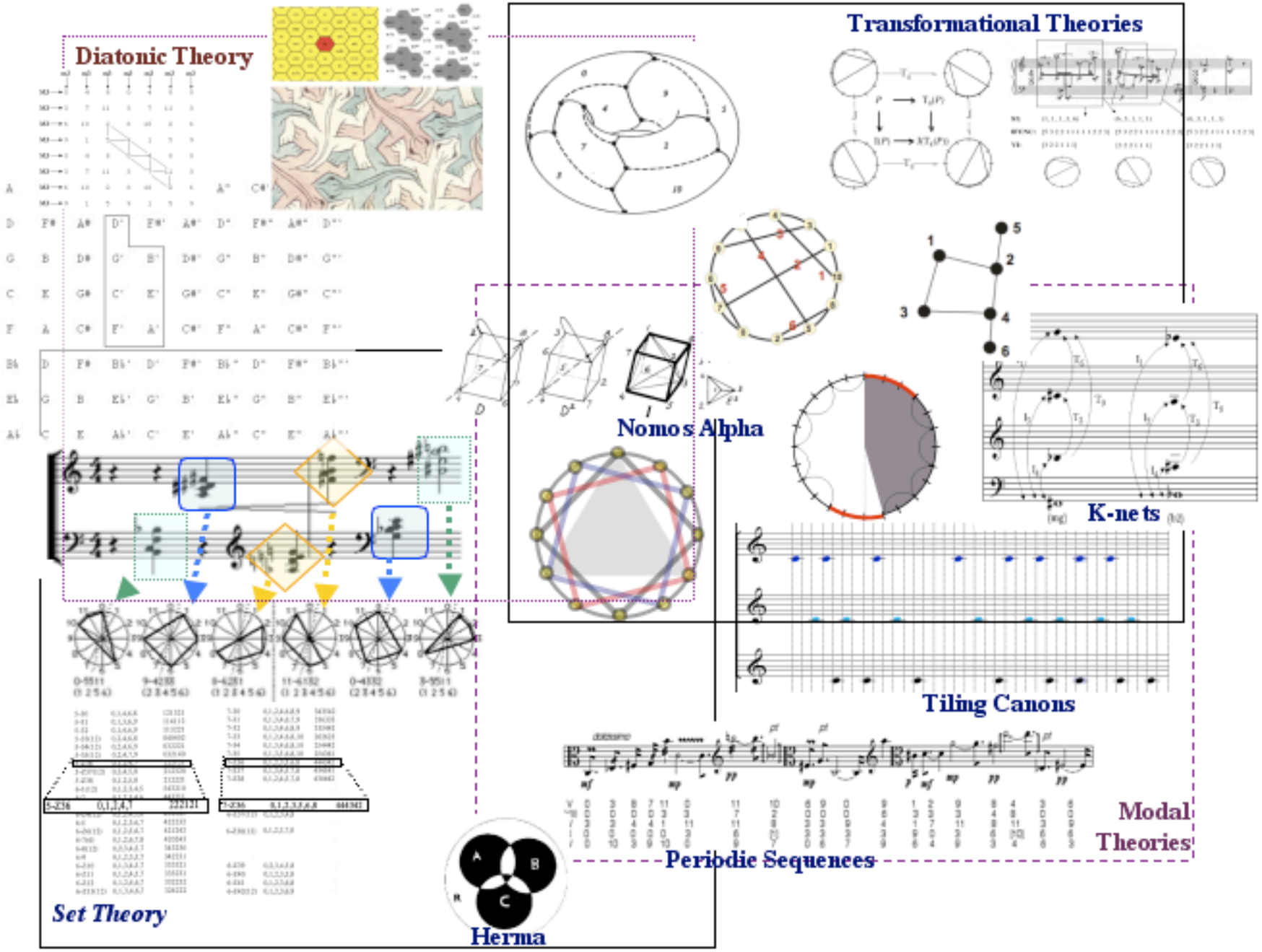






Il carattere singolare dell'esperienza musicale è dovuto in parte alle strutture particolari di ***gruppo*** *che la musica rende accessibile* [consciamente o inconsciamente] *all'ascoltatore.*

G. Balzano : « The group-theoretic description of 12-fold and microtonal pitch systems », 1980


Diatonic Theory
Transformational Theories
Nomos Alpha
K-nets
Tiling Canons
Modal Theories
Periodic Sequences
Set Theory
Hexma