Master I.C.A.

Master I.C.A.

Spécialité A.S.T.

Art Sciences Technologies

Traitement interactif de l'image et du son

Méthodes mathématiques pour la création musicale : aspects théoriques et ramifications cognitives

Moreno Andreatta - Jean Bresson

Equipe Représentations Musicales

IRCAM/CNRS UMR 9912

ircam Centre Pompidou

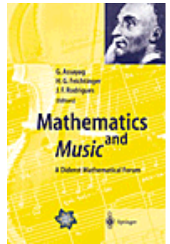
cnrs

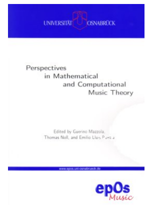
Plan du cours

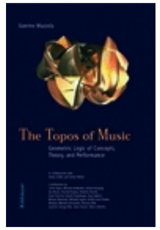
1.) Représentation, formalisation et énumération des structures musicales
2.) Théories transformationnelles, diatoniques et néo-riemanniennes
3.) Pavages en composition : la construction des canons rythmiques mosaiques

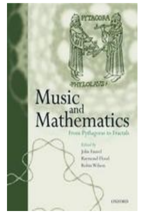
Mathématiques/Musique…une histoire récente!

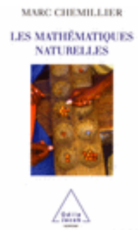
- 1999: 4e Forum Diderot (Paris, Vienne, Lisbonne), *Mathematics and Music* (Assayag et al., Springer, 2001)
- 2000-2001: Séminaire *MaMuPhi, Penser la musique avec les mathématiques ?* (Assayag, Mazzola, Nicolas ed., Coll. « Musique/Sciences », Ircam/Delatour, 2006)
- 2000-2003: International Seminar on *MaMuTh* (*Perspectives in Mathematical and Computational Music Theory* (Mazzola, Noll, Luis-Puebla, epOs, 2004)
- 2003: *The Topos of Music* (G. Mazzola et al.)
- 2003: *Music and Mathematics. From Pythagoras to Fractals* (J. Fauvel et al.)
- 2001 - 2010: Séminaire *MaMuX* de l'Ircam `http://recherche.ircam.fr/equipes/repmus/mamux/`
- 2004 - 2010 : Séminaire « Musique et Mathématique » (Ens/Ircam) `http://www.entretemps.asso.fr/maths`
- 2006: *Mathematical Theory of Music* (Franck Jedrzejewski), Ircam/Delatour
- 2007: *Music. A Mathematical Offering* (Dave Benson), Cambridge University Press
- 2007: *Les mathématiques naturelles* (Marc Chemillier), Odile Jacob
- 2007: *La vérité du beau dans la musique* (G Mazzola), Ircam/Delatour
- 2008: *Music Theory and Mathematics* (Jack Douthett et al.), Univ. of Rochester Press
- 2007: Journal of Mathematics and Music (Taylor & Francis) et SMCM
- 2011 : Conférence de la Society of Mathematics and Computation in music (Ircam)

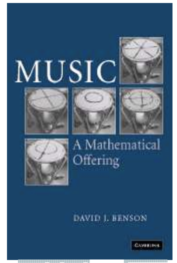



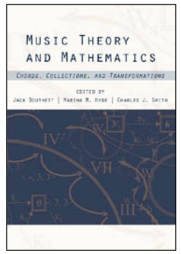


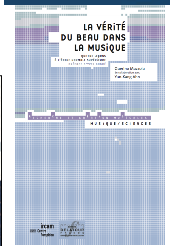


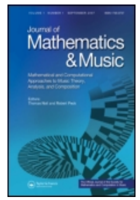





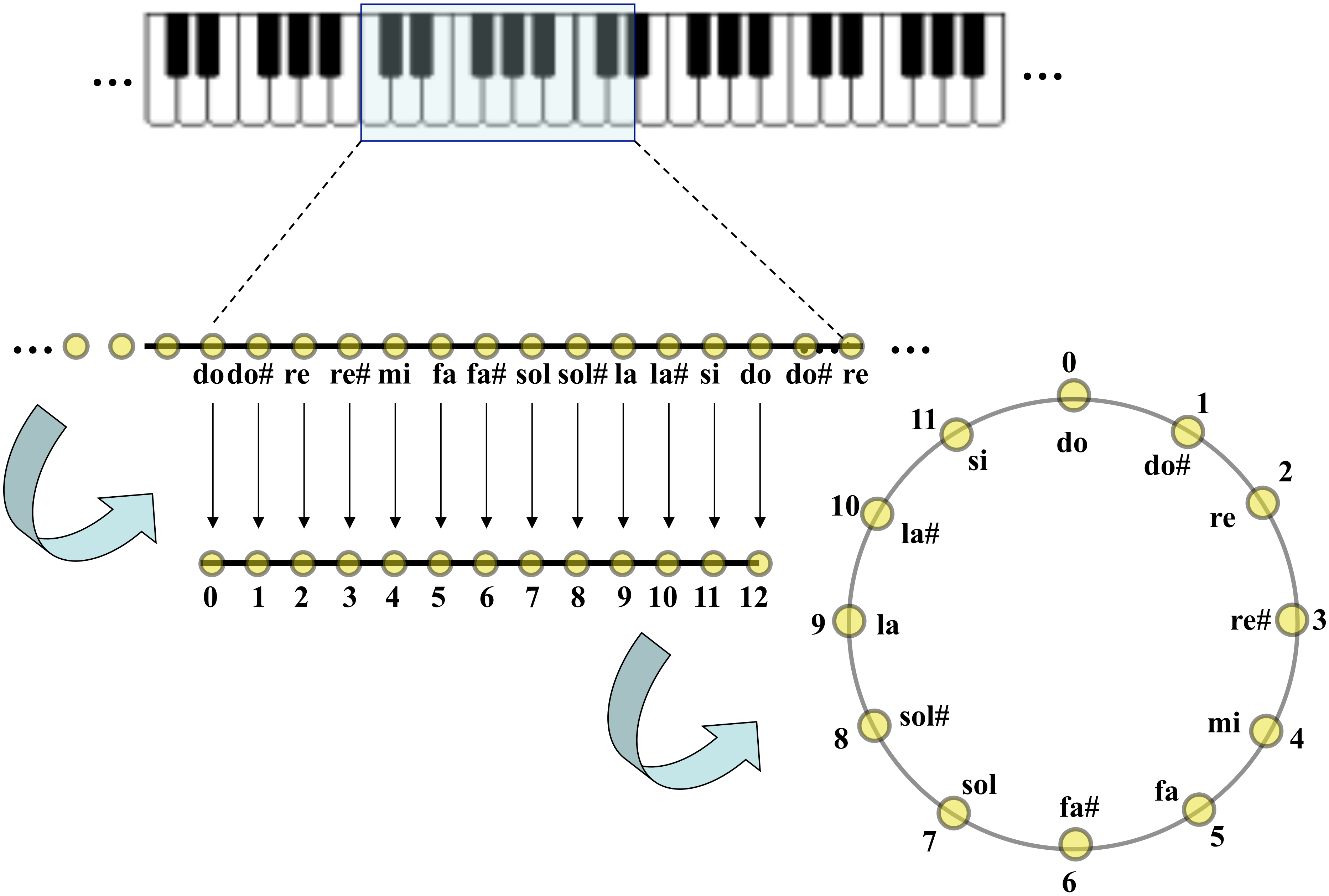




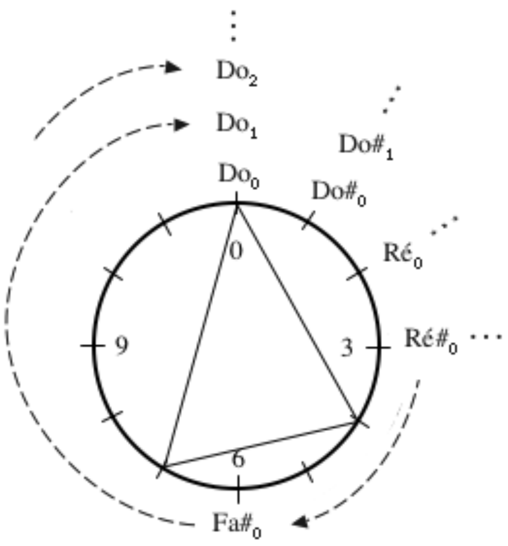




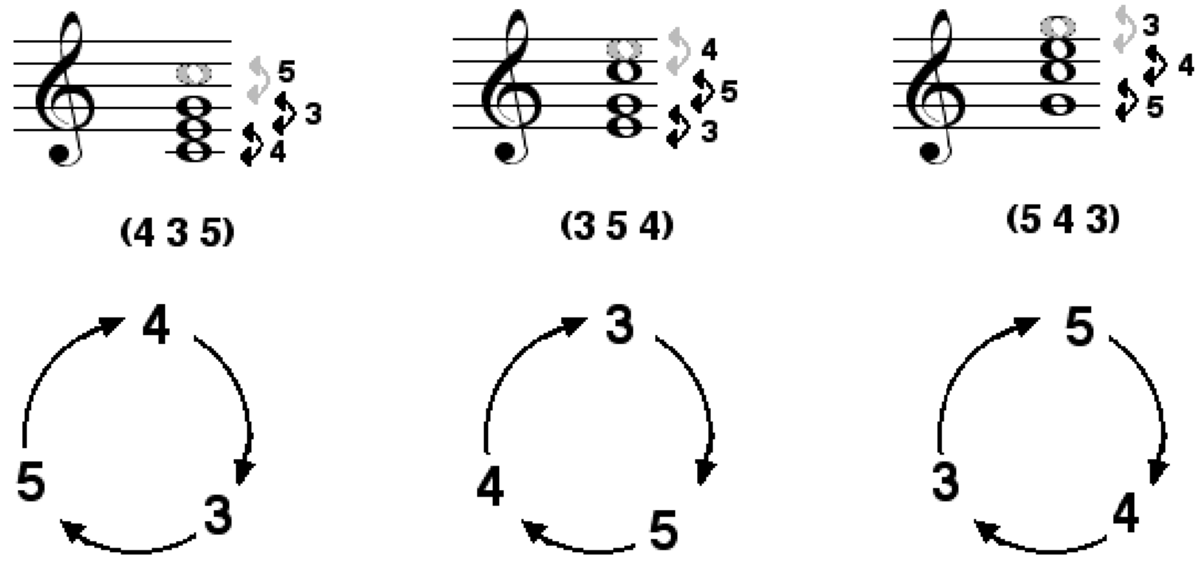
Réduction à l’octave et congruence modulo 12



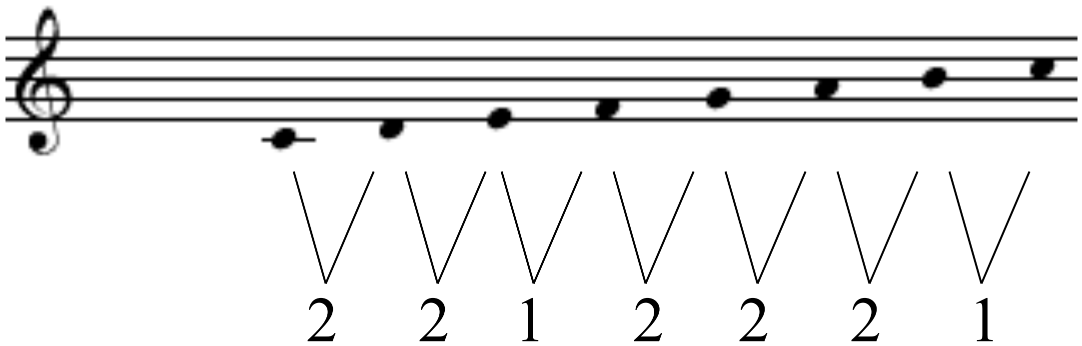
Représentation circulaire et structure intervallique

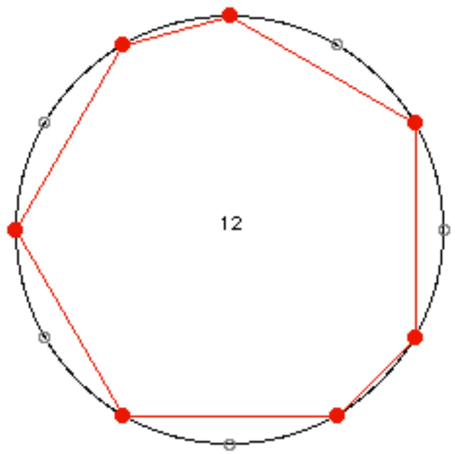


(4,3,5)

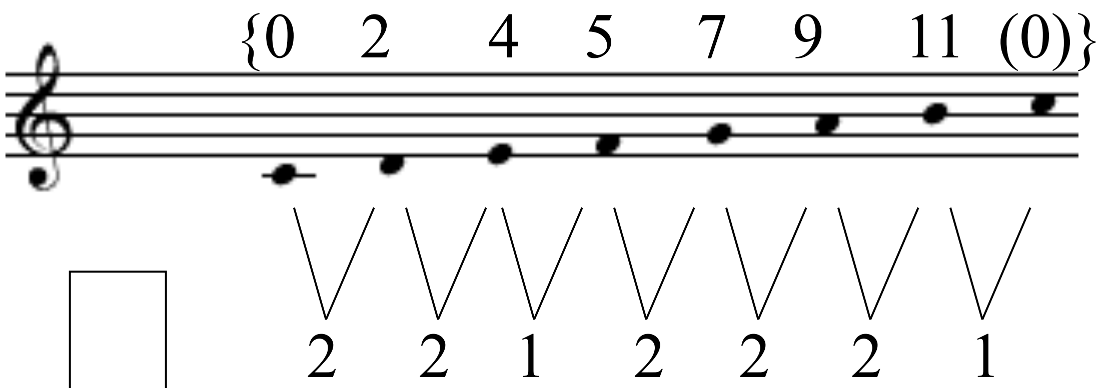


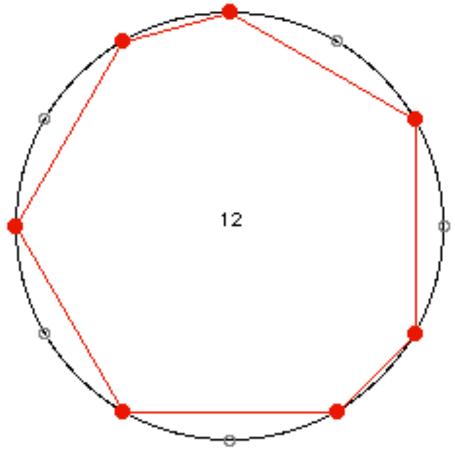
Renversements = permutations circulaires de la structure intervallique

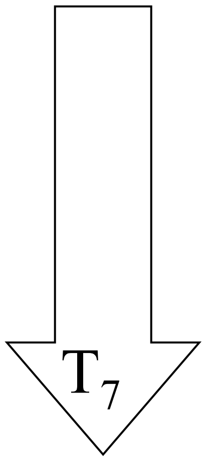




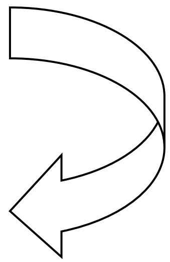
Transformations géométriques : la transposition



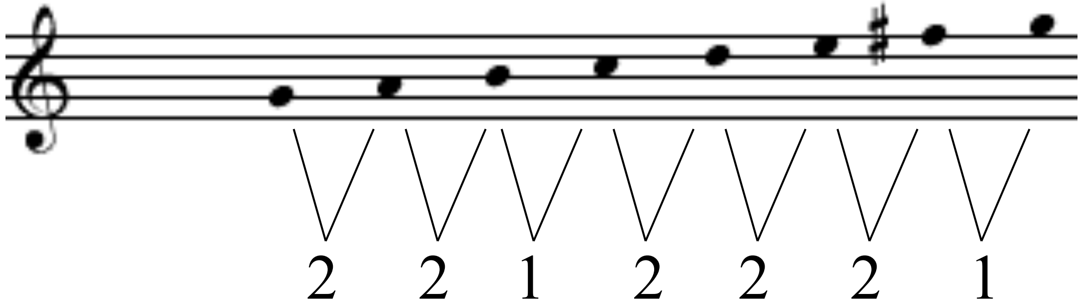


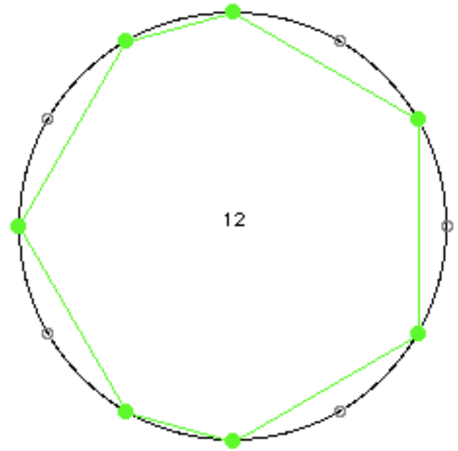


$T_7(x)=7+x \pmod{12}$



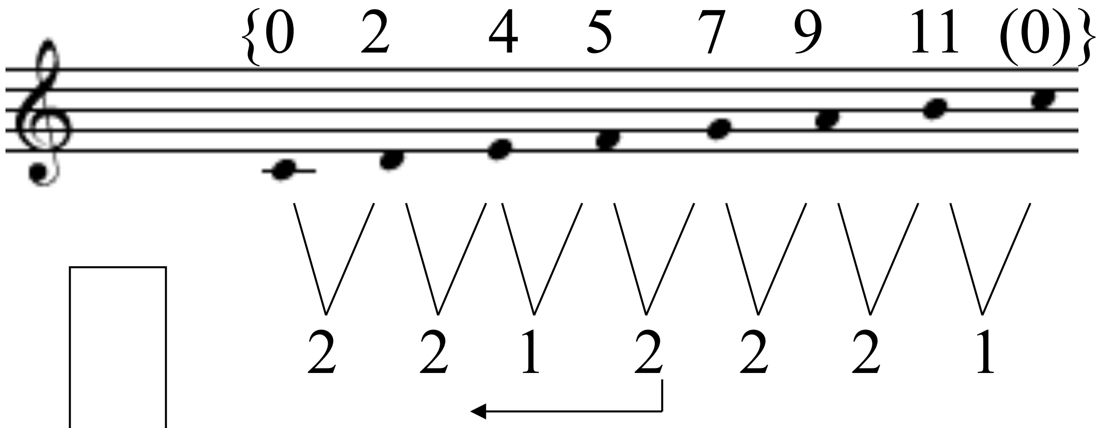
$\alpha=210°$

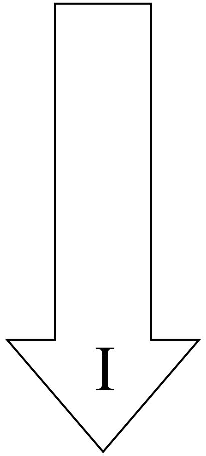




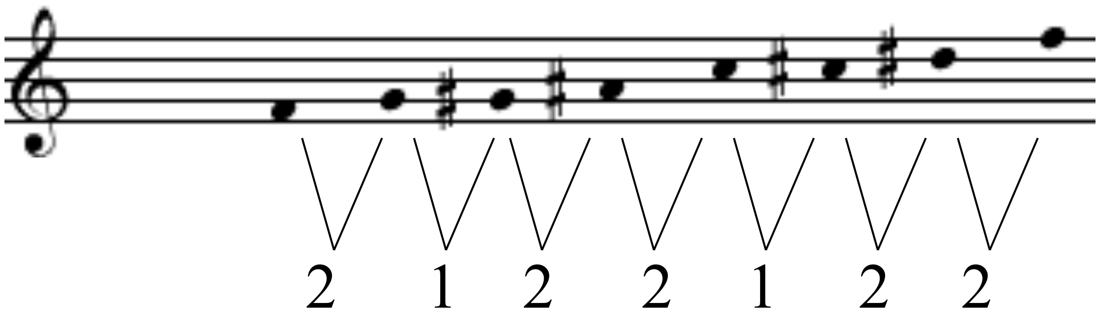
Equivalence modulo la transposition

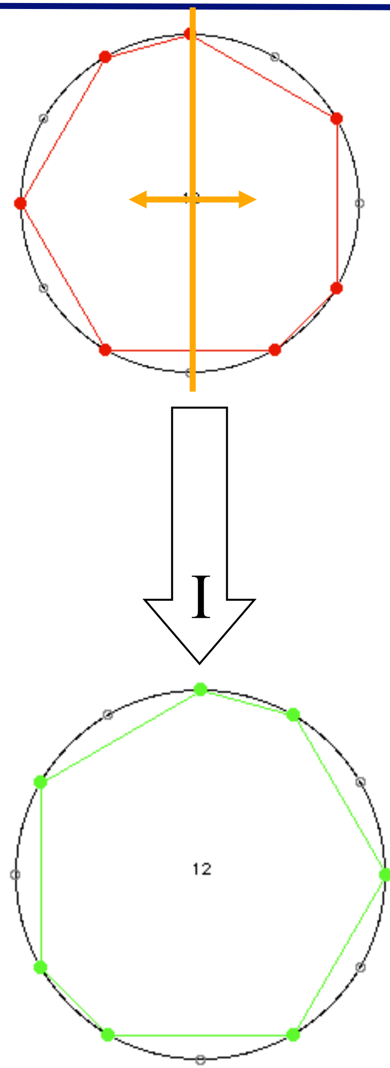
Transformations géométriques : l'inversion





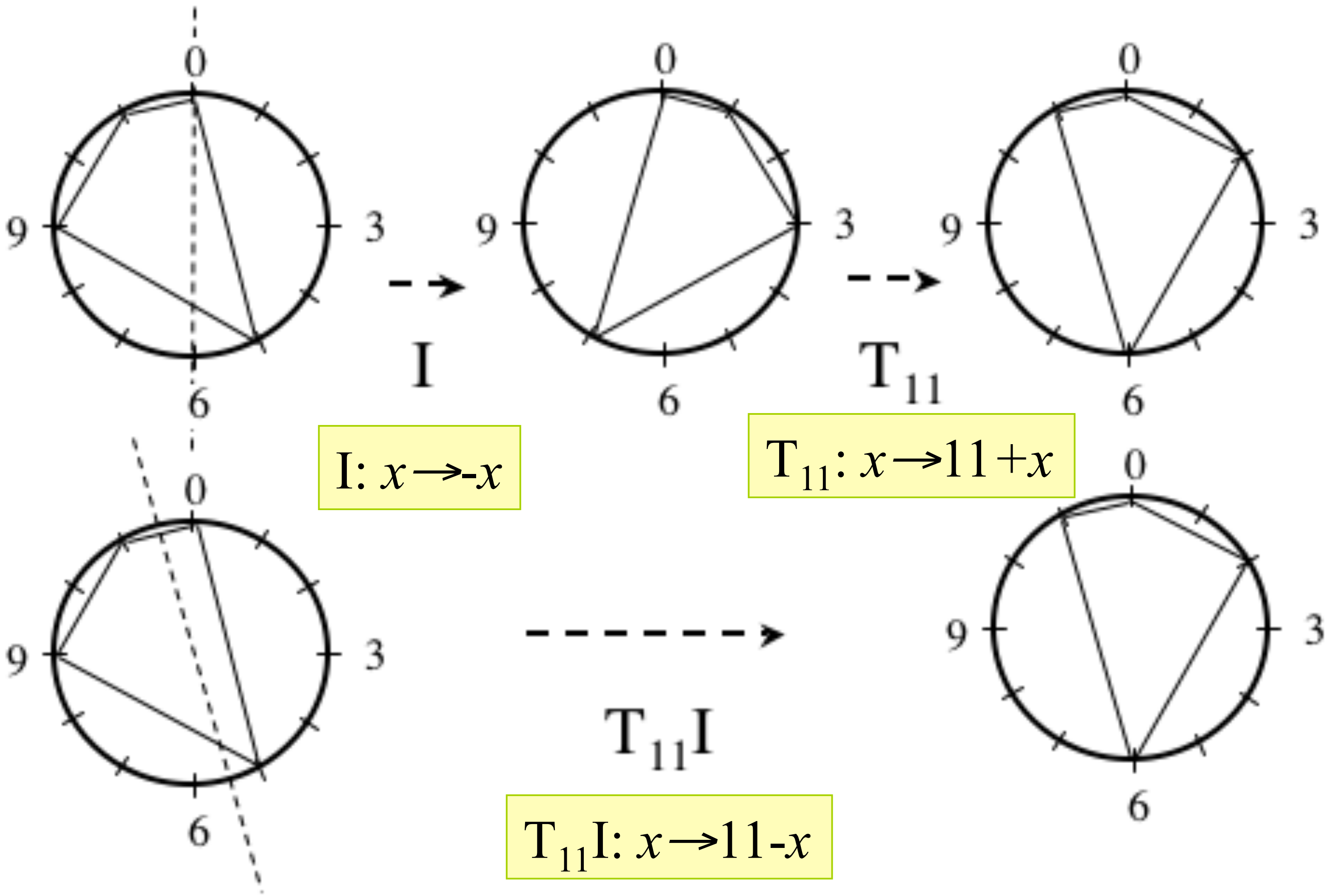
$I(x) = -x \pmod{12}$





Equivalence modulo l'inversion

La *Set Theory*: équivalence modulo transposition/inversion

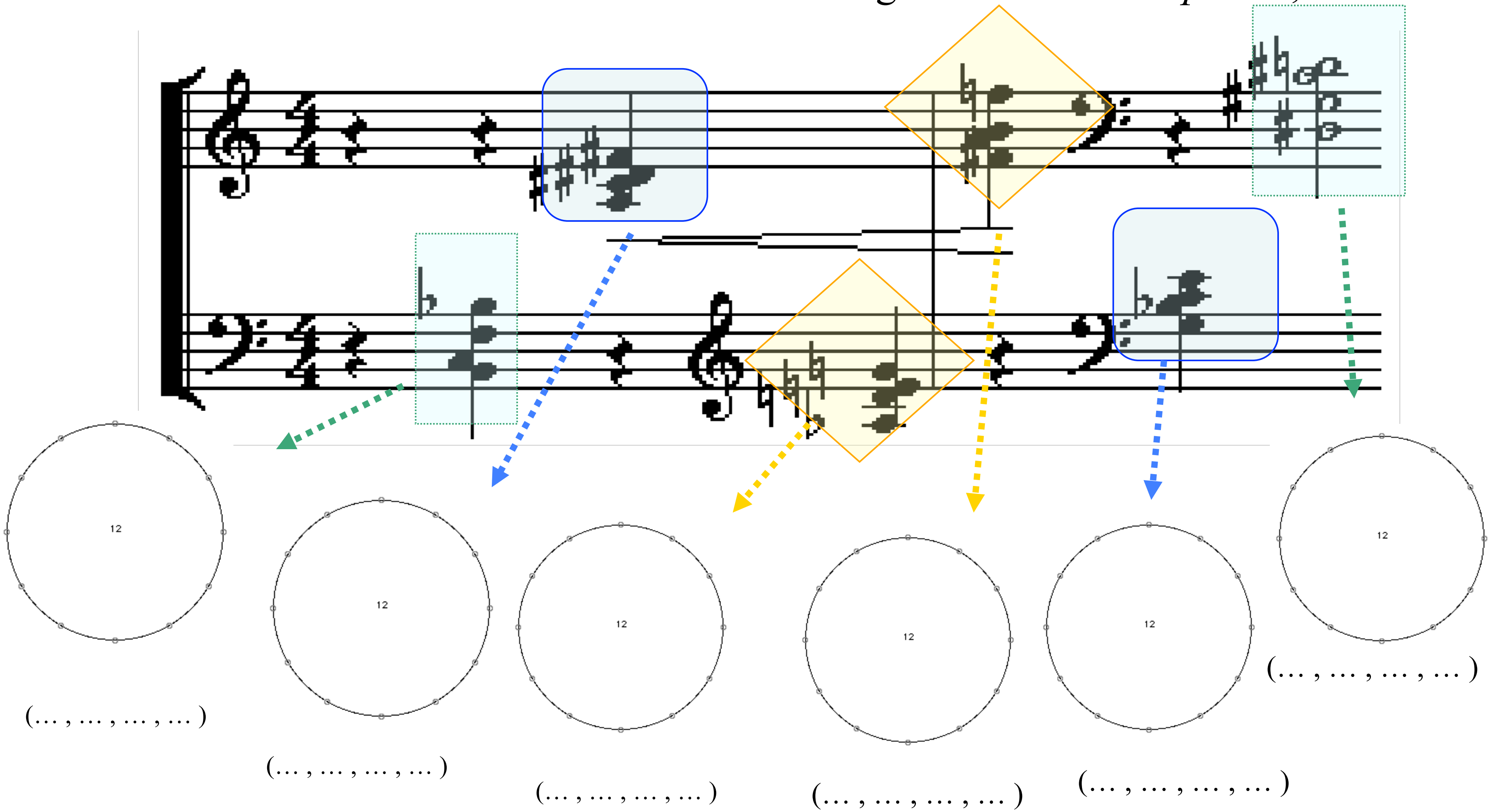


$\{0, 5, 9, 11\} \longrightarrow \{11, 6, 3, 0\}$

L'analyse formalisée ou les entités formelles en musique

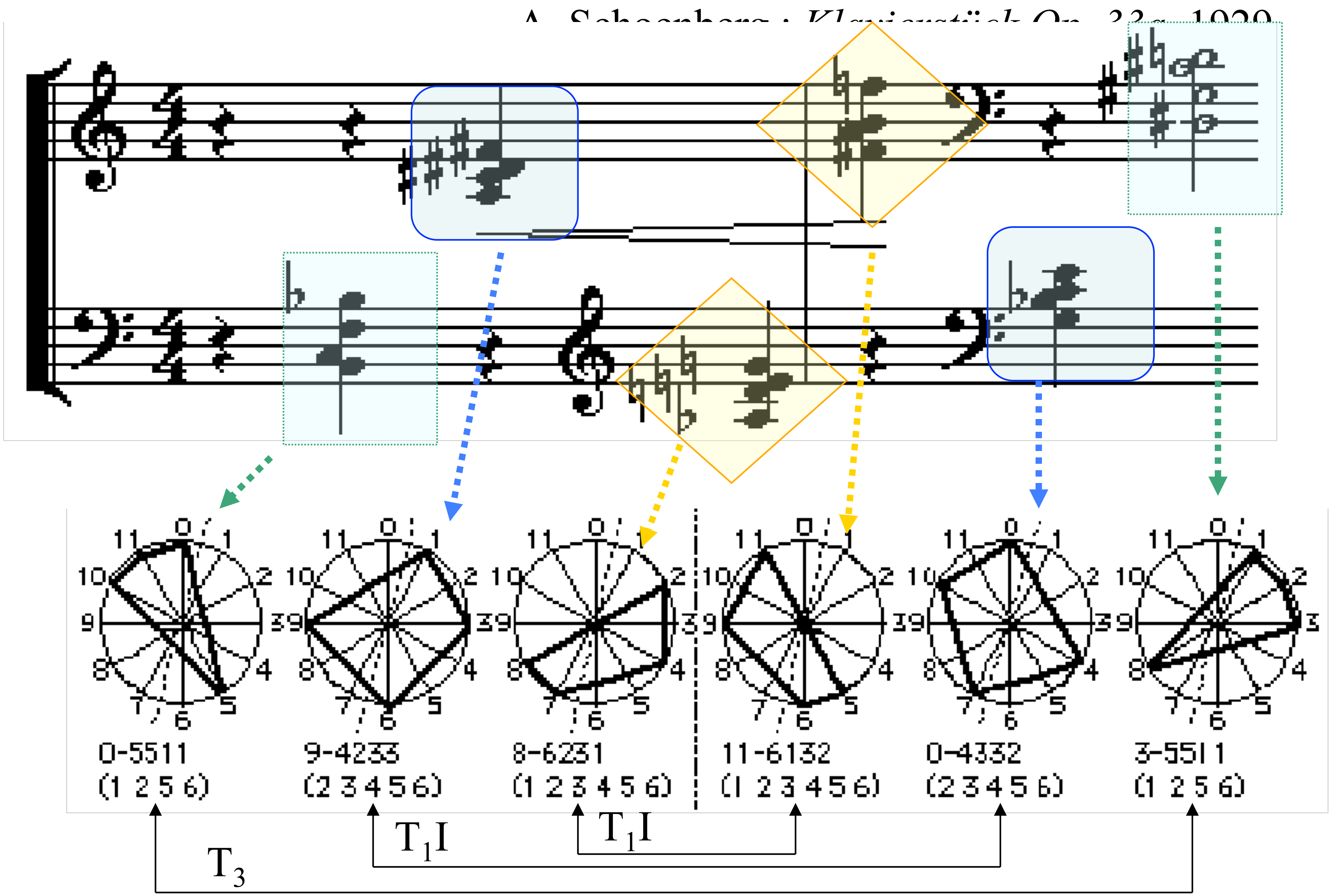
André Riotte e Marcel Mesnage

A. Schoenberg : *Klavierstück Op. 33a*, 1929



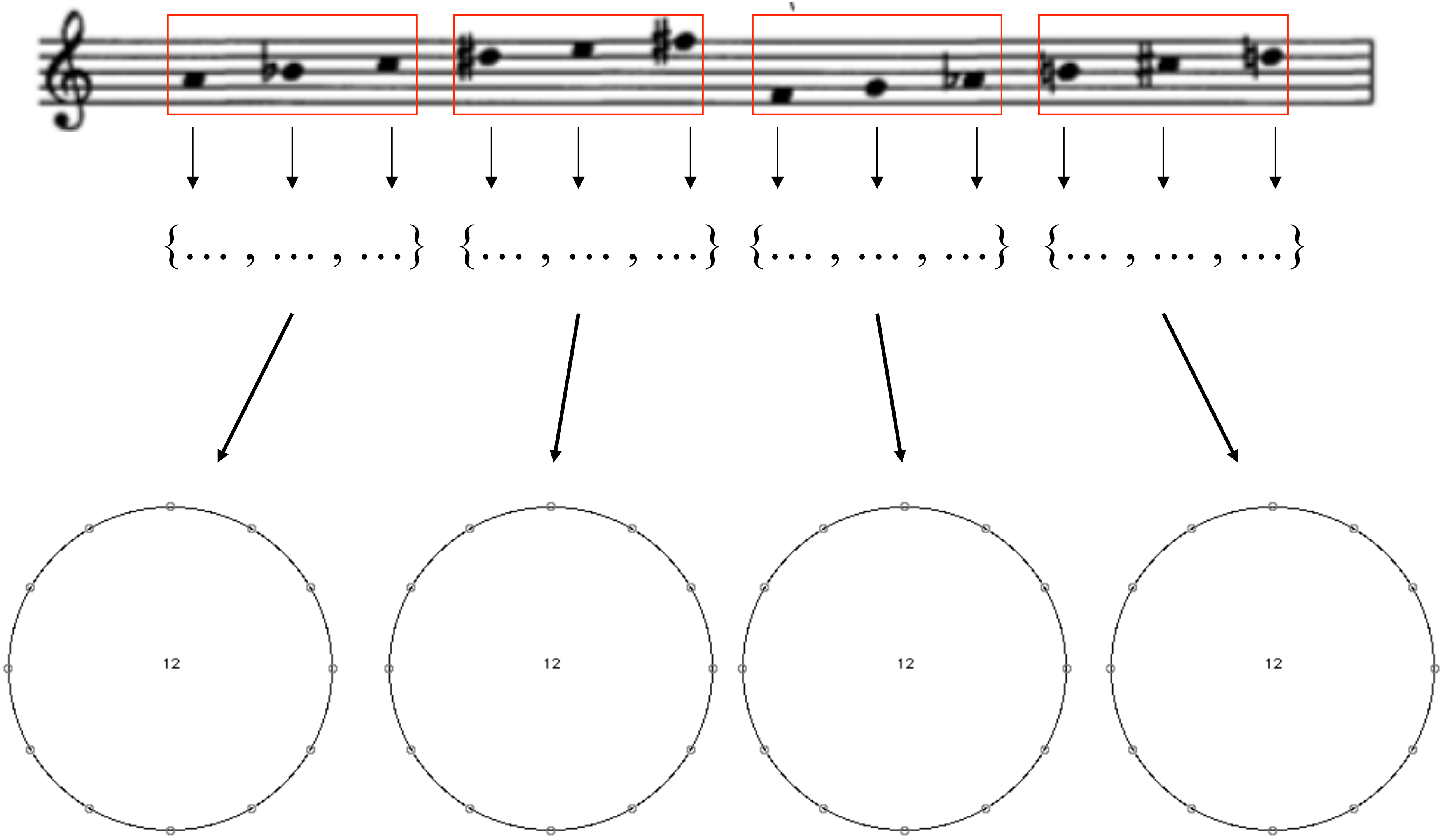
L'analyse formalisée ou les entités formelles en musique

André Riotte e Marcel Mesnage



Exercise : retrouver les symétries dans une série (I)

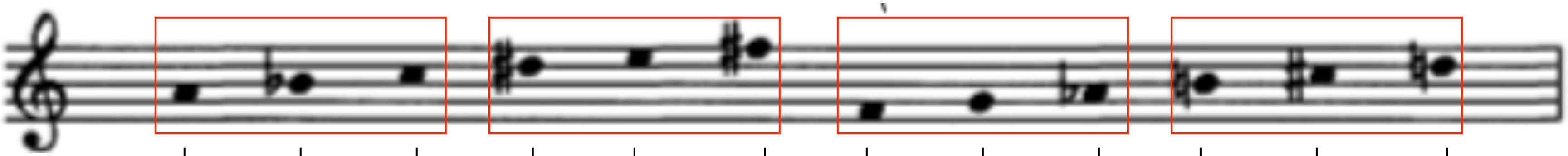
Schoenberg: Serenade Op.24, Mouvement 5



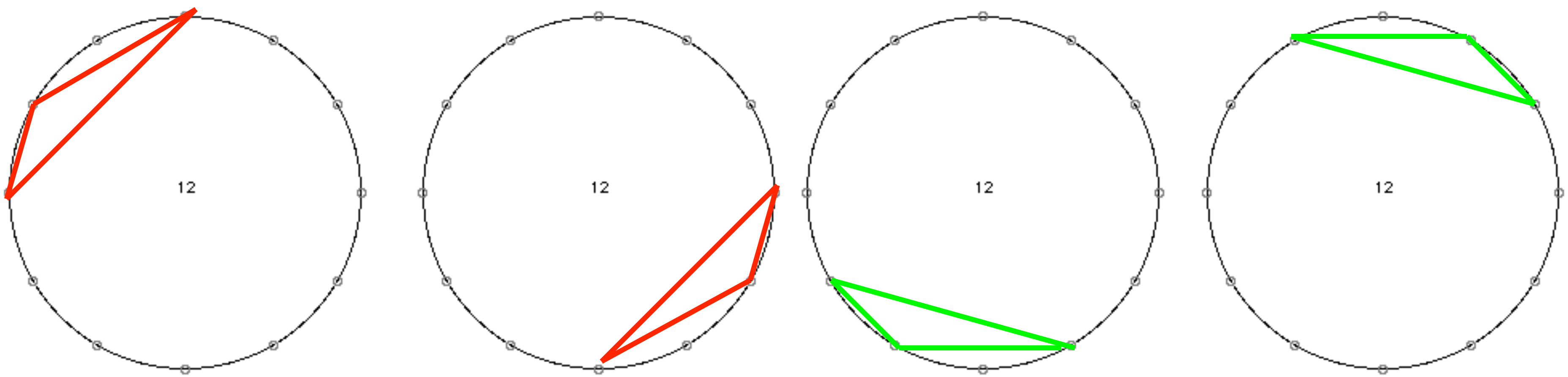
(… , … , …) (… , … , …) (… , … , …) (… , … , …)

Exercise : retrouver les symétries dans une série (I)

Schoenberg: Serenade Op.24, Mouvement 5



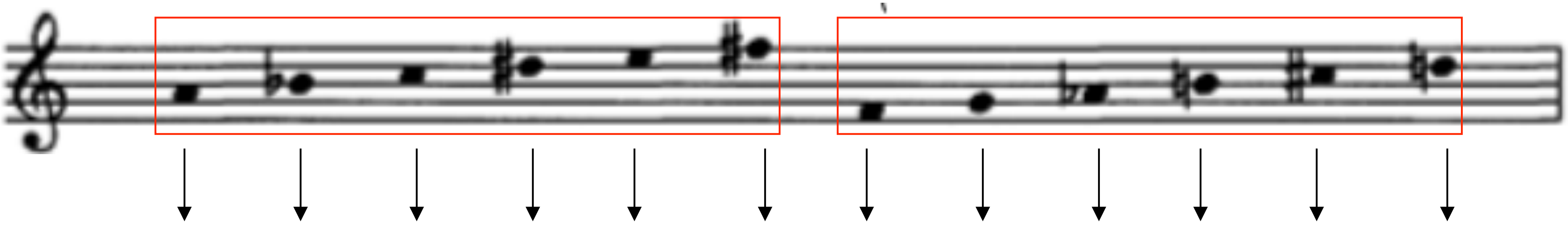
{9 , 10 , 0}   {3 , 4 , 6}   {5 , 7 , 8}   {11 , 1 , 2}



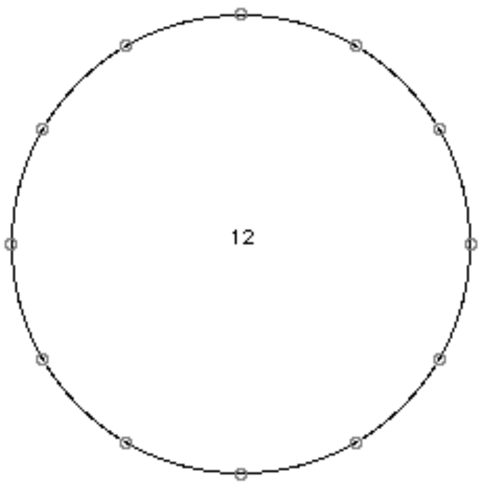
(1 , 2 , 9)   (1 , 2 , 9)   (2 , 1 , 9)   (2 , 1 , 9)

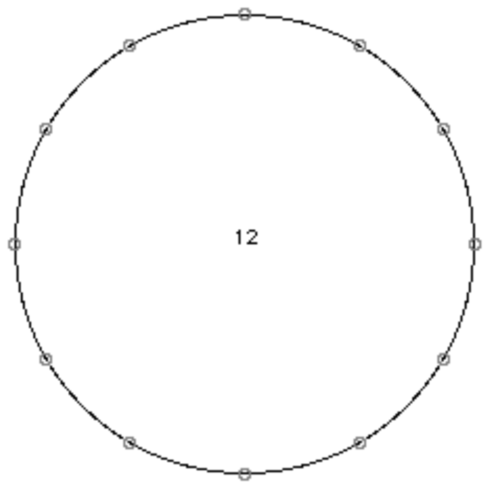
Exercise : retrouver les symétries dans une série (II)

Schoenberg: Serenade Op.24, Mouvement 5



{… , … , … , … , … , … } {… , … , … , … , … , … }

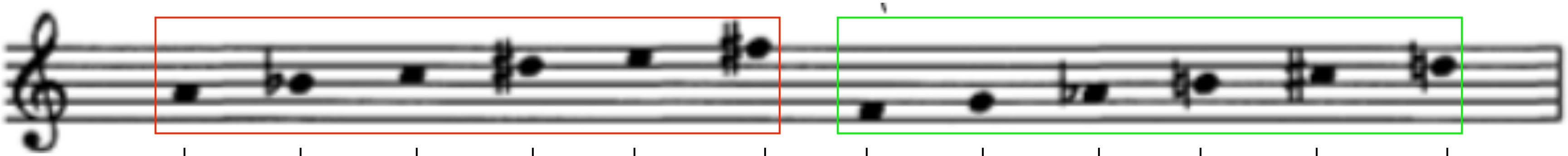




(… , … , … , … , … , …) (… , … , … , … , … , …)

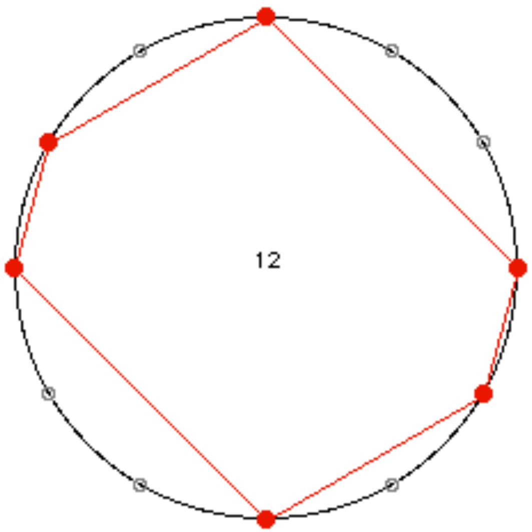
Exercise : retrouver les symétries dans une série (II)

Schoenberg: Serenade Op.24, Mouvement 5

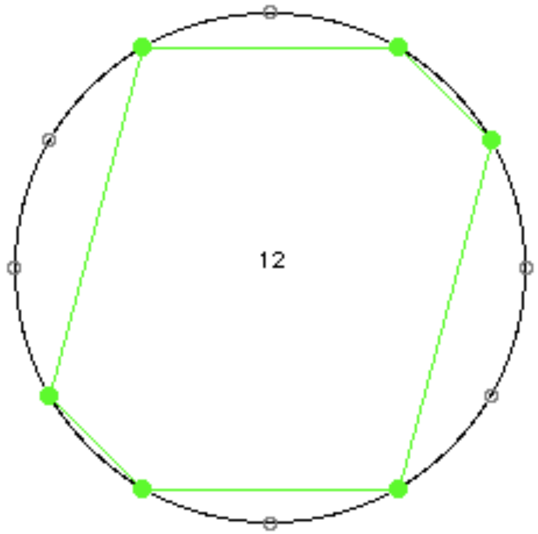


{9 , 10 , 0 , 3 , 4 , 6 }

{5 , 7 , 8 , 11 , 1 , 2 }



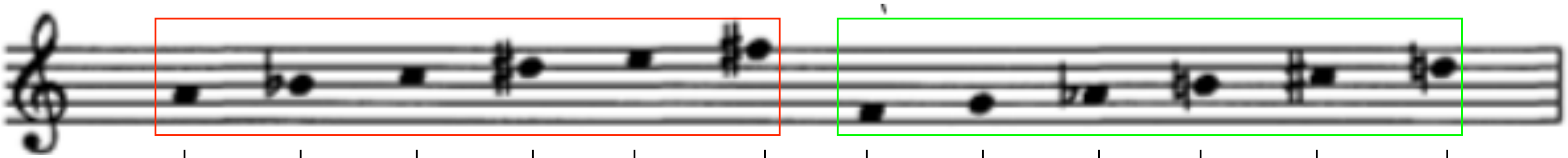
(3, 1, 2, 3, 1, 2)



(2, 1, 3, 2, 1, 3)

“Combinatorialité” et symétrie par transposition

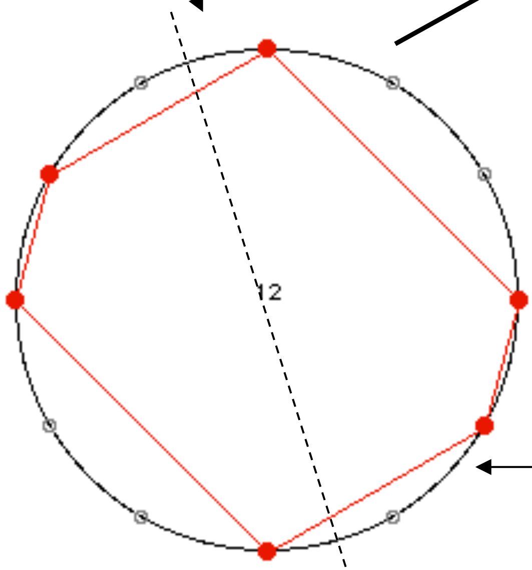
Schoenberg: Serenade Op.24, Mouvement 5



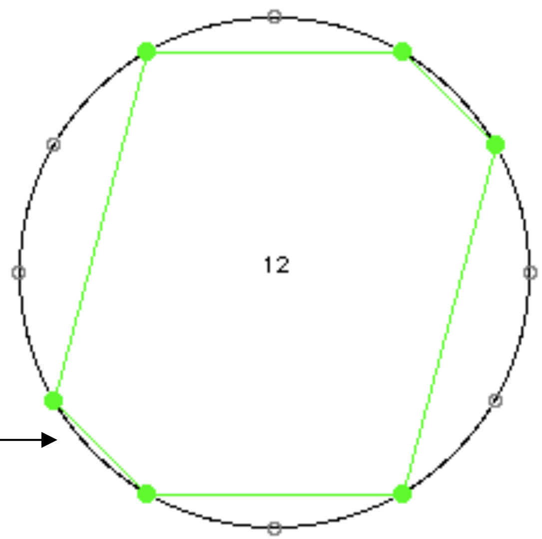
A = {9 , 10 , 0 , 3, 4 , 6} {5 , 7, 8, 11, 1, 2} =A’

$T6\{9,10,0,3,4,6\}=$
$=\{6+9,6+10,6,6+3,6+4,6+6\}=$
$=\{3,4,6,9,10,0\}$

T6(A)=A



$\mathbf{I_{11} = T_{11}\, I}$

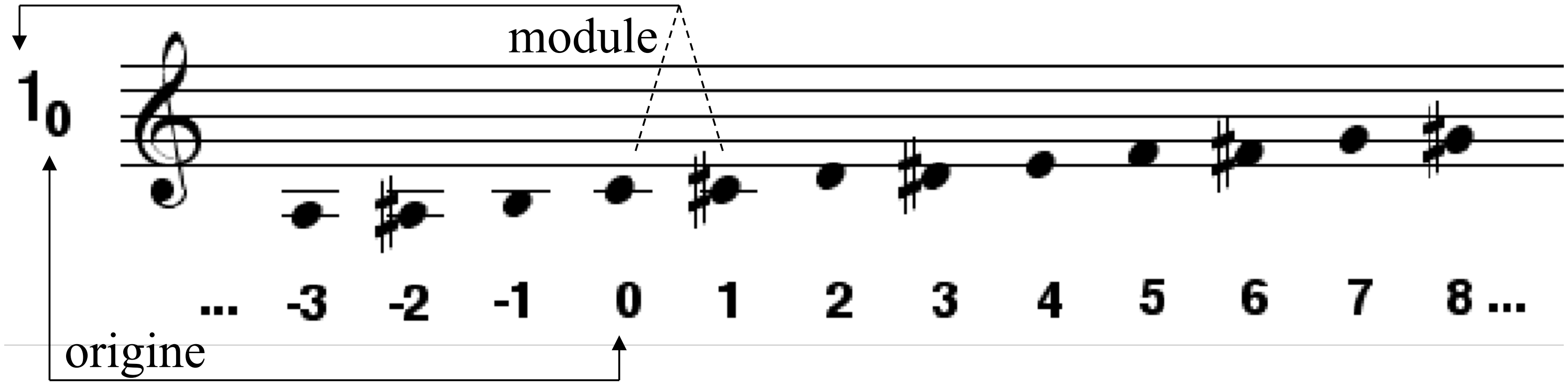


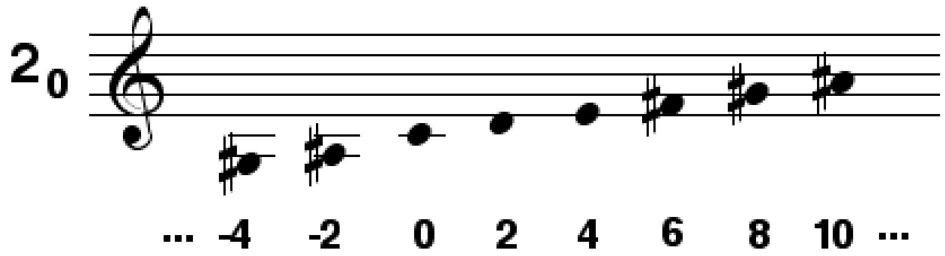
(3, 1, 2, 3, 1, 2)

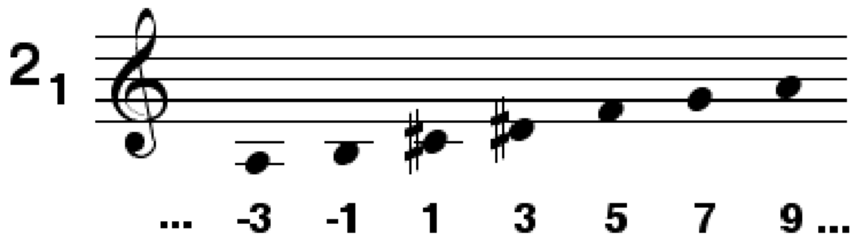
(2, 1, 3, 2, 1, 3)

La théorie des cribles

Formalisation algébrique des structures musicales selon Xenakis







$1_0 = 2_0 \cup 2_1$

$2_0 \cap 2_1 = \varnothing$

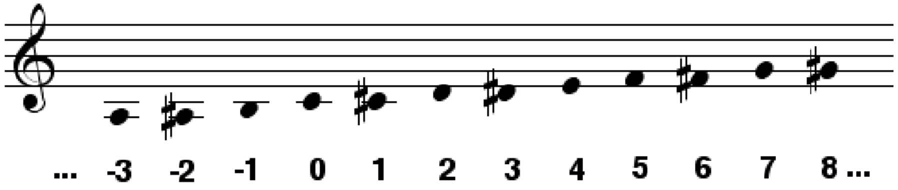
$(2_0)^c = 2_1$

$(2_1)^c = 2_0$.

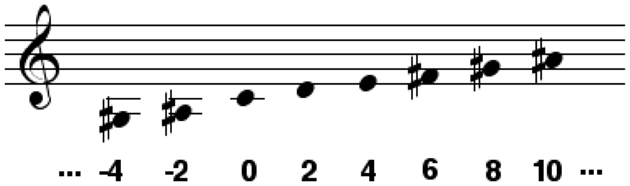
La théorie des cribles

Formalisation algébrique des structures musicales selon Xenakis

1_0



2_0

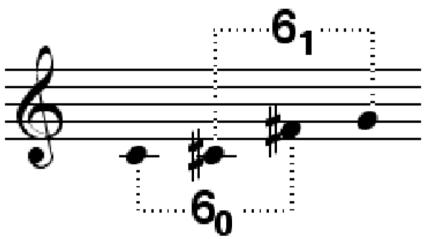


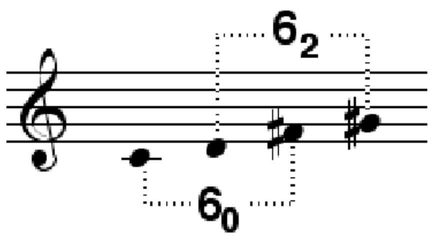
$T_m\ (A)=A$

$m \neq 0 \bmod 12$

(3_0)	Septième diminuée
(4_0)	Triade augmentée
(6_0)	Triton

$6_0 \cup 6_1$





$6_0 \cup 6_2$

$6_0 \cup 6_3$?

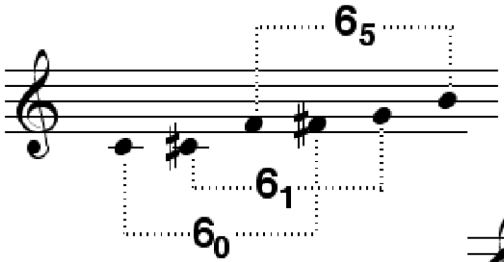
« Cribles » / Messiaen

Catalogue

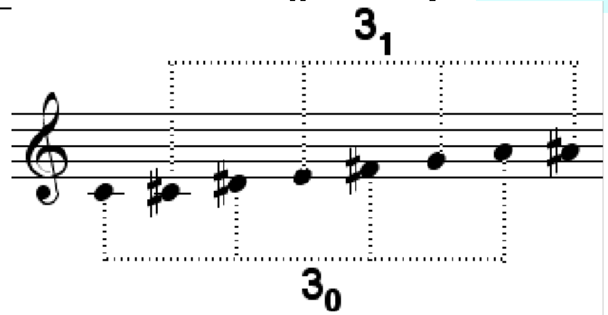
(1_0) (2_0) (3_0) (4_0) (6_0) $6_0 \cup 6_1$ $6_0 \cup 6_2$

$6_0 \cup 6_1 \cup 6_5$

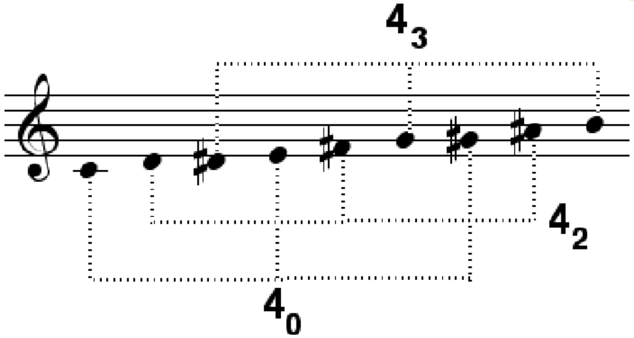
Mode n.5

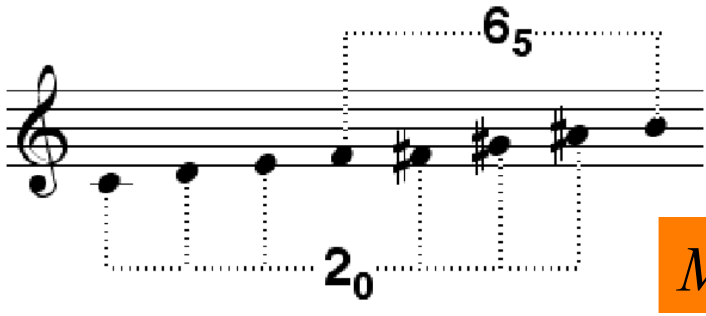


$3_0 \cup 3_1$ *Mode n.2*



$4_0 \cup 4_2 \cup 4_3$ *Mode n.3*

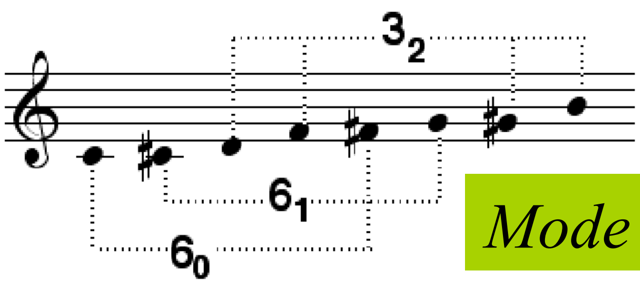




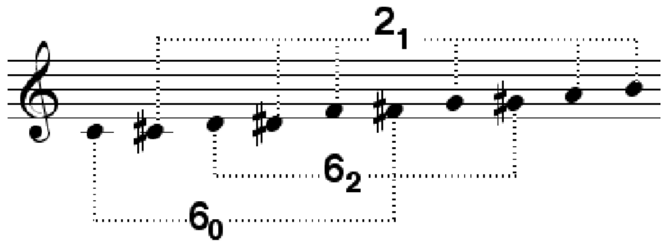
$2_0 \cup 6_5$

Mode n.6

$6_0 \cup 6_1 \cup 3_2$



Mode n.4



Mode n.7 $2_1 \cup 6_0 \cup 6_2$

$6_0 \cup 6_1$ $6_0 \cup 6_1 \cup 6_5$ $4_0 \cup 4_2 \cup 4_3$

(6_0) (2_0) $3_0 \cup 3_1$

« Cribles » / Messiaen

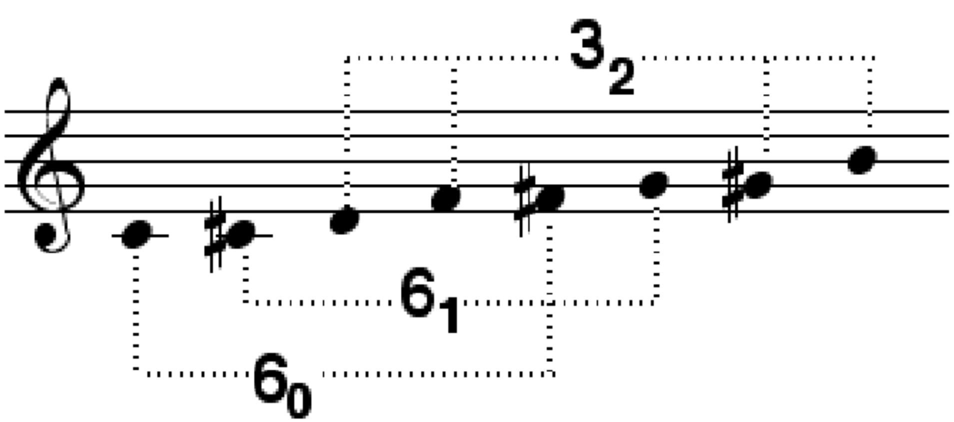
(1_0)

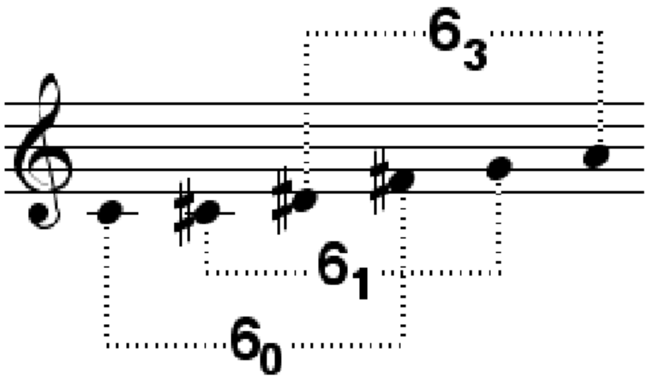
Les modes oubliés…

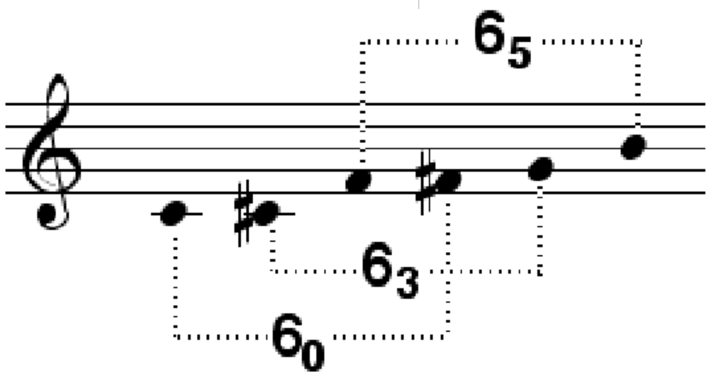
(4_0)

$6_0 \cup 6_2$ $6_0 \cup 6_1 \cup 3_2$ $2_0 \cup 6_5$ $2_1 \cup 6_0 \cup 6_2$ (3_0)

$6_0 \cup 6_1 \cup 3_2$



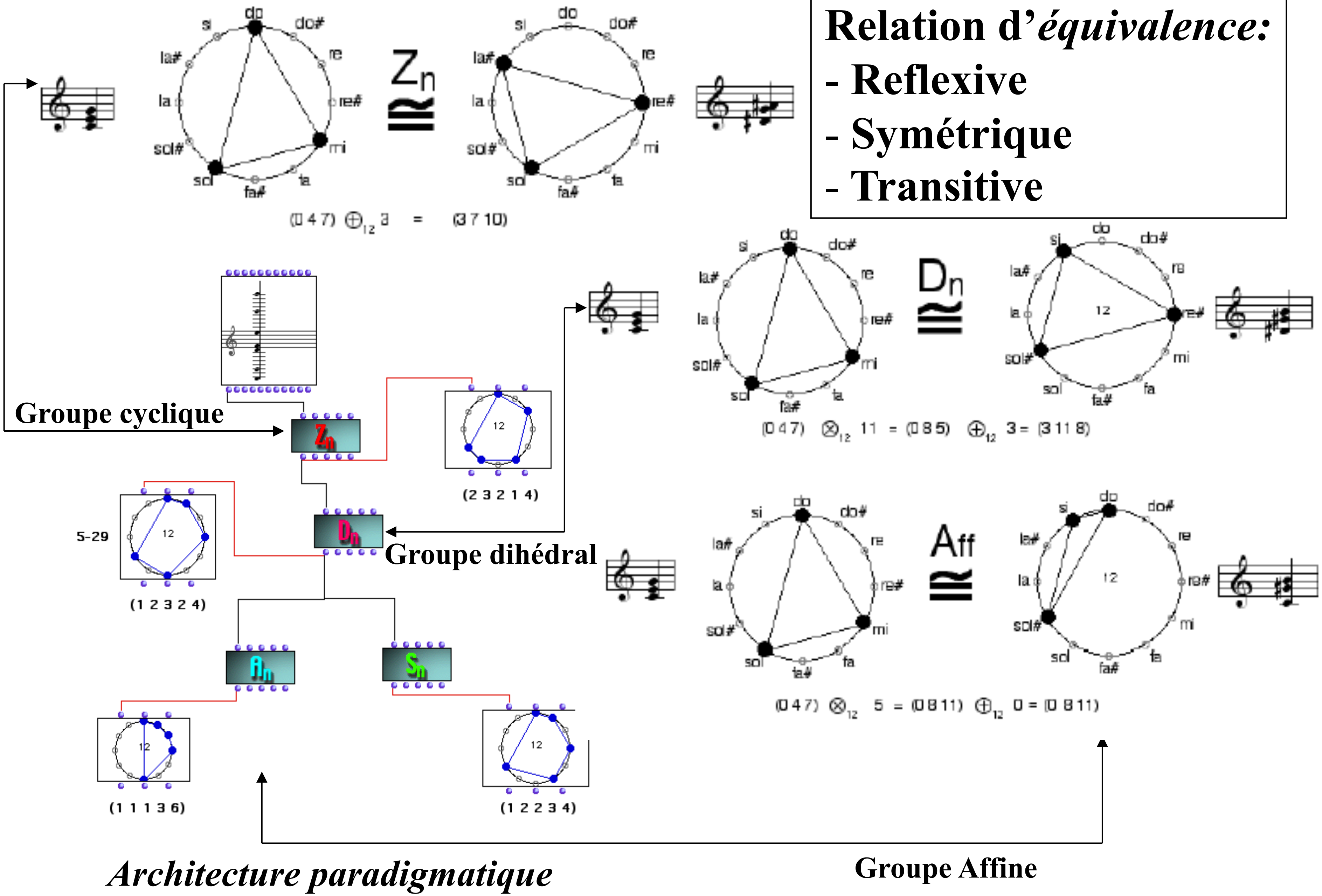




$6_0 \cup 6_1 \cup 6_3$ $6_0 \cup 6_3 \cup 6_5$

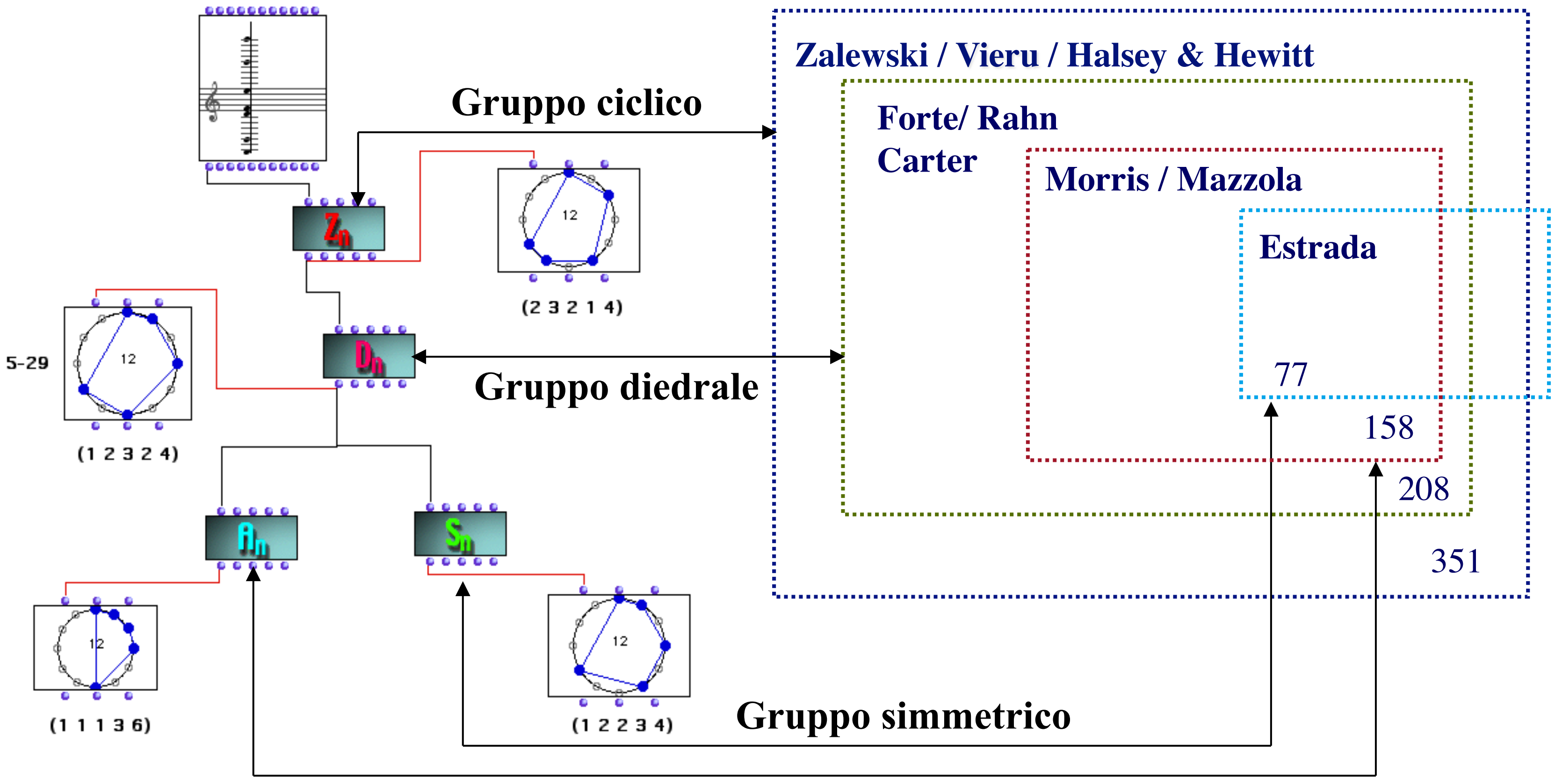
Les groupes comme “paradigmes”

Relation d’*équivalence:*
- **Reflexive**
- **Symétrique**
- **Transitive**



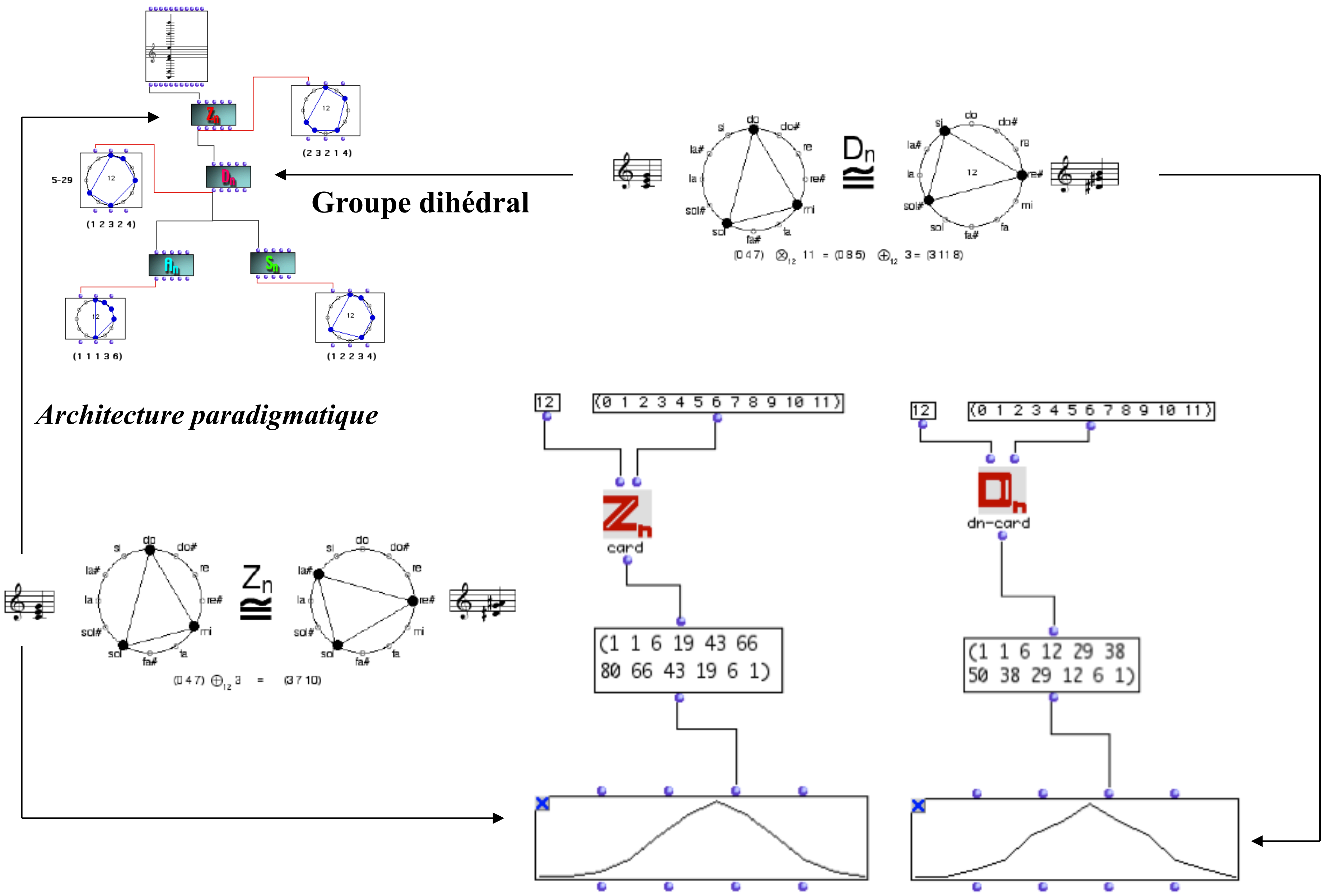
Classification paradigmatique des structures musicales

$G\backslash k$	1	2	3	4	5	6	7	8	9	10	11	12	
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1	
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1	**Set Theory**
$\mathrm{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1	



Architettura paradigmatica

Classes d'équivalence d'accords et groupes de transformations

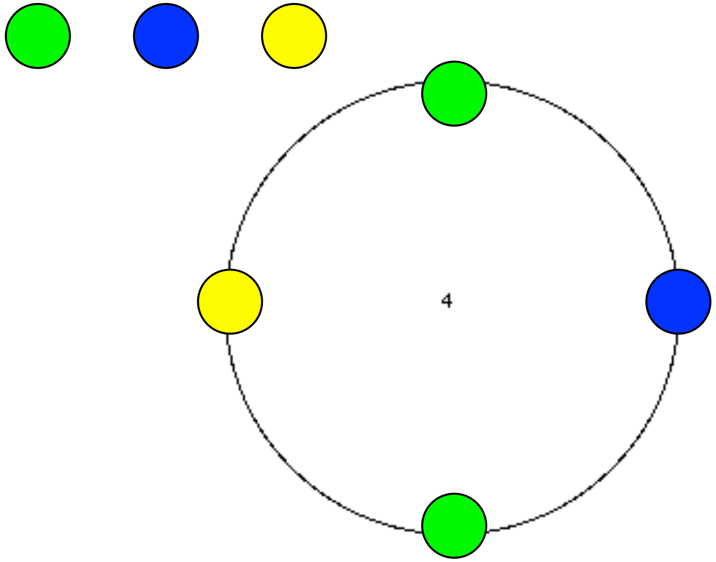


Enumeration des orbites par rapport à l'action d'un groupe

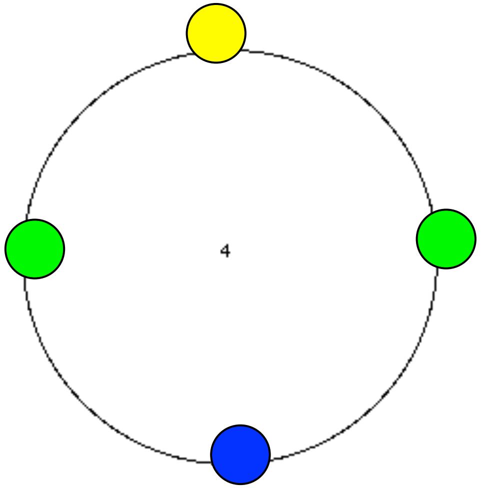
Lemme de Burnside

$$n = \frac{1}{|G|}\sum_{g\in G}|X^g|.$$

$$X^g = \{x \in X : gx = x\}$$



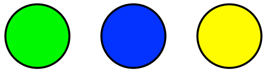
$\cong$

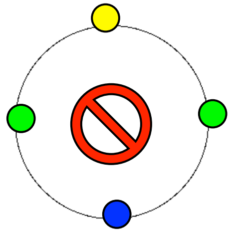


?

⇒ Trouver le nombre de configurations possibles

Enumération d'orbites par rapport à l'action d'un groupe

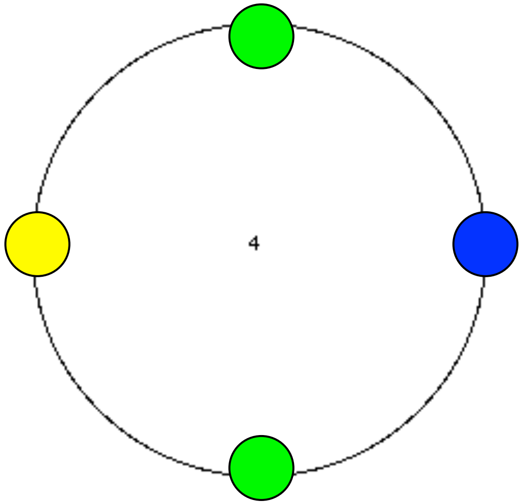




Lemme de Burnside

$$n = \frac{1}{|G|}\sum_{g\in G} |X^g|.$$

$X^g = \{x \in X : gx = x\}$



Action de **Z/4Z**

T_0 = identité
T_1 = rotation de 90°
T_2 = rotation de 180°
T_3 = rotation de 270°

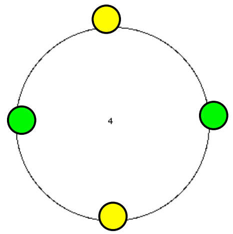
Configurations possibles = $3^4 = 81$

T_0 fixe toute configuration => $|X^{T_0}|=81$

T_1 fixe toute configuration monochromes => $|X^{T_1}|=3$

T_3 idem

T_2 fixe toute configuration «double-diamètre » => $|X^{T_1}| = 3^2 = 9$



n = 1/4 (81+3+3+9) = 24

Enumération d'accords et dans un système tempéré

(Reiner, 1985)

Z_n

$$\#\text{ of }k\text{-chords} = \frac{1}{n}\sum_{j|(n,k)} \phi(j)\binom{n/j}{k/j} = \frac{1}{n}\Phi_n(k),$$

D_n

$$\#\text{ of }k\text{-chords} = \begin{cases} \frac{1}{2n}\left[\Phi_n(k) + n\binom{(n-1)/2}{[k/2]}\right], & \text{if } n \text{ is odd,} \\ \frac{1}{2n}\left[\Phi_n(k) + n\binom{n/2}{k/2}\right], & \text{if } n \text{ is even and } k \text{ is even,} \\ \frac{1}{2n}\left[\Phi_n(k) + n\binom{(n/2)-1}{[k/2]}\right], & \text{if } n \text{ is even and } k \text{ is odd.} \end{cases}$$

D_n

$k =$	0	1	2	3	4	5	6	7	8	9	10	11	12
number	1	1	6	12	29	38	50	38	29	12	6	1	1

- D. Halsey & E. Hewitt: « Eine gruppentheoretische Methode in der Musik-theorie », *Jahresber. Der Dt. Math.-Vereinigung*, 80, 1978.
- D. Reiner: «Enumeration in Music Theory», *Amer. Math. Month*. 92:51-54, 1985
- H. Fripertinger: «Enumeration in Musical Theory», *Beiträge zur Elektr. Musik*,1,1992
- R.C. Read: « Combinatorial problems in the theory of music », *Discrete Math.*, 1997
- H. Fripertinger: « Enumeration of mosaics », *Discrete Math.*, 1999
- H. Fripertinger: « Enumeration of non-isomorphic canons », *Tatra Mt. Math. Publ.*, 2001
- M. Broué : « Les tonalités musicales vues par un mathématicien », *Le temps des savoirs, Revue de l'Institut Universitaire de France,* 2002
- David J. Hunter & Paul T. von Hippel : « How Rare Is Symmetry in Musical 12-Tone Rows? », *The American Mathematical Monthly*, Vol. 110, No. 2., Feb., 2003
- H. Fripertinger: « Tiling problems in music theory », in *Perspectives in Mathematical and Computational Music Theory* (Mazzola, Noll, Puebla ed., Epos, 2004)
- Rachel W. Hall & P. Klingsberg: « Asymmetric Rhythms, Tiling Canons, and Burnside's Lemma », *Bridge Proceedings*, 2004
- …

La *Set Theory* d'Allen Forte: catalogue des *pitch-class sets*

complementare

name	pcs	vector
5-30	0,1,4,6,8	121321
5-31	0,1,3,6,9	114112
5-32	0,1,4,6,9	113221
5-33(12)	0,2,4,6,8	040402
5-34(12)	0,2,4,6,9	032221
5-35(12)	0,2,4,7,9	032140
5-Z36	0,1,2,4,7	222121
5-Z37(12)	0,3,4,5,8	212320
5-Z38	0,1,2,5,8	212221
6-1(12)	0,1,2,3,4,5	543210
6-2	0,1,2,3,4,6	443211
6-Z4(12)	0,1,2,4,5,6	432321
6-5	0,1,2,3,6,7	422232
6-Z6(12)	0,1,2,5,6,7	421242
6-7(6)	0,1,2,6,7,8	420243
6-8(12)	0,2,3,4,5,7	343230
6-9	0,1,2,3,5,7	342231
6-Z10	0,1,3,4,5,7	333321
6-Z11	0,1,2,4,5,7	333231
6-Z12	0,1,2,4,6,7	332232
6-Z13(12)	0,1,3,4,6,7	324222

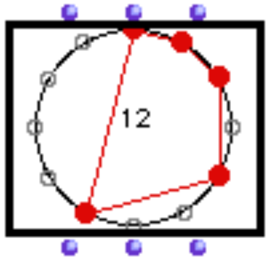
name	pcs	vector
7-30	0,1,2,4,6,8,9	343542
7-31	0,1,3,4,6,7,9	336333
7-32	0,1,3,4,6,8,9	335442
7-33	0,1,2,4,6,8,10	262623
7-34	0,1,3,4,6,8,10	254442
7-35	0,1,3,5,6,8,10	254361
7-Z36	0,1,2,3,5,6,8	444342
7-Z37	0,1,3,4,5,7,8	434541
7-Z38	0,1,2,4,5,7,8	434442
6-Z37(12)	0,1,2,3,4,8	
6-Z38(12)	0,1,2,3,7,8	
6-Z39	0,2,3,4,5,8	
6-Z40	0,1,2,3,5,8	
6-Z41	0,1,2,3,6,8	
6-Z42(12)	0,1,2,3,6,9	

5-Z36 0,1,2,4,7 222121

7-Z36 0,1,2,3,5,6,8 444342

Relation Z

Vecteur d'intervalles et relation Z

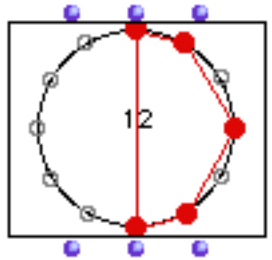


5-30	0,1,4,6,8	121321
5-31	0,1,3,6,9	114112
5-32	0,1,4,6,9	113221
5-33(12)	0,2,4,6,8	040402
5-34(12)	0,2,4,6,9	032221
5-35(12)	0,2,4,7,9	032140
5-Z36	0,1,2,4,7	222121
5-Z37(12)	0,3,4,5,8	212320
5-Z38	0,1,2,5,8	212221
6-1(12)	0,1,2,3,4,5	543210
6-2	0,1,2,3,4,6	443211
6-Z4(12)	0,1,2,4,5,6	432321
6-5	0,1,2,3,6,7	422232
6-Z6(12)	0,1,2,5,6,7	421242
6-7(6)	0,1,2,6,7,8	420243
6-8(12)	0,2,3,4,5,7	343230
6-9	0,1,2,3,5,7	342231
6-Z10	0,1,3,4,5,7	333321
6-Z11	0,1,2,4,5,7	333231
6-Z12	0,1,2,4,6,7	332232
6-Z13(12)	0,1,3,4,6,7	324222

5-Z36 0,1,2,4,7 222121

7-30	0,1,2,4,6,8,9	343542
7-31	0,1,3,4,6,7,9	336333
7-32	0,1,3,4,6,8,9	335442
7-33	0,1,2,4,6,8,10	262623
7-34	0,1,3,4,6,8,10	254442
7-35	0,1,3,5,6,8,10	254361
7-Z36	0,1,2,3,5,6,8	444342
7-Z37	0,1,3,4,5,7,8	434541
7-Z38	0,1,2,4,5,7,8	434442

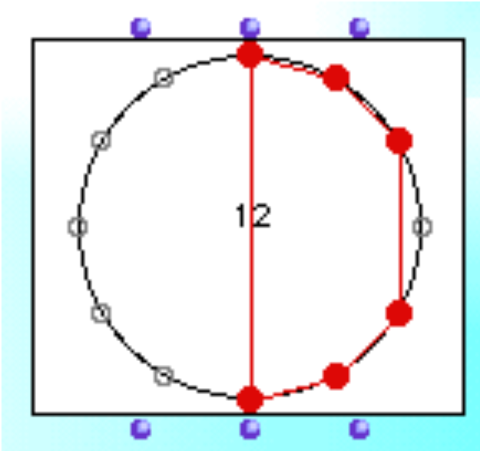
6-Z36	0,1,2,3,4,7
6-Z37(12)	0,1,2,3,4,8
6-Z38(12)	0,1,2,3,7,8
6-Z39	0,2,3,4,5,8
6-Z40	0,1,2,3,5,8
6-Z41	0,1,2,3,6,8
6-Z42(12)	0,1,2,3,6,9



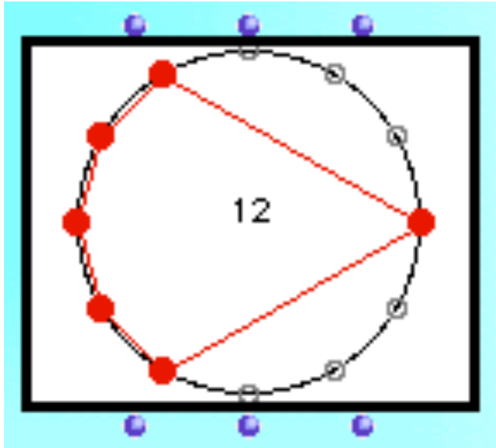
5-Z12

Théorème de l'hexacorde (ou théorème de Babbitt)

(Wilcox, Ralph Fox (?), Chemillier, Lewin, Mazzola, Schaub, …, Amiot [2006])



A



A'

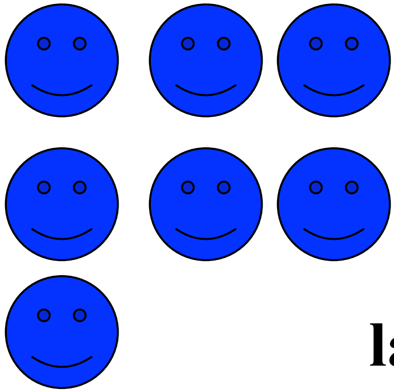
IV(*A*) = [4, 3, 2, 3, 2, 1] = [4, 3, 2, 3, 2, 1] = IV(*A'*)

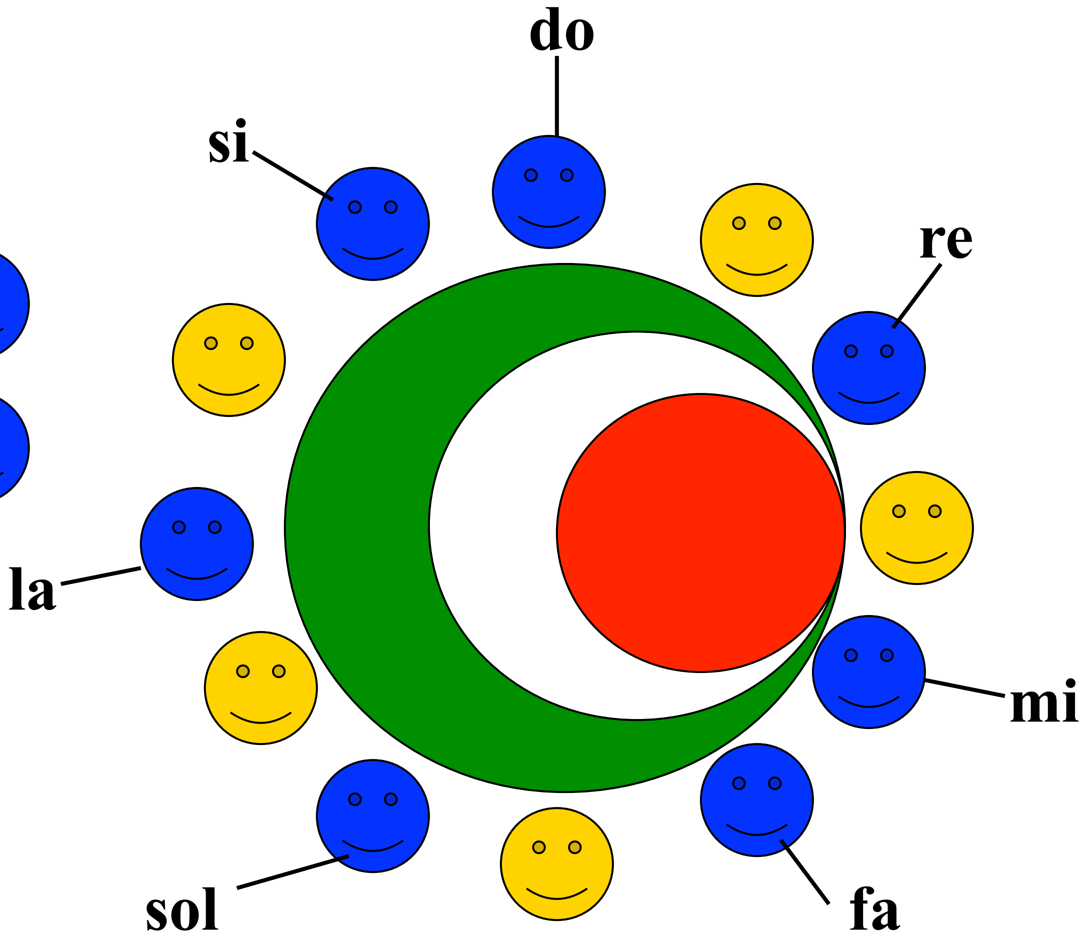
Un hexacorde et son complementaire ont le même vecteur d'intervalles

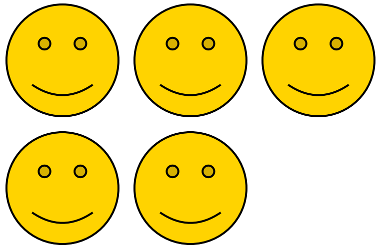
=> *OpenMusic*

La gamme diatonique comme *ME-Set*

<u>The dinner table problem</u> (Italian version)

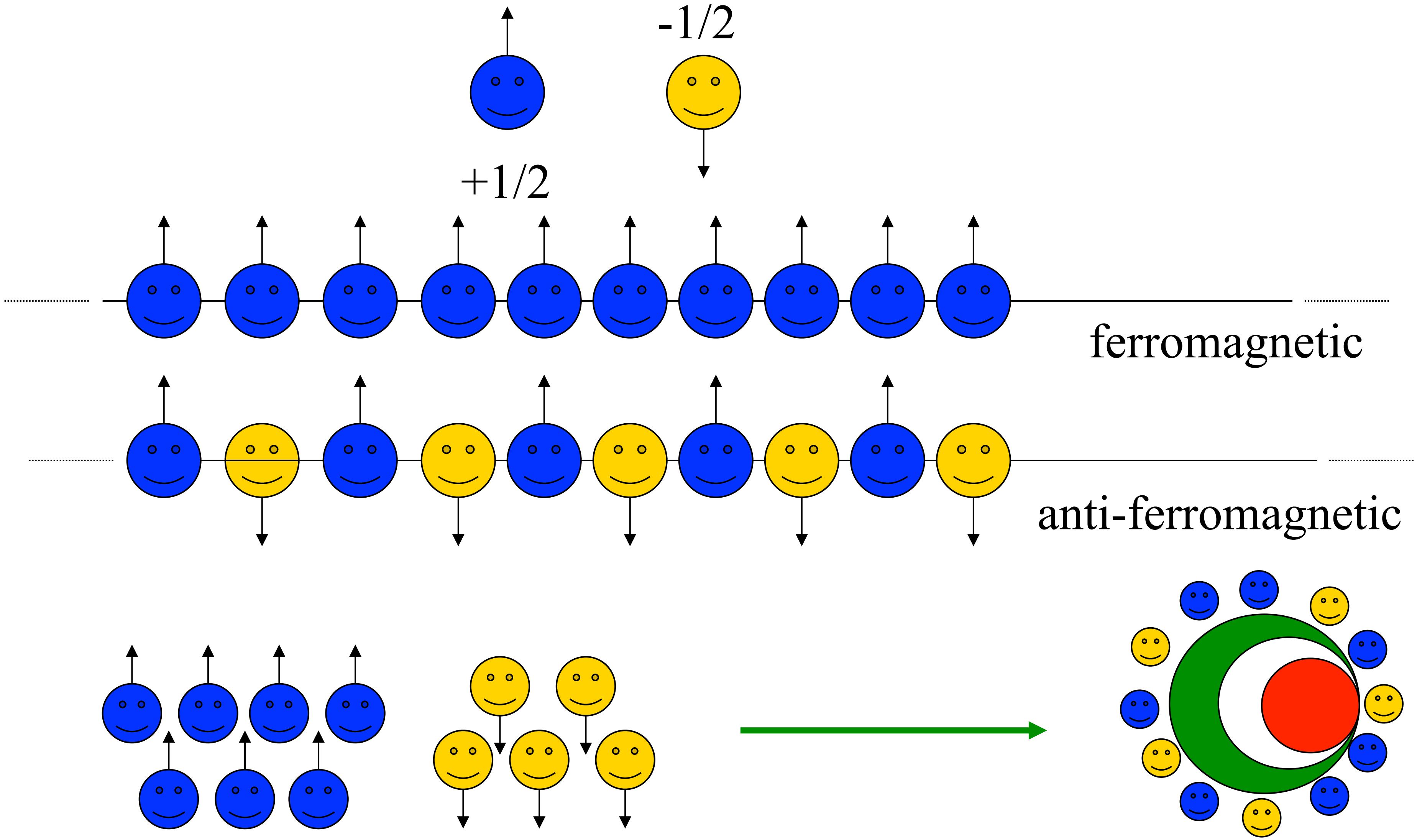






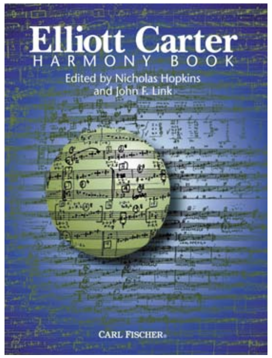
The one-dimensional antiferromagnetic spin-1/2 Ising Model

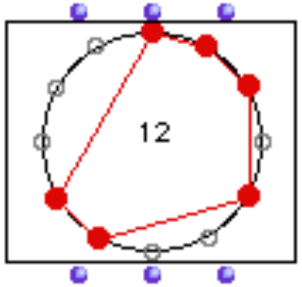
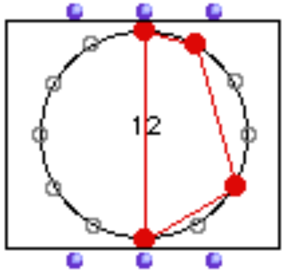
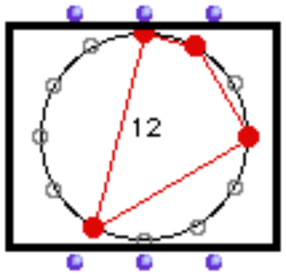
Jack Douthett &Richard Krantz, “Energy extremes and spin configurations for the one-dimensional antiferromagnetic Ising model with arbitrary-range interaction”, *J. Math. Phys.* 37 (7), July 1996



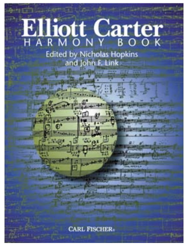
Elliott Carter: 90+ (1994)

« From about 1990, I have reduced my vocabulary of chords more and more to the six note chord n° 35 and the four note chords n° 18 and 23, which encompass all the intervals » (*Harmony Book*, 2002, p. ix)



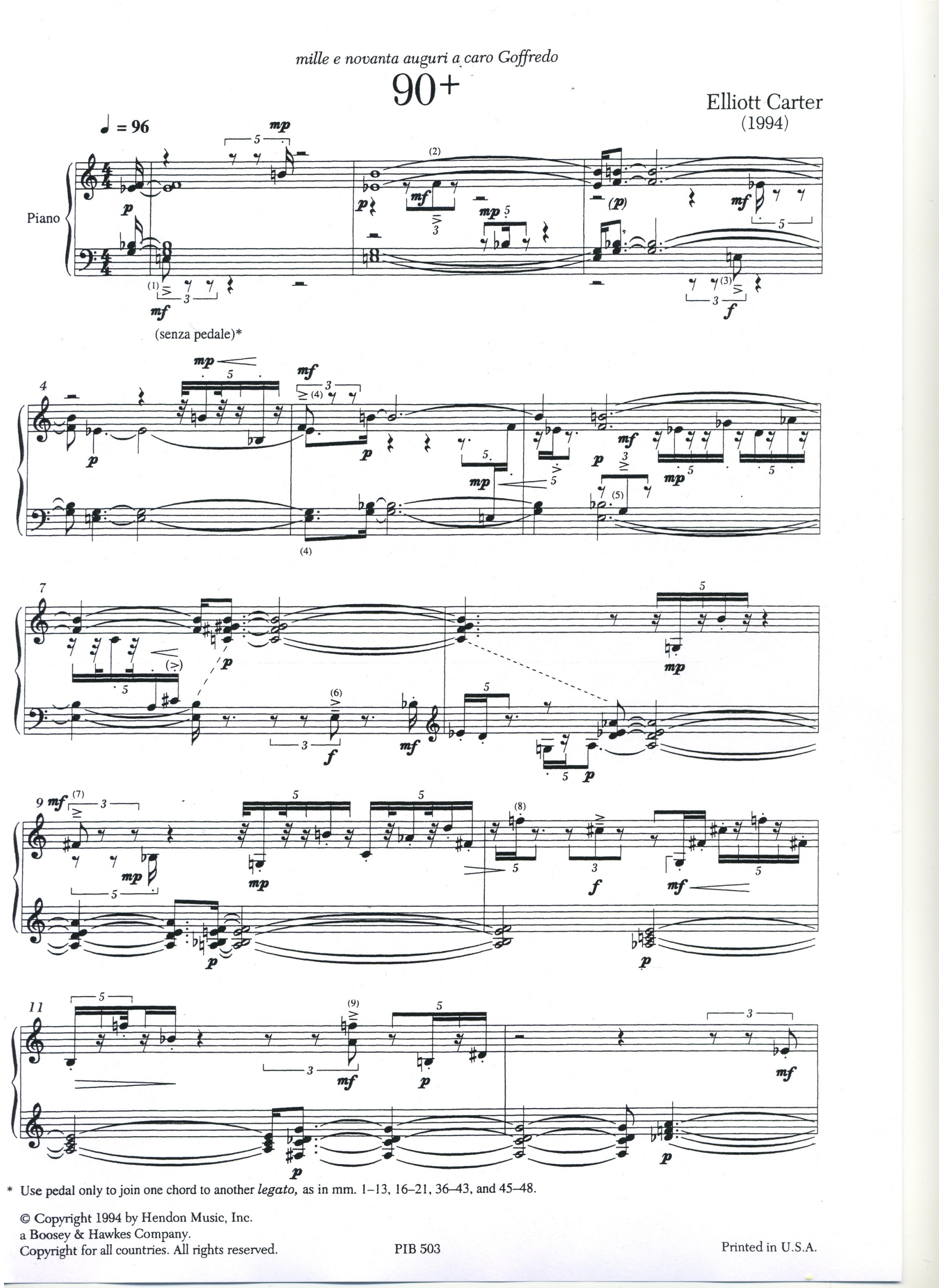
		pcset	*vector*	*name*
35		(0 1 2 4 7 8)	[322332]	6-Z17
		All-triad hexachord		
18		(0 1 4 6)	[111111]	4-Z15
23		(0 1 3 7)	[111111]	4-Z29

Elliott Carter : 90+ (1994)



- **Combinatoire d'accords**
  - **Hexacordes**
  - **Tetracordes**
  - **Triades**
- **Séries tous–intervalles**
  - ***Link–chords***
- **Polyrythmie et modulations métriques**

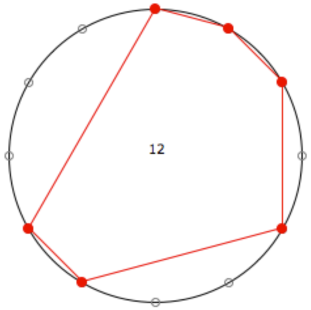
(piano: John Snijders)



Elliott Carter: 90+ (1994)

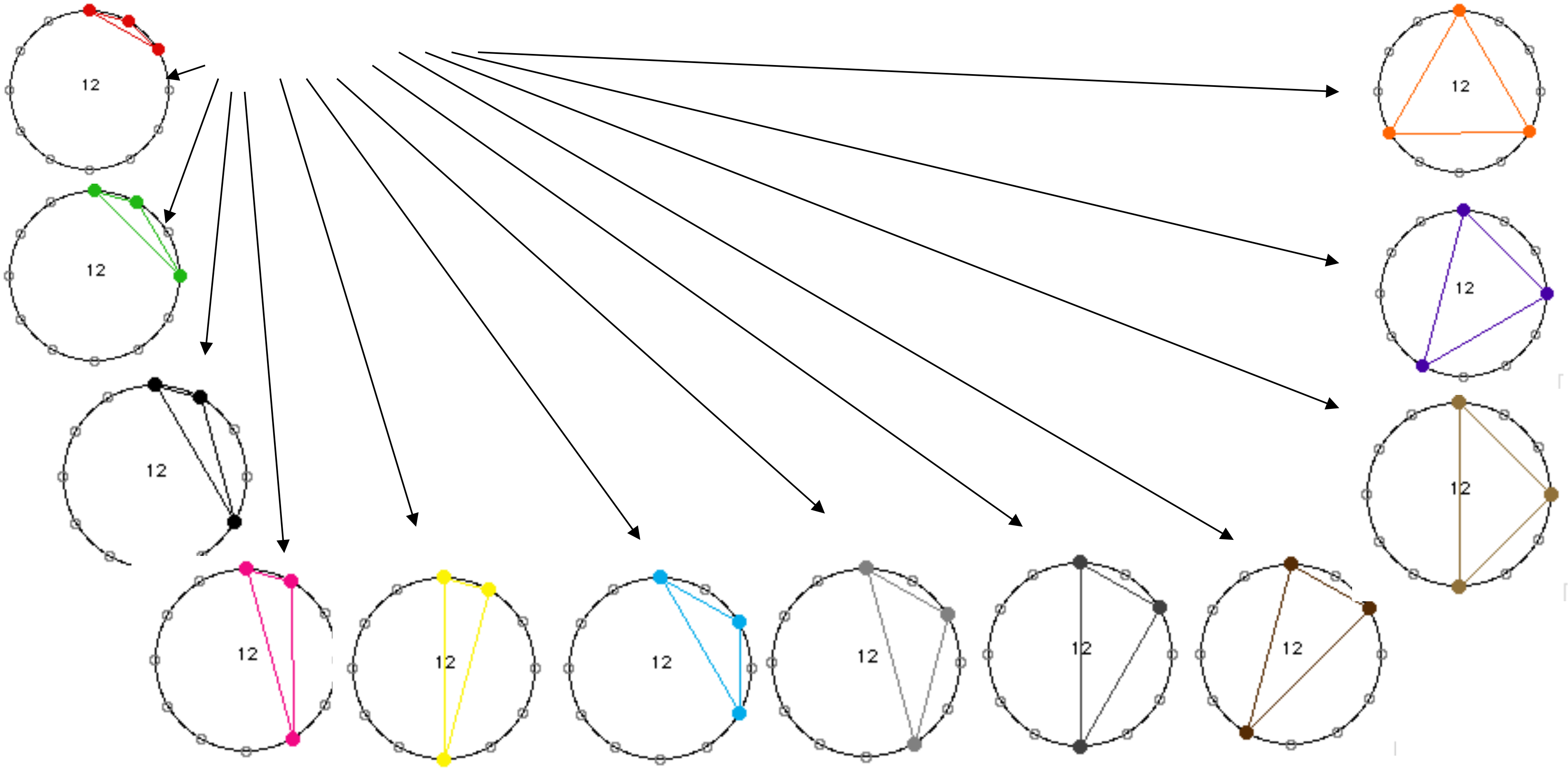
$G\backslash k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1
$\mathrm{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1

35

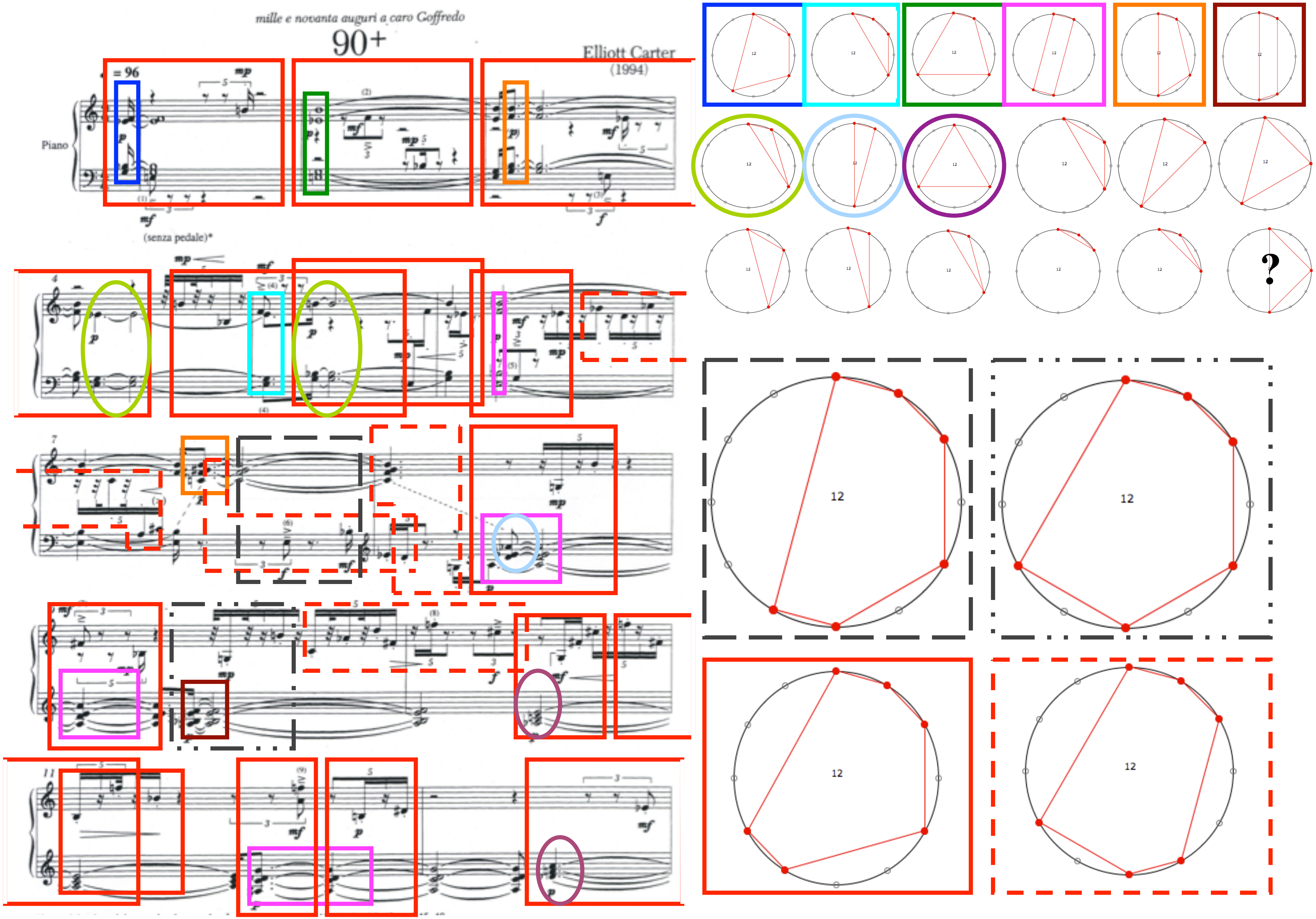


pcset	*vector*	*name*
(0 1 2 4 7 8)	[322332]	6-Z17

All-triad hexachord



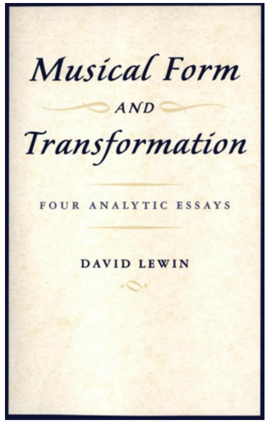
Elliott Carter : 90+ (1994) : combinatoire tetra/tricordale

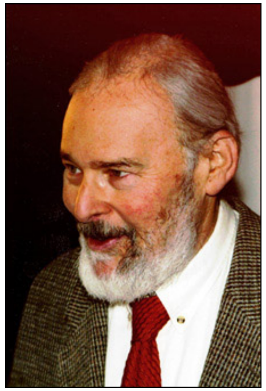


« *Making* and *Using* a Pcset Network for Stockhausen's Klavierstück III »

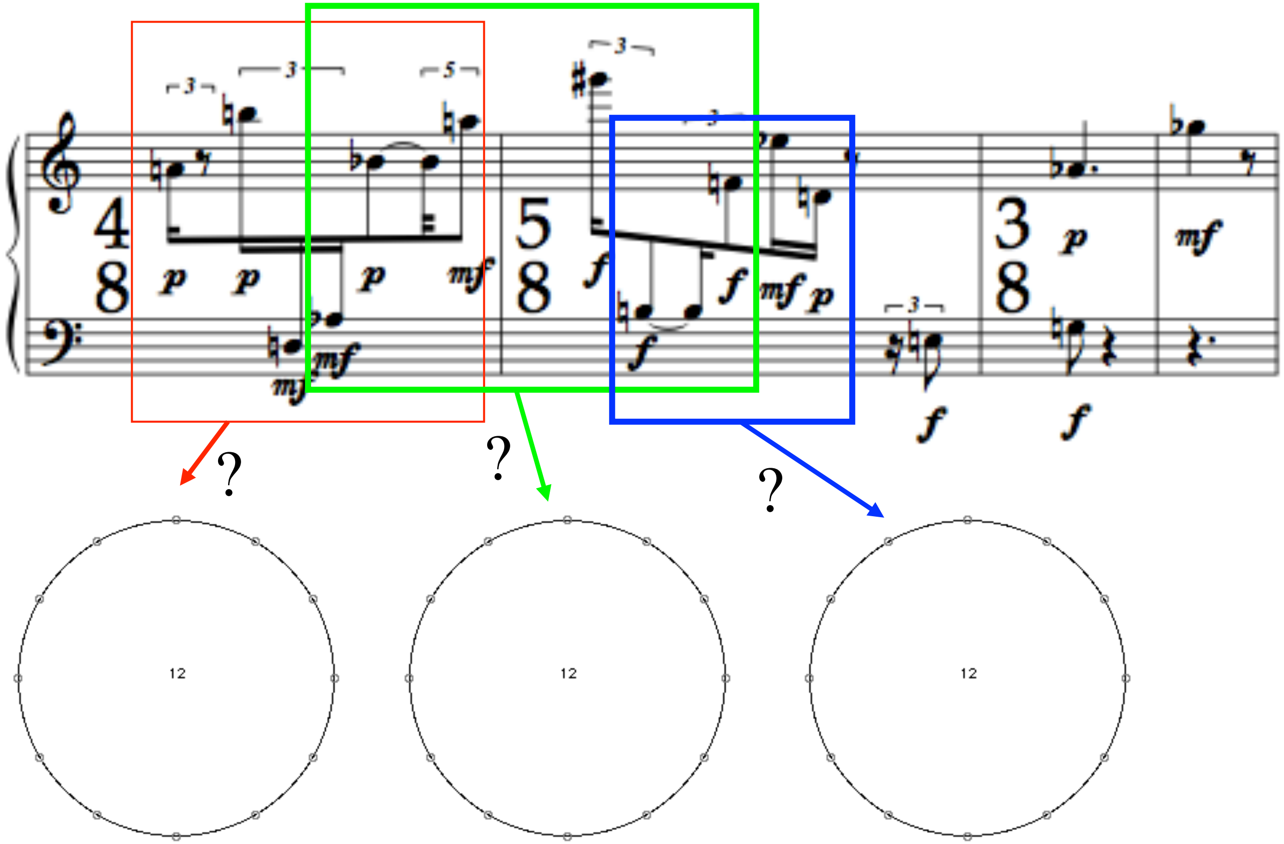
Trois interprétations :

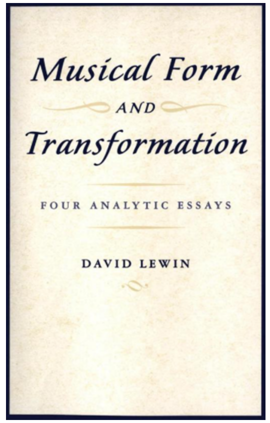
- **Henck**
- **Kontarsky**
- **Tudor**

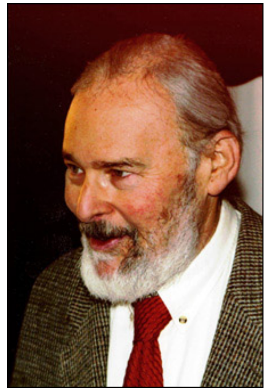




« *Making* and *Using* a Pcset Network for Stockhausen's Klavierstück III »



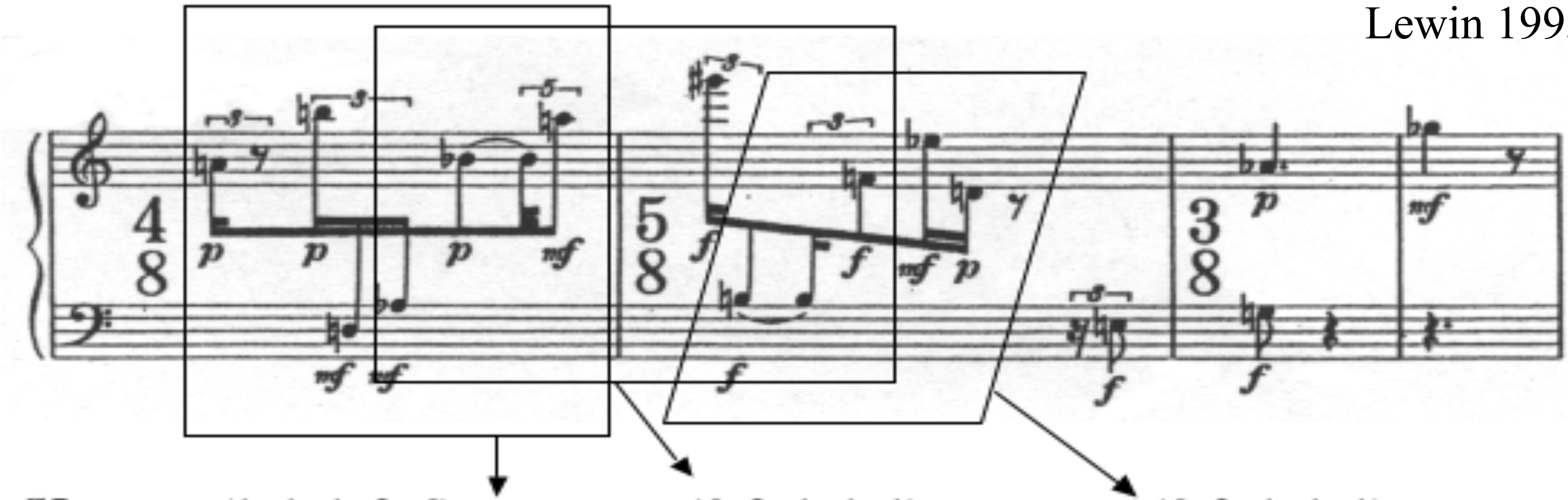




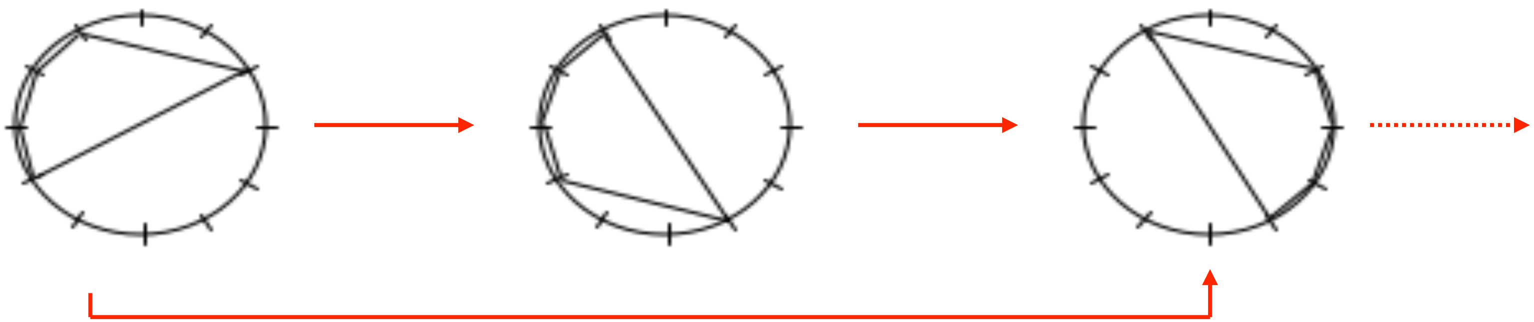
« The most ‘theoretical’ of the four essays, it focuses on the forms of one pentachord reasonably ubiquitous in the piece. A special **group of transformations** is developed, one suggested by the musical interrelations of the pentachord forms. Using that group, the essay arranges **all pentachord forms** of the music into a **spatial configuration** that illustrates network structure, for this particular phenomenon, over the entire piece. »

David Lewin, *Musical Form and Transformation,* YUP 1993

Lewin 1993

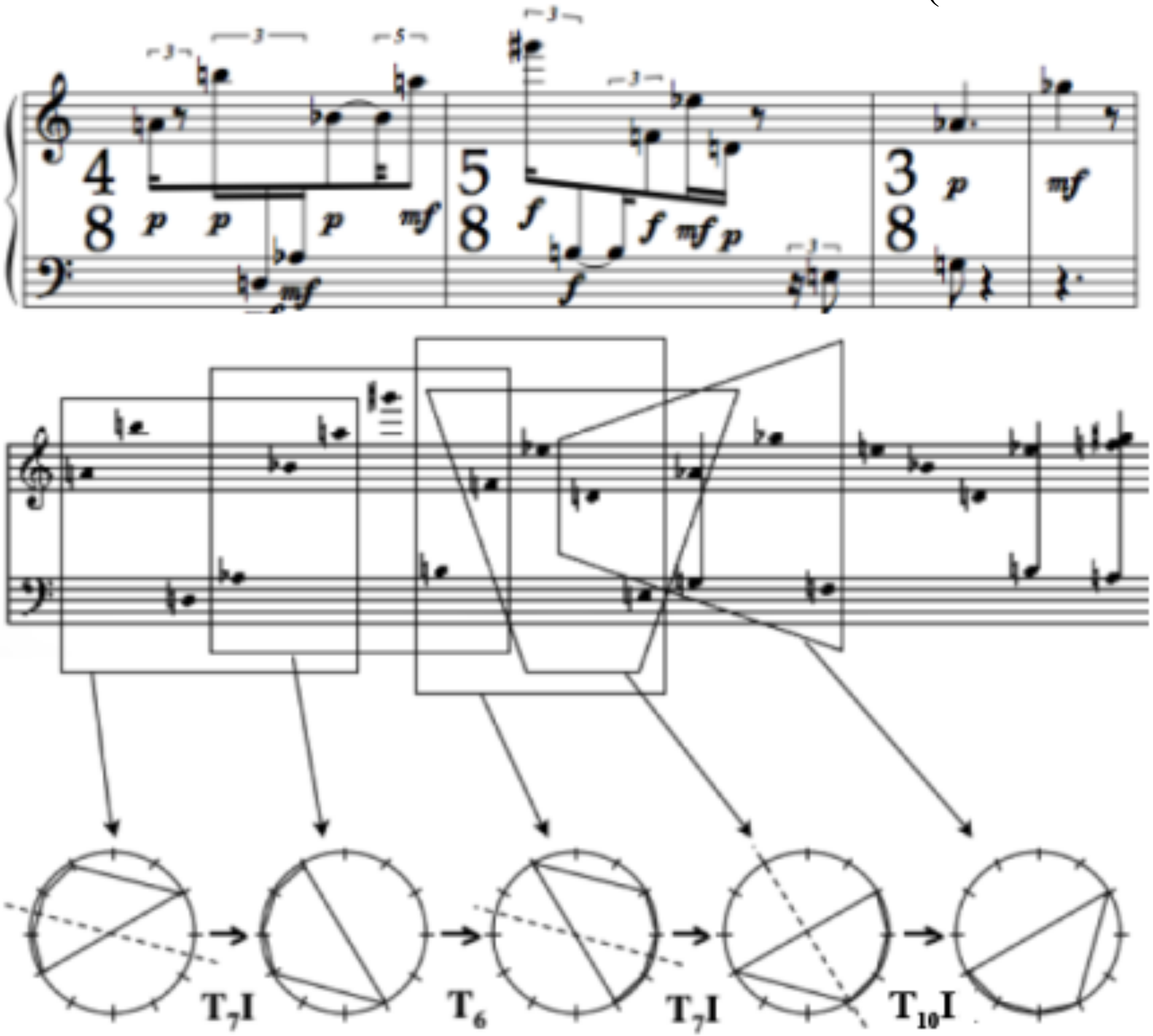


SI:	(1, 1, 1, 3, 6)	(6, 3, 1, 1, 1)	(6, 3, 1, 1, 1)
IFUNC:	[5 3 2 2 1 1 1 1 1 2 2 3]	[5 3 2 2 1 1 1 1 1 2 2 3]	[5 3 2 2 1 1 1 1 1 2 2 3]
VI:	[3 2 2 1 1 1]	[3 2 2 1 1 1]	[3 2 2 1 1 1]

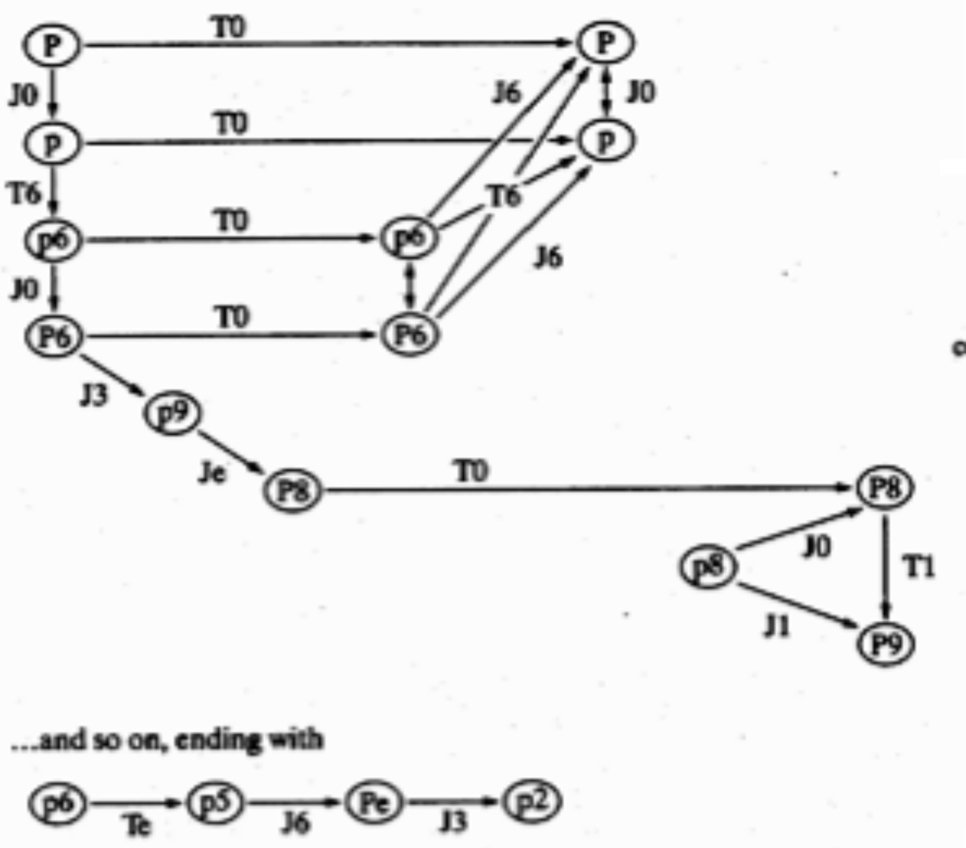


Segmentation par « imbriction »: progression transformationnelle

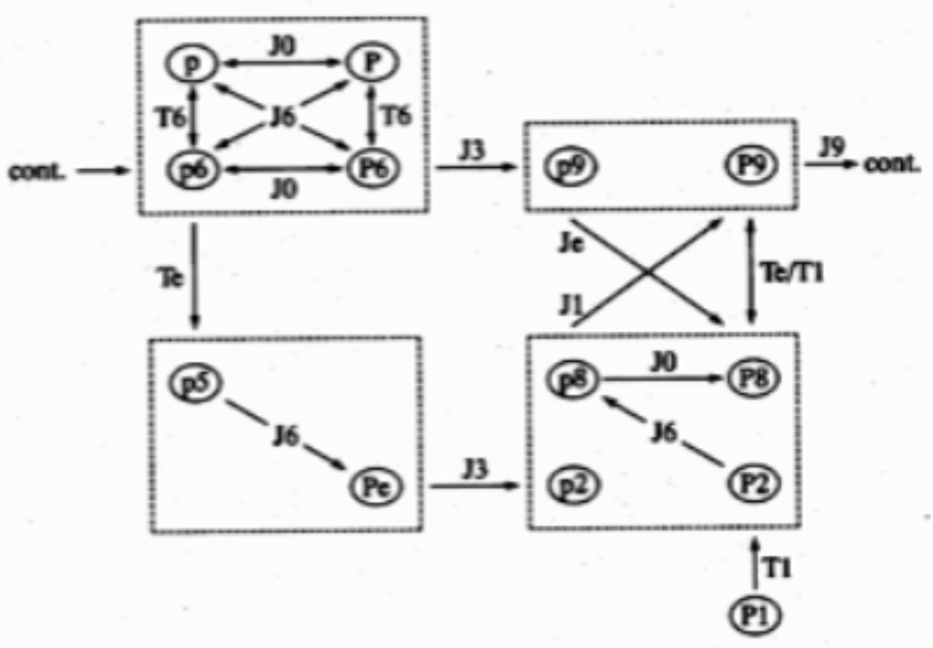
Stockhausen: *Klavierstück III* (Analisi di D. Lewin)



Progression transformationnelle *vs* réseau transformationnel



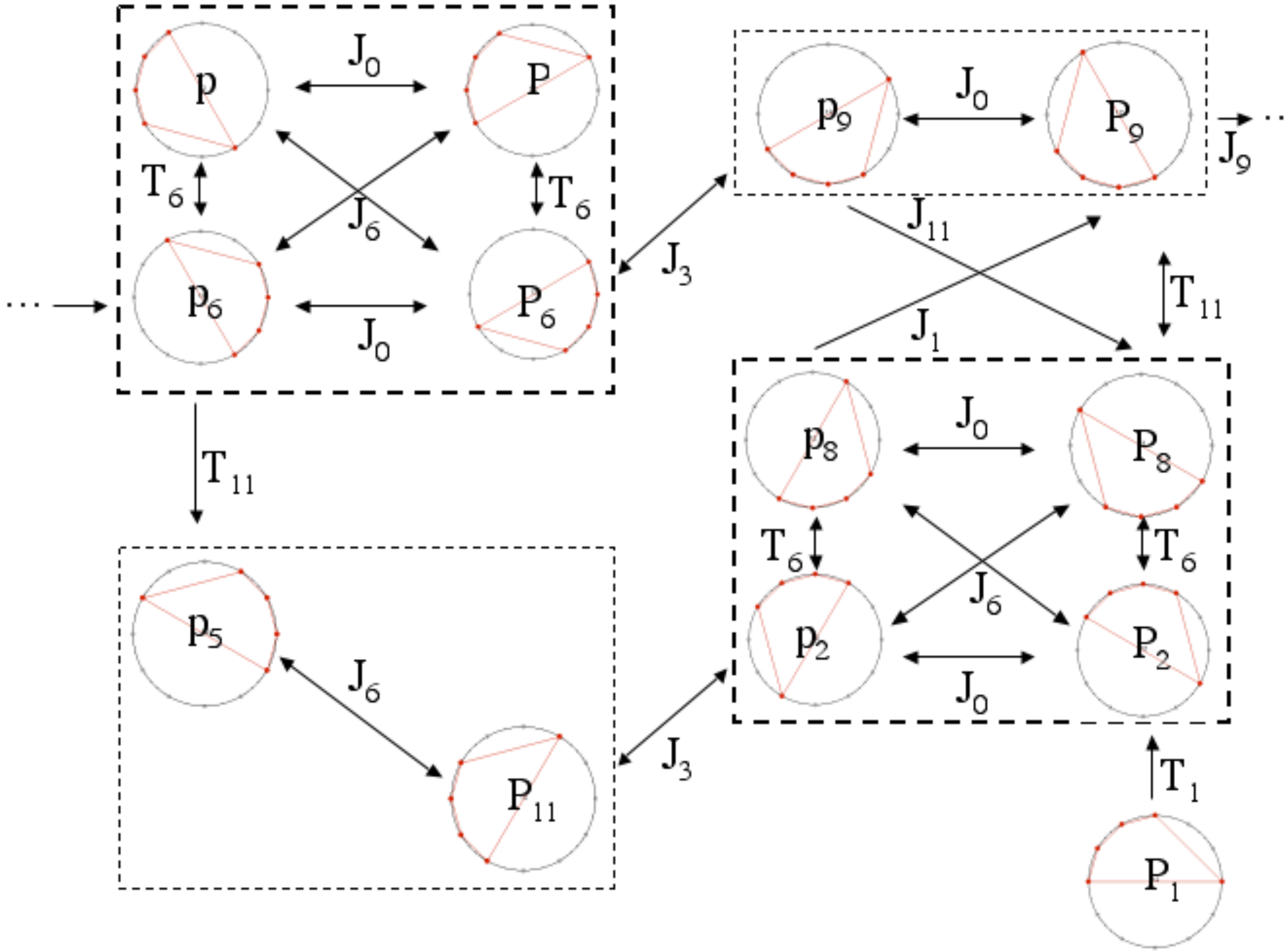
Example 2.4. A network whose left-to-right layout reflects the chronological progress of the piece through P/p forms.



« Rather than asserting a network that follows pentachord relations one at a time, according to the chronology of the piece, I shall assert instead a network that displays all the pentachord forms used and all their potentially functional interrelationships, in a very compactly organized little spatial configuration. »

Reseau transformationnel

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

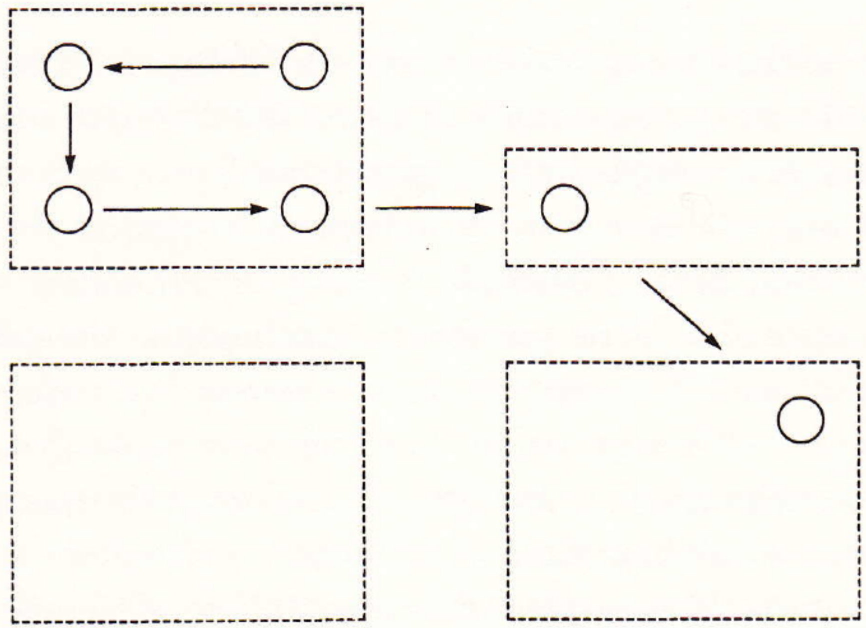


« […] the sequence of events moves within a clearly defined world of possible relationships, and because - in so moving - it makes the abstract space of such a world accessible to our sensibilities. That is to say that the story projects what one would traditionally call form. »

Parcours multiples d'écoute dans un réseau transformationnel

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

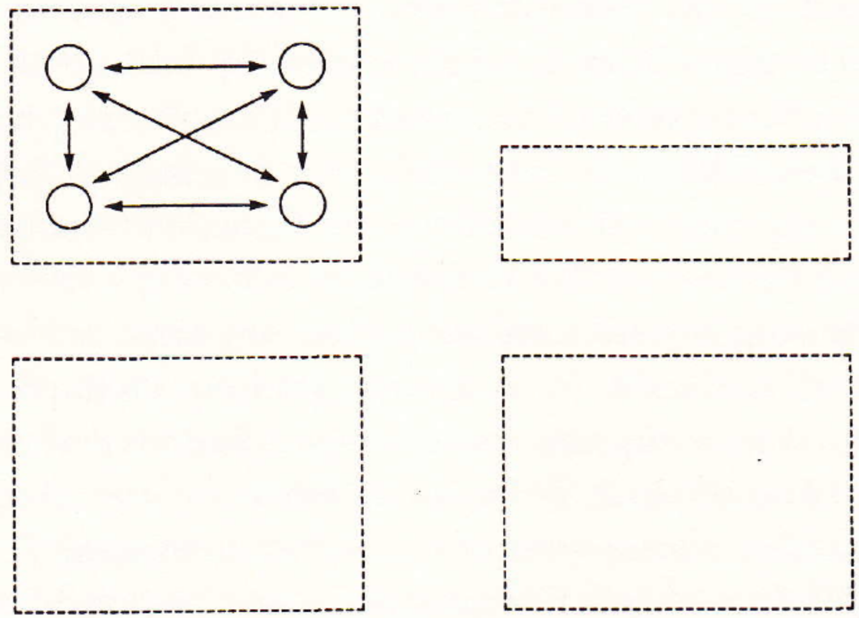
Pass 1 (mm. 1-5).



a

horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1

Pass 2 (mm. 5-8) goes back and elaborates the beginning area of pass 1.



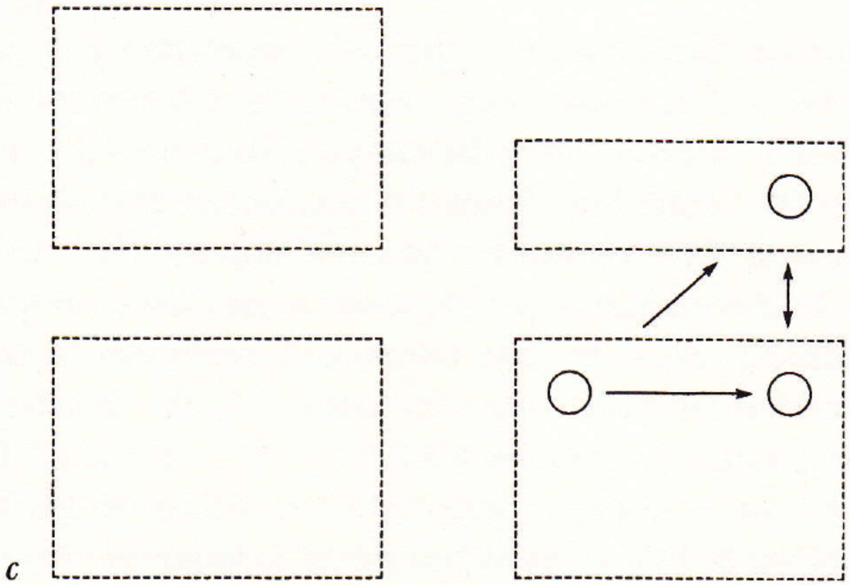
b

horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1

Parcours multiples d'écoute dans un réseau transformationnel

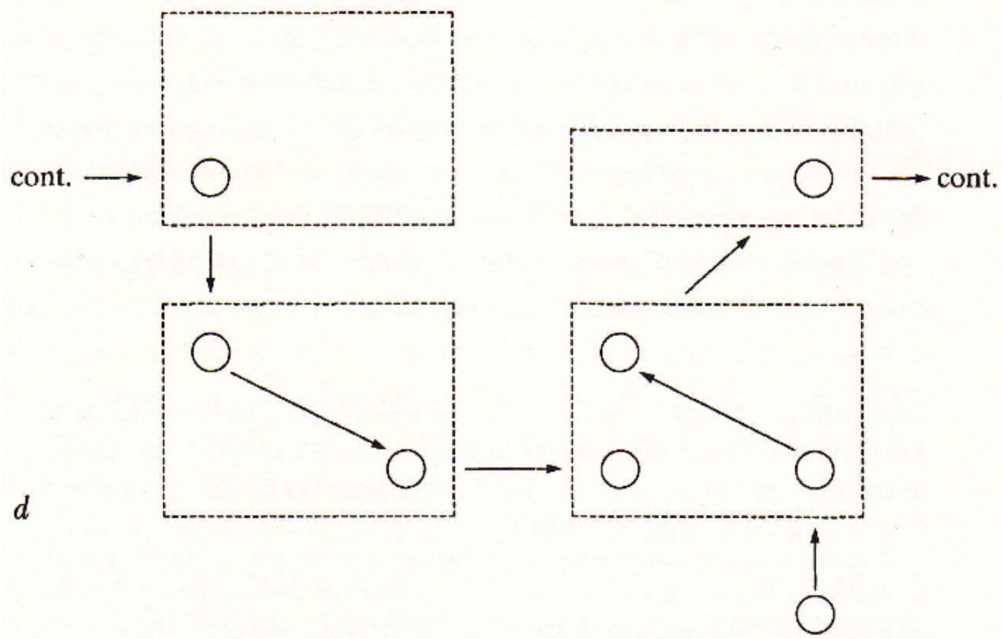
Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

Pass 3 (mm. 8-10) picks up and elaborates the ending area of pass 1.



horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1

Pass 4 (mm. 9-16) expands the p8 + P8 area of pass 3 to activate P2 and p2 as well. P2 is the "essential" incipit of pass 4; p2 is the end of the pass, and of the piece.



horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1

Exercices d'écoute

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

Example 2.7. An ear-training aid for listening to P/p forms and their interrelations.

« I doubt that [Nicholas] Cook [cf. *A Guide to Music Analysis*, 1987] would have much patience with my network analysis; I suspect he would read it as yet one more exercise in what he calls "cracking the code." Let me be the first to say emphatically that the network analysis is very far from an analysis of the piece, that I find it problematical, and that it took some effort for me to develop the aural agenda of [the ear-training exercises in] example 2.7. »

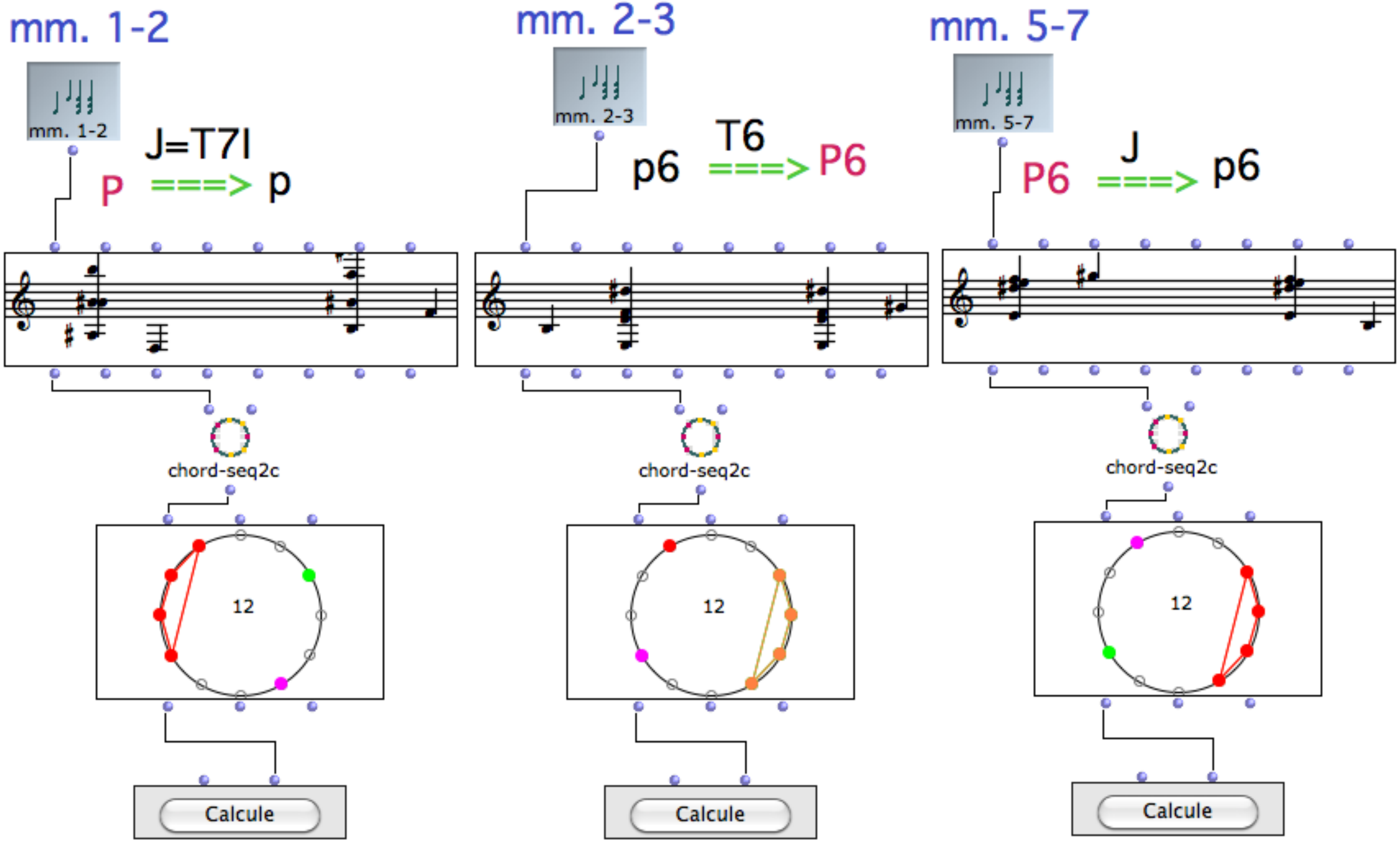
Exercices d’écoute

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

Example 2.7. An ear-training aid for listening to P/p forms and their interrelations.

« However, I must say that I enjoyed developing that agenda, which of course I did gradually as my work developed, and not in so neatly packaged a way as in this essay. I felt I was getting at something in the piece that very much involved “what the music did to me,” if only in one of its aspects. I felt I was responding in some measure to a strong sense of challenge I felt about the piece. No matter to what degree I am deluding myself, I miss in Cook the sense of <u>having to extend my ear in response to a sense of challenge</u> ».

Computer-Aided Transformational Analysis in OpenMusic



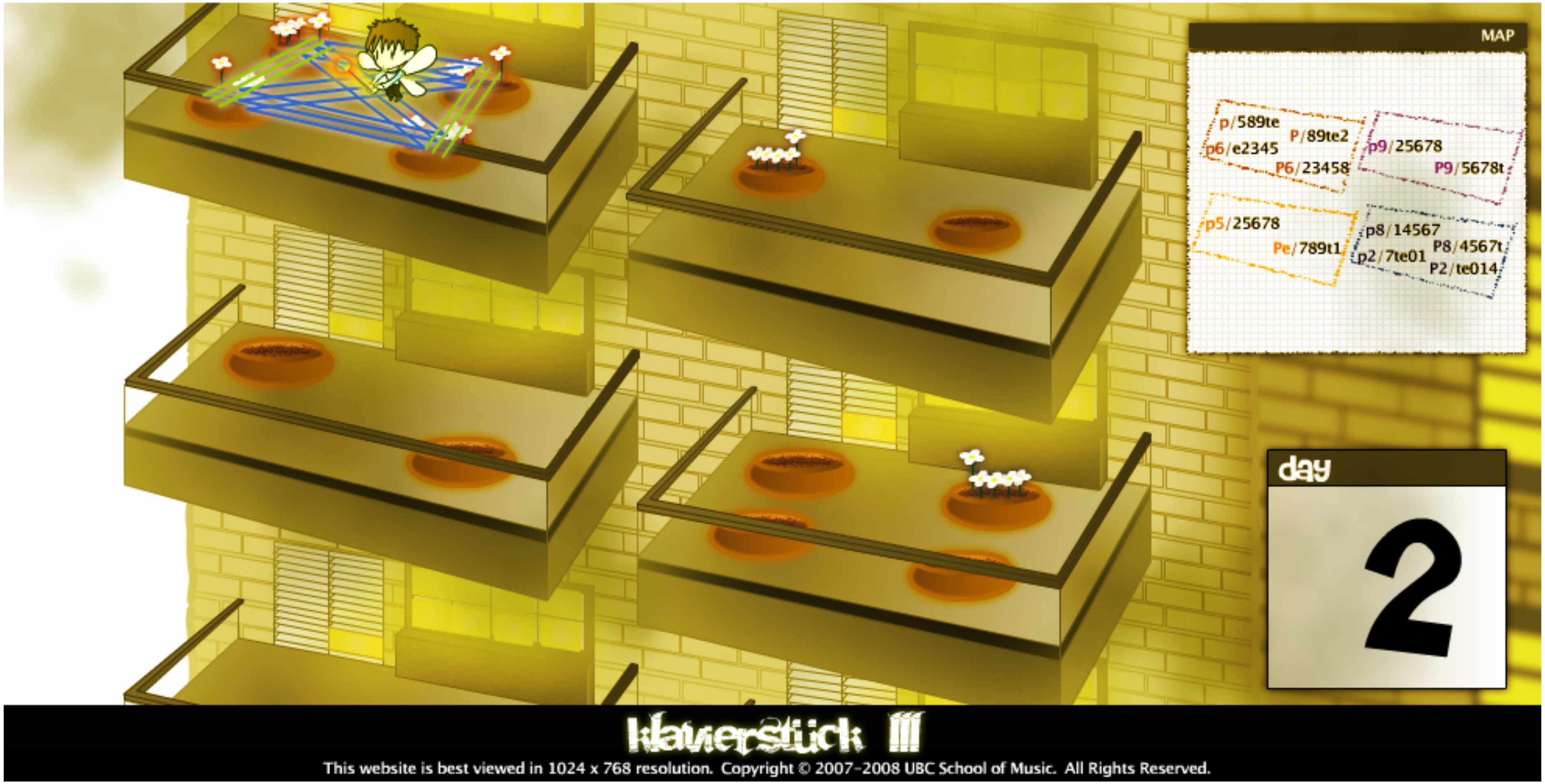
→ **http://recherche.ircam.fr/equipes/repmus/OpenMusic/**

→ **OpenMusic**

Visualisations multimédia de l'analyse transformationnelle

R. Attas : Metaphors in Motion: Agents and Representation in Transformational Analysis, *MTO,* 15(1), 2009
http://mto.societymusictheory.org/issues/mto.09.15.1/mto.09.15.1.attas.html

Animation 1. Klavierstück III



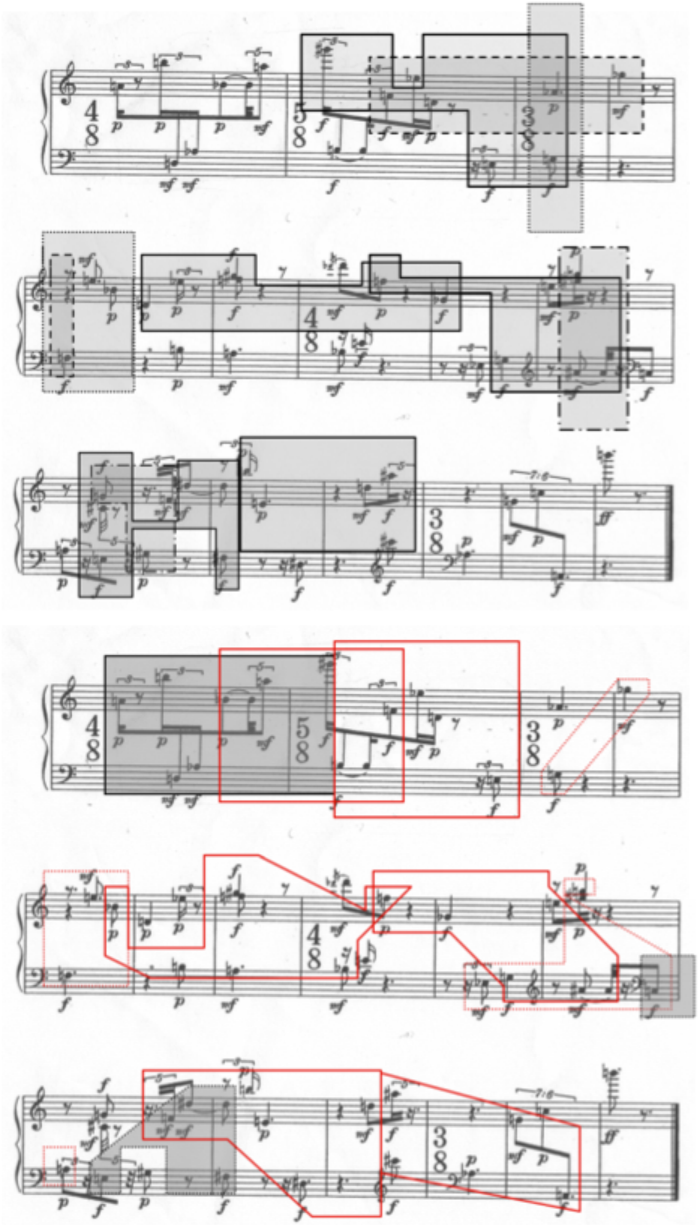
Visualisations multimédia de l'analyse transformationnelle

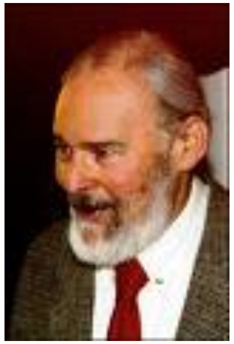
R. Attas : Metaphors in Motion: Agents and Representation in Transformational Analysis, *MTO*, 15(1), 2009
http://mto.societymusictheory.org/issues/mto.09.15.1/mto.09.15.1.attas.html

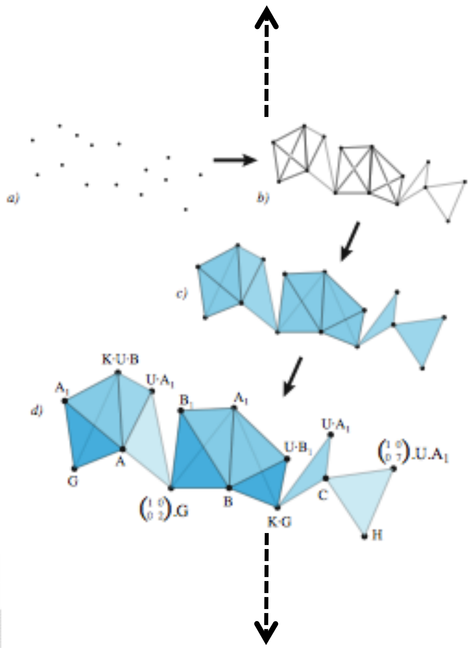
Animation 2. Grow Your Own Pentachord

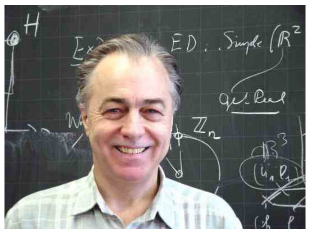
Vers une modélisation informatique de l'analyse transformationnelle

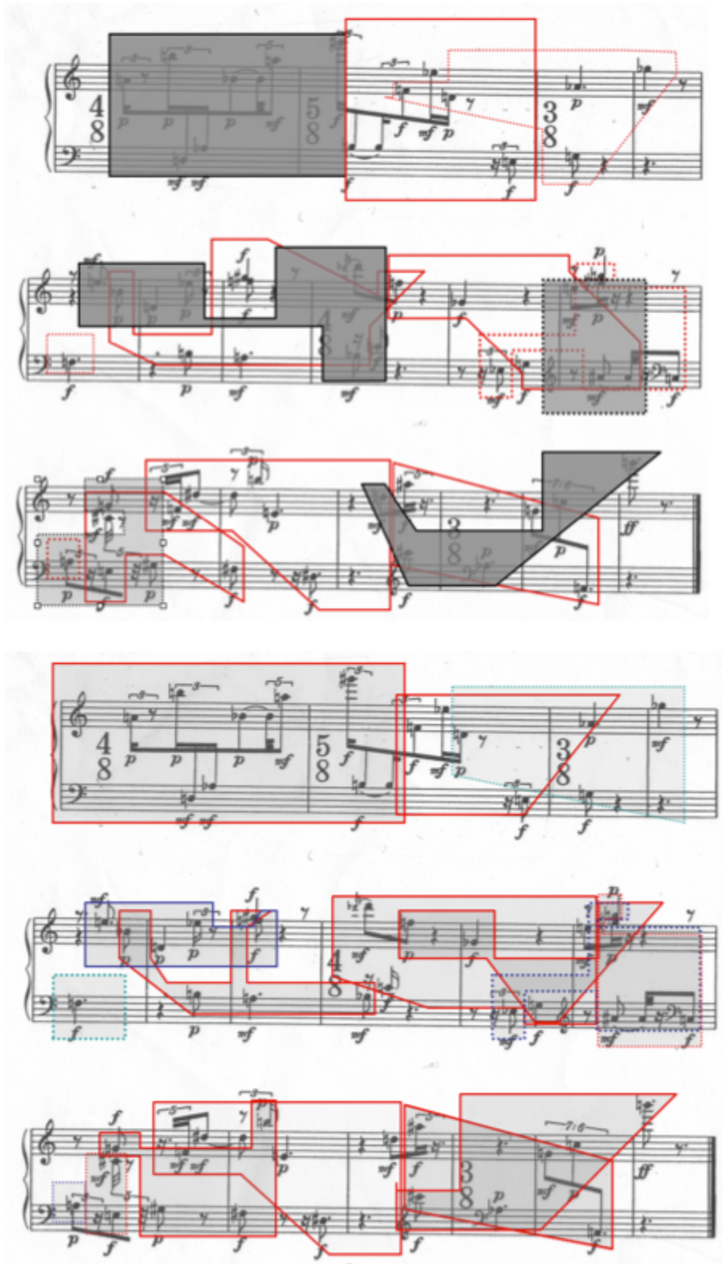
YunKang Ahn, L'analyse musicale computationnelle, thèse, Université de Paris VI / Ircam, déc 2009

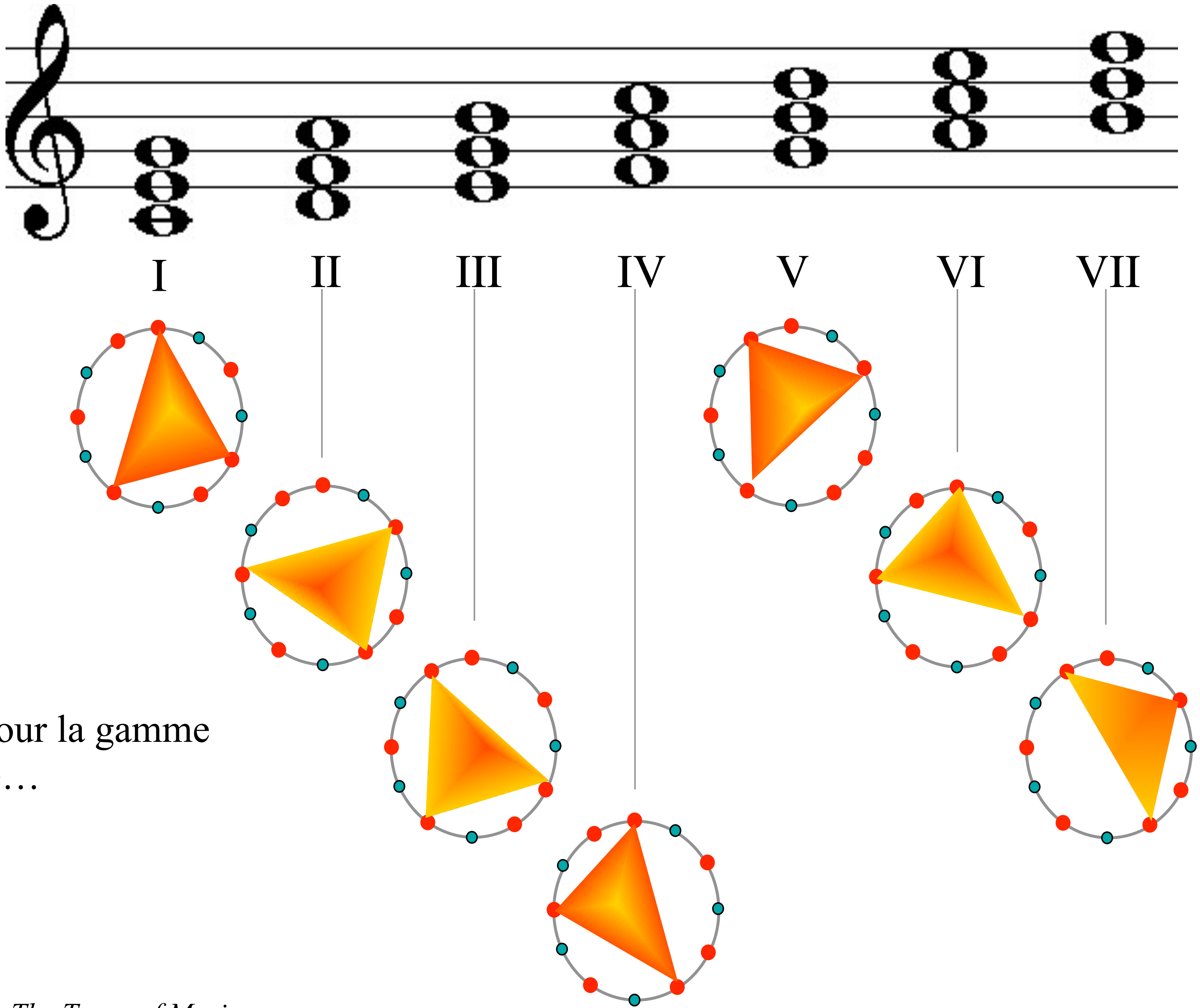




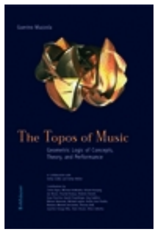




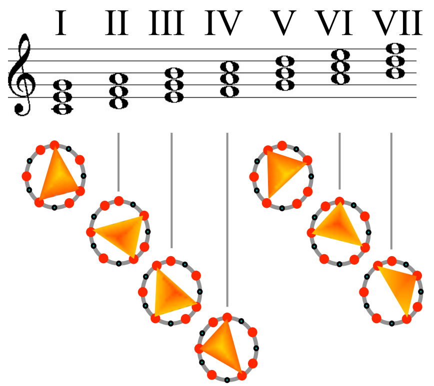




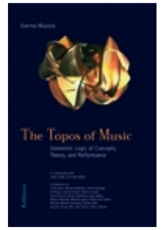
Un atlas pour la gamme diatonique…

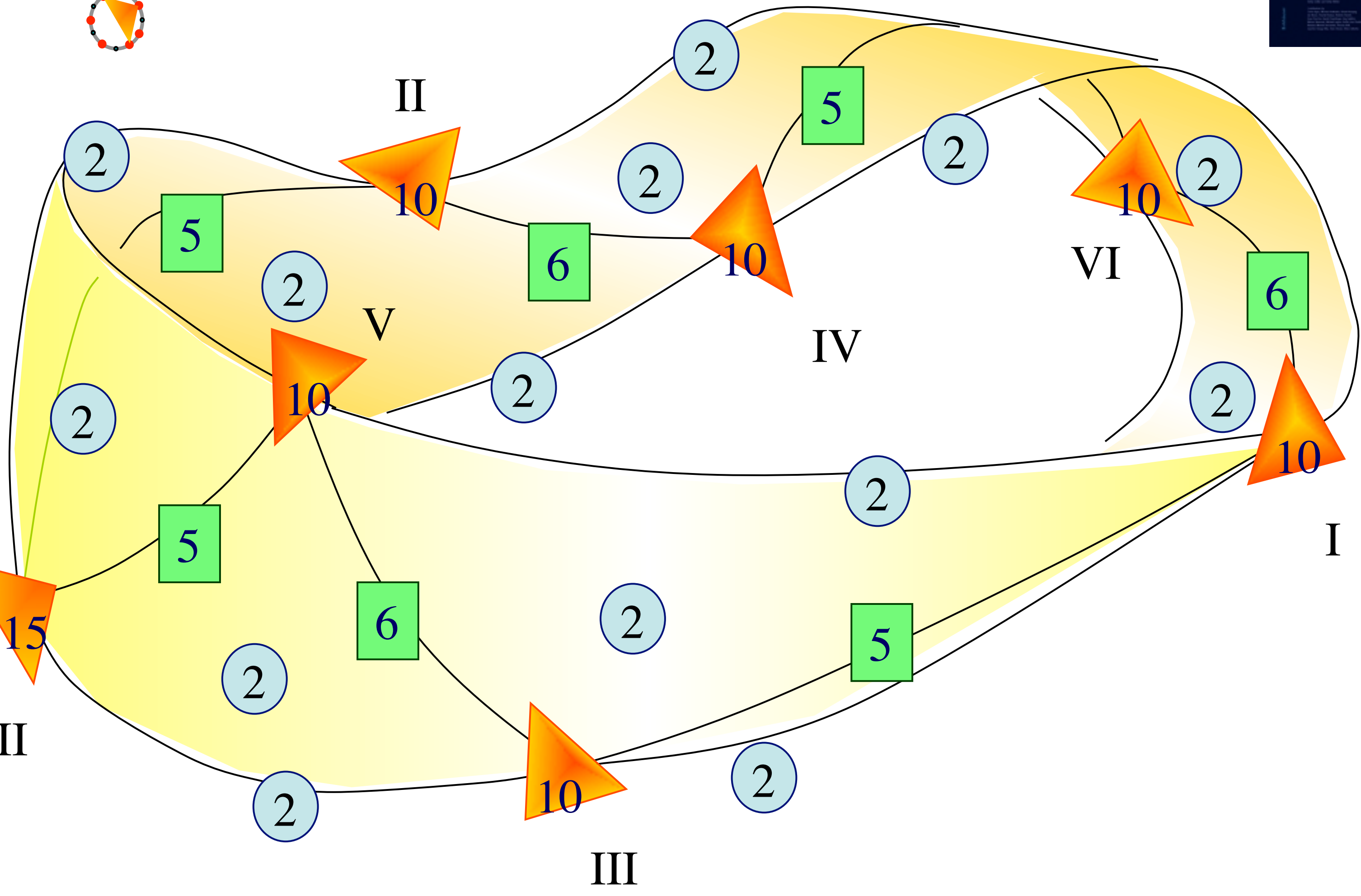


G. Mazzola, *The Topos of Music*



G. Mazzola, *The Topos of Music*

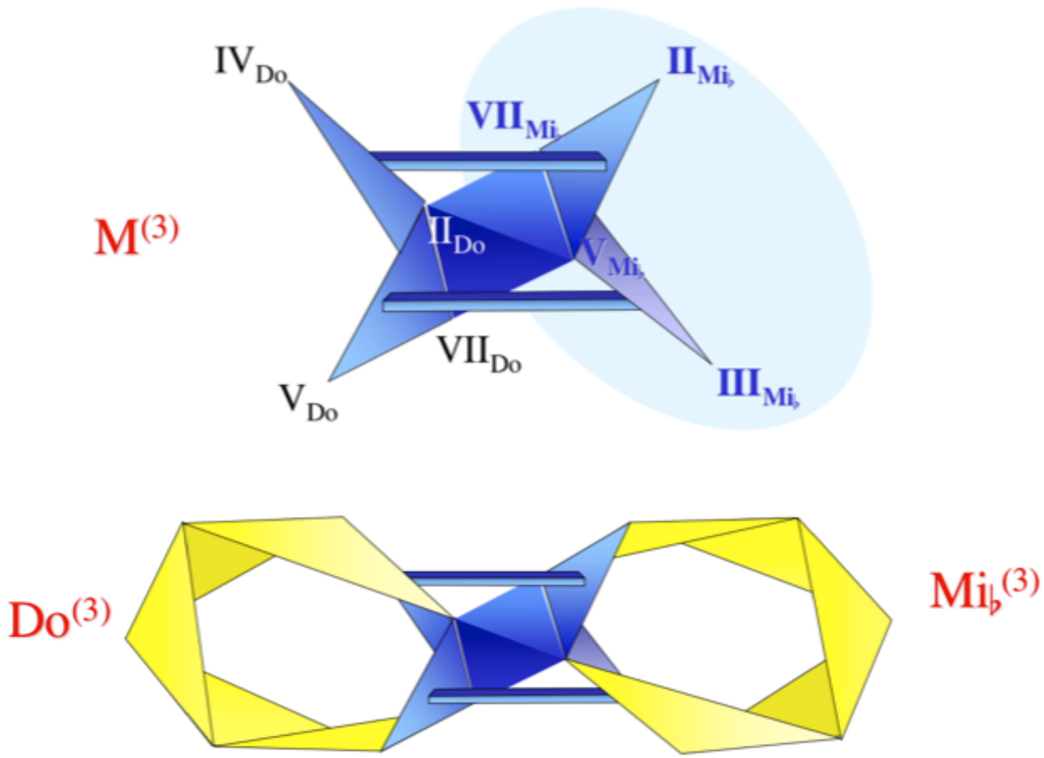




…et le nerf topologique associé

Nerf topologique et théorie de la modulation

G. Mazzola, *La vérite du beau dans la musique*, Collection « Musique/Sciences », IRCAM/Delatour, 2007.



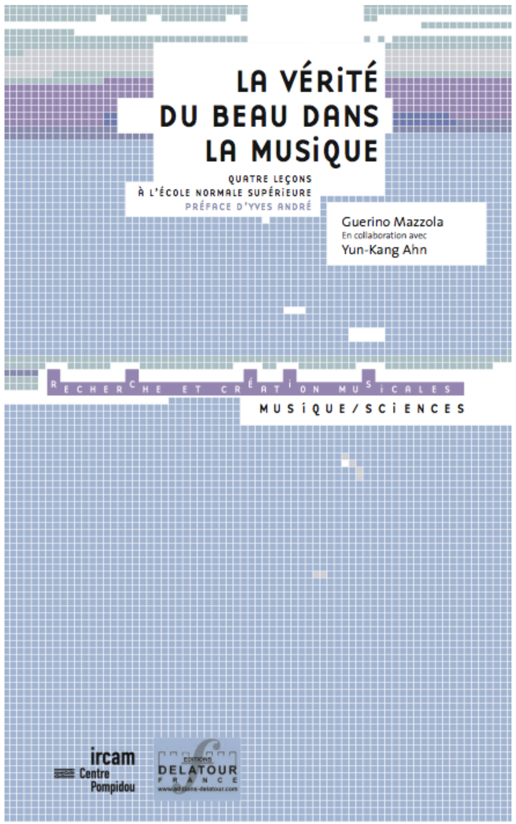
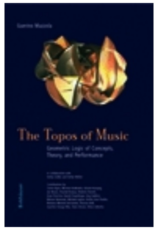
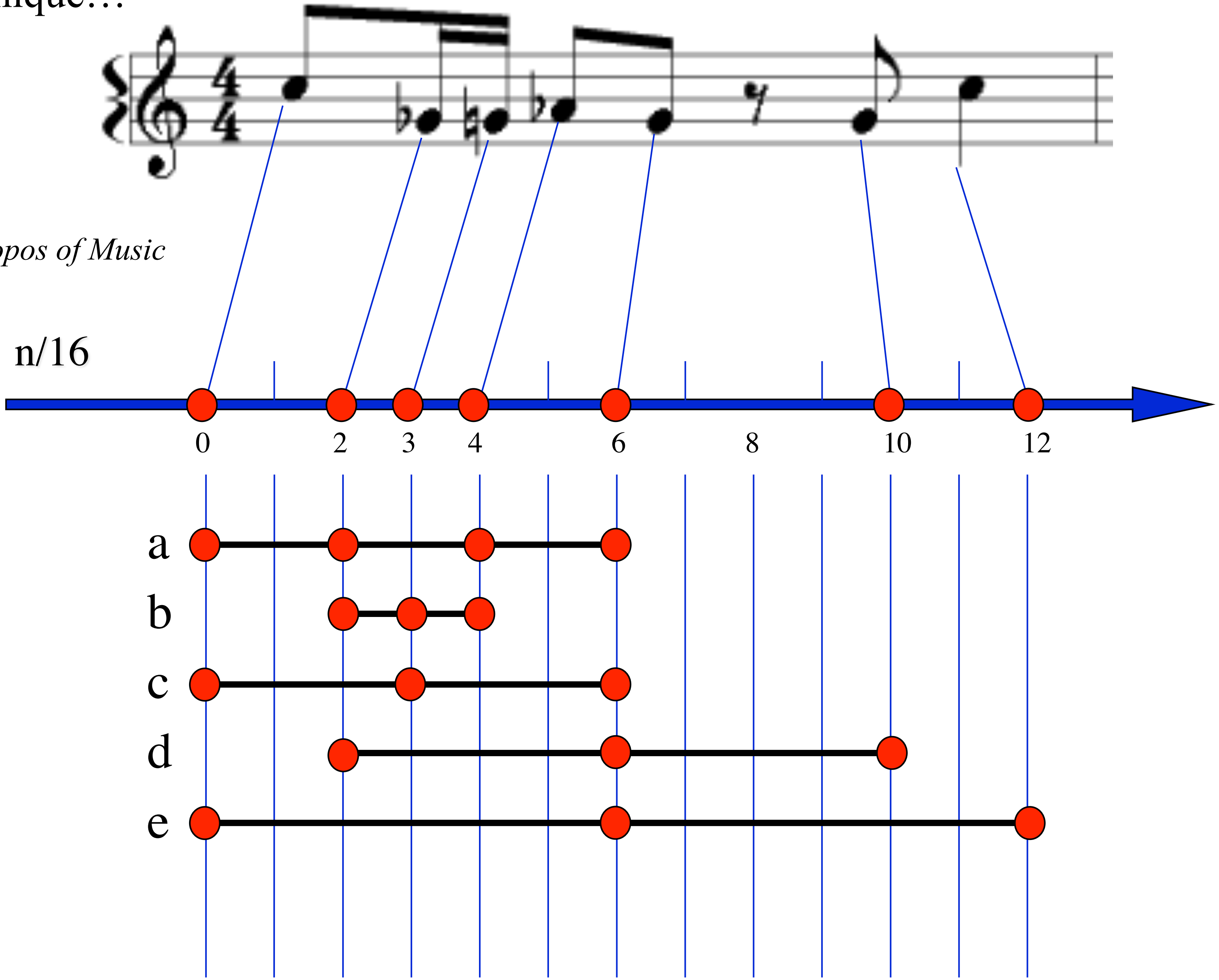


FIG. 8.6 – Le quantum de modulation de la tonalité $Do^{(3)}$ dans la tonalité $Mi_\flat^{(3)}$ dans sa représentation topologique via le nerf. Le nerf est de dimension cinq (six sommets en position générale), ce qui est symbolisé par les barres horizontales reliant six degrés chacune. En haut à droite sont représentés les pivots II, III, V, VII.

Un atlas pour
un pattern rythmique…

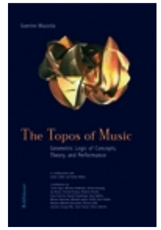


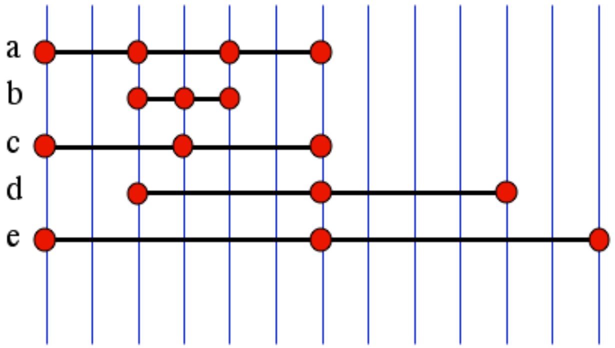
G. Mazzola, *The Topos of Music*

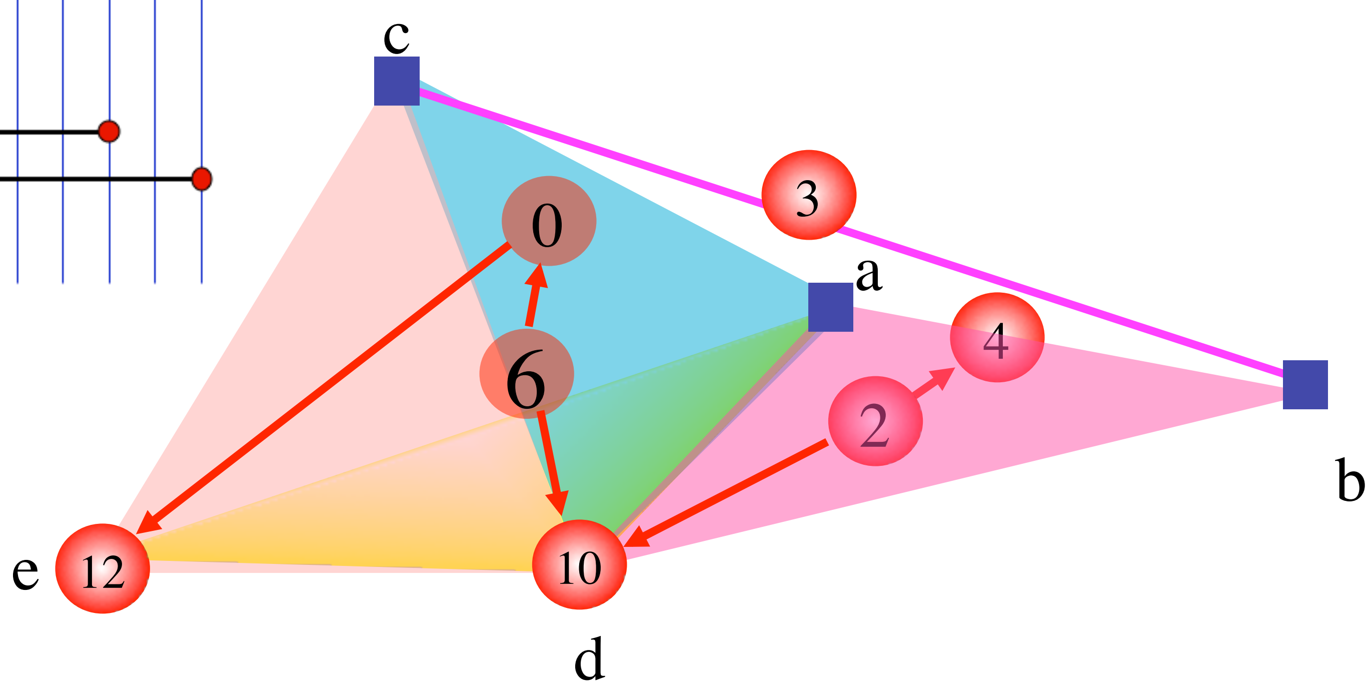


…et le nerf topologique associé

G. Mazzola, *The Topos of Music*



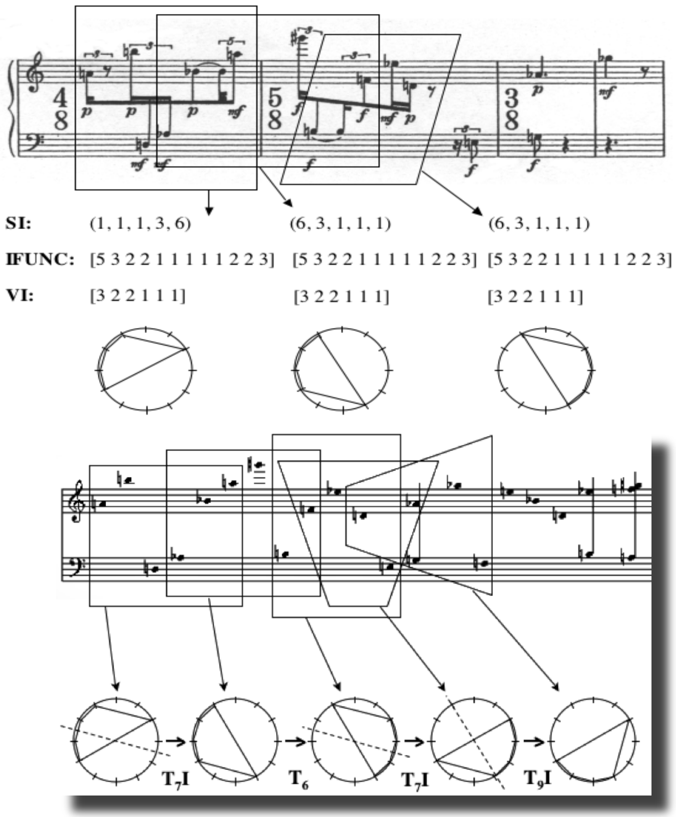


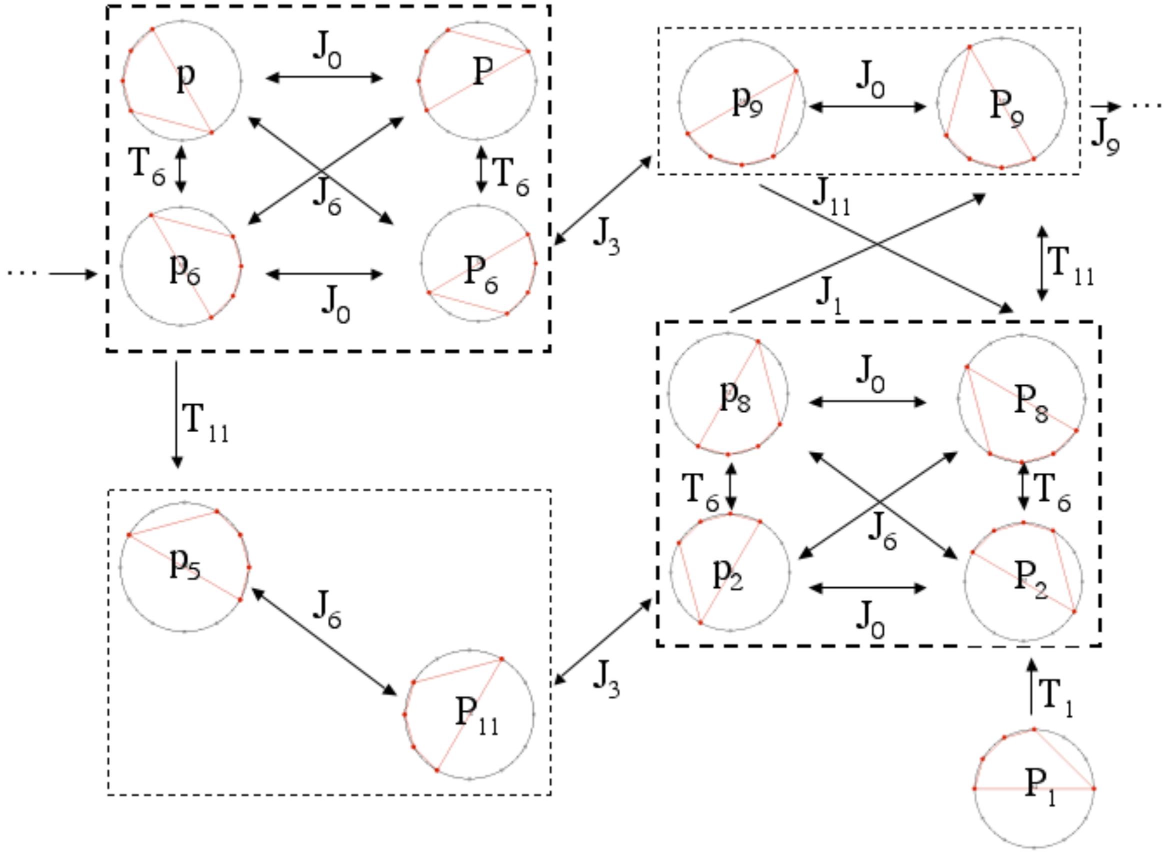


Nerf du recouvrement {a,b,c,d,e}

x domine y ssi simplex(y) ⊆ simplex(x)

Progression transformationnelle *vs* réseau transformationnel

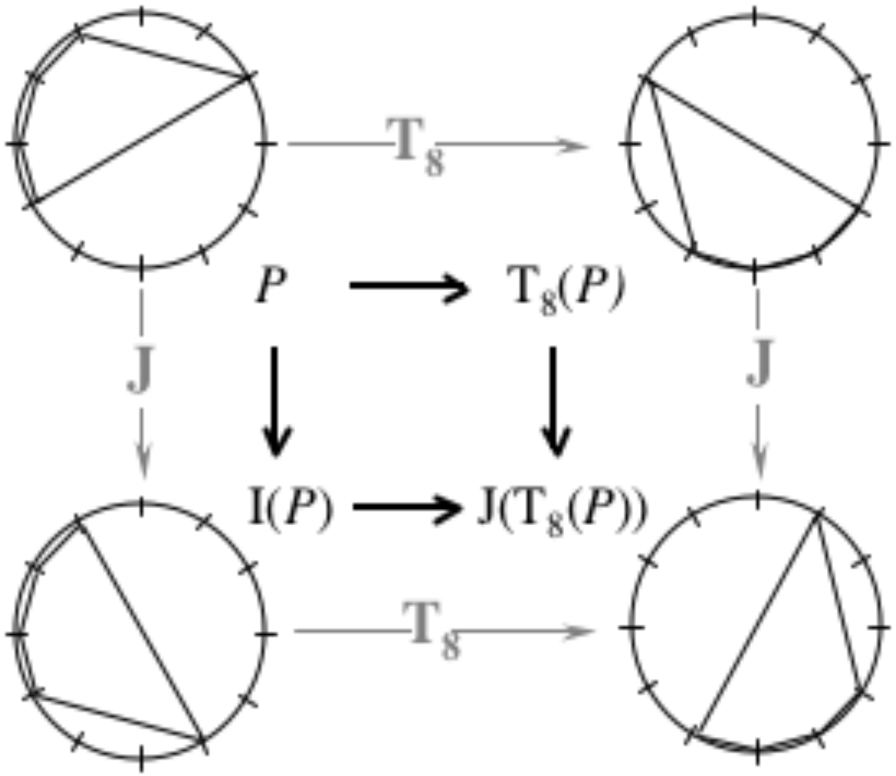


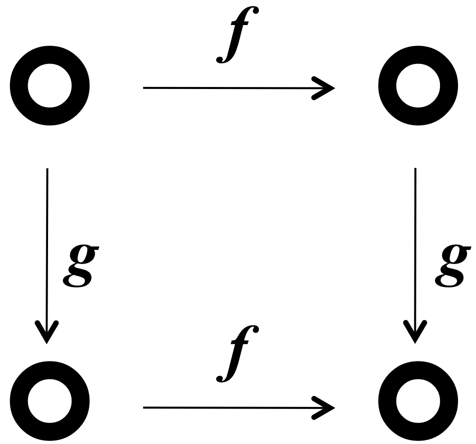


« A rational reconstruction of a work or works, which is a theory of the work or works, is an explanation not, assuredly, of the 'actual' process of construction, but of ***how the work or works may be construed by a hearer****, how the 'given' may be 'taken' »*

M. Babbitt : « Contemporary Music Composition and Music Theory as Contemporary Intellectual History », 1972

Inversions « contextuelles » et commutativité





Tout diagramme commute

$\forall f, g \in < T, J >$

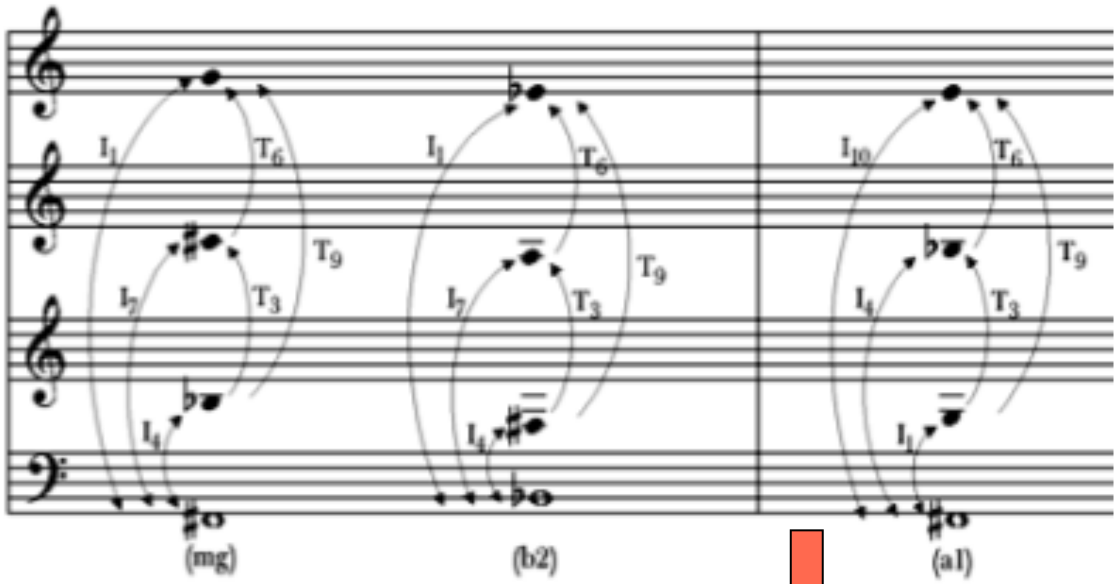
Le groupe des 24 transformations σ = {T_0, T_1,…, T_{11}, T_0J, T_1J, …, T_{11}J} est commutatif et opère de manière simplement transitive sur l'espace *S* des 24 formes du pentacorde de base (i.e. l'ensemble de ses 12 transpositions et de ses 12 inversions)

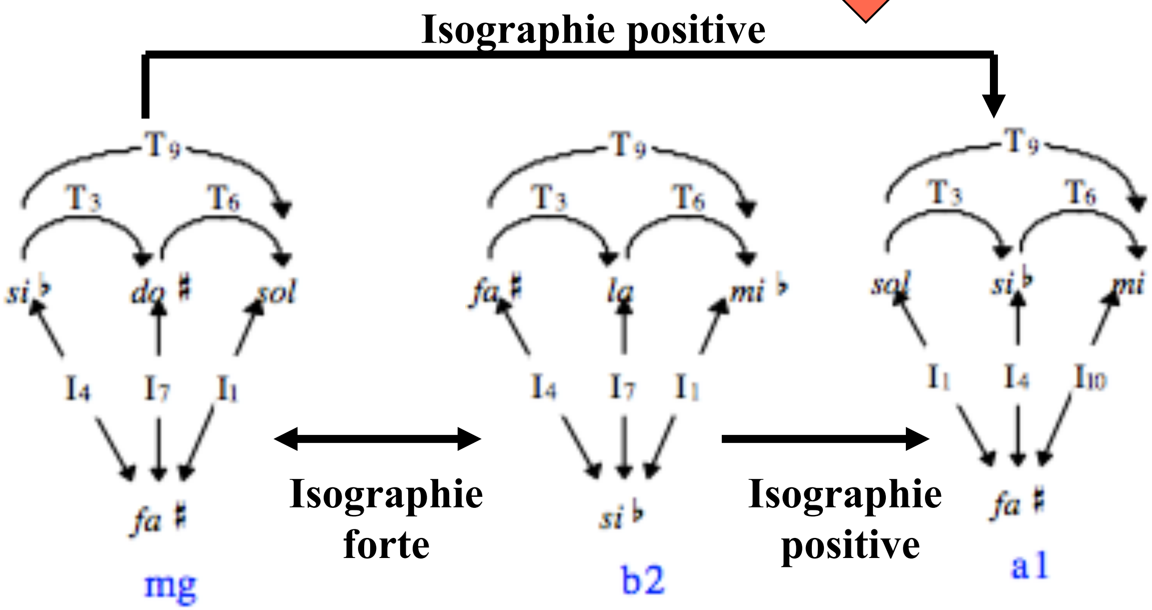
⇒ (*S*, σ, int) est un GIS

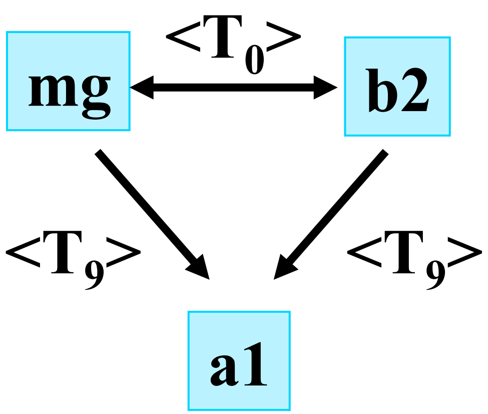
Klumpenhouwer Networks (K-réseaux)

Xavier Hascher: « Liszt et les sources de la notion d'agrégat », Analyse Musicale, 43, 2002

Ex. 1 - « Ladislaus Teleki » (*Historische ungarische Bildnisse* n° 4), mes. 1-7
Les agrégats dans la classification de Forte





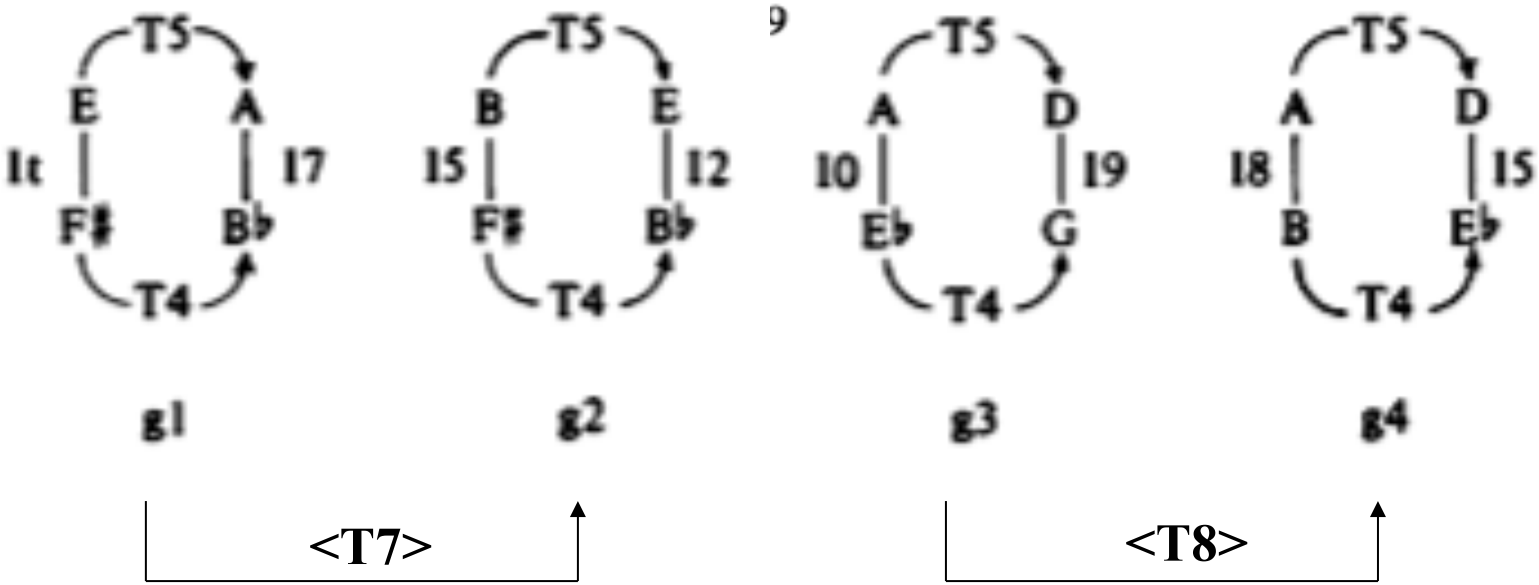


Klumpenhouwer Networks (K-réseaux)

$\langle T_k \rangle : T_m \to T_m$

$I_m \to I_{k+m}$

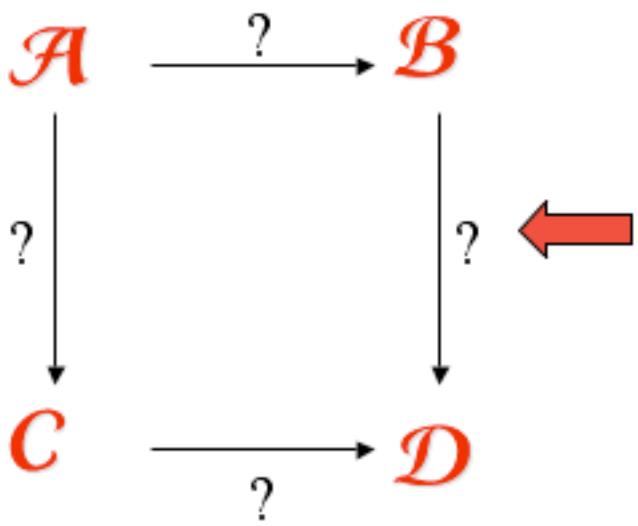
David Lewin: «A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », JMT, 1994

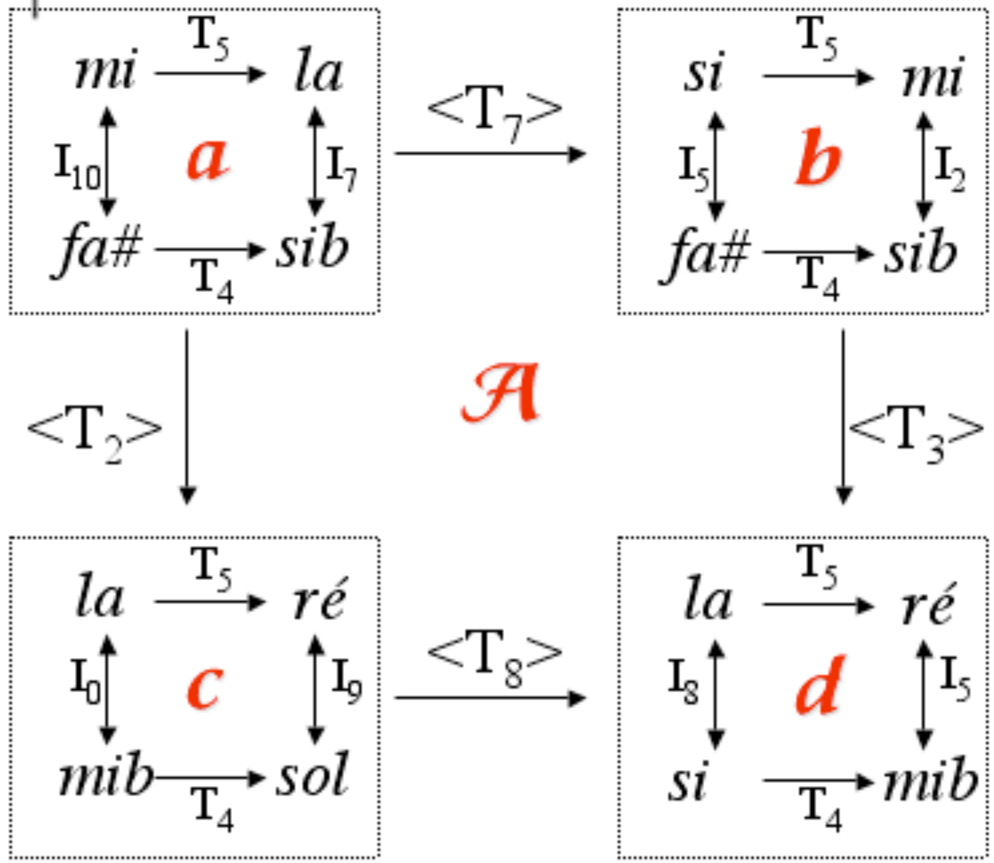


Isographie positive **Isographie positive**

Klumpenhouwer Networks (K-réseaux) : recursivité

David Lewin: «A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », JMT, 1994

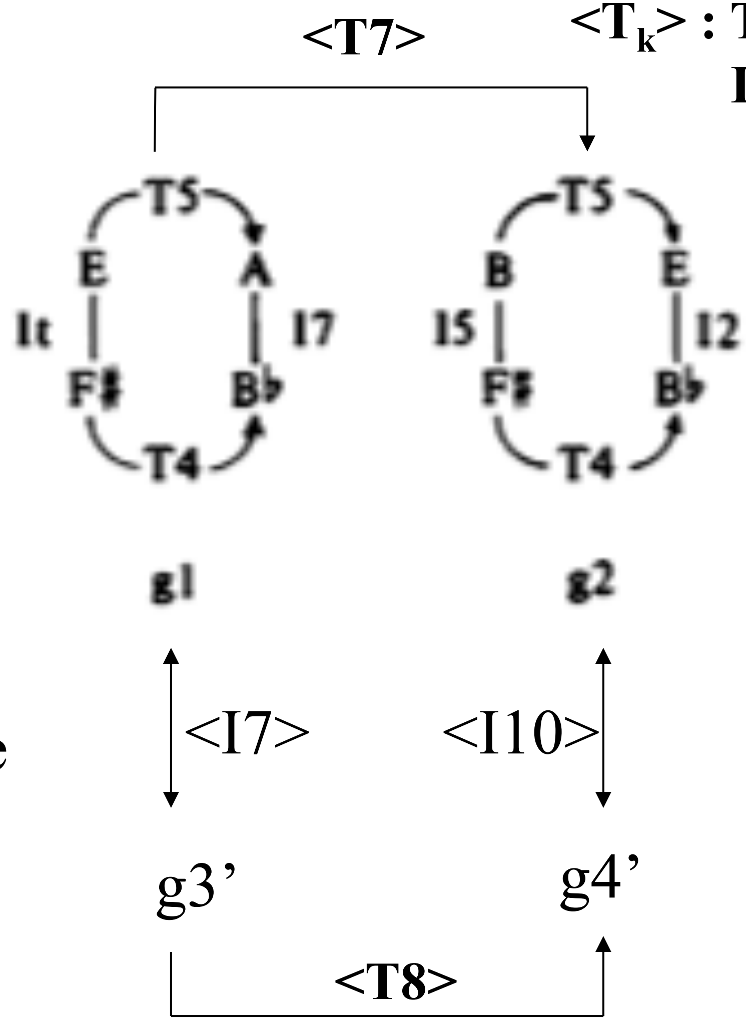




Klumpenhouwer Networks (K-nets)

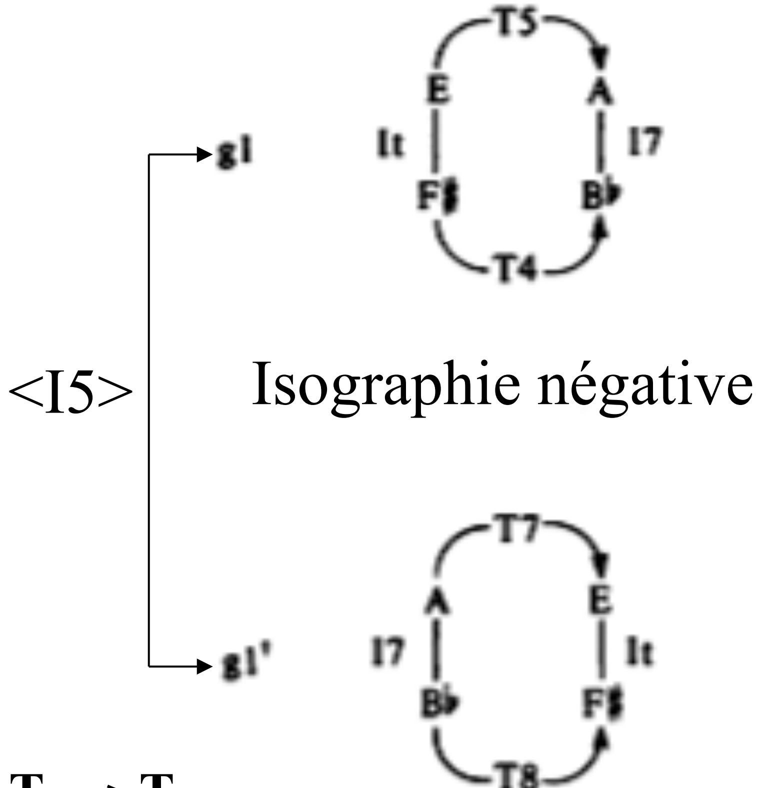
David Lewin: «A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », JMT, 1994

Example 9



$\langle T_k \rangle : T_m \rightarrow T_m$

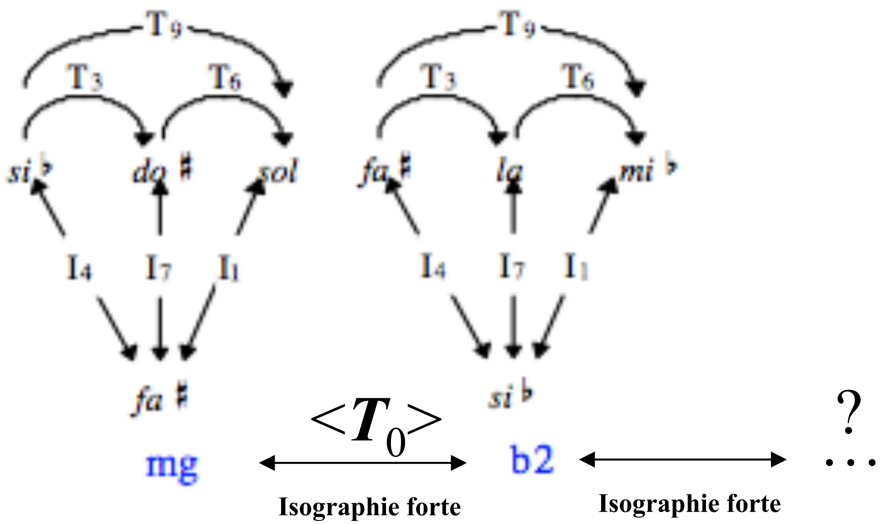
$I_m \rightarrow I_{k+m}$

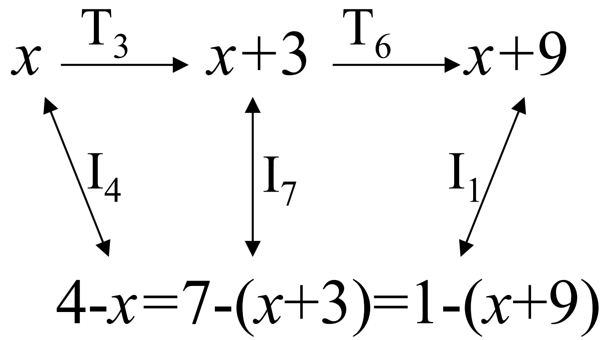


$\langle I_k \rangle : T_m \rightarrow T_{-m}$

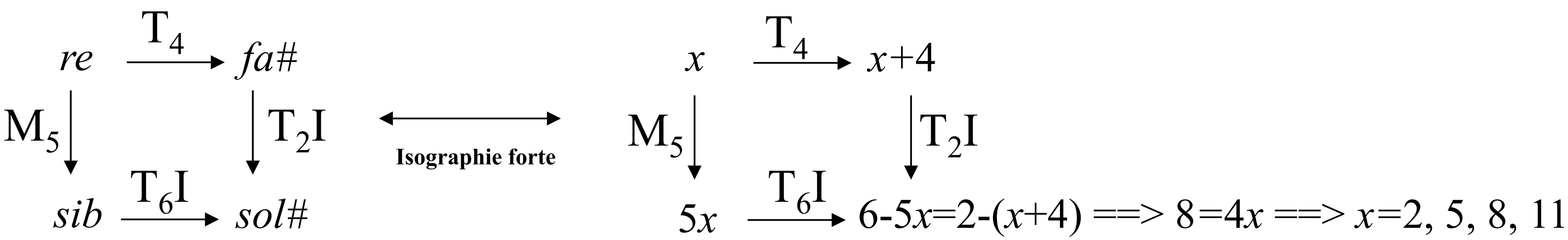
$I_m \rightarrow I_{k-m}$

Enumération des K-nets en relation d'isographie forte

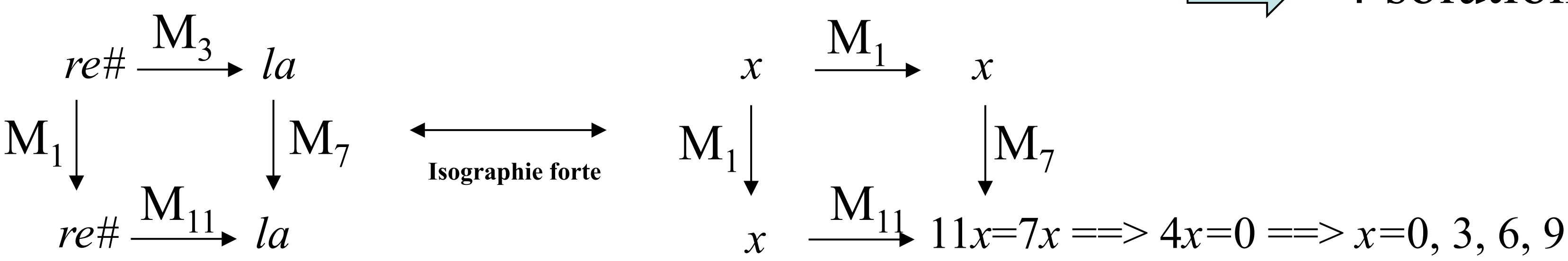




⟹ 12 solutions

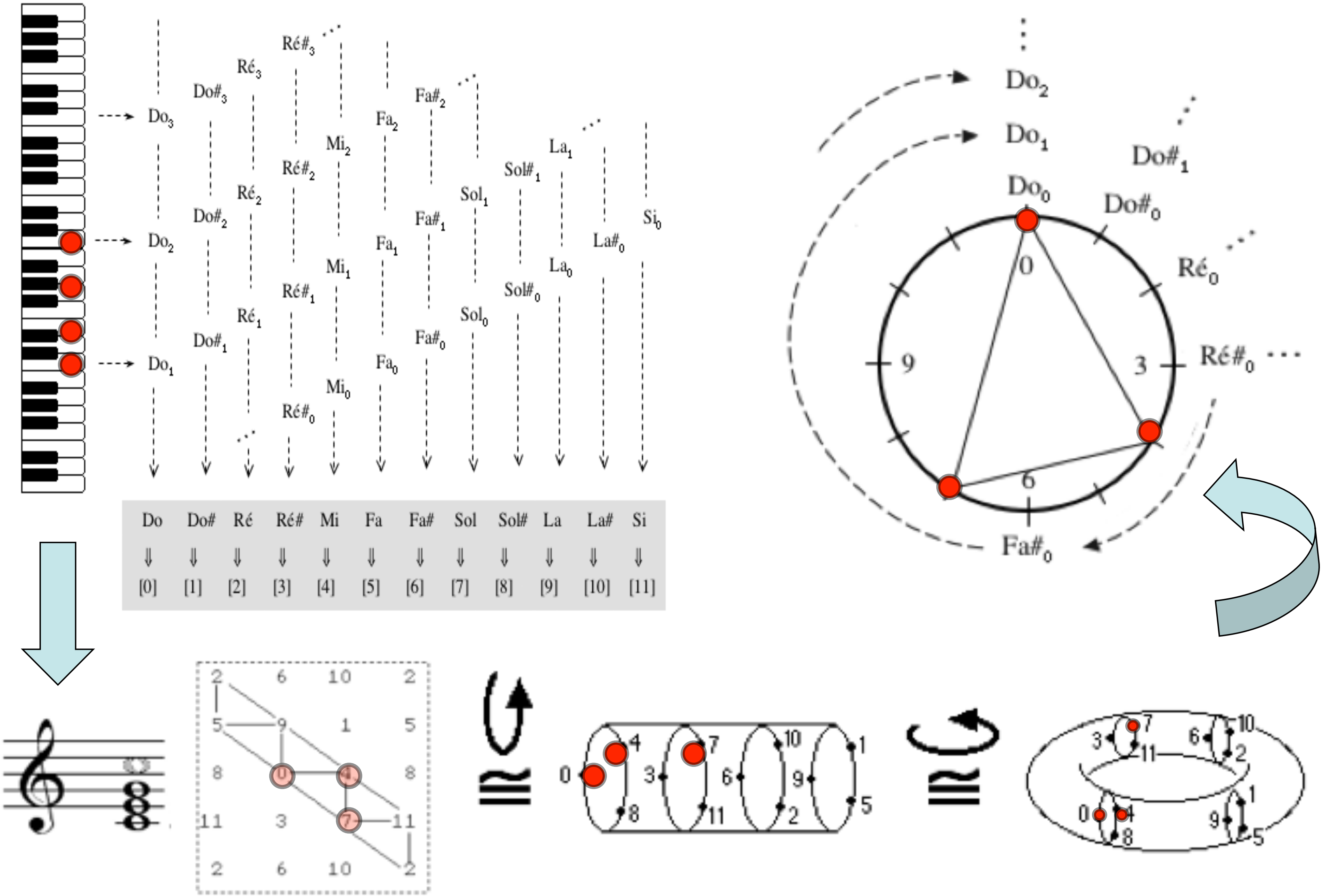


⟹ 4 solutions

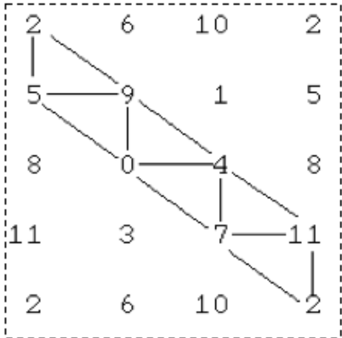


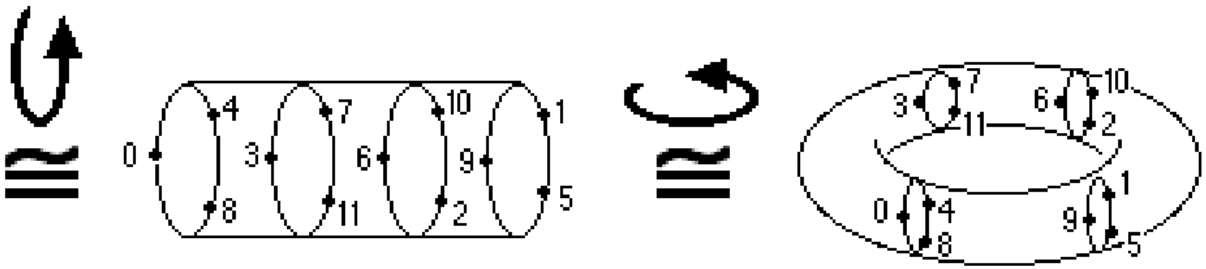
⟹ 4 solutions

Reduction à l'octave et congruence modulo 12

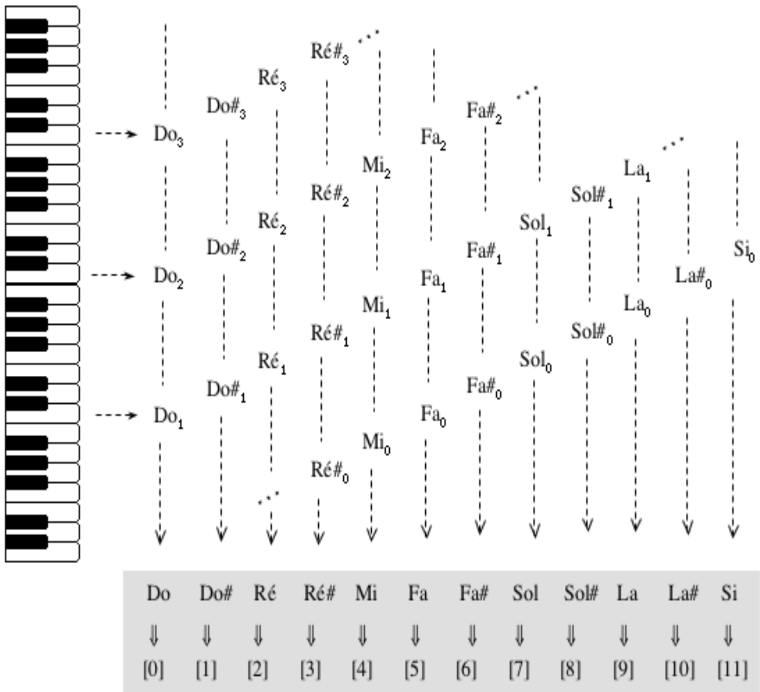


Equivalence algébrique entre représentations géométriques

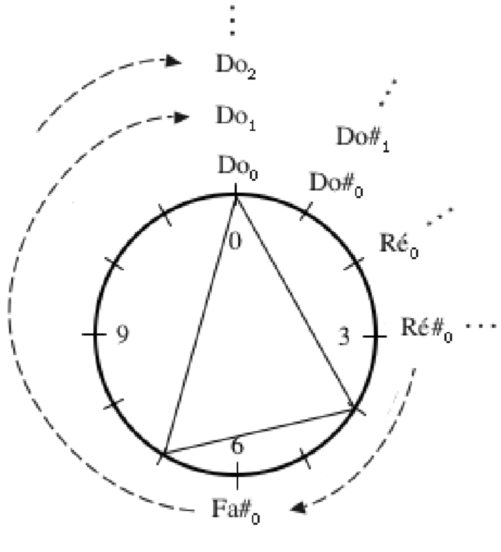




Représentation toroidale



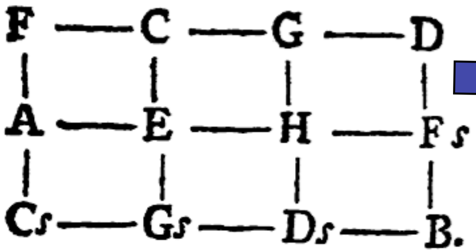
$$Z_{12} = Z_3 \times Z_4$$



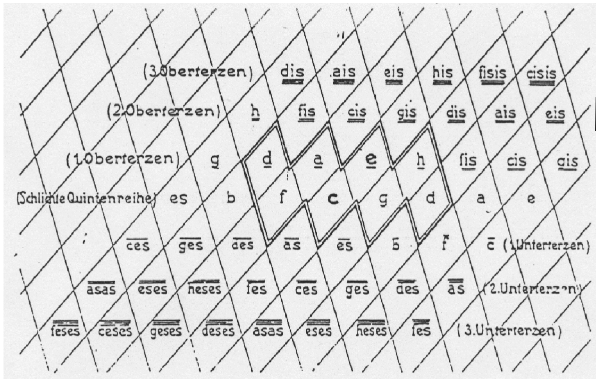
(4,3,5)

Représentation circulaire

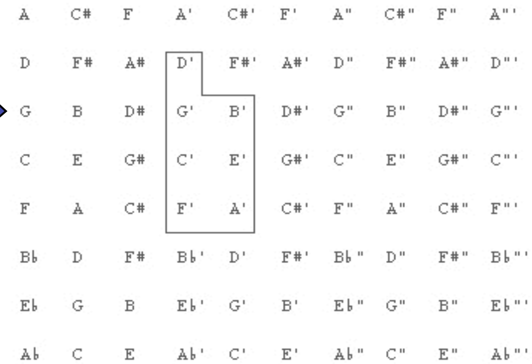
Approches transformationnelles diatoniques et néo-riemanniennes



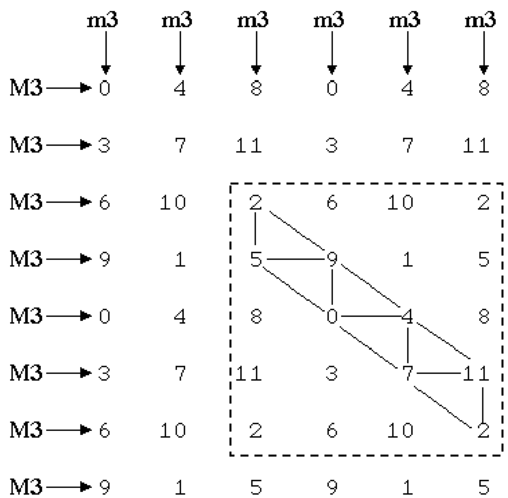
Euler : *Speculum musicum*, 1773



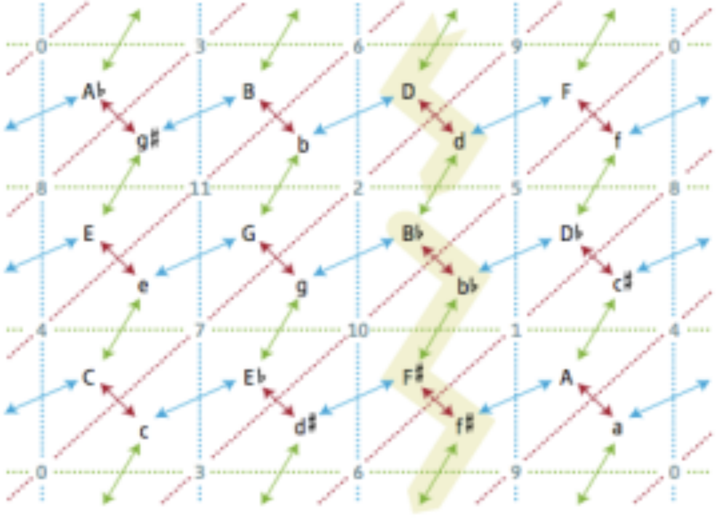
Hugo Riemann : « Ideen zu einer Lehre von den Tonvorstellung », 1914



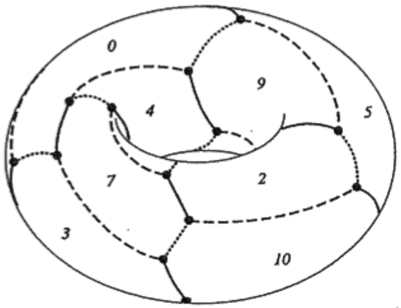
Longuet-Higgins (1962)



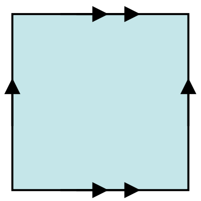
Balzano (1980)



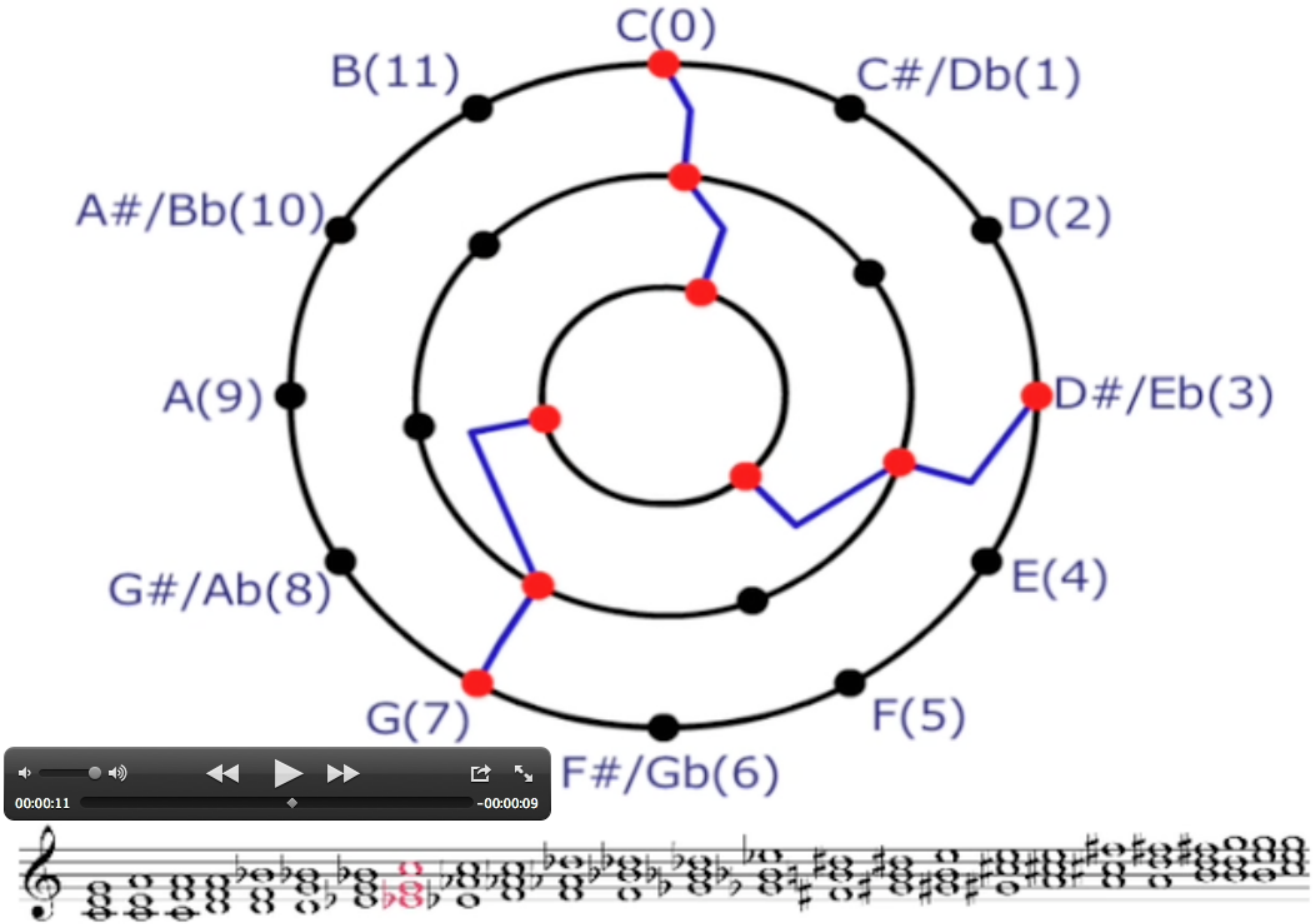
J. Hook, « Exploring Musical Space », *Science, 2006*



Douthett & Steinbach, *JMT*, 1998

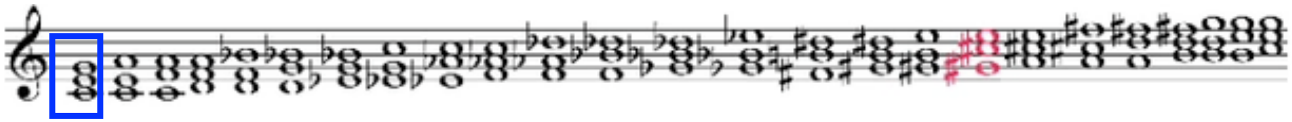


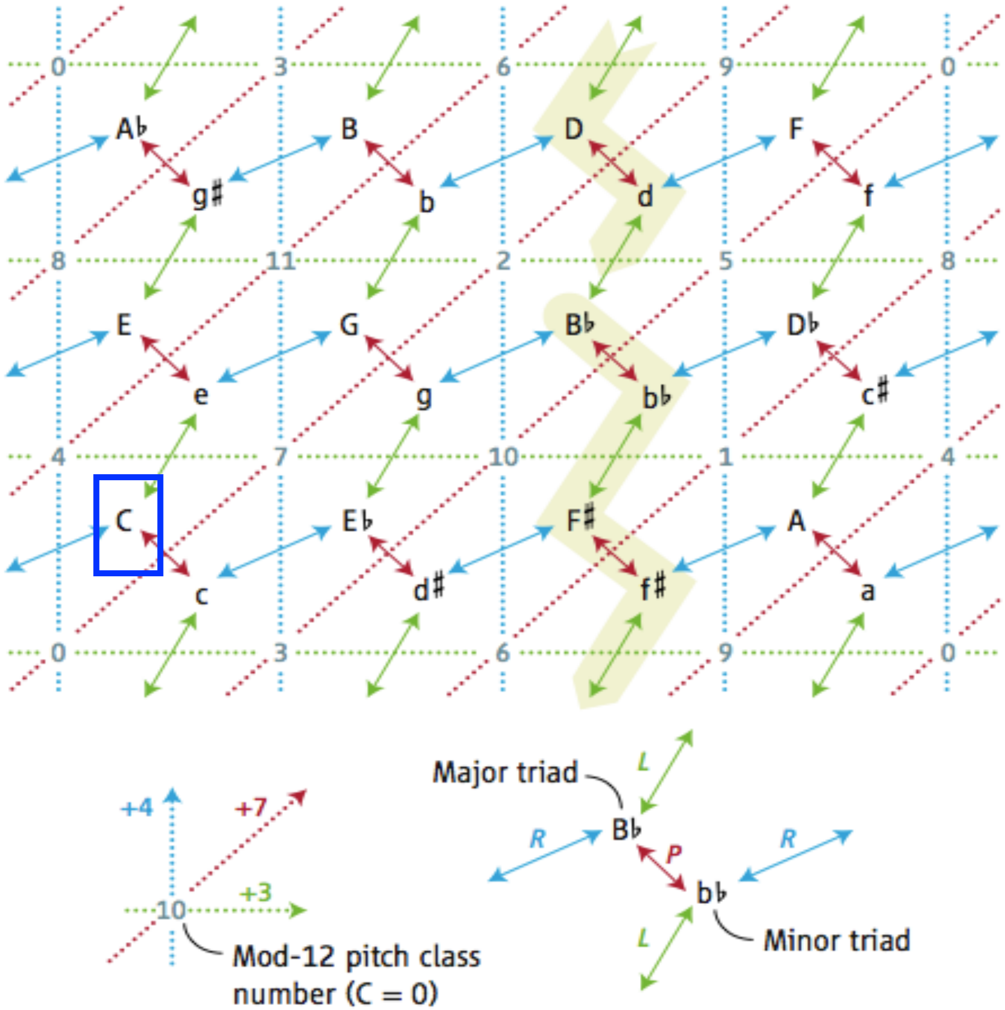
Beethoven's 9th Symphony
Mvmt. 2, mm. 143-176



Cf. Jack Douthett, "Filtered Point-Symmetry and Dynamical Voice-Leading", dans *Music Theory and Mathematics. Chords, Collections, and* Transformations, edited by J. Douthett, M. M. Hyde, C. J. Smith, URP, 2008.

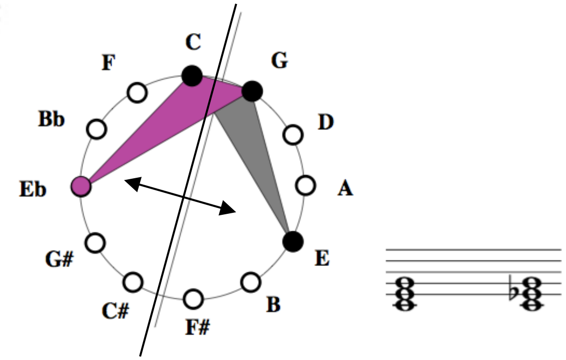
Le *Tonnetz* d’Oettingen/Riemann



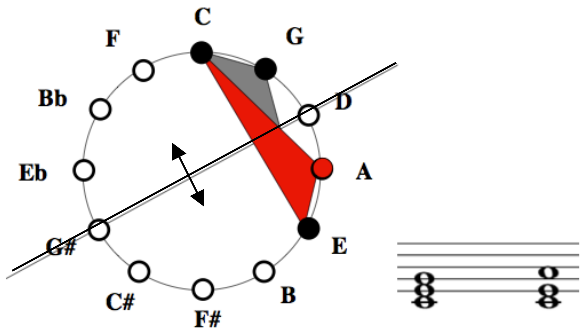


[J. Hook 06]

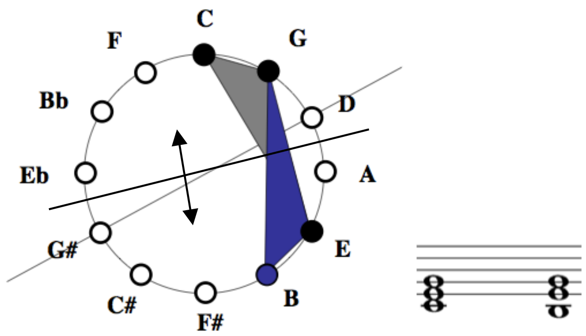
(Neo-)Riemannian Operation **P** = „**Parallel**“ **[Noll04]**



(Neo-)Riemannian Operation **R** = „**Relative**“



(Neo-)Riemannian Operation **L** = „**Leading-Tone**“



Recherche des cycles hamiltoniens dans **LPR**

C	e	G	b	D	f#	A	c#	E	g#	B	d#	F#	a#	C#	f	G#	c	D#	g	A#	d	F	a	#41
C	e	E	g#	G#	c	D#	g	G	b	B	d#	F#	a#	A#	d	D	f#	A	c#	C#	f	F	a	#62
C	c	G#	f	C#	c#	A	a	F	d	A#	a#	F#	f#	D	b	G	g	D#	d#	B	g#	E	e	#13
C	c	G#	g#	E	c#	A	a	F	f	C#	a#	F#	f#	D	d	A#	g	D#	d#	B	b	G	e	#4
C	e	E	g#	G#	c	D#	d#	B	b	G	g	A#	a#	F#	f#	D	d	F	f	C#	c#	A	a	#58
C	c	D#	g	G	b	B	d#	F#	f#	D	d	A#	a#	C#	c#	A	a	F	f	G#	g#	E	e	#19
C	c	G#	g#	B	d#	D#	g	G	b	D	d	A#	a#	F#	f#	A	a	F	f	C#	c#	E	e	#7
C	c	G#	g#	E	e	G	b	B	d#	D#	g	A#	a#	F#	f#	D	d	F	f	C#	c#	A	a	#27
C	c	D#	d#	B	b	G	g	A#	d	D	f#	F#	a#	C#	c#	A	a	F	f	G#	g#	E	e	#21
C	c	G#	g#	B	b	G	g	D#	d#	F#	a#	A#	d	D	f#	A	a	F	f	C#	c#	E	e	#8
C	c	G#	g#	E	e	G	g	D#	d#	B	b	D	f#	F#	a#	A#	d	F	f	C#	c#	A	a	#30
C	c	D#	d#	B	b	G	g	A#	a#	F#	f#	D	d	F	a	A	c#	C#	f	G#	g#	E	e	#22
C	c	G#	g#	B	b	G	g	D#	d#	F#	f#	D	d	A#	a#	C#	f	F	a	A	c#	E	e	#10
C	c	G#	g#	E	e	G	g	D#	d#	B	b	D	d	A#	a#	F#	f#	A	c#	C#	f	F	a	#31
C	c	G#	f	F	a	A	c#	C#	a#	A#	d	D	f#	F#	d#	D#	g	G	b	B	g#	E	e	#9
C	c	G#	g#	E	c#	C#	f	F	a	A	f#	F#	a#	A#	d	D	b	B	d#	D#	g	G	e	#6
C	c	D#	d#	F#	f#	A	c#	E	e	G	g	A#	a#	C#	f	G#	g#	B	b	D	d	F	a	#33
C	e	G	g	A#	a#	C#	c#	E	g#	B	b	D	d	F	f	G#	c	D#	d#	F#	f#	A	a	#44
C	c	D#	g	A#	a#	C#	c#	E	e	G	b	D	d	F	f	G#	g#	B	d#	F#	f#	A	a	#40
C	c	D#	d#	F#	a#	C#	c#	E	e	G	g	A#	d	F	f	G#	g#	B	b	D	f#	A	a	#38
C	c	D#	d#	F#	a#	C#	f	G#	g#	B	b	D	f#	A	c#	E	e	G	g	A#	d	F	a	#34
C	e	G	g	A#	a#	C#	f	G#	c	D#	d#	F#	f#	A	c#	E	g#	B	b	D	d	F	a	#42
C	e	G	b	D	d	F	f	G#	c	D#	g	A#	a#	C#	c#	E	g#	B	d#	F#	f#	A	a	#43
C	c	D#	g	A#	d	F	f	G#	g#	B	d#	F#	a#	C#	c#	E	e	G	b	D	f#	A	a	#39
C	e	G	b	B	d#	F#	a#	A#	g	D#	c	G#	g#	E	c#	C#	f	F	d	D	f#	A	a	#50
C	c	D#	g	A#	d	D	f#	A	c#	C#	a#	F#	d#	B	b	G	e	E	g#	G#	f	F	a	#37
C	c	G#	f	F	d	A#	g	D#	d#	B	g#	E	e	G	b	D	f#	F#	a#	C#	c#	A	a	#25
C	c	D#	d#	B	g#	G#	f	C#	a#	F#	f#	D	b	G	g	A#	d	F	a	A	c#	E	e	#16
C	c	G#	g#	B	b	G	e	E	c#	A	f#	D	d	A#	g	D#	d#	F#	a#	C#	f	F	a	#23
C	c	G#	f	F	a	A	f#	F#	a#	C#	c#	E	g#	B	d#	D#	g	A#	d	D	b	G	e	#1
C	e	E	g#	B	b	G	g	A#	a#	F#	d#	D#	c	G#	f	C#	c#	A	f#	D	d	F	a	#59
C	e	G	b	B	d#	F#	f#	D	d	F	f	C#	a#	A#	g	D#	c	G#	g#	E	c#	A	a	#48
C	c	D#	g	A#	d	D	f#	A	a	F	f	G#	g#	E	c#	C#	a#	F#	d#	B	b	G	e	#18
C	e	E	c#	A	f#	D	d	A#	g	G	b	B	g#	G#	c	D#	d#	F#	a#	C#	f	F	a	#60
C	c	G#	f	C#	c#	E	g#	B	d#	D#	g	A#	a#	F#	f#	A	a	F	d	D	b	G	e	#2

#41	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R
#62	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R
#13	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L
#4	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L
#58	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P	R
#19	P	R	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P	L
#7	P	L	P	R	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L
#27	P	L	P	L	P	R	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R
#21	P	R	P	L	P	L	P	R	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L
#8	P	L	P	R	P	L	P	L	P	R	L	P	L	P	L	R	P	L	P	L	P	R	P	L
#30	P	L	P	L	P	R	P	L	P	L	P	R	L	P	L	P	L	R	P	L	P	L	P	R
#22	P	R	P	L	P	L	P	R	P	L	P	L	P	R	L	P	L	P	L	R	P	L	P	L
#10	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P	R	L	P	L	P	L	R	P	L
#31	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P	R	L	P	L	P	L	R
#9	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P	R	L	P	L
#6	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P	R	L
#33	P	R	P	R	P	R	L	R	P	R	P	R	P	R	L	R	P	R	P	R	P	R	L	R
#44	L	R	P	R	P	R	P	R	L	R	P	R	P	R	P	R	L	R	P	R	P	R	P	R
#40	P	R	L	R	P	R	P	R	P	R	L	R	P	R	P	R	P	R	L	R	P	R	P	R
#38	P	R	P	R	L	R	P	R	P	R	P	R	L	R	P	R	P	R	P	R	L	R	P	R
#34	P	R	P	R	L	R	L	R	P	R	P	R	L	R	L	R	P	R	P	R	L	R	L	R
#42	L	R	P	R	P	R	L	R	L	R	P	R	P	R	L	R	L	R	P	R	P	R	L	R
#43	L	R	L	R	P	R	P	R	L	R	L	R	P	R	P	R	L	R	L	R	P	R	P	R
#39	P	R	L	R	L	R	P	R	P	R	L	R	L	R	P	R	P	R	L	R	L	R	P	R
#50	L	R	L	P	L	R	L	P	R	L	R	L	P	L	R	P	L	P	R	P	L	R	P	R
#37	P	R	L	R	L	P	L	R	L	P	R	L	R	L	P	L	R	P	L	P	R	P	L	R
#25	P	L	R	P	R	L	R	L	P	L	R	L	P	R	L	R	L	P	L	R	P	L	P	R
#16	P	R	P	L	R	P	R	L	R	L	P	L	R	L	P	R	L	R	L	P	L	R	P	L
#23	P	L	P	R	P	L	R	P	R	L	R	L	P	L	R	L	P	R	L	R	L	P	L	R
#1	P	L	R	P	L	P	R	P	L	R	P	R	L	R	L	P	L	R	L	P	R	L	R	L
#59	L	P	L	R	P	L	P	R	P	L	R	P	R	L	R	L	P	L	R	L	P	R	L	R
#48	L	R	L	P	L	R	P	L	P	R	P	L	R	P	R	L	R	L	P	L	R	L	P	R
#18	P	R	L	R	L	P	L	R	P	L	P	R	P	L	R	P	R	L	R	L	P	L	R	L
#60	L	P	R	L	R	L	P	L	R	P	L	P	R	P	L	R	P	R	L	R	L	P	L	R
#2	P	L	R	L	P	R	L	R	L	P	L	R	P	L	P	R	P	L	R	P	R	L	R	L

G. Albini, *Modelli matematici per l'analisi e la composizione musicale. Uno studio sul* Tonnetz *e sulle teorie neo-riemanniane,* Tesi di Laurea, Università di Pavia, 2007-2008.

Recherche des cycles hamiltoniens dans **LPR**

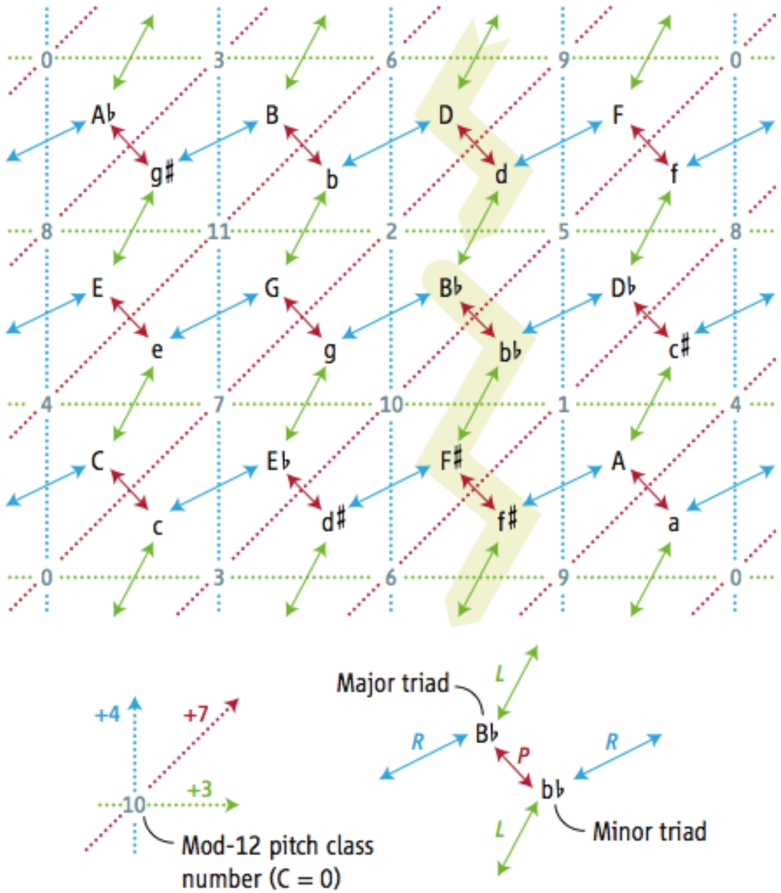
C	e	E	g#	B	d#	D#	c	G#	f	C#	c#	A	f#	F#	a#	A#	g	G	b	D	d	F	a	#53
C	c	D#	g	A#	a#	C#	f	G#	g#	B	d#	F#	f#	A	c#	E	e	G	b	D	d	F	a	#32
C	e	G	g	A#	d	F	f	G#	c	D#	d#	F#	a#	C#	c#	E	g#	B	b	D	f#	A	a	#45
C	c	G#	f	C#	c#	A	f#	D	b	B	g#	E	e	G	g	D#	d#	F#	a#	A#	d	F	a	#29
C	e	G	g	D#	c	G#	g#	E	c#	A	f#	F#	d#	B	b	D	d	A#	a#	C#	f	F	a	#49
C	c	G#	f	C#	a#	A#	d	F	a	A	c#	E	g#	B	b	D	f#	F#	d#	D#	g	G	e	#11
C	e	E	g#	B	d#	F#	f#	D	b	G	g	D#	c	G#	f	F	d	A#	a#	C#	c#	A	a	#54
C	c	D#	g	G	b	D	f#	A	a	F	d	A#	a#	F#	d#	B	g#	G#	f	C#	c#	E	e	#14
C	c	G#	g#	B	d#	D#	g	A#	d	F	f	C#	a#	F#	f#	D	b	G	e	E	c#	A	a	#24
C	c	D#	d#	B	b	D	f#	F#	a#	C#	f	G#	g#	E	c#	A	a	F	d	A#	g	G	e	#20
C	e	E	c#	C#	f	F	d	A#	a#	F#	d#	B	g#	G#	c	D#	g	G	b	D	f#	A	a	#55
C	c	D#	g	G	e	E	g#	G#	f	C#	c#	A	f#	D	b	B	d#	F#	a#	A#	d	F	a	#36
C	e	G	g	A#	d	D	b	B	d#	D#	c	G#	g#	E	c#	A	f#	F#	a#	C#	f	F	a	#47
C	c	G#	f	C#	a#	A#	g	D#	d#	F#	f#	D	d	F	a	A	c#	E	g#	B	b	G	e	#5
C	e	E	g#	B	d#	F#	f#	A	c#	C#	a#	A#	d	D	b	G	g	D#	c	G#	f	F	a	#57
C	c	G#	f	F	a	A	f#	D	d	A#	g	D#	d#	F#	a#	C#	c#	E	g#	B	b	G	e	#3
C	e	E	g#	B	b	G	g	A#	d	D	f#	A	c#	C#	a#	F#	d#	D#	c	G#	f	F	a	#61
C	e	E	c#	A	f#	F#	d#	B	g#	G#	c	D#	g	G	b	D	d	A#	a#	C#	f	F	a	#56
C	c	D#	g	A#	a#	F#	d#	B	b	G	e	E	g#	G#	f	C#	c#	A	f#	D	d	F	a	#35
C	c	D#	g	G	b	D	f#	F#	d#	B	g#	G#	f	C#	a#	A#	d	F	a	A	c#	E	e	#15
C	c	G#	f	C#	c#	E	g#	B	b	D	f#	A	a	F	d	A#	a#	F#	d#	D#	g	G	e	#12
C	c	G#	g#	B	d#	D#	g	A#	d	D	b	G	e	E	c#	A	f#	F#	a#	C#	f	F	a	#26
C	c	D#	g	A#	a#	C#	f	G#	g#	E	c#	A	a	F	d	D	f#	F#	d#	B	b	G	e	#17
C	e	G	g	D#	c	G#	g#	E	c#	C#	f	F	d	A#	a#	F#	d#	B	b	D	f#	A	a	#51
C	e	E	g#	B	d#	D#	c	G#	f	F	d	A#	g	G	b	D	f#	F#	a#	C#	c#	A	a	#52
C	e	G	g	A#	d	F	f	C#	a#	F#	f#	D	b	B	d#	D#	c	G#	g#	E	c#	A	a	#46
C	c	G#	f	C#	c#	A	f#	F#	a#	A#	g	D#	d#	B	g#	E	e	G	b	D	d	F	a	#28

#53	L	P	L	R	L	P	R	L	R	L	P	L	R	P	L	P	R	P	L	R	P	R	L	R
#32	**P**	**R**	**L**	**R**	P	R	L	R	P	R	L	R	P	R	L	R	P	R	L	R	P	R	L	R
#45	L	R	**P**	**R**	**L**	**R**	P	R	L	R	P	R	L	R	P	R	L	R	P	R	L	R	P	R
#29	P	L	R	L	P	L	R	L	R	P	R	L	P	R	P	L	P	R	L	P	L	R	L	R
#49	L	R	P	L	R	L	P	L	R	L	R	P	R	L	P	R	P	L	P	R	L	P	L	R
#11	P	L	R	L	R	P	L	R	L	P	L	R	L	R	P	R	L	P	R	P	L	P	R	L
#54	L	P	L	R	L	R	P	L	R	L	P	L	R	L	R	P	R	L	P	R	P	L	P	R
#14	P	R	L	P	L	R	L	R	P	L	R	L	P	L	R	L	R	P	R	L	P	R	P	L
#24	P	L	P	R	L	P	L	R	L	R	P	L	R	L	P	L	R	L	R	P	R	L	P	R
#20	P	R	P	L	P	R	L	P	L	R	L	R	P	L	R	L	P	L	R	L	R	P	R	L
#55	L	P	R	P	L	P	R	L	P	L	R	L	R	P	L	R	L	P	L	R	L	R	P	R
#36	P	R	L	P	R	P	L	P	R	L	P	L	R	L	R	P	L	R	L	P	L	R	L	R
#47	L	R	P	R	L	P	R	P	L	P	R	L	P	L	R	L	R	P	L	R	L	P	L	R
#5	P	L	R	L	R	P	R	L	P	R	P	L	P	R	L	P	L	R	L	R	P	L	R	L
#57	L	P	L	R	L	R	P	R	L	P	R	P	L	P	R	L	P	L	R	L	R	P	L	R
#3	P	L	R	P	L	P	R	L	P	L	R	L	P	R	L	R	P	R	L	R	P	L	R	L
#61	L	P	L	R	P	L	P	R	L	P	L	R	L	P	R	L	R	P	R	L	R	P	L	R
#56	L	P	R	L	R	P	R	L	R	P	L	R	L	P	L	R	P	L	P	R	L	P	L	R
#35	P	R	L	R	P	L	R	L	P	L	R	P	L	P	R	L	P	L	R	L	P	R	L	R
#15	P	R	L	P	L	R	L	P	R	L	R	P	R	L	R	P	L	R	L	P	L	R	P	L
#12	P	L	R	L	P	R	L	R	P	R	L	R	P	L	R	L	P	L	R	P	L	P	R	L
#26	P	L	P	R	L	P	L	R	L	P	R	L	R	P	R	L	R	P	L	R	L	P	L	R
#17	P	R	L	R	P	R	L	R	P	L	R	L	P	L	R	P	L	P	R	L	P	L	R	L
#51	L	R	P	L	R	L	P	L	R	P	L	P	R	L	P	L	R	L	P	R	L	R	P	R
#52	L	P	L	R	L	P	R	L	R	P	R	L	R	P	L	R	L	P	L	R	P	L	P	R
#46	L	R	P	R	L	R	P	L	R	L	P	L	R	P	L	P	R	L	P	L	R	L	P	R
#28	P	L	R	L	P	L	R	P	L	P	R	L	P	L	R	L	P	R	L	R	P	R	L	R

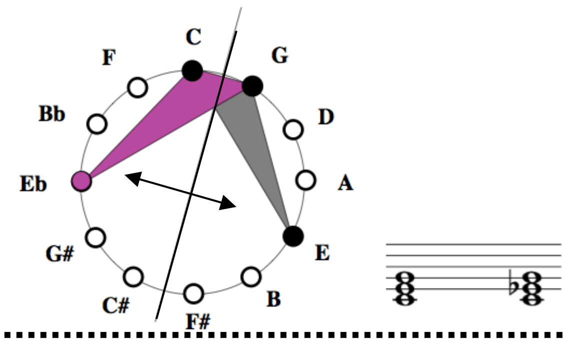
62 cycles hamiltoniens classés en 8 types

[Albini 2007-2008]

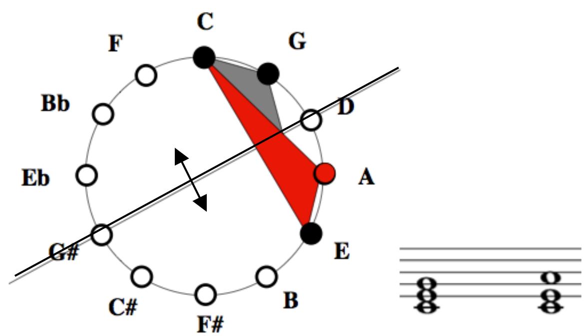
Le *Tonnetz* en tant que GIS



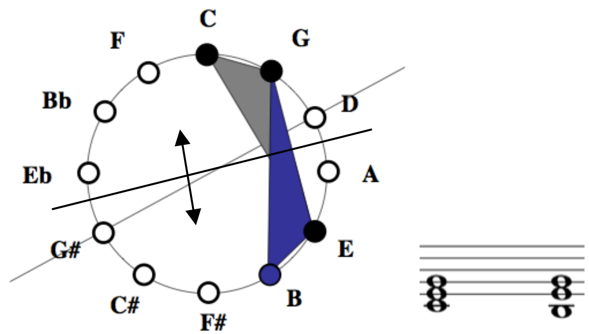
(Neo-)Riemannian Operation **P = „Parallel“** **[Noll04]**



(Neo-)Riemannian Operation **R = „Relative“**



(Neo-)Riemannian Operation **L = „Leading-Tone“**



$$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 \; ; \; LRL = L(LR)^{-1} \rangle$$

• **ρ opère de façon simplement transitive sur l'ensemble *S* des 24 triades consonantes**

⇒ (*S*, ρ, int) est un GIS

Une autre structure de GIS sur l’espace *S*

L		R		RL		I_{11}	
C	→ e	C	→ a	C	→ G	C	→ e
c	→ A♭	c	→ E♭	c	→ f	c	→ E
D♭	→ f	D♭	→ b♭	D♭	→ A♭	D♭	→ e♭
c♯	→ A	c♯	→ E	c♯	→ f♯	c♯	→ E♭
D	→ f♯	D	→ b	D	→ A	D	→ d
d	→ B♭	d	→ F	d	→ g	d	→ D
E♭	→ g	E♭	→ c	E♭	→ B♭	E♭	→ c♯
d♯	→ B	d♯	→ F♯	d♯	→ g♯	d♯	→ C♯
E	→ g♯	E	→ c♯	E	→ B	E	→ c
e	→ C	e	→ G	e	→ a	e	→ C
F	→ a	F	→ d	F	→ C	F	→ b
f	→ D♭	f	→ A♭	f	→ b♭	f	→ B
F♯	→ a♯	F♯	→ d♯	F♯	→ C♯	F♯	→ b♭
f♯	→ D	f♯	→ A	f♯	→ b	f♯	→ B♭
G	→ b	G	→ e	G	→ D	G	→ a
g	→ E♭	g	→ B♭	g	→ c	g	→ A
A♭	→ c	A♭	→ f	A♭	→ E♭	A♭	→ a♭
g♯	→ E	g♯	→ B	g♯	→ c♯	g♯	→ G♯
A	→ c♯	A	→ f♯	A	→ E	A	→ g
a	→ F	a	→ C	a	→ d	a	→ G
B♭	→ d	B♭	→ g	B♭	→ F	B♭	→ f♯
a♯	→ F♯	a♯	→ C♯	a♯	→ d♯	a♯	→ F♯
B	→ e♭	B	→ g♯	B	→ F♯	B	→ f
b	→ G	b	→ D	b	→ e	b	→ F

[Satyendra 2004]

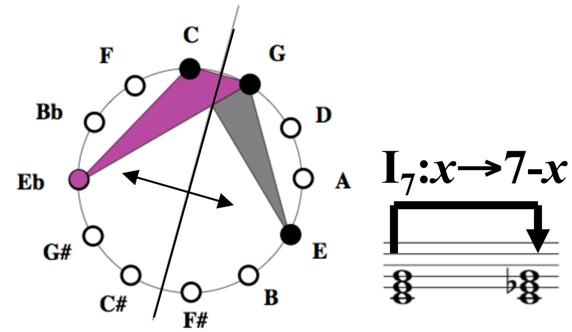
$D_{12} = \langle I, T \mid I^2 = T^{12} = 1 \,;\, ITI = I(IT)^{-1} \rangle$

• **D_{12} opère de façon simplement transitive sur l’ensemble *S* des 24 triades consonantes**

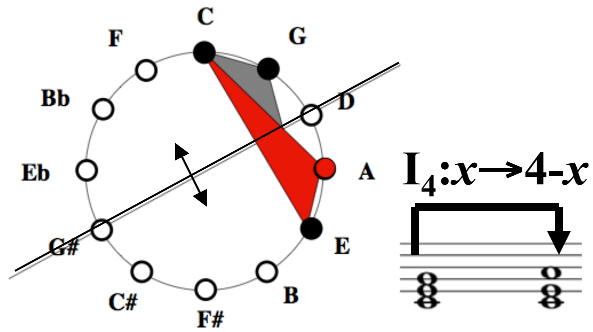
⇒ (*S*, D_{12}, int) est un GIS

(Neo-)Riemannian Operation **P = „Parallel“**

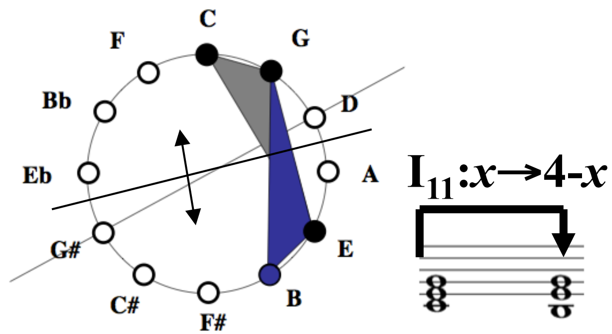
[Noll04]



(Neo-)Riemannian Operation **R = „Relative“**



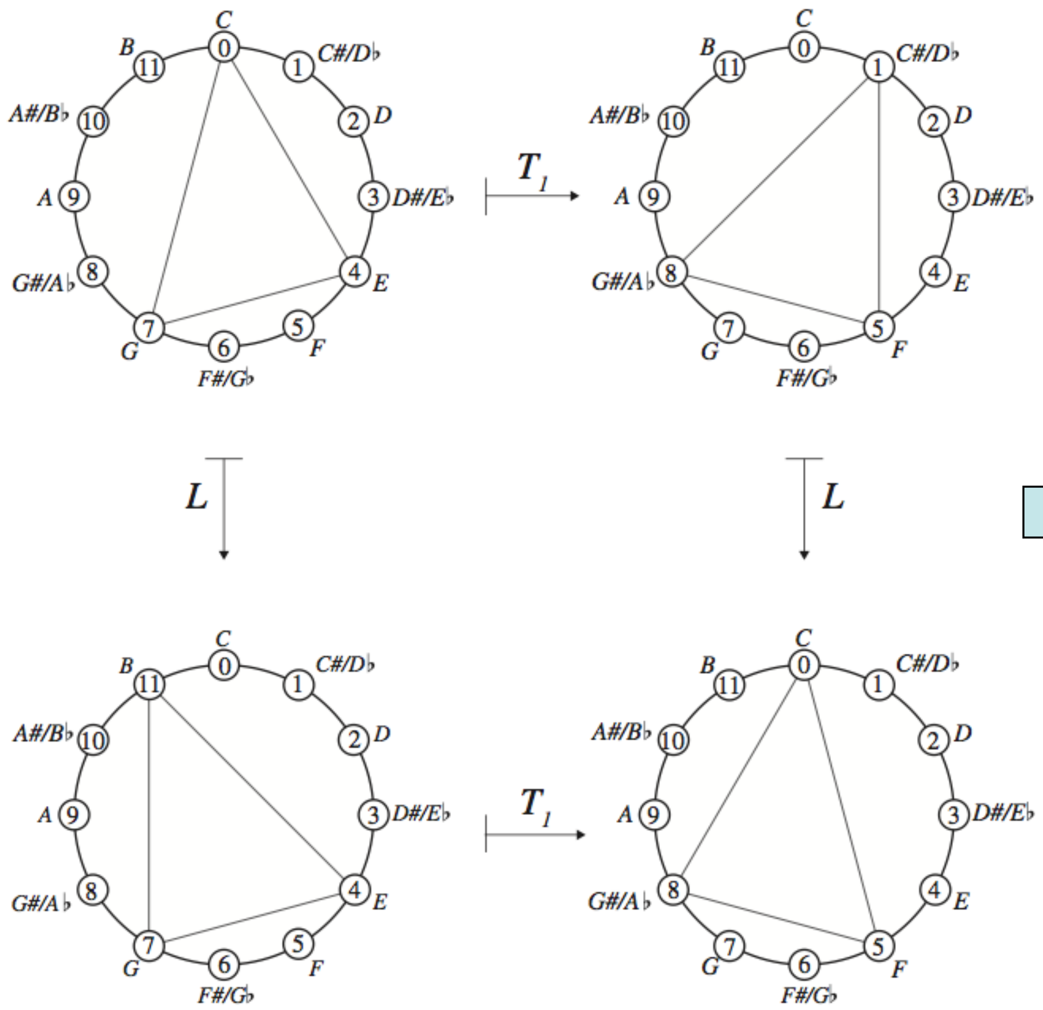
(Neo-)Riemannian Operation **L = „Leading-Tone“**



Dualité entre (S, ρ, int) et (S, D_{12}, int)

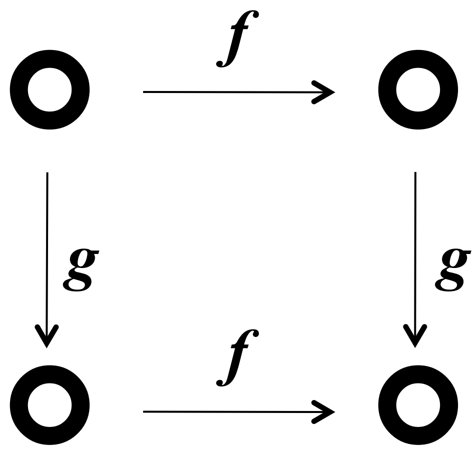
$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 \,;\, LRL = L(LR)^{-1} \rangle$ ↔ **$D_{12} = \langle I, T \mid I^2 = T^{12} = 1 \,;\, ITI = I(IT)^{-1} \rangle$**

⇒ ρ et D_{12} sont l'un le *centralisateur* de l'autre (dans le groupe symétrique *Sym*(S))



[Crans, Fiore & Satyendra, 2008]

(S, ρ, int) ≠ (S, D_{12}, int)
[cf. équivalence entre GIS]

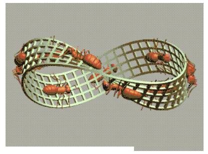


Tout diagramme commute

$\forall f \in D_{12}$

$\forall g \in \rho$

Se (dé)placer dans un *orbifold*



$T^2 = \mathbf{R/12Z} \times \mathbf{R/12Z} \longrightarrow T^2 / S_2$

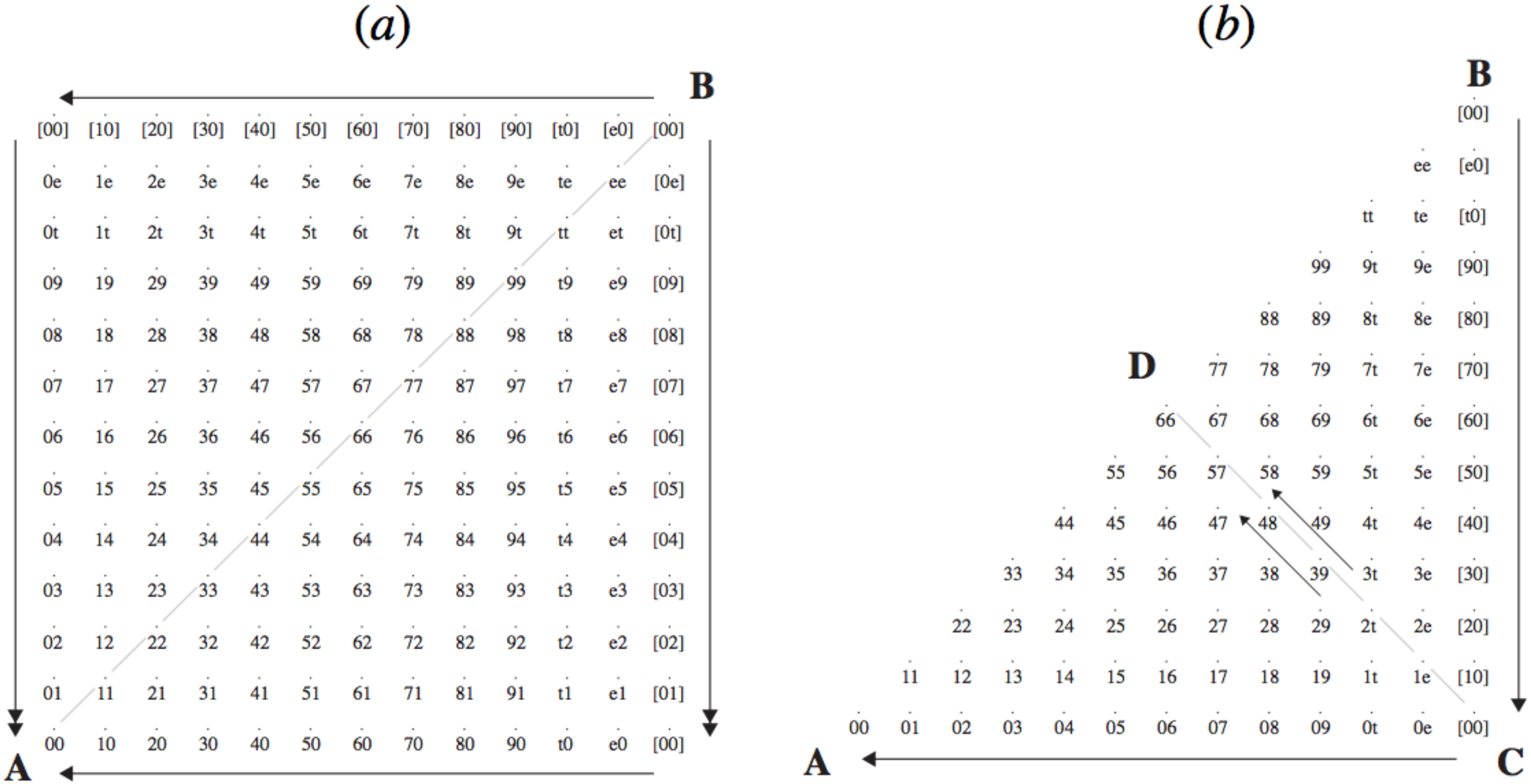
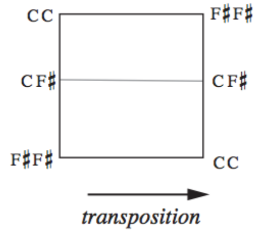
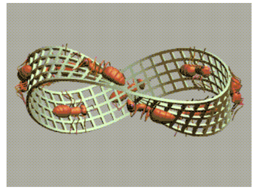


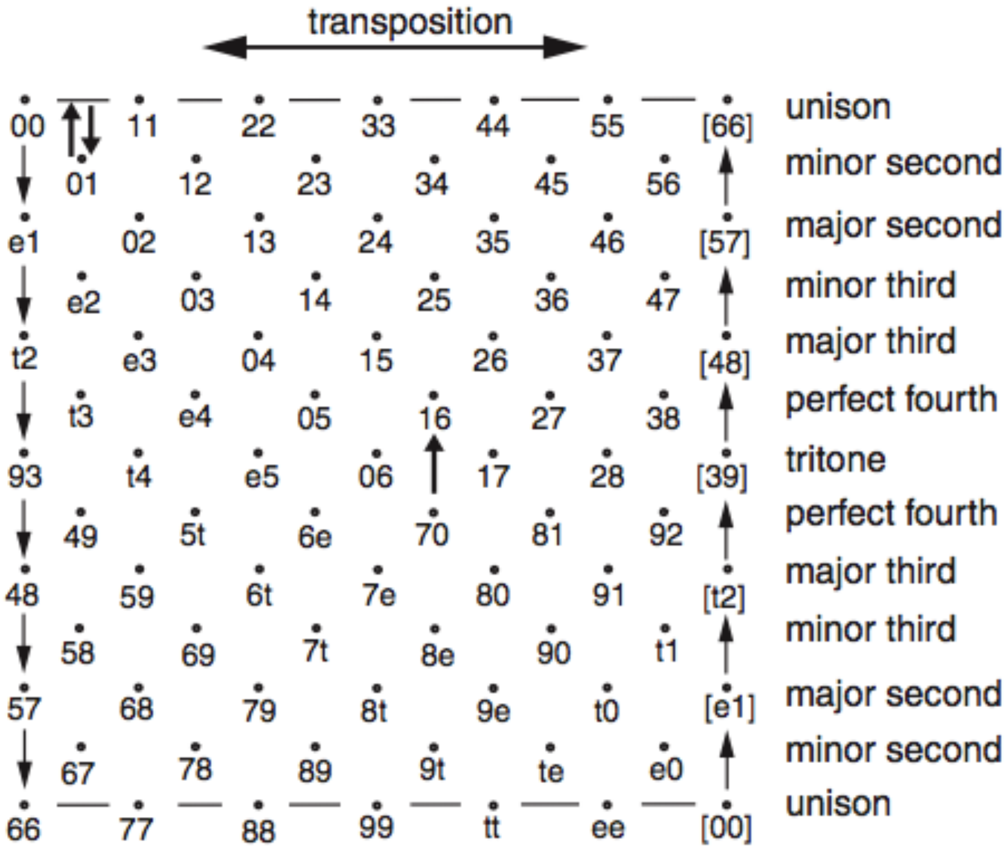
Figure S9. (*a*) ***The space of ordered two-note chords of pitch-classes is a 2-torus. To identify points (x, y) and (y, x), we need to fold the torus along the AB diagonal. The resulting figure, shown in (b), is a triangle with two of its sides identified. This is a Möbius strip. To see why, cut figure (b) along the line CD and glue AC to CB. (To make this identification in Euclidean 3-space, you will need to turn over one of the pieces of paper.) The resulting is a square with opposite sides identified, as in Figure 2 of the main paper.***Dmitri Tymoczko :« The Geometry of Musical Chords », *Science*, 313, 2006

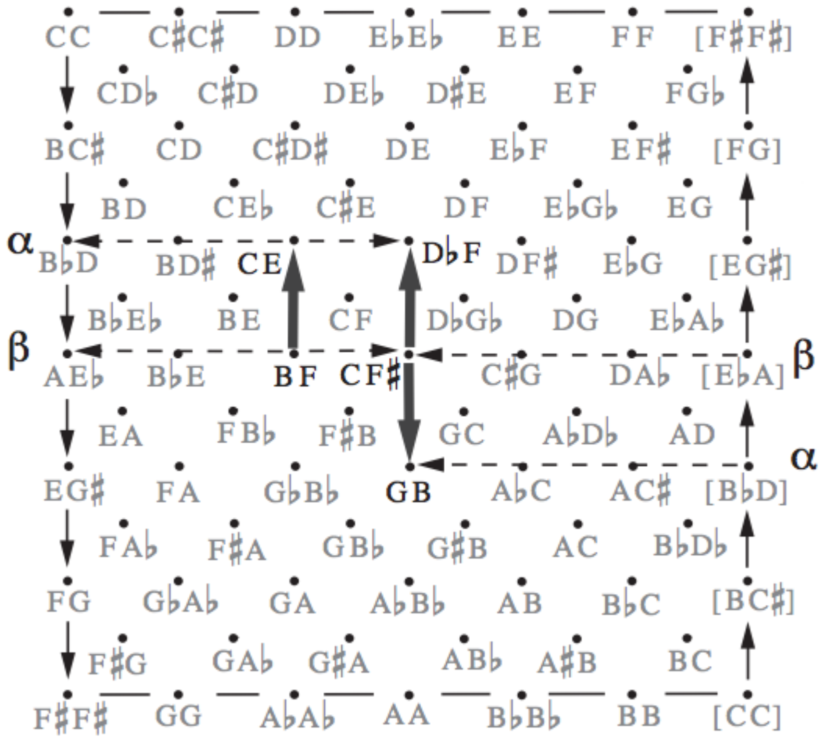
Se (dé)placer dans un *orbifold*

$T^2 = \mathbf{R/12Z} \times \mathbf{R/12Z} \longrightarrow T^2/S_2$





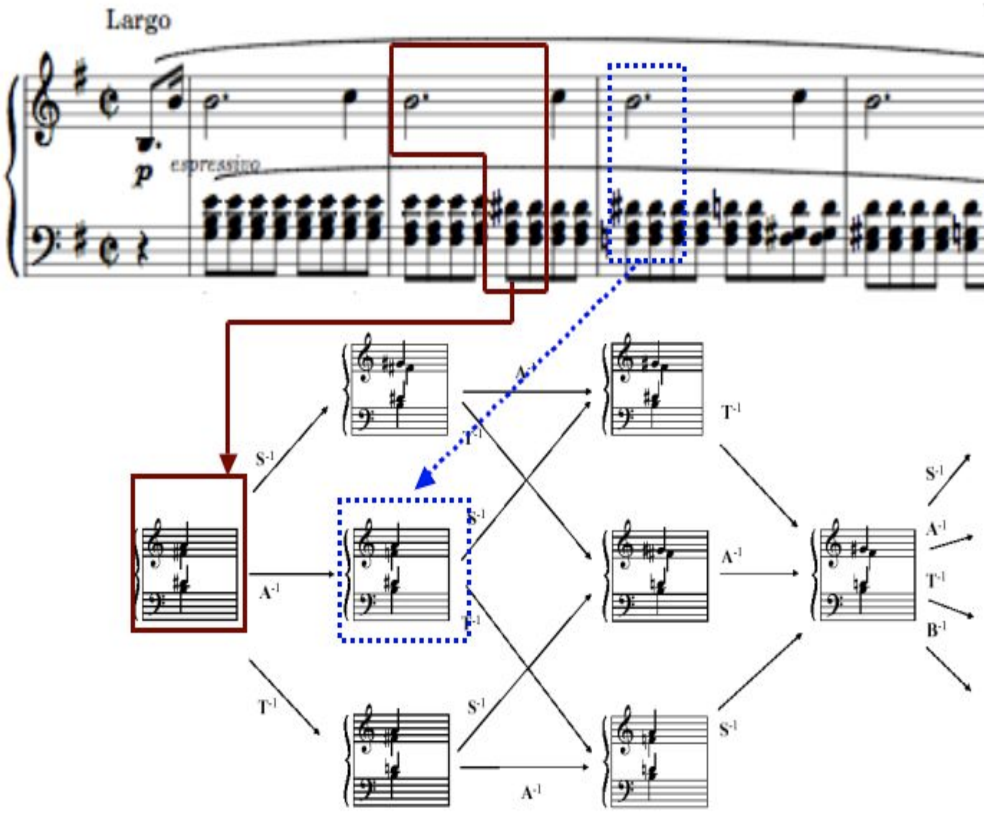


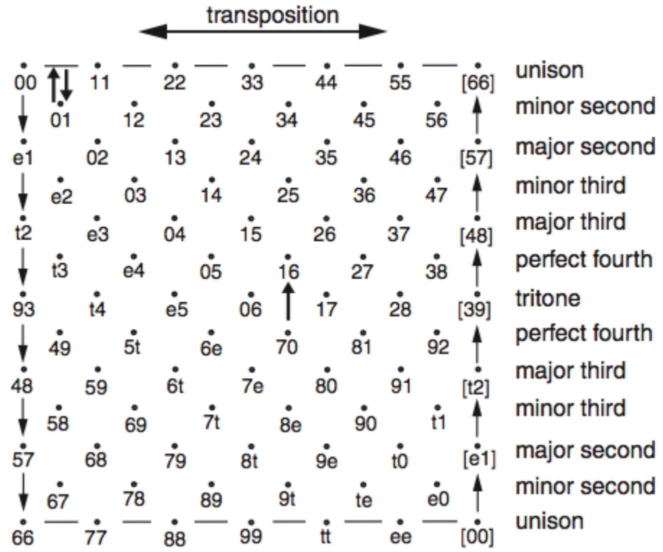


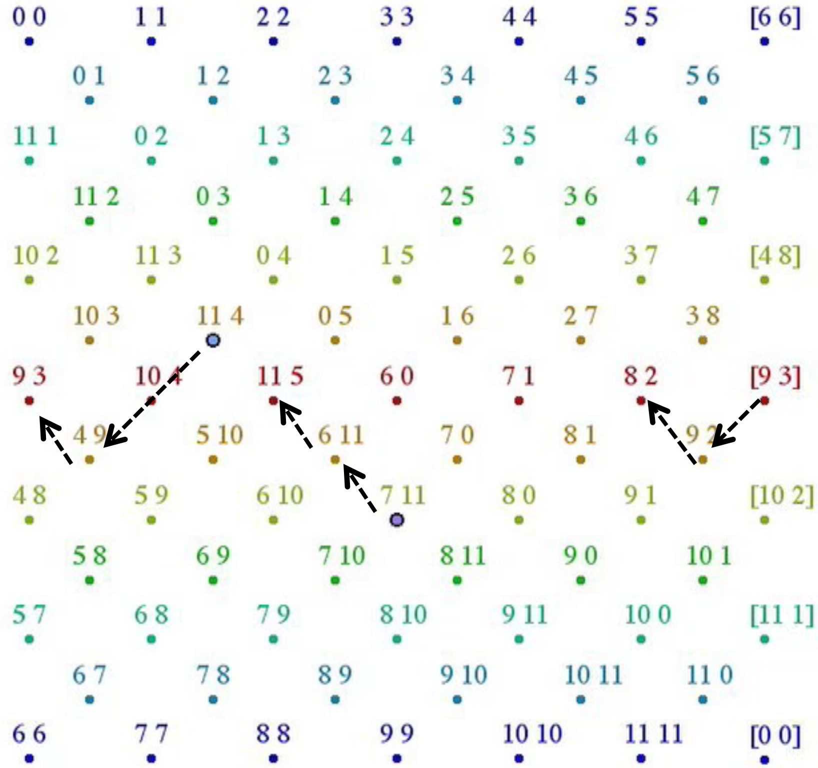
Dmitri Tymoczko :
« The Geometry of Musical Chords »,
Science, 313, 2006

[Tymoczko 2010]

$$T^2 = \mathbf{R/12Z} \times \mathbf{R/12Z} \longrightarrow T^2/S_2$$

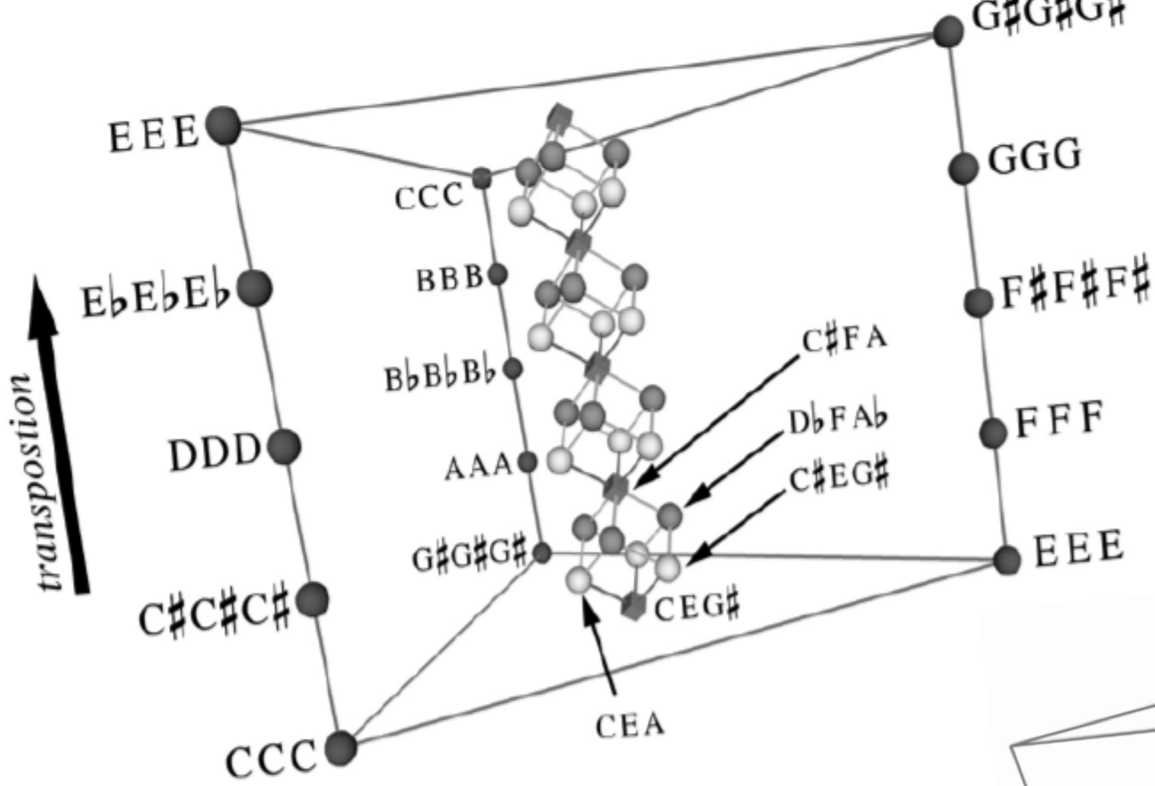


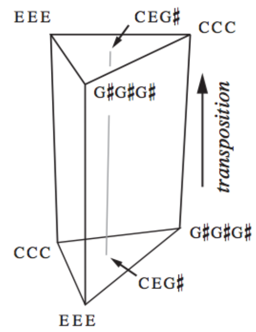


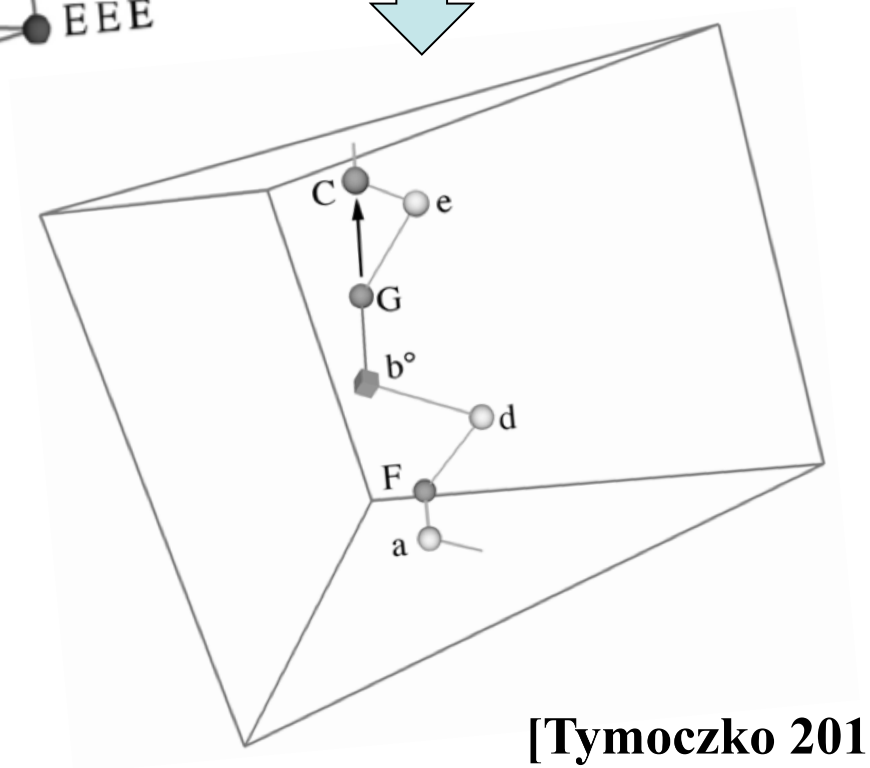


Dmitri Tymoczko :
« The Geometry of Musical Chords »,
Science, 313, 2006

$T^3 = (\mathbf{R}/\mathbf{12Z})^3 \longrightarrow T^3/S_3$





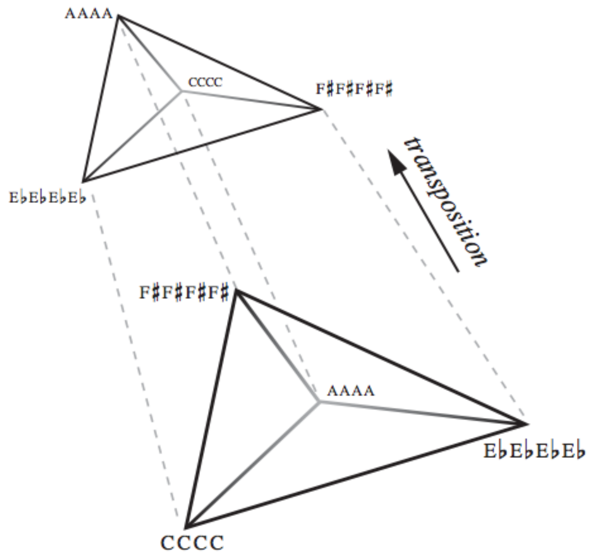


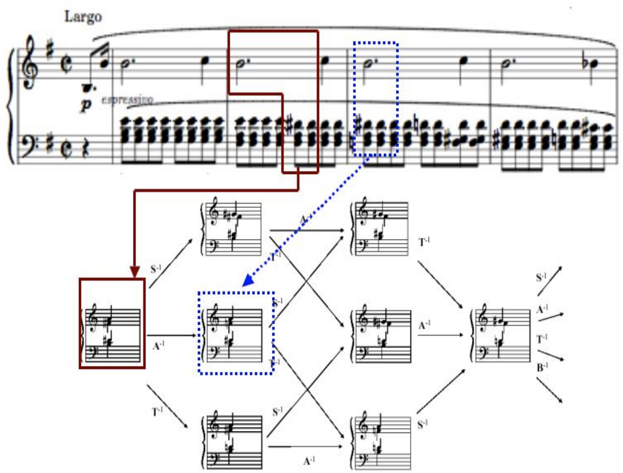
Dmitri Tymoczko :
« The Geometry of Musical Chords »,
Science, 313, 2006

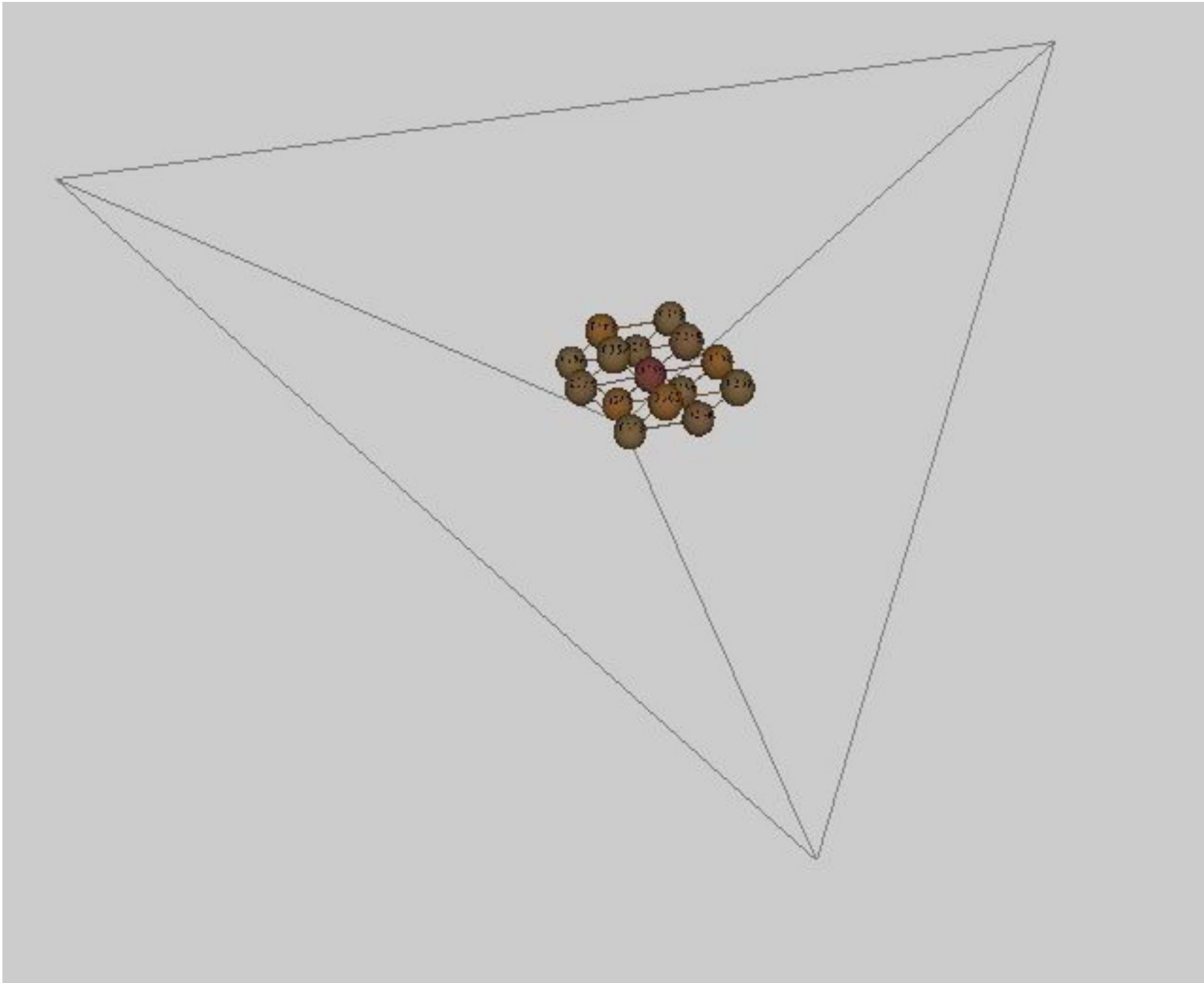
[Tymoczko 2010]

Se (dé)placer dans un espace de dim. 4

$T^4 = (\mathbf{R/12Z})^4 \longrightarrow T^4/S_4$



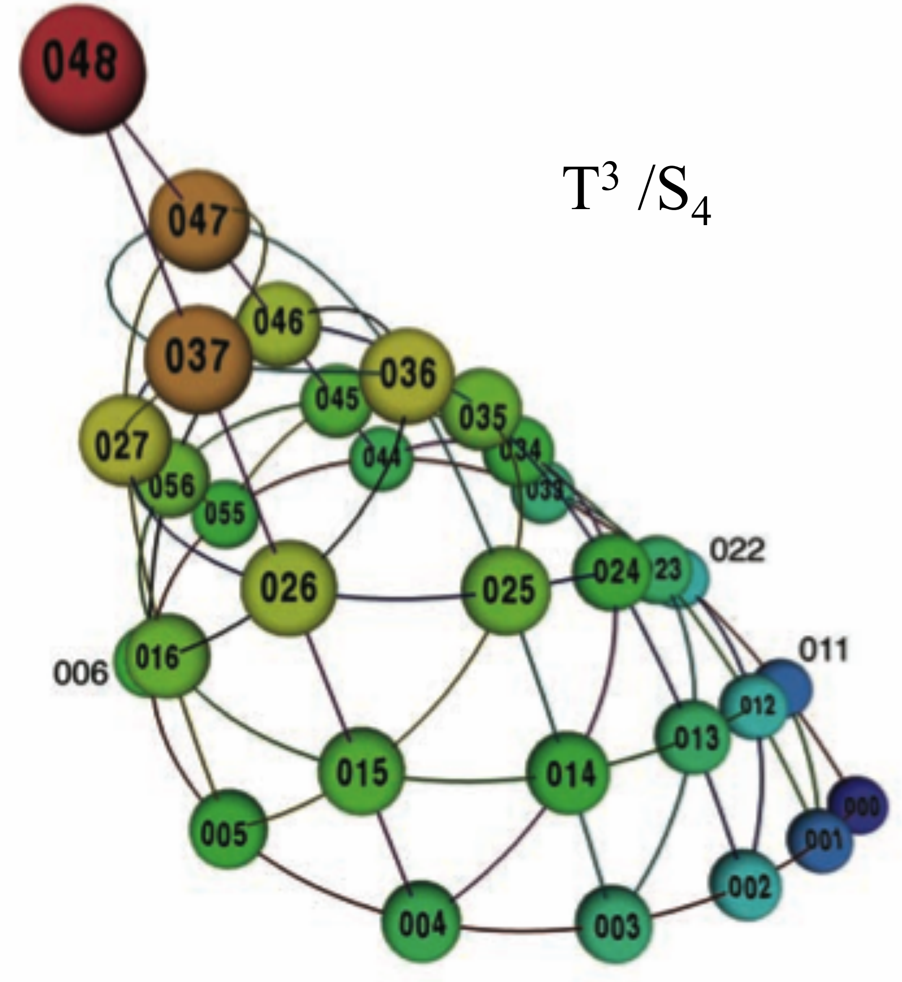




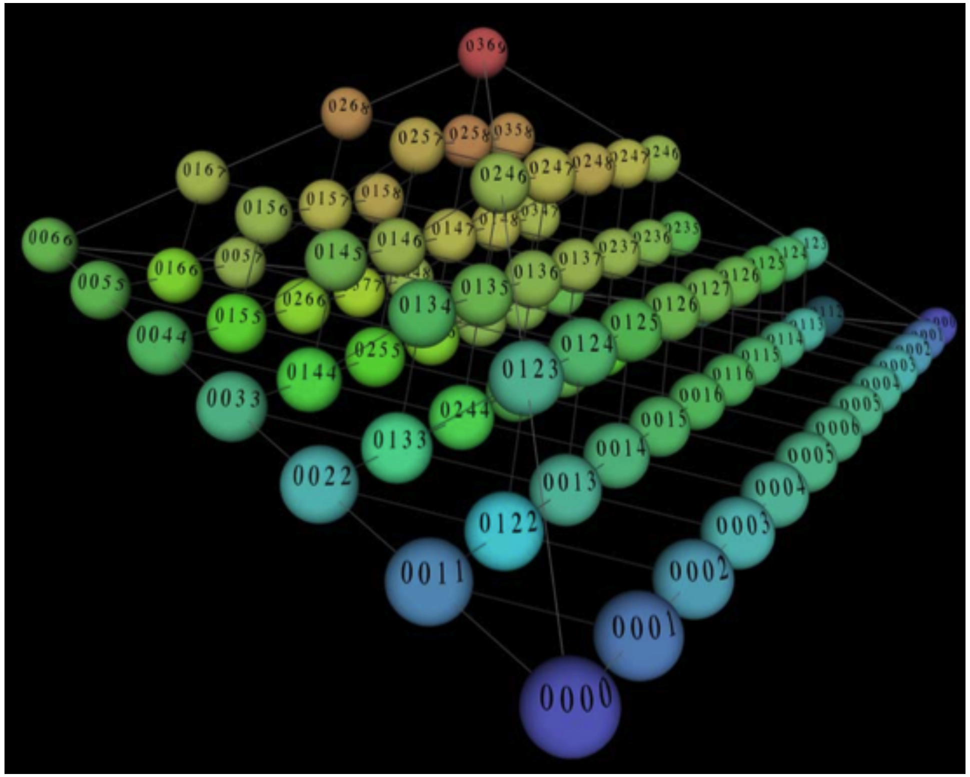
Dmitri Tymoczko, « The Geometry of Musical Chords », *Science*, 313, 2006

Autres exemples d'*orbifolds*

T^3/S_4

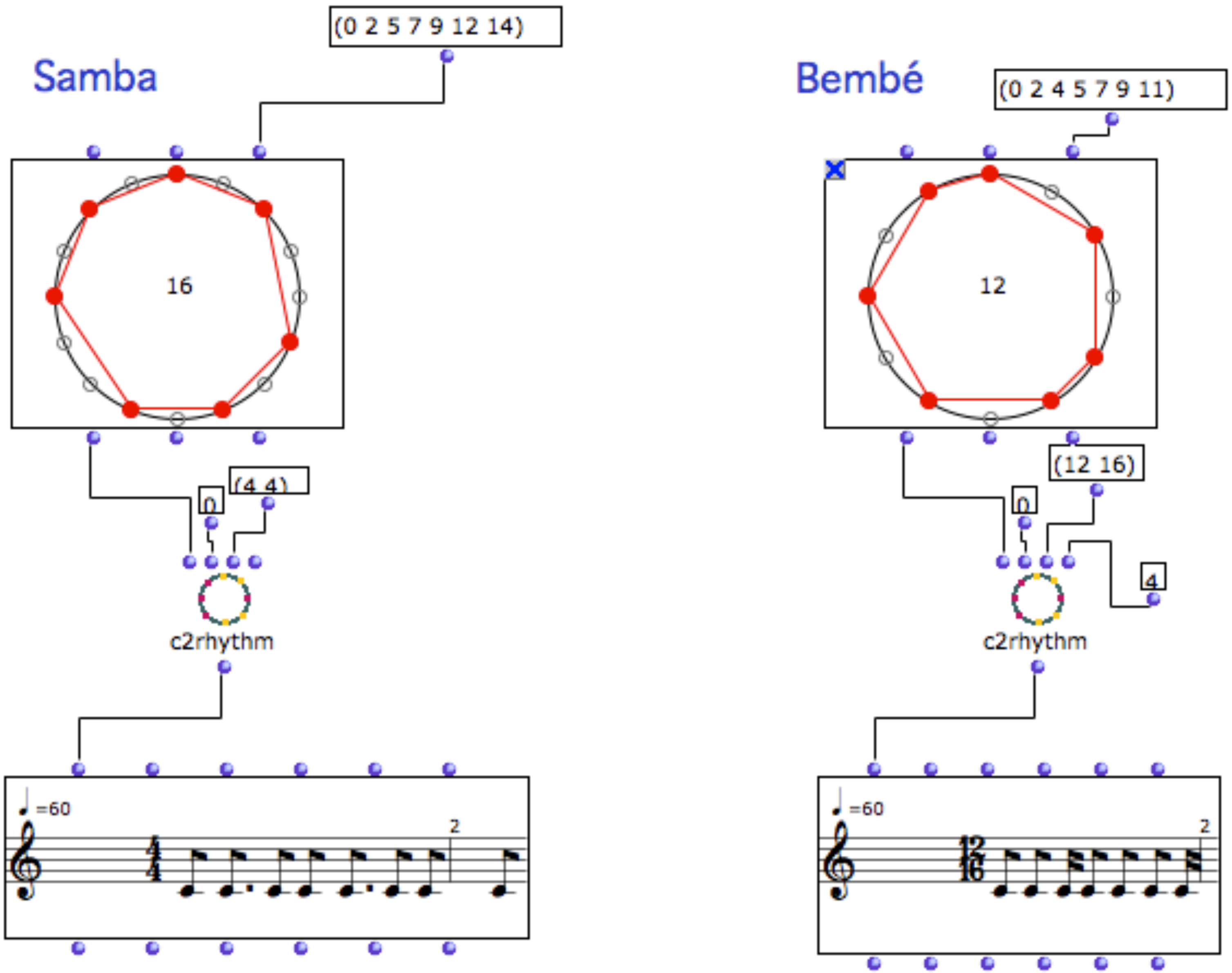


$T^3/(S_4 \times Z_2)$



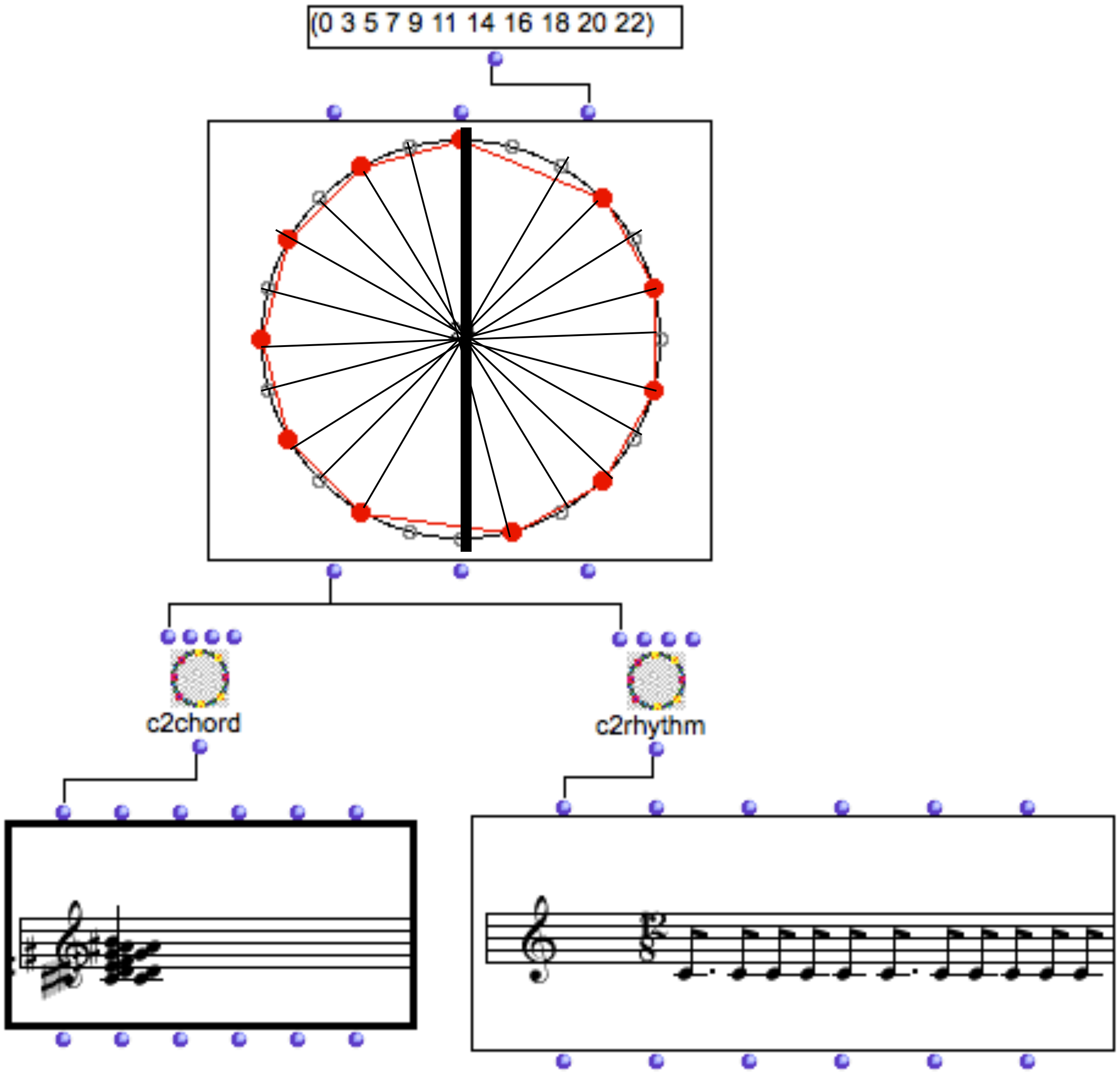
C. Callender, I. Quinn & D. Tymoczko, « Generalized Voice-Leading Spaces », *Science*, 320, 2008

Théorie du rythme périodique

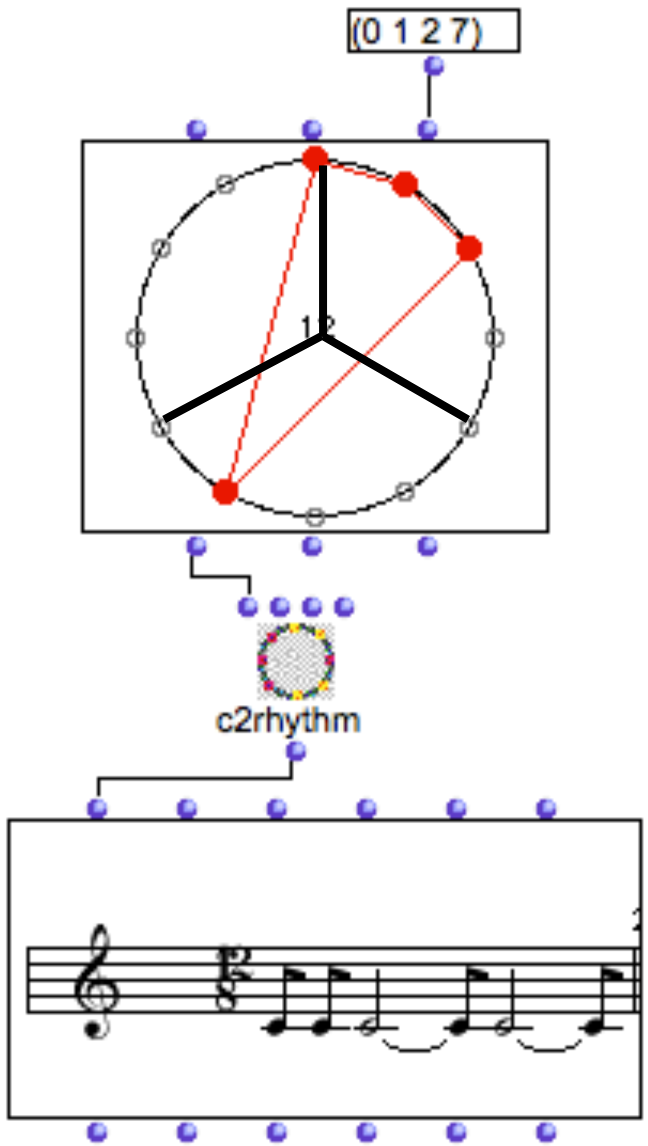


The Oddity Property and a generalization

(Simha Arom & Marc Chemillier)

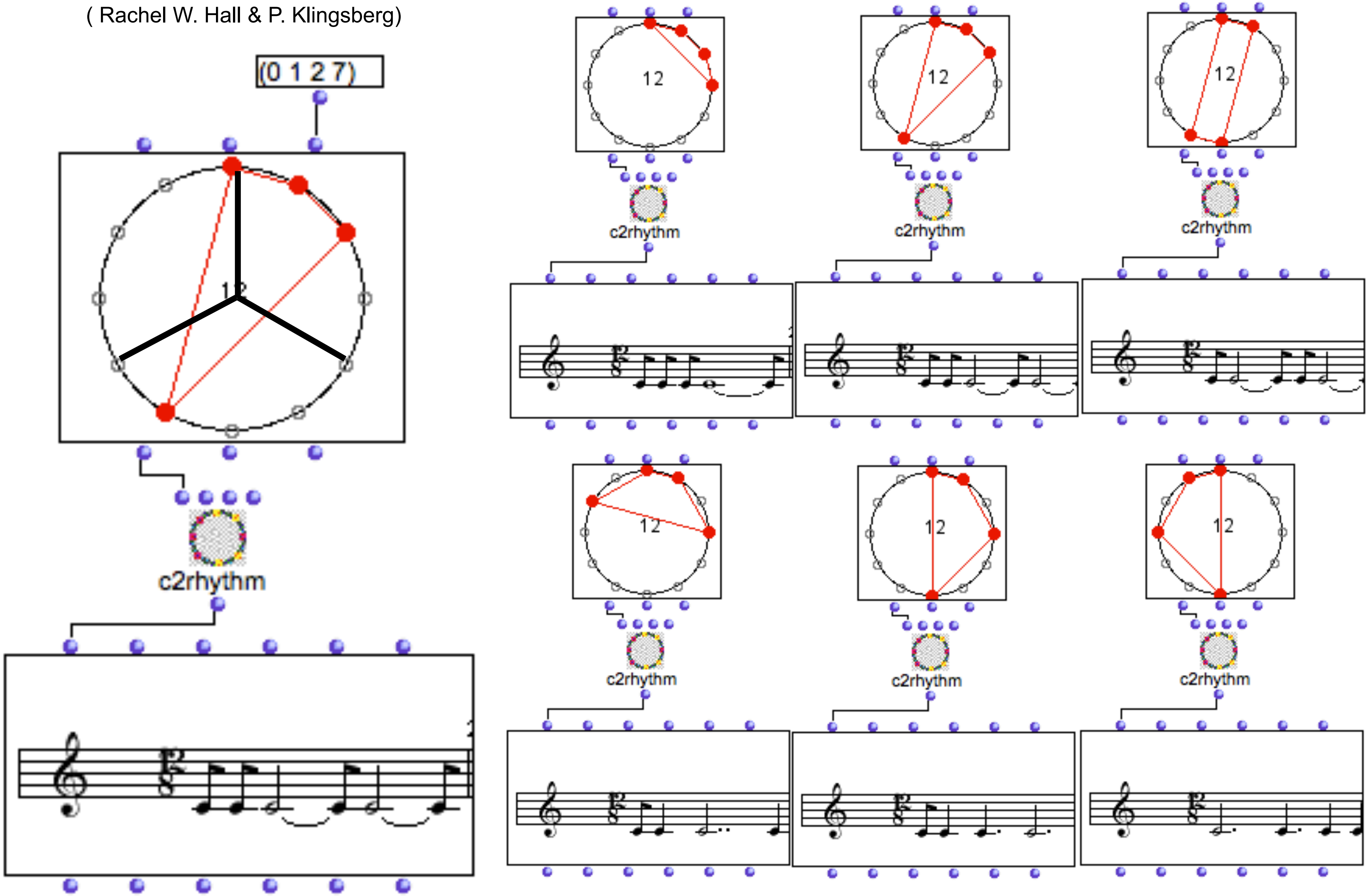


(Rachel W. Hall & P. Klingsberg)

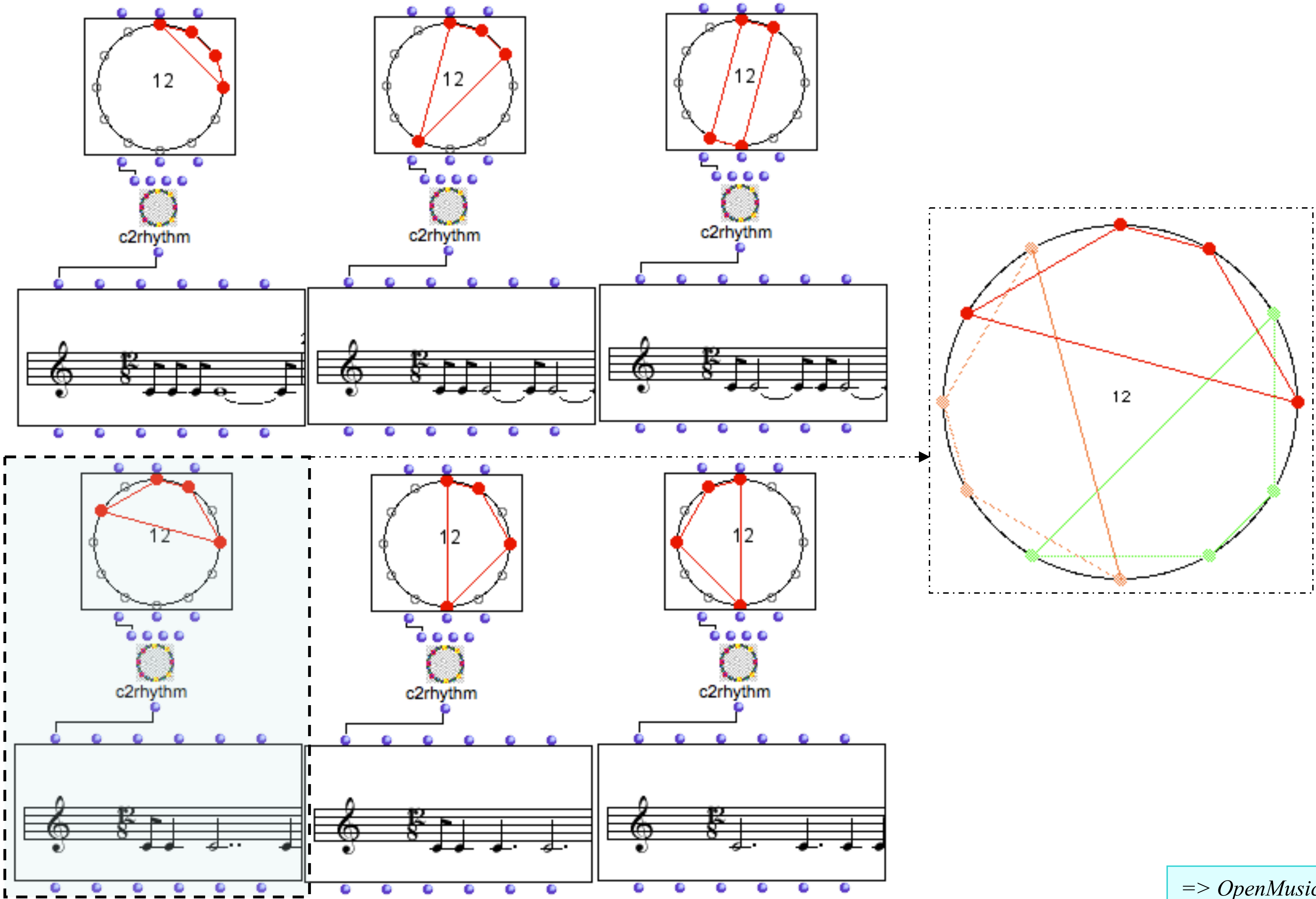


The 3-Oddity Property

(Rachel W. Hall & P. Klingsberg)



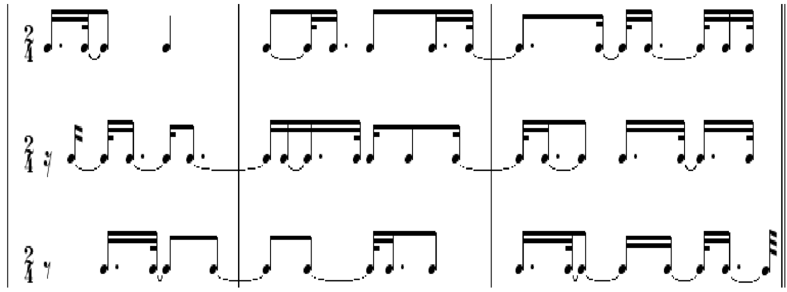
3-asymmetric rhythmic pattern and tiling process



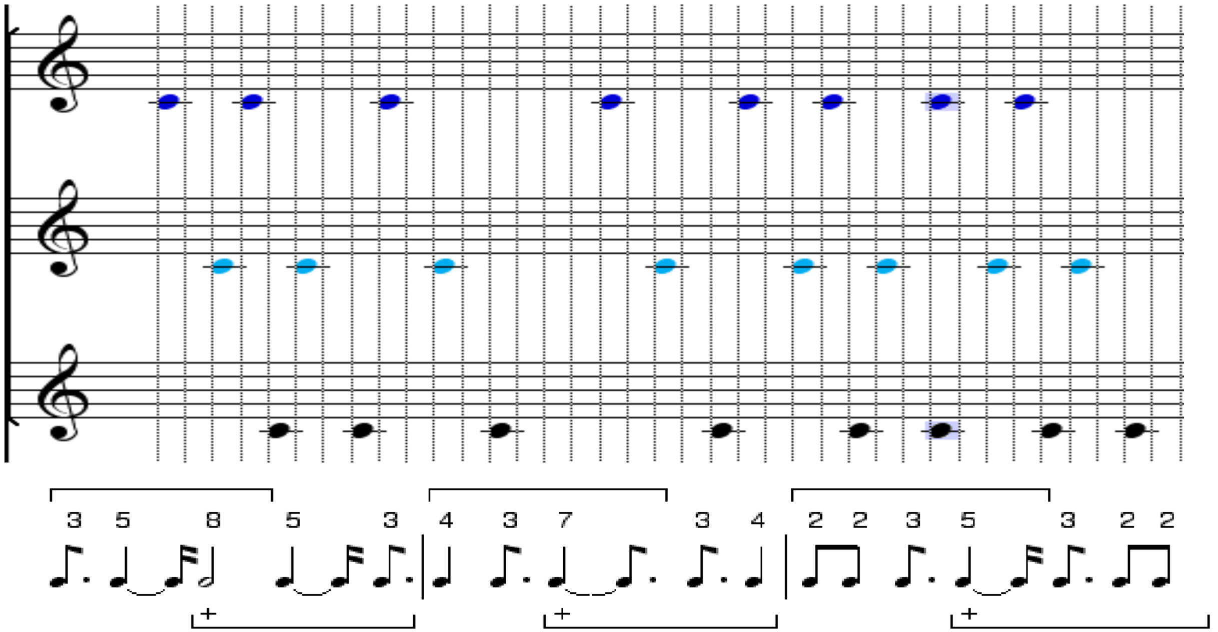
=> *OpenMusic*

Olivier Messiaen et les canons rythmiques

Harawi (1945)



Visions de l'Amen (1943)



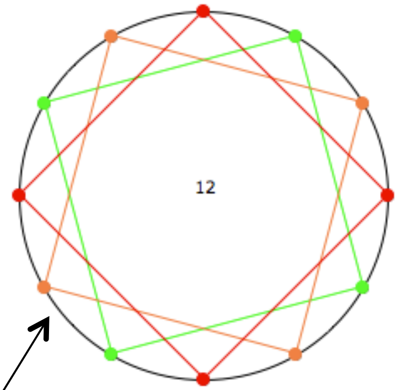
Modèle rythmique

« …il résulte de tout cela que les différentes sonorités se mélangent ou s'opposent de manières très diverses, **jamais au même moment ni au même endroit […]. C'est du désordre organisé** »

O. Messiaen : *Traité de Rythme, de Couleur et d'Ornithologie,* tome 2, Alphonse Leduc, Paris, 1992

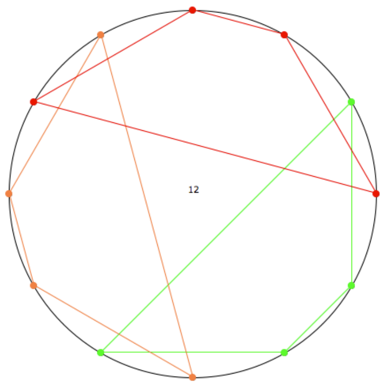
Quatre types de canons rythmiques mosaïques

$Z_n = A+B$



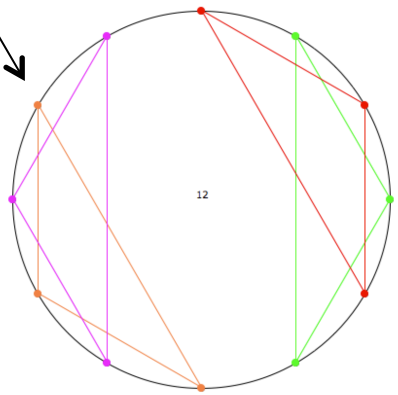
$A<Z_n$

$B<Z_n$



$A\subset Z_n$

$B<Z_n$

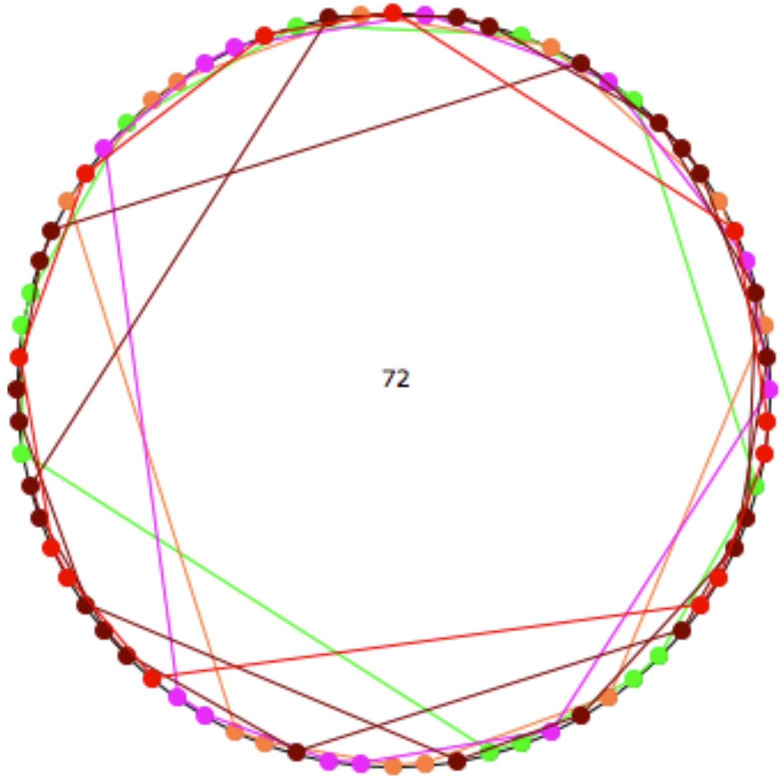


$A\subset Z_n$

$B\subset Z_n$

sous-ensemble périodique

Canon de Vuza



$A,\ B \subset Z_n$

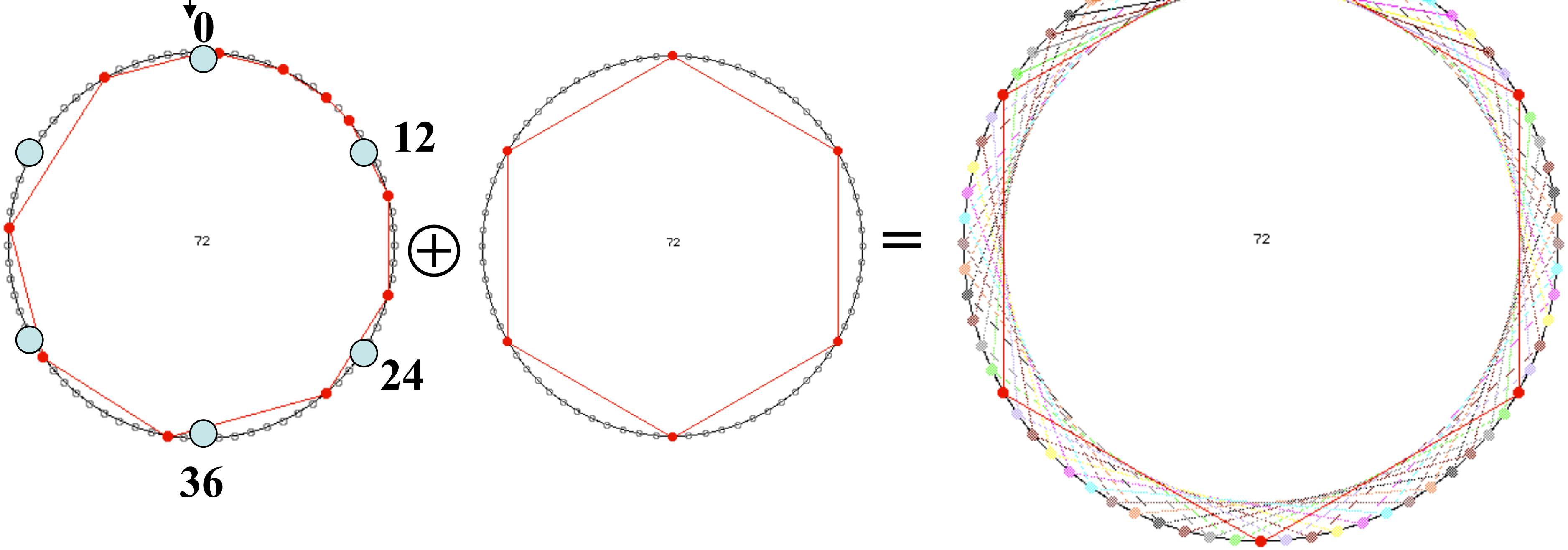
sous-ensembles périodiques

Canons rythmiques mosaïques et séries tous-intervalles

S={0, 1, 3, 6, 7, 9, 8, 10, 11, 2, 4, 5)

*S**=(1,4,3,2,5,6,7,10,9,8,11)

mod 72



n-asymmetric rhythmic pattern and tiling process

…
…
25
24
13
12

S={0, 1, 3, 6, 7, 9, 8, 10, 11, 2, 4, 5)

*S**=(1,2,3,1,2,11,2,1,3,2,1,45) ←

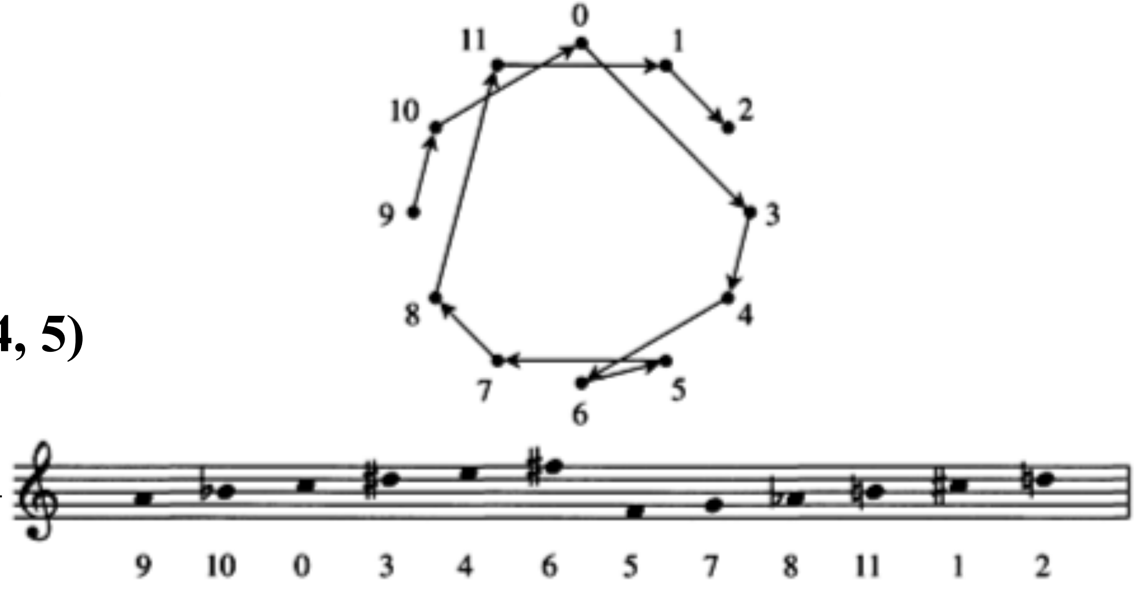
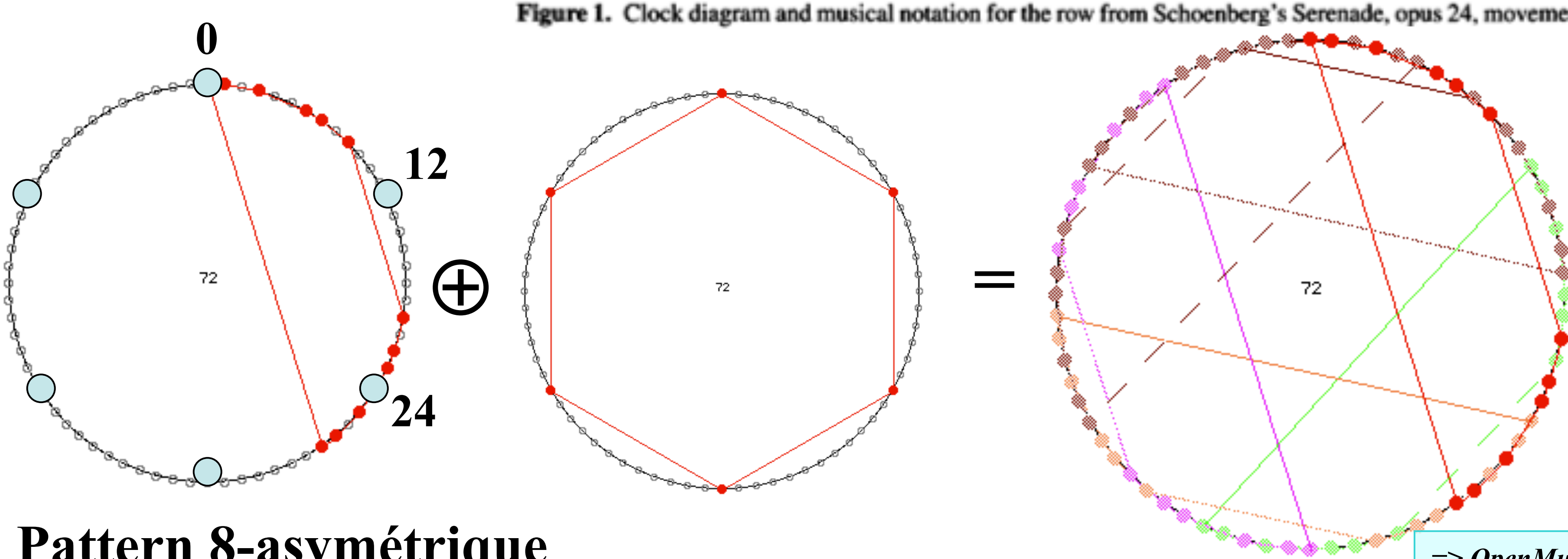


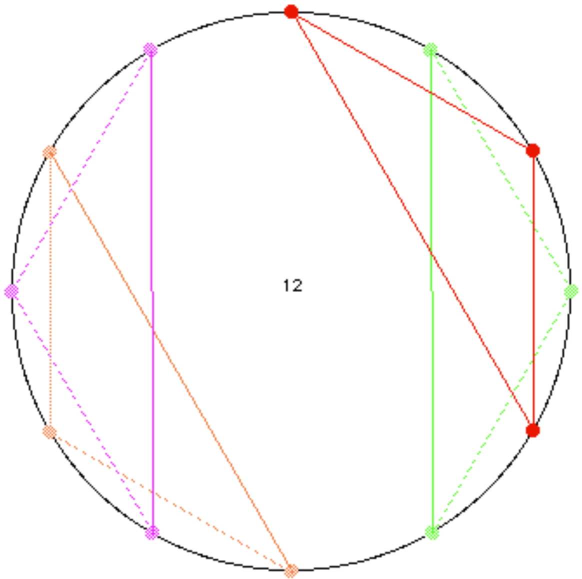
Figure 1. Clock diagram and musical notation for the row from Schoenberg's Serenade, opus 24, movement 5.

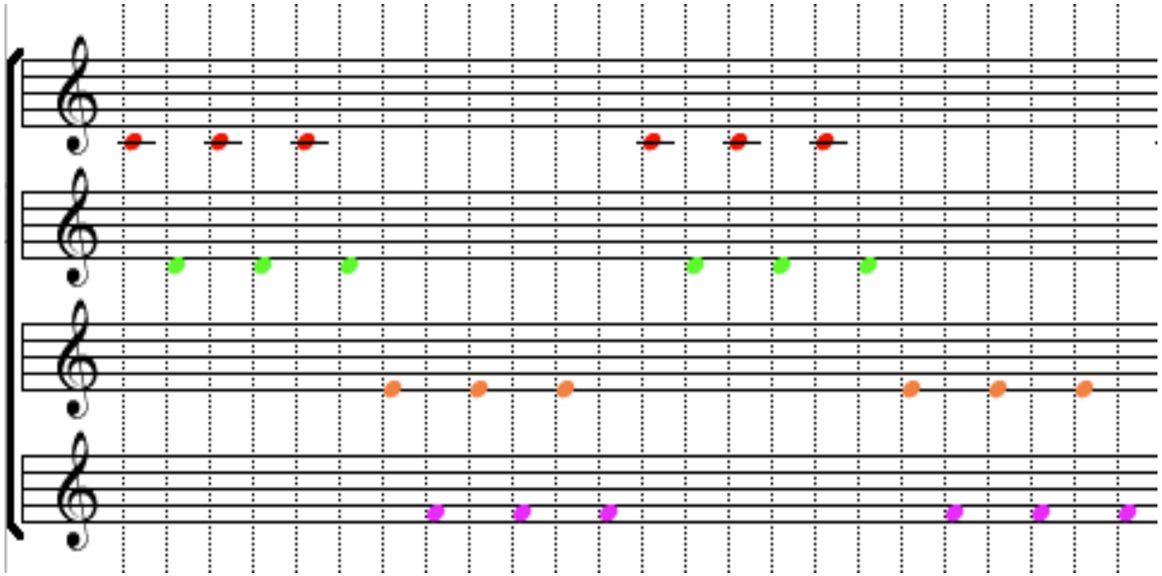


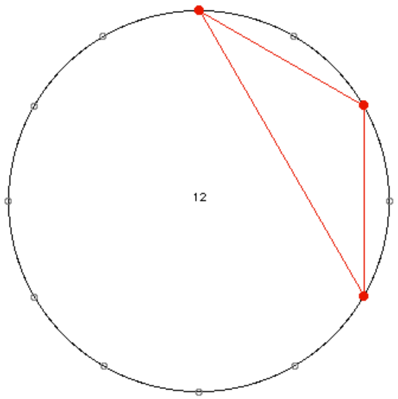
Pattern 8-asymétrique

=> *OpenMusic*

Canons mosaïques avec symétrie transpositionnelle



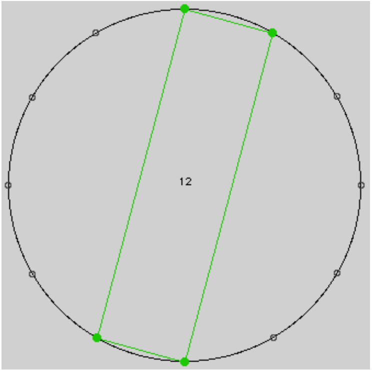




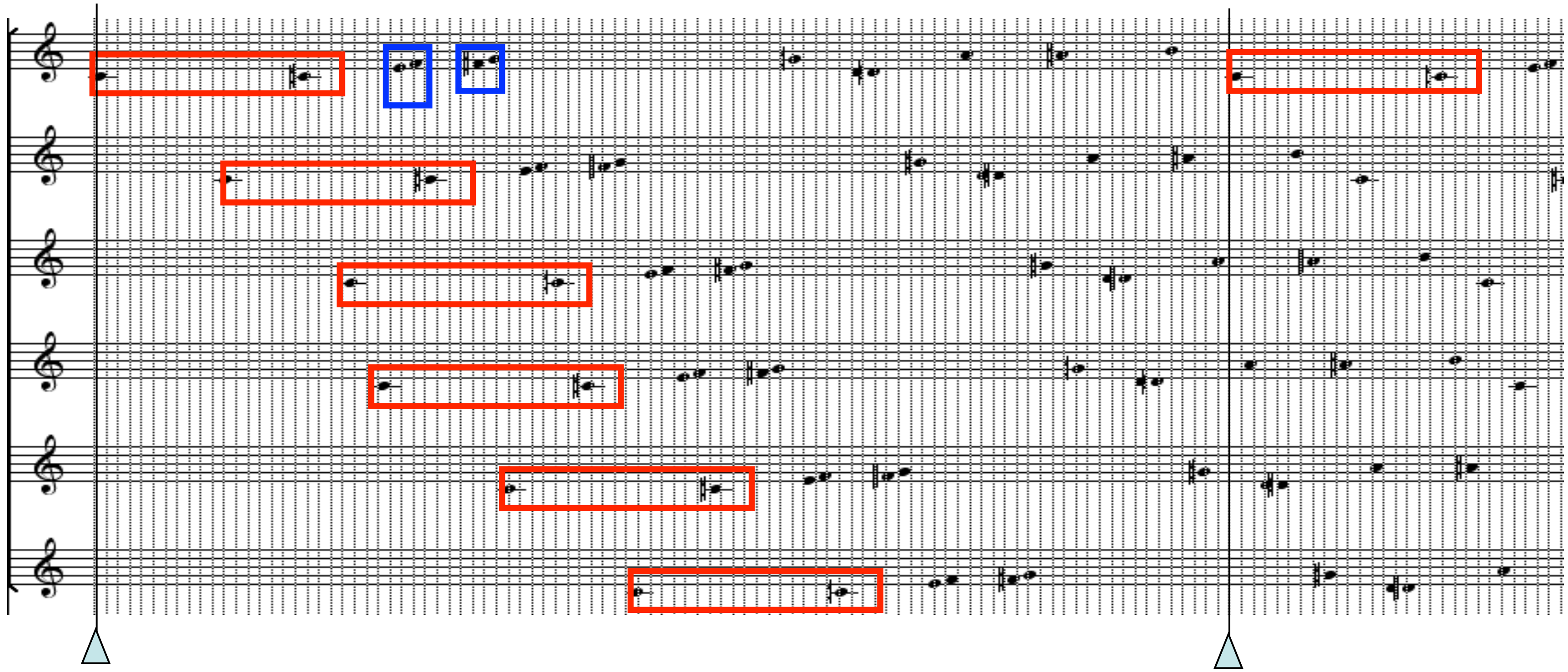
$\mathbf{Z}_{12} = A \oplus B$

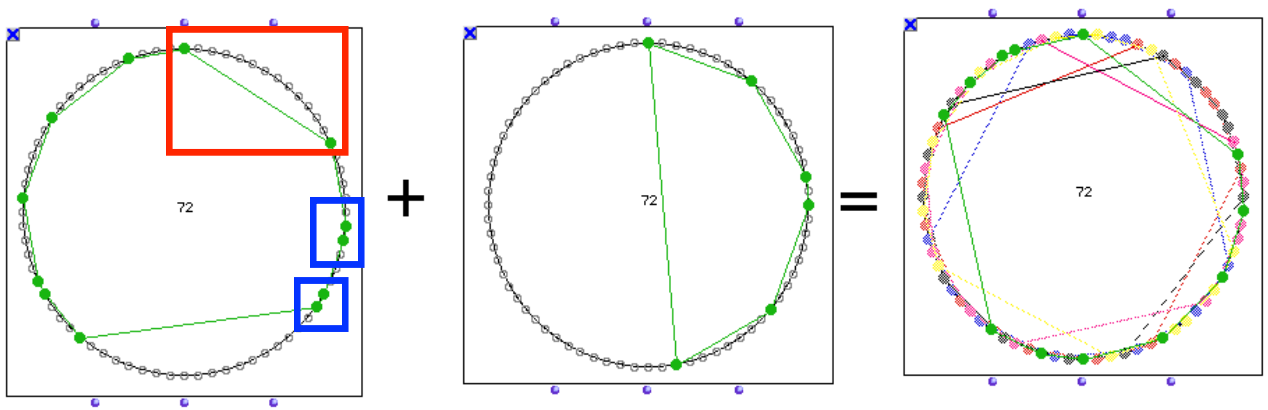
A= {0,2,4}

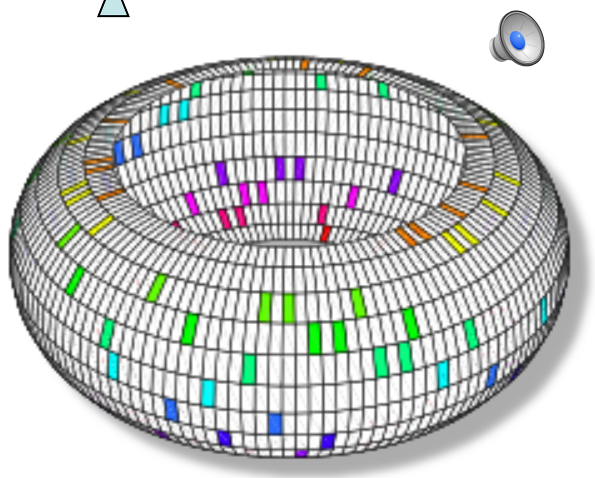
B= {0,1,6,7}



Vuza Canons : canons mosaïques sans périodicité interne







Classification paradigmatique des canons de période 72

{ Z_n }

R
(1 3 3 6 11 4 9 6 5 1 3 20)
(20 3 1 5 6 9 4 11 6 3 3 1)
(1 4 1 19 4 1 6 6 7 4 13 6)
(6 13 4 7 6 6 1 4 19 1 4 1)
(1 5 15 4 5 6 6 3 4 17 3 3)
(3 3 17 4 3 6 6 5 4 15 5 1)

S
(8 8 2 8 8 38)
(16 2 14 2 16 22)
(14 8 10 8 14 18)

{ D_n }

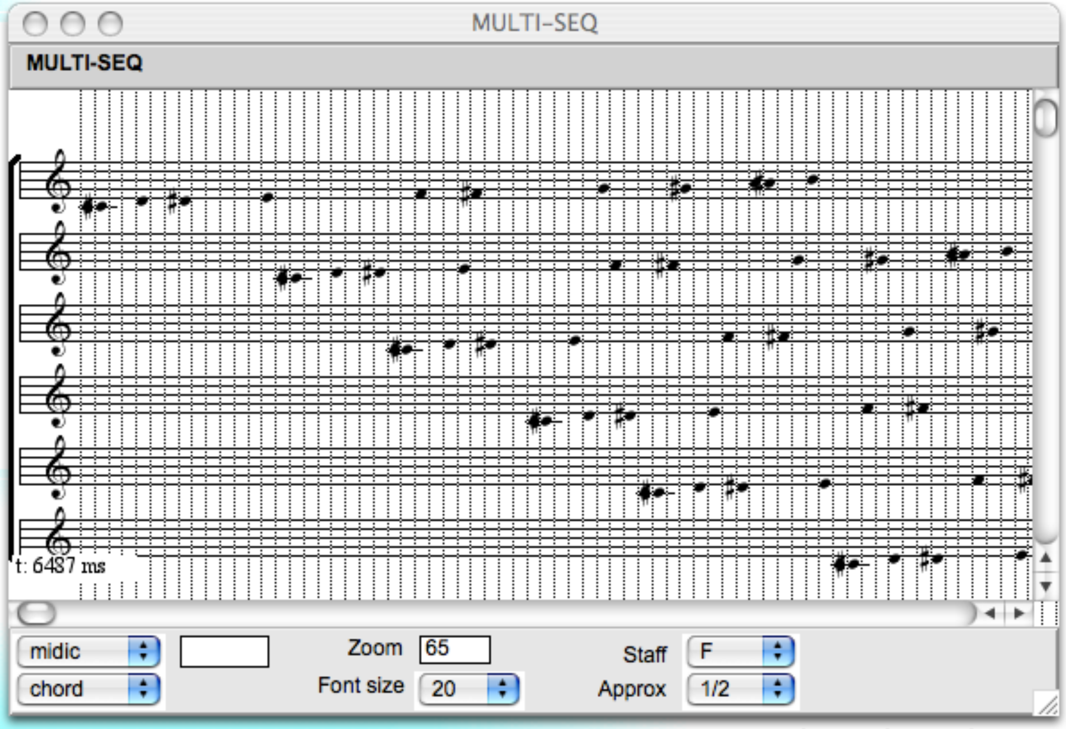
R
(1 3 3 6 11 4 9 6 5 1 3 20)
(1 4 1 19 4 1 6 6 7 4 13 6)
(1 5 15 4 5 6 6 3 4 17 3 3)

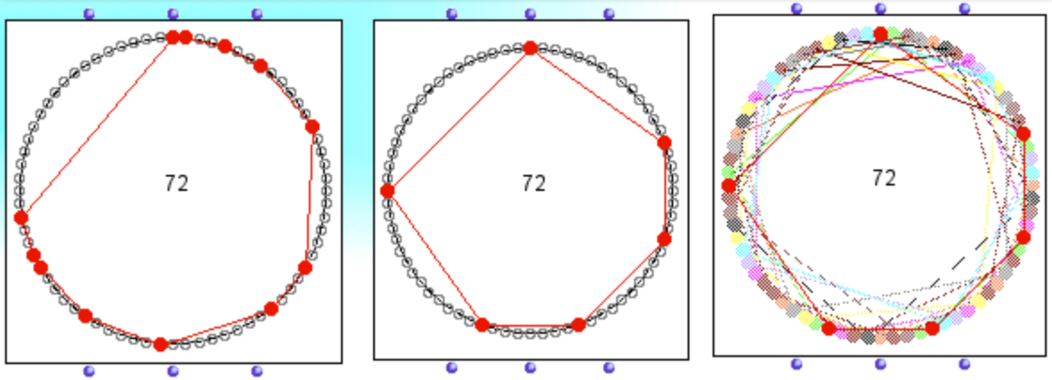
S
(8 8 2 8 8 38)
(16 2 14 2 16 22)
(14 8 10 8 14 18)

{ Af_n }

R
(1 3 3 6 11 4 9 6 5 1 3 20)
(1 4 1 19 4 1 6 6 7 4 13 6)

S
(14 8 10 8 14 18)



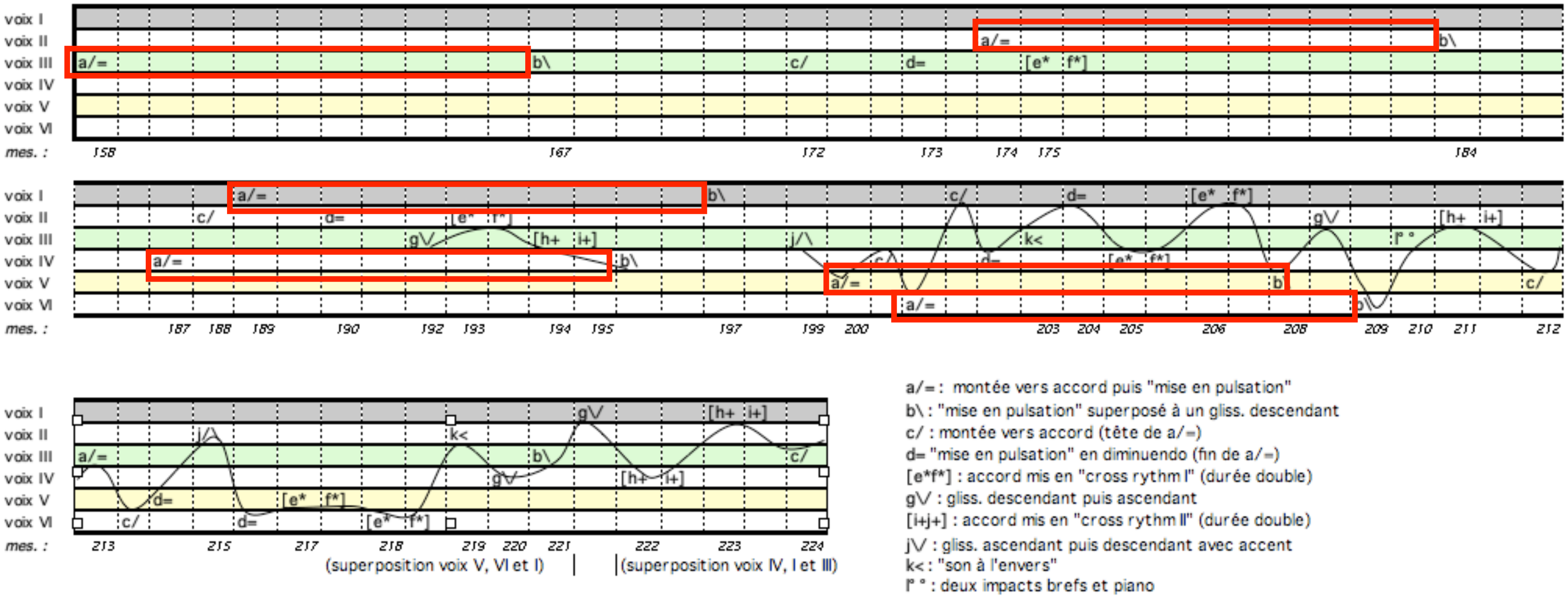


Résultat : uniquement deux « types » de canons différents (à une transformation affine près, i.e. $f : Z_{72} \rightarrow Z_{72}$ t.q. $f(x)=ax+b$ avec $a \in (Z_{72})^*$ et $b \in Z_{72}$

Fabien Lévy

Première utilisation des canons de Vuza

• *Coïncidences* (pour 33 musiciens, 1999-2007)



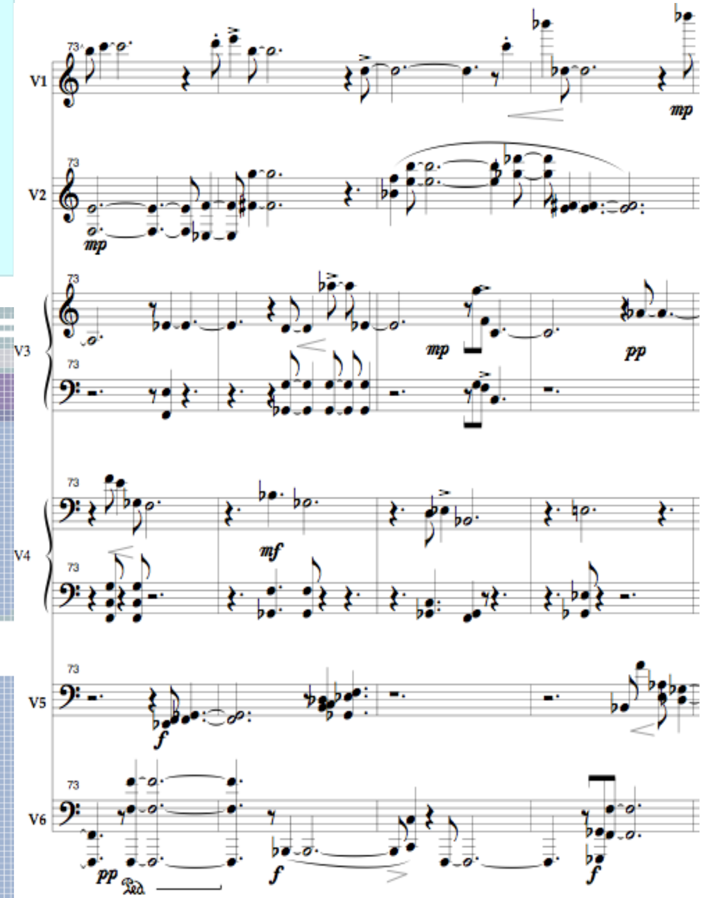
Coïncidences - Fabien Levy : déroulement du canon (mes. 158 à 226)
(chaque impact fait 3 temps)

Interprètes : Tokyo Symphony Orchestra, Dir.: Kazuyoshi Akiyama, 05/09/2007, Suntory Hall, Tokyo, Japon

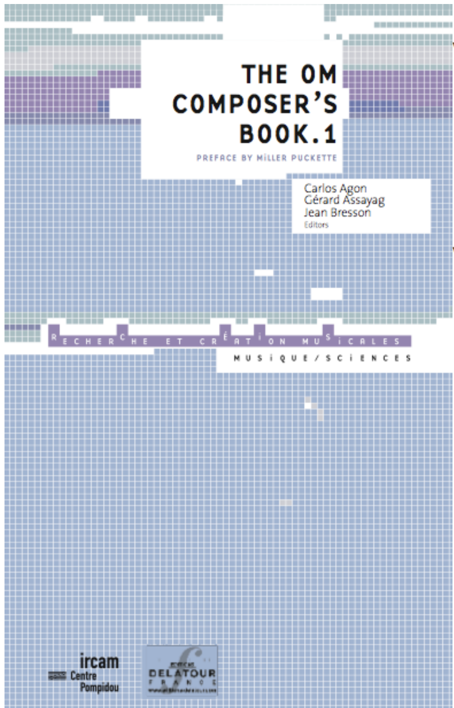
Georges Bloch

Stratégies compositionnelles à partir d'un modèle formel

- **Organisation métrique d'un canon mosaïque**
- **Réduction d'un canon par auto-similarité**
- **Modulation métrique entre canons**
- **Transformation d'un canon dans une texture**
- **Canons mosaïques et IAO (*OMax*)**



- ***Projet Beyeler* (2001)**
- ***Projet Hitchcock***
- ***Visite des tours de la cathédrale de Reims***
- ***Noël des Chasseurs***
- ***Canons à marcher***
- ***Canon à eau***
- ***Harawun* (2004)**
- ***L'Homme du champ* (2005)**
- ***A piece based on Monk* (2007)**
- ***Peking Duck Soup* (2008)**



- ***A piece based on Monk* (2007) (« Well You Need'nt »)**

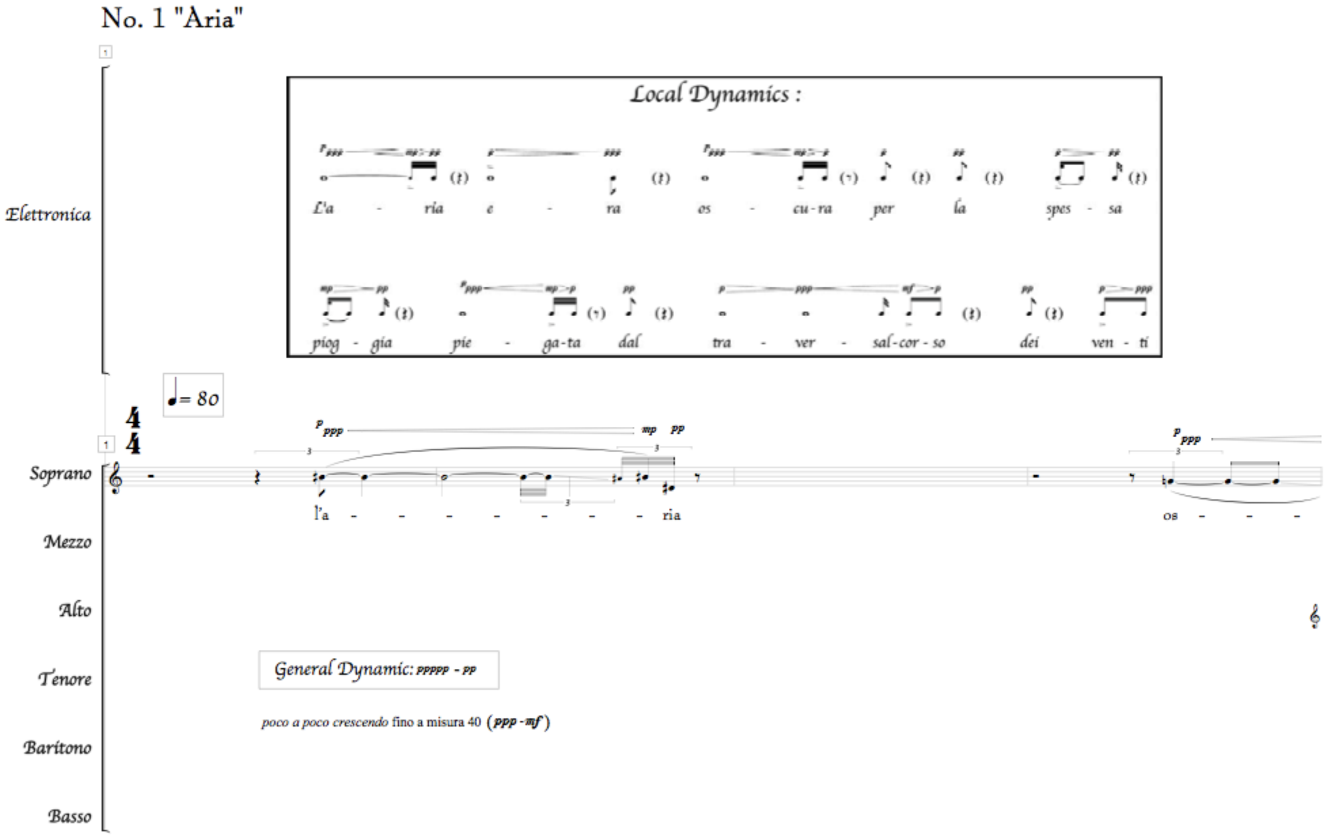
Mauro Lanza

Canons de Vuza et périodicités locales

• *La descrizione del diluvio* (Ricordi, 2007-2008)

Canon à 14 voix sur le pattern rythmique :

(1 3 25 27 1 3 11 14 27 1 3 25 27 4 25 27 1 3 25 14 13 1 3 25 27 1 3 52)



6 voix sont en live et 8 dans l'électronique. L'unité est la double-croche de triolet. Le choix des notes et des durées est fait en cherchant à souligner certaines ***quasi-périodicités*** *du canon de Vuza, et cela donne à chaque voix un caractère beaucoup plus "redondant".*

Mauro Lanza
Canons de Vuza et périodicités locales

• *La descrizione del diluvio* (Ricordi, 2007-2008)

[…] Le choix des notes et des durées est fait en cherchant à souligner certaines quasi-périodicités du canon de Vuza, et cela donne à chaque voix un caractère beaucoup plus "redondant".

(1 3 25 27 1 3 11 14 27 1 3 25 27 4 25 27 1 3 25 14 13 1 3 25 27 1 3 52)

(*1 3 25 27 1 3* 11 14 27 1 3 25 27 4 25 27 1 3 25 14 13 *1 3 25 27 1 3* 52)

(*1 3 25 27* 1 3 11 14 27 *1 3 25 27* 4 25 27 1 3 25 14 13 *1 3 25 27* 1 3 52)

(*1 3 25* 27 1 3 11 14 27 *1 3 25* 27 4 25 27 *1 3 25* 14 13 *1 3 25* 27 1 3 52)

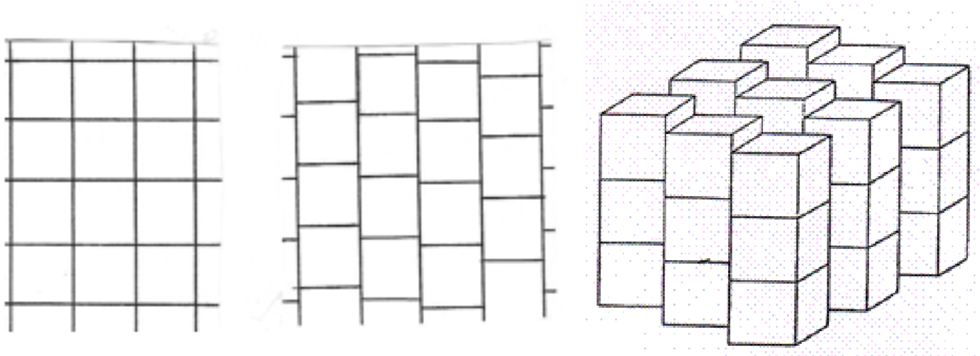
(*1 3* 25 27 *1 3* 11 14 27 *1 3* 25 27 4 25 27 *1 3* 25 14 13 *1 3* 25 27 *1 3* 52)

=> *OpenMusic*

Les canons rythmiques mosaïques comme un problème « mathémusical »

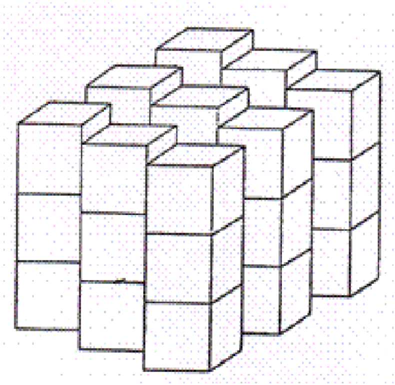
- Olivier Messiaen's 'formalization' of rhythmic canons
- Dan Tudor Vuza's model of Regular Complementary Canons of Maximal Category (*Perspectives of New Music*, 1991-1993)
- The computer-aided model of Vuza Canons and first catalogues of solutions (Agon&Andreatta, 1999)
- Compositional applications of the model (by Fabien Levy, Georges Bloch)
- Enumeration and classifications of Vuza canons (Fripertinger, Amiot, Noll, Andreatta, Tangian, Jedrzejewski)
- Thomas Noll's generalized model of augmented tiling canons
- Emmanuel Amiot's model of cyclotomic tiling canons
- The *MathTools* environment in *OpenMusic* (Agon&Andreatta)

- Minkowski's Conjecture (1896/1907)
- Hajos algebraic solution (1942)
- The classification of Hajos groups (Hajos, de Bruijn, Sands, …)
- The Tiling of the line problem and Fuglede's Conjecture (Tijdeman,Lagarias, Laba, Coven-Meyerowitz, Kolountzakis…)
- Fuglede's Conjecture and Vuza's Canons (Amiot)



In a simple lattice tiling of the n-dimensional space by unit cubes, at least one couple of cubes share a complete n-1 dimensional face
(Cf. S. Stein, S. Szabó : *Algebra and Tiling*, 1994)

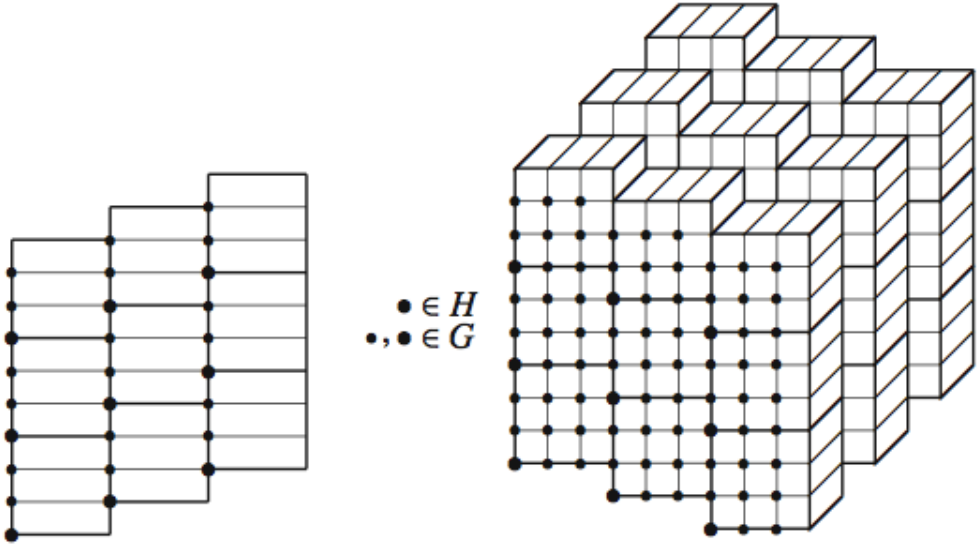
Conjecture de Minkowski et formalisation algébrique



***Conjecture de Minkowski* (1896/1907)**

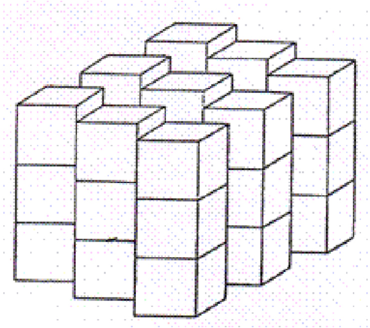
In a simple lattice tiling of the n-dimensional space by unit cubes, at least one couple of cubes share a complete n-1 dimensional face

G. Fidanza, *Canoni ritmici a mosaico*, tesi di laurea, 2006/2007



- H = réseau formé des sommets de coordonnées inférieures (à valeurs dans **Q** sans perte de généralité)
- G = sommets de coordonnées inférieures qui divisent chaque cube dans un nombre fini de tranches
- $H<G$
- $\exists\{a_1, \ldots, a_n\}$ basa de G telle que $m_i a_i = e_i$ $\forall$i=1,…,n où m_i est le nombre de tranches dans lequel chaque cube est divisé tout au long de la i-ème coordonnée
- On construit le quotient G/H et pour chaque i on considère l'ensemble $A_i = \{0, a_i, 2a_i, \ldots, (m_i-1)a_i\}$
- $G/H = A_1 \oplus A_2 \oplus \ldots \oplus A_n$

Conjecture de Minkowski et théorème de Hajós



Conjecture de Minkowski (1896/1907)

Dans un pavage simple [simple lattice tiling] d'un espace à n dimensions par des cubes unités, il y a au moins un couple de cubes qui ont en commun une face entière de dimension n-1.

Théorème de Hajós (1942)

Soit G un groupe abélien fini et soient $a_1, a_2, \ldots, a_n$ n éléments de G. Si l'on suppose que le groupe admet comme factorisation la somme directe des sous-ensembles $A_1 \ldots A_n$

$$A_1 = \{1, a_1, \ldots, a_1^{m_1-1}\}, A_2 = \{1, a_2, \ldots, a_2^{m_2-1}\}, \ldots, A_n = \{1, a_n, \ldots, a_n^{m_n-1}\}$$

avec $m_i > 0$ pour tout i=1, 2, …, n, alors un des A_i est un groupe

Théorème de Redei (1965)

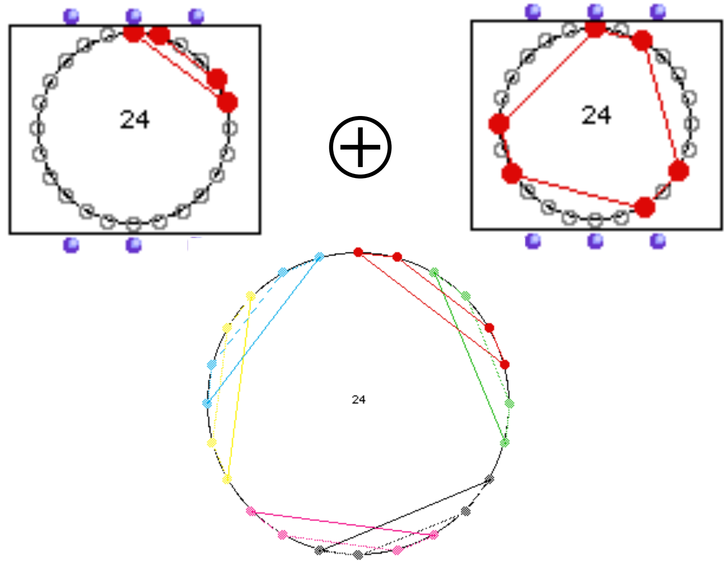
Soit G un groupe abélien fini et soient $A_1, A_2, \ldots, A_n$ n sous-ensembles de G, chacun contenant l'élément neutre du groupe et chacun ayant un nombre premier d'éléments et supposons que le groupe admette comme factorisation la somme directe des sous-ensembles A_i, i=1, …, n.
Alors, un des sous-ensembles A_i est **périodique**

Cf. S. Stein, S. Szabó : *Algebra and Tiling. Homomorphisms in the service of Geometry,* Carus Math. Monographs, 1994.

Groupes de Hajos et périodicité des facteurs

A group G is an "Hajós group" if for all factorisation of G into a direct sum of subsets $A_1, A_2, \ldots, A_k$, at least one of the factors is periodic.

Rédei 1947	(p, p)
Hajós 1950	**Z**
	Z/nZ avec n= p^α
De Brujin 1953	(p^α, q)
	(p, q, r)
Sands 1957	(p^2, q^2)
	(p^2, q, r)
	(p, q, r, s)
Sands 1959	$(2^2, 2^2)$
	$(3^2, 3)$
	$(2^n, 2)$
Sands 1962	$(p, 3, 3)$
	$(p, 2^2, 2)$
	$(p, 2, 2, 2, 2)$
	$(p^2, 2, 2, 2)$
$(p^3, 2, 2)$	
	$(p, q, 2, 2)$
Sands 1964	**Q**
	Z+Z/pZ
	Q+Z/pZ



Groupes non-Hajós (bad groups)

72
108 120 144 168 180
200 216 240 252 264 270 280 288
300 312 324 336 360 378 392 396
400 408 432 440 450 456 468 480
500 504 520 528 540 552 560 576 588 594
600 612 616 624 648 672 675 680 684 696
700 702 720 728 744 750 756 760 784 792
800 810 816 828 864 880 882 888…

Problème ouvert en théorie des canons rythmiques mosaïques

Groupes non-Hajós (bad groups)

72
108 120 144 168 180
200 216 240 252 264 270 280 288
300 312 324 336 360 378 392 396
400 408 432 440 450 456 468 480
500 504 520 528 540 552 560 576 588 594
600 612 616 624 648 672 675 680 684 696
700 702 720 728 744 750 756 760 784 792
800 810 816 828 864 880 882 888…

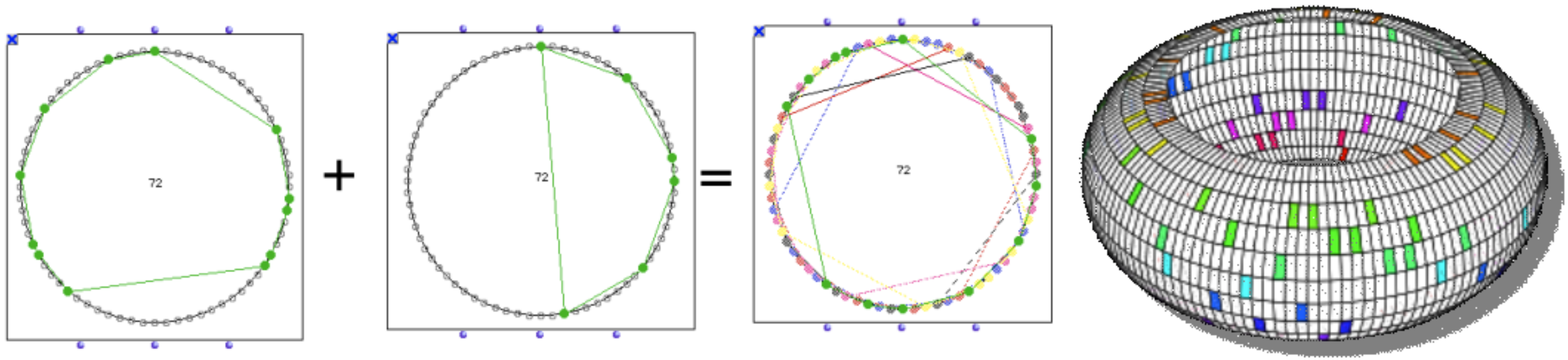
Groupes Hajós (good groups)

Z/nZ avec

n $\in \{p^{\alpha}, p^{\alpha}q, pqr, p^2q^2, p^2qr, pqrs\}$

où p, q, r, s, sont des nombres premiers distincts

Trouver un algorithme qui permet d'obtenir toutes le factorisations d'un groupe cyclique non-Hajos en somme directe de deux sous-ensembles non périodiques (i.e. classifier tous les Canons de Vuza, ou Canons sans périodicité interne)



Conjecture de Fuglede et canons rythmiques mosaïques

Canons de Vuza de période *n*

72
108 120 144 168 180
200 216 240 252 264 270 280 288
300 312 324 336 360 378 392 396
400 408 432 440 450 456 468 480
500 504 520 528 540 552 560 576 588 594
600 612 616 624 648 672 675 680 684 696
700 702 720 728 744 750 756 760 784 792
800 810 816 828 864 880 882 888…

Fuglede Conjecture (1974):
SPECTRAL ⇔ TILING

(Conjecture ouverte en dim. 1 et 2)

DEFINITION 6 *A subset A of some vector space (say $\mathbb{R}^n$) is spectral iff it admits a Hilbert base of exponentials, i.e. if any map $f \in L^2(A)$ can be written*

$$f(x) = \sum f_k \exp(2i\pi\lambda_k.x)$$

for some fixed family of vectors $(\lambda_k)_{k\in\mathbb{Z}}$ where the maps $e_k : x \mapsto \exp(2i\pi\lambda_k.x)$ are mutually orthogonal (i.e. $\int_A \overline{e_k} e_j = 0$ whenever $k \neq j$).

Les conjectures de Minkowski/Fuglede et les canons rythmiques

- Minkowski's conjecture (1896/1907)
- Hajos algebraic solution (1942)
- Hajos quasi-periodic conjecture
- The classification of Hajos groups (Hajos, de Bruijn, Sands, …)
- Classification of factorizations for non-Hajos groups (Vuza, Andreatta, Agon, Amiot, Fripertinger, …)
- …

- The Tiling of the line problem and Fuglede's Conjecture (**Tijdeman**, **Coven-Meyerowitz**, Lagarias, **Laba**, Kolountzakis…)
- Given a finite set that tiles **Z**, what will be the period (Kolountzakis, Steinberger, …)
- Fuglede's Conjecture and Vuza's Canons (Amiot, 2004)
- …

• R. Tijdeman: "Decomposition of the Integers as a direct sum of two subsets", *Number Theory*, Cambridge University Press, 1995. The fundamental Lemma:
A tiles $\mathbf{Z}_n$ => pA tiles $\mathbf{Z}_n$ when $<p,n>=1$

• I. Laba : "The spectral set conjecture and multiplicative properties of roots of polynomials", *J. Lond Math Soc*, 2002
T1 + T2 => spectral
T2 => spectral
spectral => T1

• E. Coven & A. Meyerowitz: "Tiling the integers with translates of one finite set", *J. Algebra*, 212, pp.161-174, 1999
T1 + T2 => tile
Tile => T1

• E. Amiot : "A propos des canons rythmiques", *Gazette des Mathématiciens*, n°106, Octobre 2005.
if A tiles with period n and $\mathbf{Z}_n$ is Hajos => A has T2 (=> A is spectral)

Si A pave mais il n'est pas spéctral => A est le rythme d'un canon de Vuza

Racines de l'unité et polynomes cyclotomiques

Racines *n*-ièmes de l'unité : $z^n = 1$

n=3 ⟶ $\left\{1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}\right\}$

n=4 ⟶ $\{1, +i, -1, -i\}$

Le racines *n*-ièmes de l'unité peuvent s'écrire sous la forme :

$$e^{\frac{2k\pi i}{n}} = \cos\left(\frac{2k\pi}{n}\right) + i\sin\left(\frac{2k\pi}{n}\right) \qquad (k, n \in \mathbb{N} \text{ et } 0 \leq k < n)$$

Elles sont exactement les racines du polynôme : $P(X) = X^n - 1$

Le racines *n*-ièmes primitives de l'unité : $e^{\frac{2ki\pi}{n}}$ $(n,k)=1$

Elles sont exactement les racines du polynôme cyclotomique :

$$\Phi_n(X) = \prod_{k=1}^{\varphi(n)} (X - z_k) \longleftrightarrow X^n - 1 = \prod_{d|n} \Phi_d(X).$$

Pavage de la ligne et polynômes cyclotomiques

$$\Phi_n(X) = \prod_{k=1}^{\varphi(n)} (X - z_k) \longleftrightarrow X^n - 1 = \prod_{d|n} \Phi_d(X).$$

$\Phi_1(X) = X - 1 \longleftrightarrow (-1, 1)$

$\Phi_2(X) = 1 + X \longleftrightarrow (1, 1)$

$\Phi_3(X) = 1 + X + X^2 \longleftrightarrow (1, 1, 1)$

$\Phi_4(X) = 1 + X^2 \longleftrightarrow (1, 0, 1)$

$\Phi_5(X) = 1 + X + X^2 + X^3 + X^4 \longleftrightarrow (1, 1, 1, 1, 1)$

$\Phi_6(X) = 1 - X + X^2 \longleftrightarrow (1, -1, 1)$

$$\Delta_n = 1 + X + X^2 + \ldots + X^{n-1} = \prod_{\substack{d | n \\ d \neq 1}} \Phi_d(X)$$

$$\Delta_4 = 1 + X + X^2 + X^3 = \Phi_2(X) \times \Phi_4(X)$$

$$A(x) \times B(x) = (A \oplus B)(x) \equiv 1 + x + \ldots x^{n-1} \pmod{X^n - 1}$$

Bonnes et mauvaises factorisations

$$\Delta_n = 1 + X + X^2 + \ldots + X^{n-1} = \prod_{\substack{d \mid n \\ d \neq 1}} \Phi_d(X)$$

$$\Phi_2(X) = 1 + X \quad \longleftrightarrow \quad (1, 1)$$

$$\Phi_3(X) = 1 + X + X^2 \quad \longleftrightarrow \quad (1, 1, 1)$$

$$\Phi_4(X) = 1 + X^2 \quad \longleftrightarrow \quad (1, 0, 1)$$

$$\Phi_6(X) = 1 - X + X^2 \quad \longleftrightarrow \quad (1, -1, 1)$$

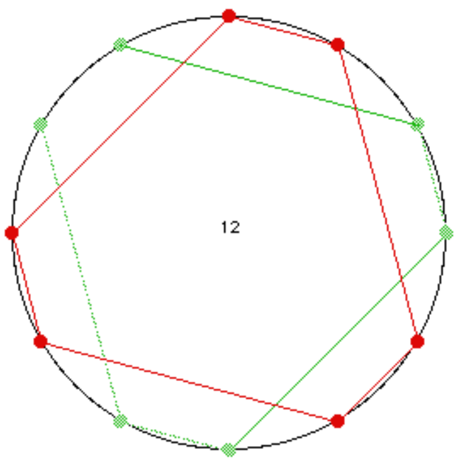
$$\Delta_{12} = 1 + X + \ldots + X^{11} = \Phi_2 \times \Phi_3 \times \Phi_4 \times \Phi_6 \times \Phi_{12}$$

$$A(X) = \Phi_2 \times \Phi_3 \times \Phi_6 \times \Phi_{12} = 1 + X + X^4 + X^5 + X^8 + X^9$$

$$B(X) = \Phi_4 = 1 + X^2$$

$$\to S = \{0, 2\}$$

$$\to R = \{0, 1, 4, 5, 8, 9\}$$



$$A^*(X) = \Phi_2 \times \Phi_3 \times \Phi_{12}$$

$$B^*(X) = \Phi_4 \times \Phi_6$$

Cette décomposition ne marche pas

Les conditions de Coven-Meyerowitz

- E. Coven & A. Meyerowitz : “Tiling the integers with translates of one finite set”, *J. Algebra*, 212, pp.161-174, 1999

There is no loss of generality in restricting attention to translates of a finite set A of *nonnegative* integers. Then $A(x) = \sum_{a\in A} x^a$ is a polynomial such that $\#A = A(1)$. Let S_A be the set of prime powers s such that the s-th cyclotomic polynomial $\Phi_s(x)$ divides $A(x)$. Consider the following conditions on $A(x)$.

(T1) $A(1) = \prod_{s\in S_A} \Phi_s(1)$.

(T2) If $s_1, \ldots, s_m \in S_A$ are powers of distinct primes, then $\Phi_{s_1\cdots s_m}(x)$ divides $A(x)$.

Theorem A. *If $A(x)$ satisfies (T1) and (T2), then A tiles the integers.*

Theorem B1. *If A tiles the integers, then $A(x)$ satisfies (T1).*

Theorem B2. *If A tiles the integers and $\#A$ has at most two prime factors, then $A(x)$ satisfies (T2).*

Corollary. *If $\#A$ has at most two prime factors, then A tiles the integers if and only if $A(x)$ satisfies (T1) and (T2).*

Les conditions de Coven-Meyerowitz

(T1) $A(1) = \prod_{s \in S_A} \Phi_s(1)$.

(T2) If $s_1, \ldots, s_m \in S_A$ are powers of distinct primes, then $\Phi_{s_1 \cdots s_m}(x)$ divides $A(x)$.

Theorem A. *If $A(x)$ satisfies (T1) and (T2), then A tiles the integers.*

$$A(X) = \Phi_2 \times \Phi_3 \times \Phi_6 \times \Phi_{12} = 1 + X + X^4 + X^5 + X^8 + X^9$$

$$\Phi_2(X) = 1 + X$$

$$\Phi_3(X) = 1 + X + X^2$$

(T1) $A(1) = 6 = \Phi_2(1) \times \Phi_3(1) = 2 \times 3$

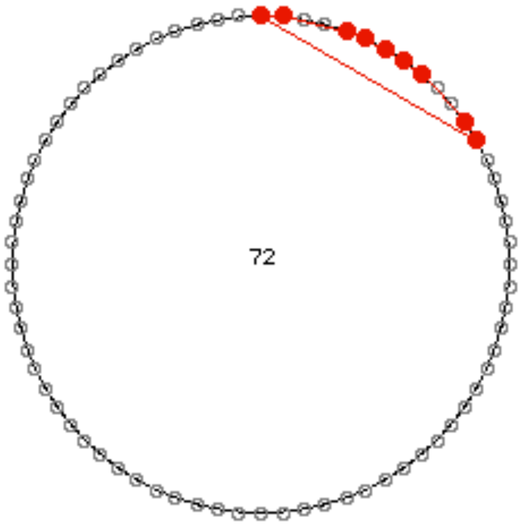
(T2) $\Phi_2 \mid A(X)$ et $\Phi_3 \mid A(X) \Rightarrow \Phi_{2\times 3} \mid A(X)$

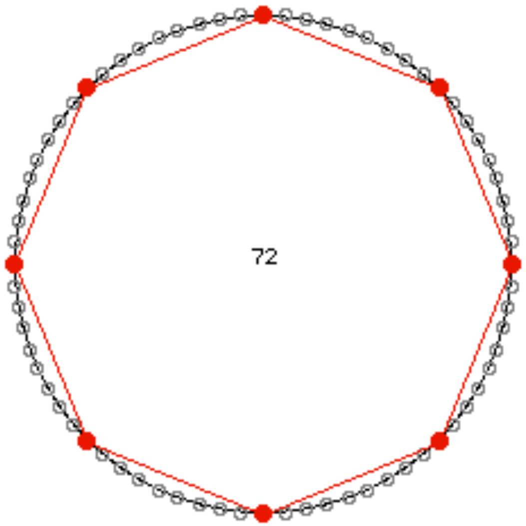
Theorem B1. *If A tiles the integers, then $A(x)$ satisfies (T1).*

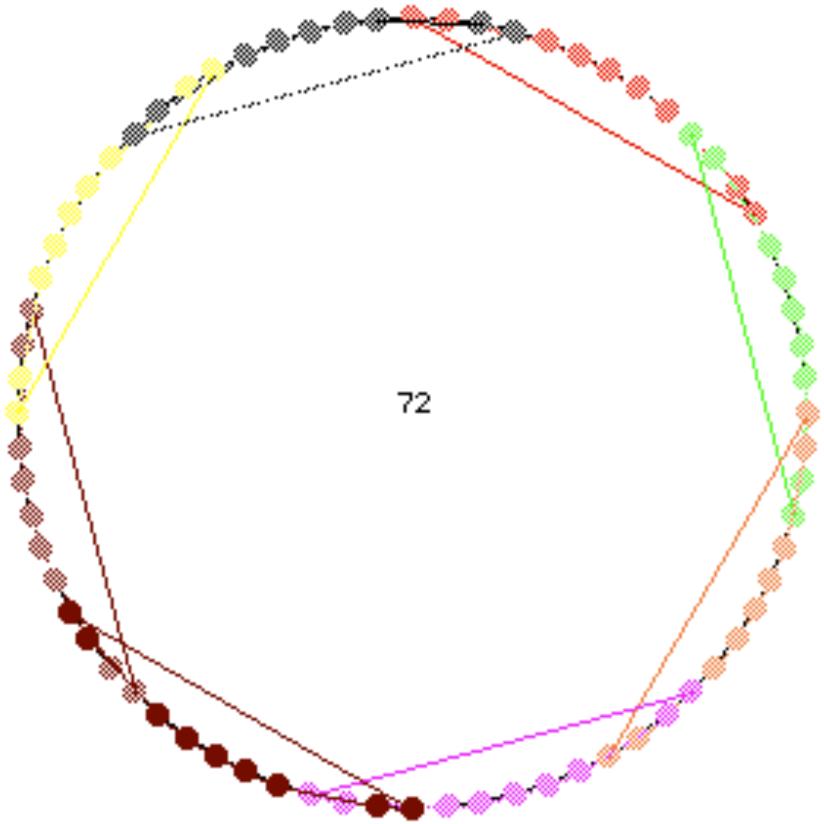
$$A^*(X) = \Phi_2 \times \Phi_3 \times \Phi_{12} = 1 + 2X + 2X^2 - X^3 - X^4 + X^5 + 2X^6 + X^7$$

$$A^*(1) = 7 \neq \Phi_2 \times (1) \times \Phi_3(1) = 6$$

La famille des « canons cyclotomiques »







- E. Amiot, M. Andreatta, C. Agon: « Tiling the (musical) line with polynomial: some theoretical and implementational aspects », *ICMC*, Barcelona, 2005, pp.227-230.

Transformée de Fourier discrète et pavage

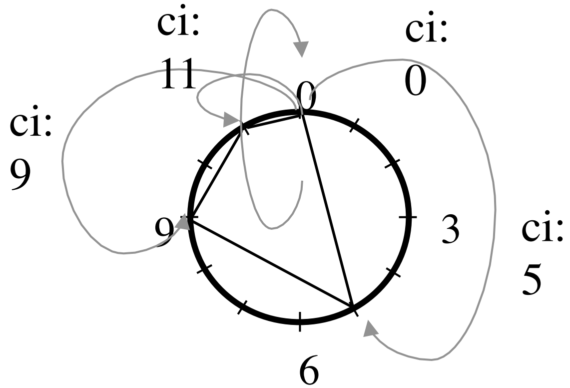
Tiling

Let $Z_A = \{ t \in \mathbb{Z}_c, F_A(t)=0 \}$

A tiles $\mathbb{Z}_c$ when equivalently:

- There exists B, $A \oplus B = \mathbb{Z}_c$
- $1_A \star 1_B = 1$
- $F_A \times F_B(t) = 1 + e^{-2i\pi t/c} + \ldots e^{-2i\pi t(c-1)/c}$ (0 unless t=0)
- $Z_A \cup Z_B = \{1,2 \ldots c-1\}$ AND Card A × Card B = c
- $IC_A \star IC_B = IC(\mathbb{Z}_c)=c$ and Card A × Card B = c

$$\mathcal{F}_A : t \mapsto \sum_{k\in A} e^{-2i\pi kt/c}$$



$A = \{0, 5, 9, 11\}$

$IC_A(k) = 1 \ \forall\ k = 1\ldots11$

$$IC_A(k) = \text{Card}\{(x,y) \in A \times A \mid x+k=y\}$$

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

E. Amiot, « New Perspectives on Rhythmic Canons and the Spectral Conjecture », *Journal of Mathematics and Music*, 2009.

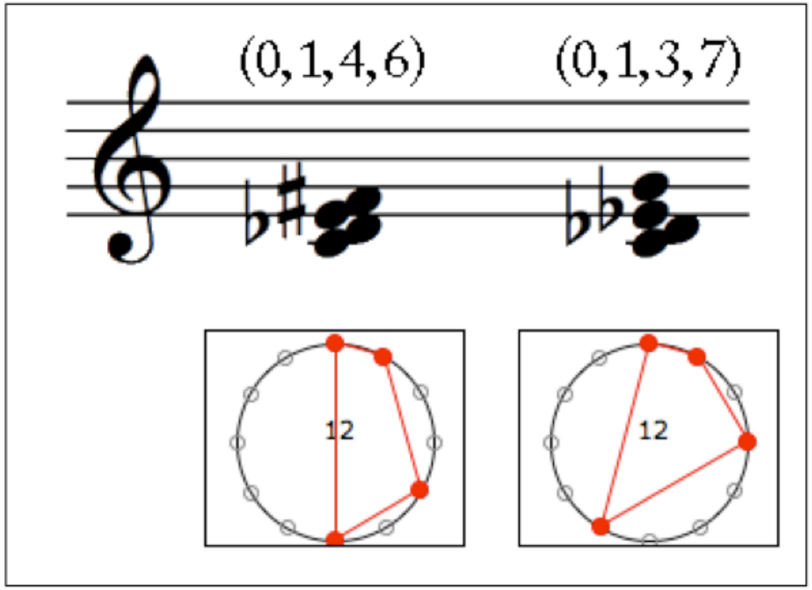
Pavage et Z-relation (homométrie)

- Deux ensembles sont en Z-relations s'ils ont le même module de la DFT

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$

$$\mathcal{F}(IC_A) = \mathcal{F}_A \times \mathcal{F}_{-A} = |\mathcal{F}_A|^2$$



➔ Reconstruction de la phase

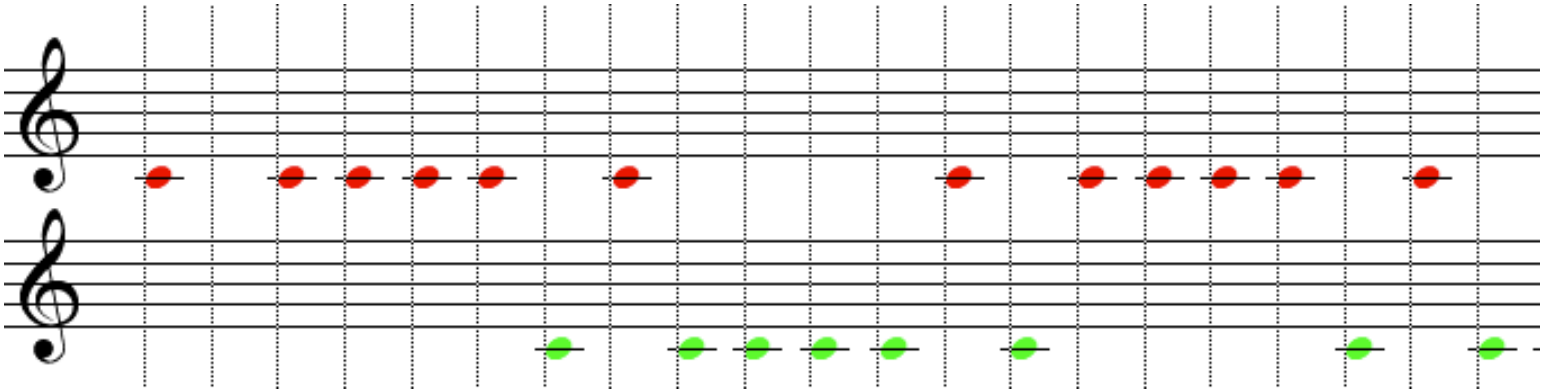
Cf. D. Ghisi, J. Mandereau, E. Amiot, C. Agon, M. Andreatta, "Generalized Z-relation and Homometric Theory", paper in progress, to be submitted to the *Journal of Mathematics and Music*

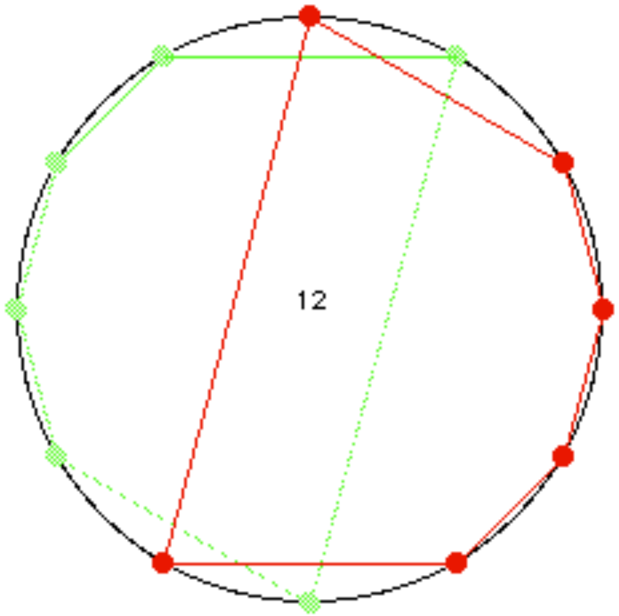
A musical offering:

Theorem:
If A tiles with B and A' has the same IC, then A' tiles with B, too.

E. Amiot, « New Perspectives on Rhythmic Canons and the Spectral Conjecture », *Journal of Mathematics and Music*, 2009.

Canons mosaïques par translation et augmentation

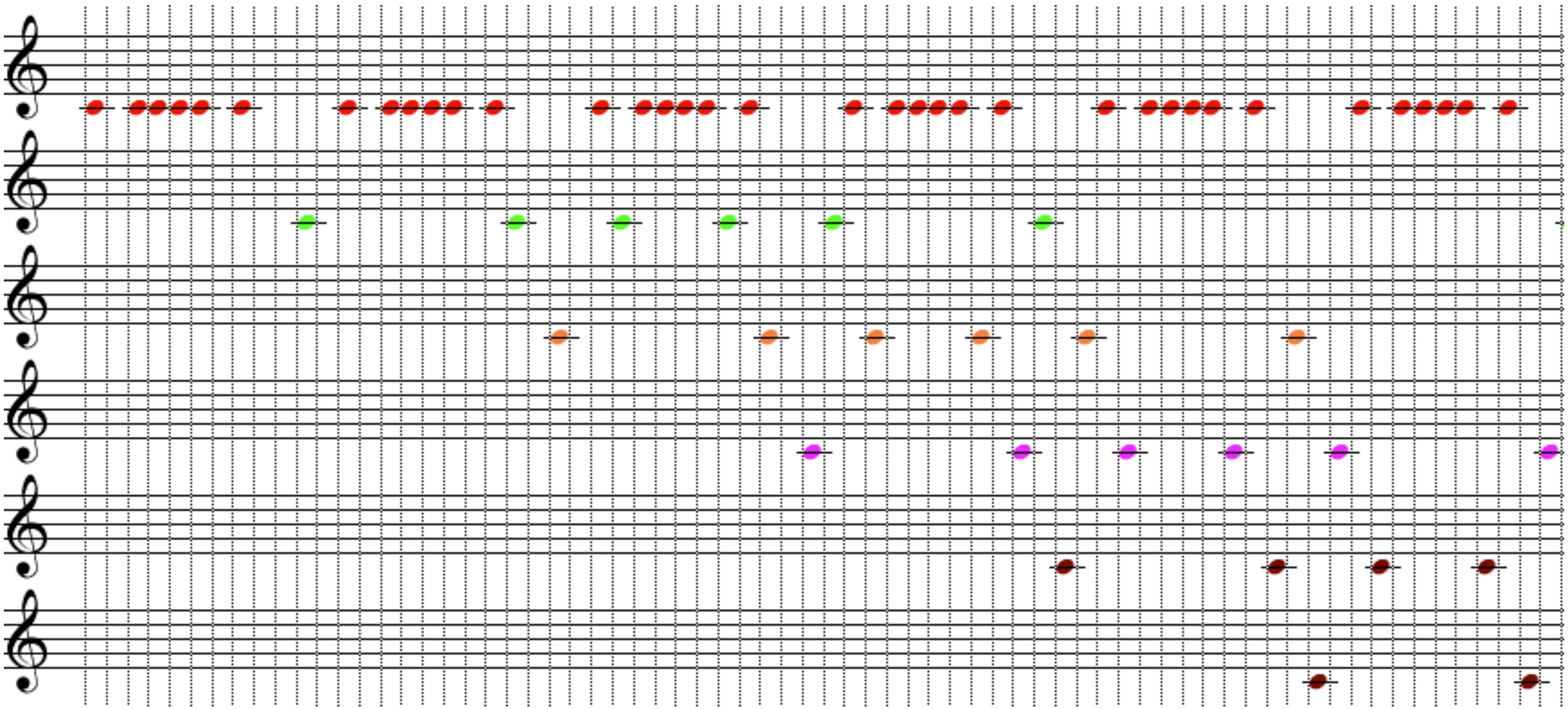





```
(((0 1 2 3 4 6) ((1 11)))
((0 1 2 3 4 5) ((1 11) (1 1)))
((0 1 2 3 5 7) ((1 11) (1 7)))
((0 1 3 4 7 8) ((1 5)))
((0 1 2 3 6 7) ((1 11)))
((0 1 3 4 6 9) ((1 11) (1 5)))
((0 1 3 6 7 9) ((1 11) (1 5)))
((0 1 2 6 7 8) ((1 11) (1 7) (1 5) (1 1)))
((0 1 4 5 8 9) ((1 11) (1 7) (1 5) (1 1)))
((0 1 2 5 6 7) ((1 7) (1 5)))
((0 2 3 4 5 7) ((1 11) (1 7) (1 5) (1 1)))
((0 1 4 5 6 8) ((1 11) (1 7)))
((0 1 2 4 5 7) ((1 5)))
((0 1 3 4 5 8) ((1 5) (1 1)))
((0 1 2 4 5 8) ((1 11)))
((0 1 2 4 6 8) ((1 11) (1 7)))
((0 2 3 4 6 8) ((1 11)))
((0 2 4 6 8 10) ((1 11) (1 7) (1 5) (1 1))))
```

Augmented Tiling Canons ou l'action du groupe affine

(en collaboration avec Thomas Noll)

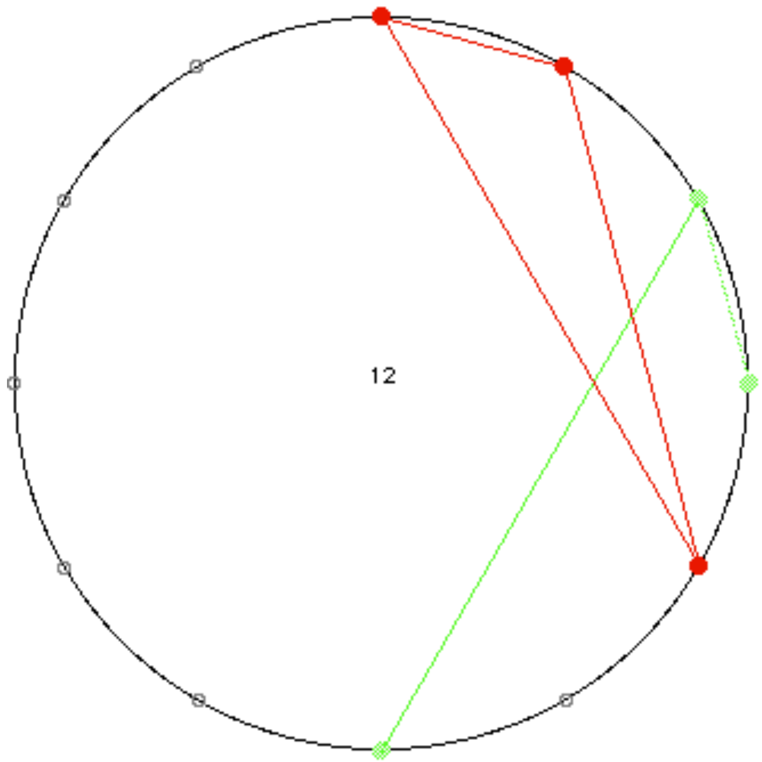


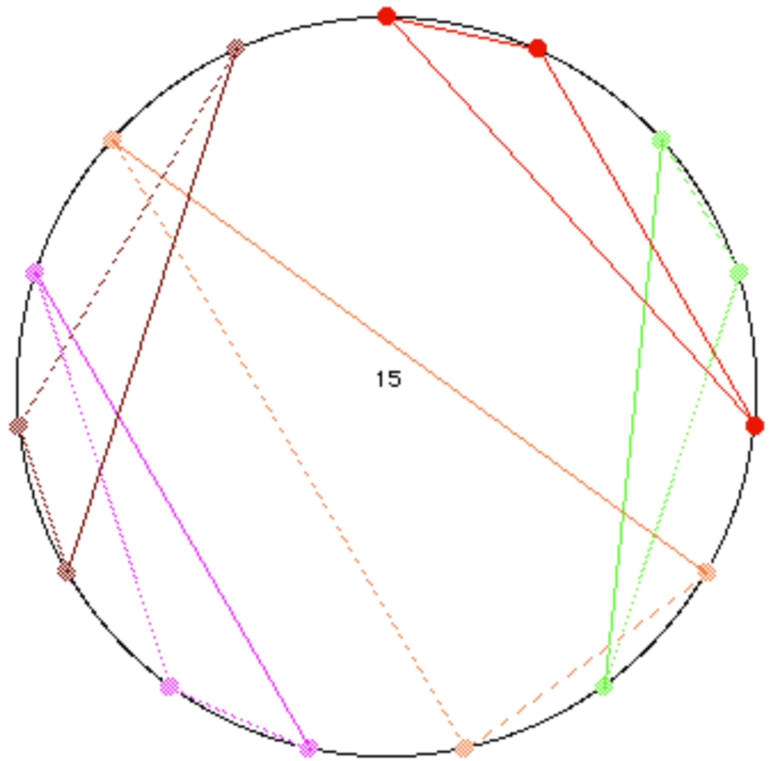
=> *OpenMusic*

Tiling the line and/or circle with augmentations

- **Tom Johnson (2001): tiling the line with a given rhythmic pattern**
  - **ex. (0 1 4). Does it tile? With augmentations? With which period?**

• Theorem (Amiot, 2002) : ***Any tiling of the line with the pattern* (0 1 4) *and its augmentations is periodic and the period is equal to a multiple of 15***



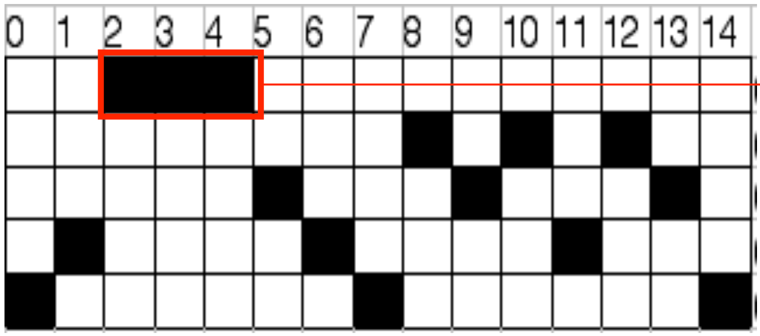


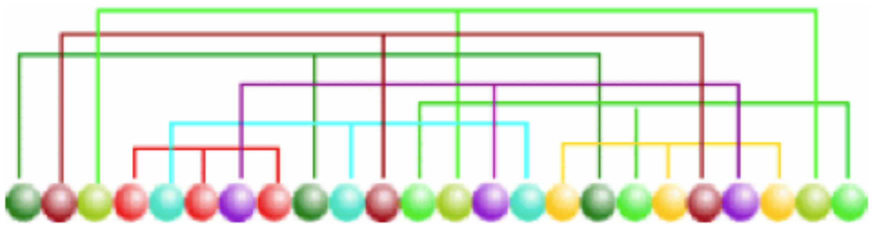
$n = 12$

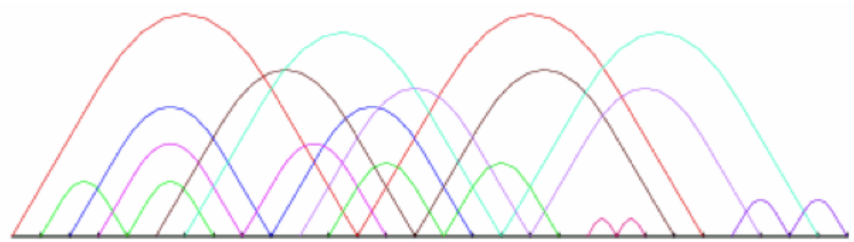
$n = 15$

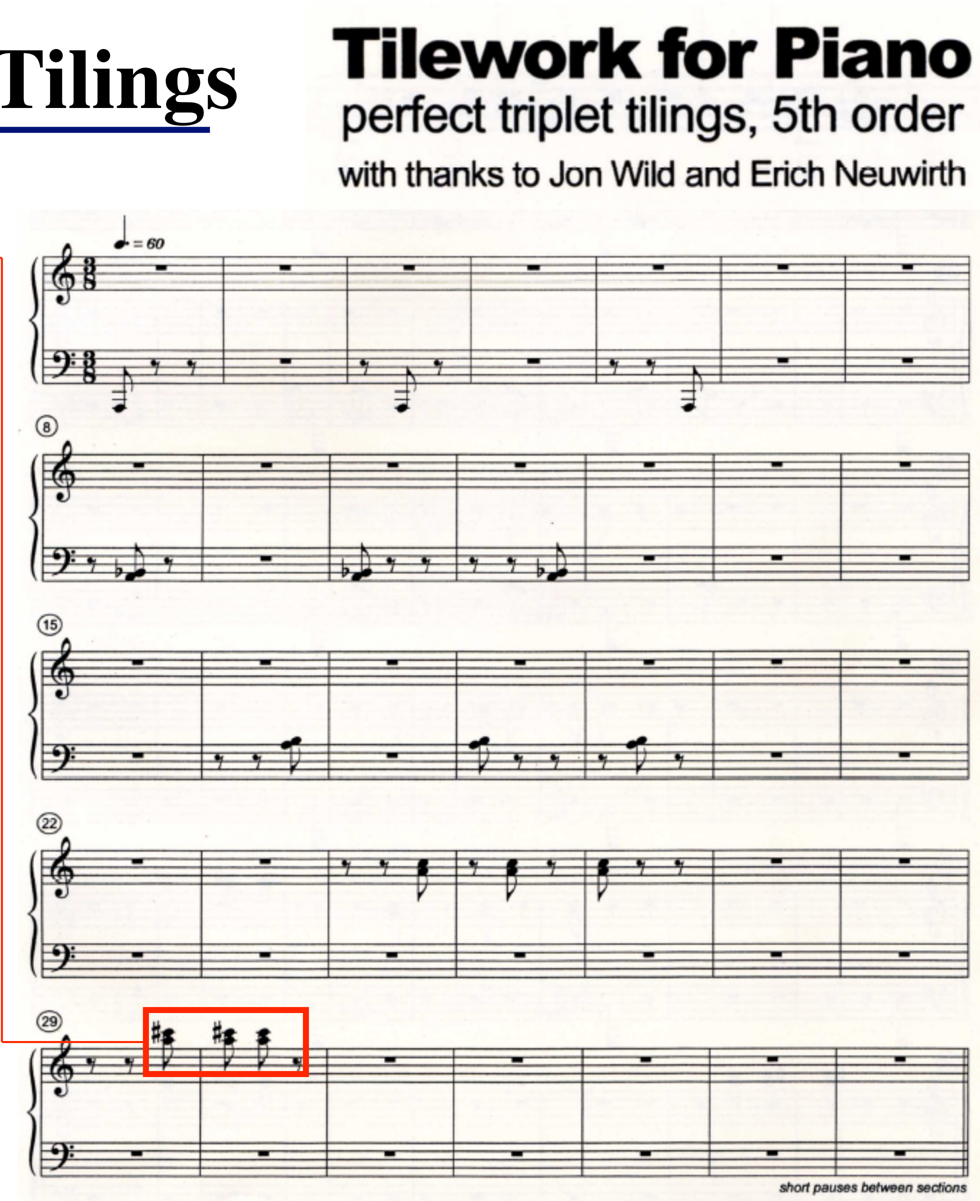
Tom Johnson's « Self-Similar Melodies »

Tom Johnson's Perfect Tilings



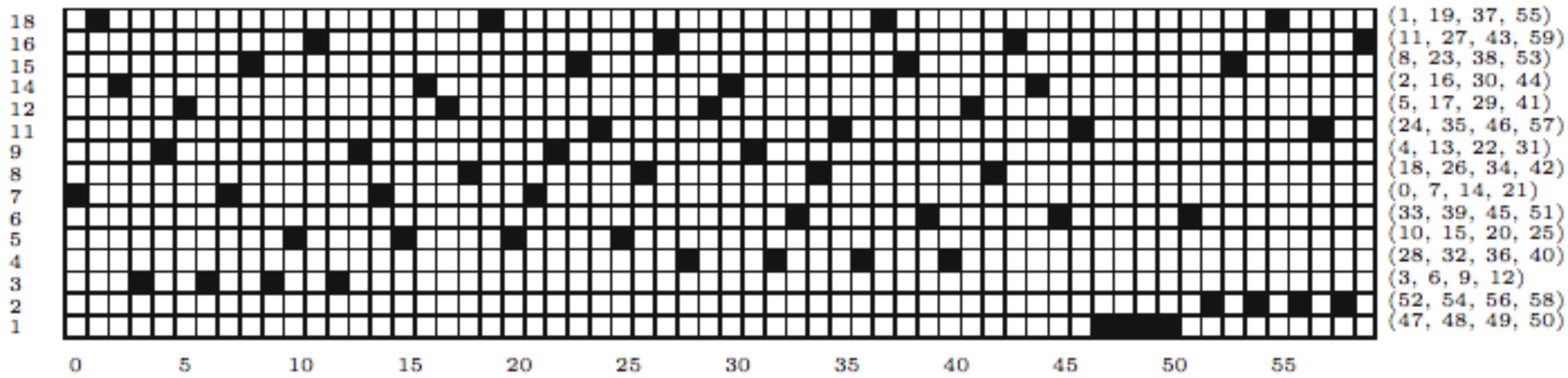


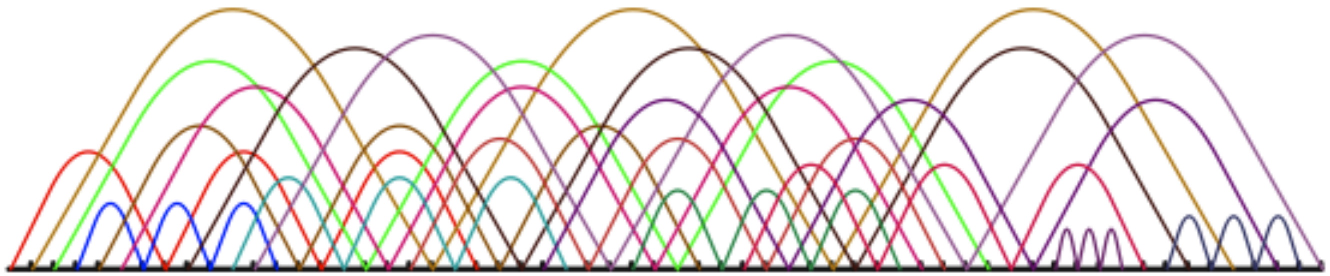




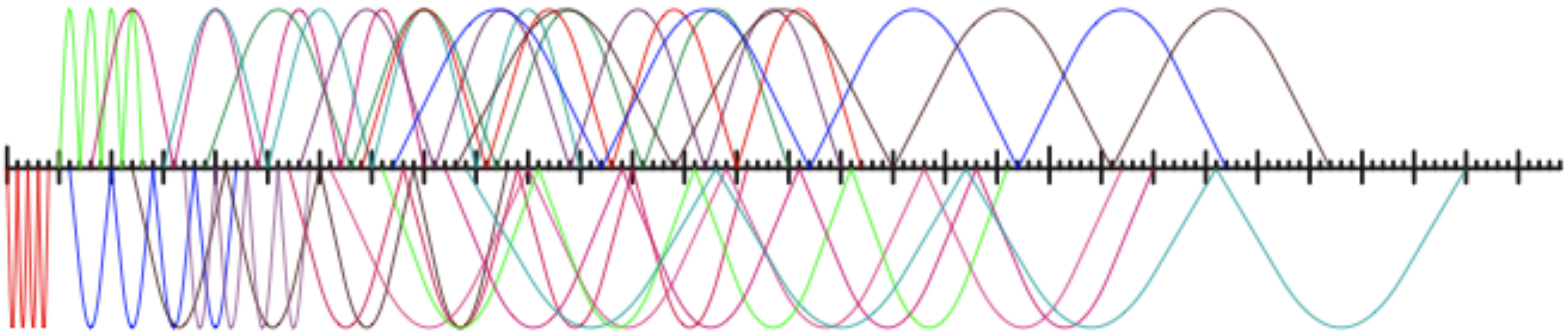
Jean-Paul Davalan, « Perfect Rhythmic Tilings » (to appear in *Tiling Problems in Music*, M. Andreatta & C. Agon eds., Collection « Musique/Sciences », 2008)

Perfect Rhythmic Tilings and open problems





Does it exist a quintuple perfect tiling canon?



Periodic sequences and finite difference calculus

$$Df(x)=f(x)-f(x-1).$$

$$f \quad = \quad \mathbf{7\ 11\ 10\ 11\ 7\ 2}\ 7\ 11\ 10\ 11\ 7\ 2\ \ 7\ 11\ldots$$

$$Df \quad = \quad \mathbf{4\ 11\ 1\ 8\ 7\ 5}\ 4\ 11\ 1\ 8\ 7\ 5\ \ 4\ 11\ldots$$

$$D^2 f \quad = \quad \mathbf{7\ 2\ 7\ 11\ 10}\ 11\ 7\ 2\ 7\ 11\ 10\ 11\ldots$$

$$D^3 f \quad = \quad \mathbf{7\ 5\ 4\ 11\ 1\ 8}\ 7\ 5\ 4\ 11\ 1\ 8\ldots$$

$$D^k f \quad = \quad \ldots\ldots$$

V	0	3	8	7	11	0	11	10	6	9	0	9	1	2	9	8	4	3	6
VIII	0	0	0	0	3	3	7	2	0	0	0	6	3	3	3	4	8	0	0
IV	3	3	4	4	1	11	11	8	3	3	9	4	1	7	11	8	11	3	9
IX	0	0	0	0	0	3	6	[1]	3	3	3	3	9	0	3	6	[10]	6	6
IV	0	10	3	9	10	0	9	7	0	6	7	9	6	4	9	3	4	6	3

Anatol Vieru: *Zone d'oubli* pour alto (1973)

Periodic sequences and finite difference calculus

=> *OpenMusic*

f = 11 6 7 2 3 10 11 6 …

Df = 7 1 7 1 7 1 7 1…

D^2f = 6 6 6 6 6…

D^4f = 0 0 0

Reducible sequences:

∃ $k \geq 1$ such that $D^k f = 0$

Théorème de décomposition: Toute sequence périodique (à valeurs dans un groupe cyclique **Z/nZ**) peut se décomposer de façon unique en somme d'une sequence reductible et d'une sequence reproductible (2001)

f = 7 11 10 11 7 2 7 11 …

Df = 4 11 1 8 7 5 4 11 1 …

D^2f = 7 2 7 11 10 11 7 2 7 …

D^3f = 7 5 4 11 1 8 7 5 4 11 1 8…

D^4f = 10 11 7 2 7 11 10 11 …

D^5f = 1 8 7 5 4 11 1 8 …

D^6f = 7 11 10 11 7 2 7 11 …

Reproducible sequences:

∃ $k \geq 1$ such that

$D^k f = f$

Growing by additions

A^3f = **0 1 5 …**

A^2f = **1 4 9 4 1 0 1 4…**

Af = **3 5 7 9 11 1 3 5 …**

f = **2 2 2 2 2 2 2**

=> ***OpenMusic***

Growing by additions and proliferation of values

A^3g = **8 4 8 10 11 1 5 1 2 10 2 4 5 7 11 7** 8 4 …

A^2g = **8 4 2 1 2 4 8 1** 8 4…

Ag = **8 10 11 1 2 4 5 7** 8 10 …

g = **2 1 2 1 2 1 2 1**

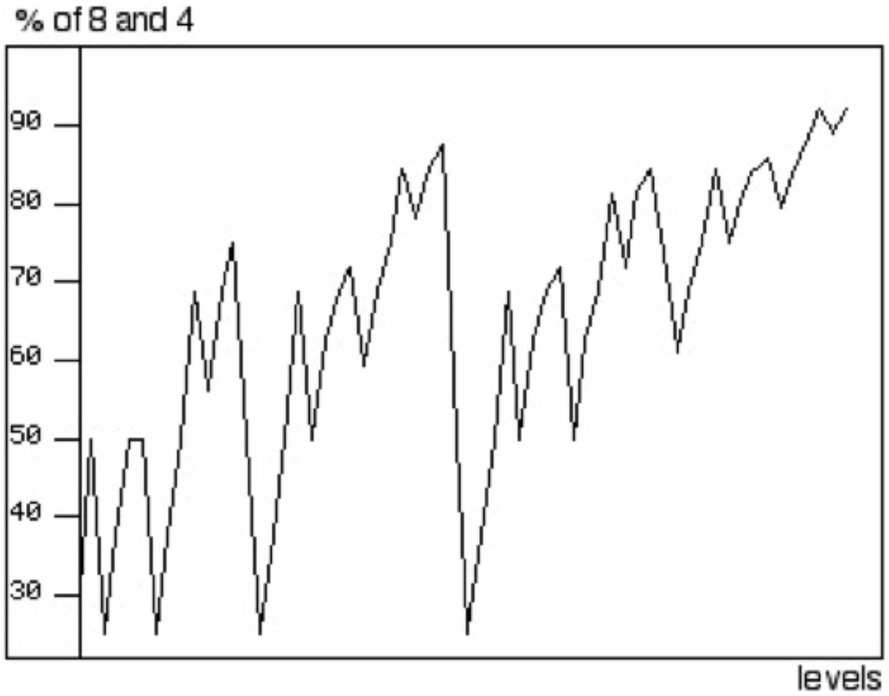


Fig. 11 Initial values equal to 8

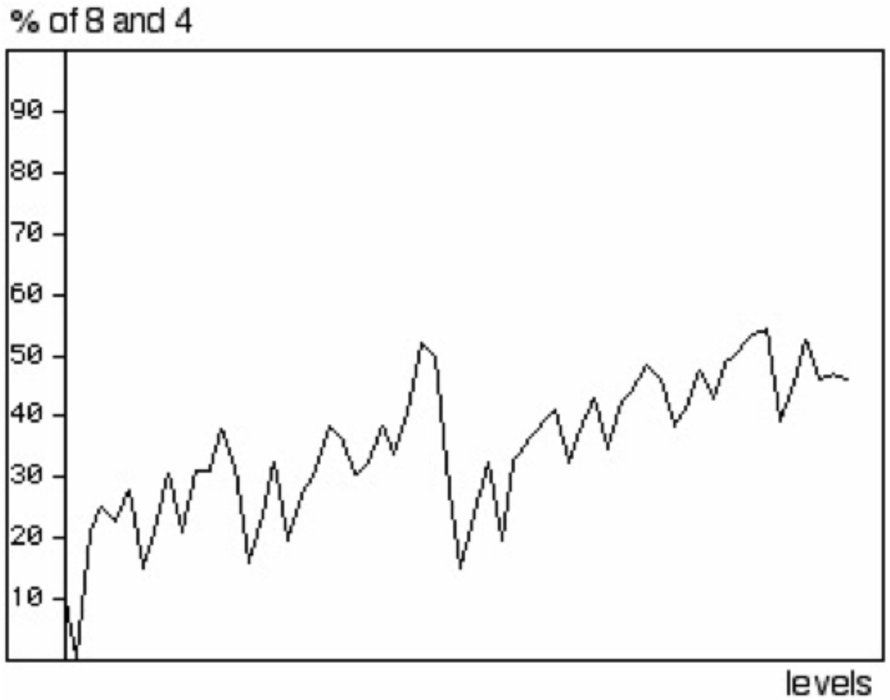
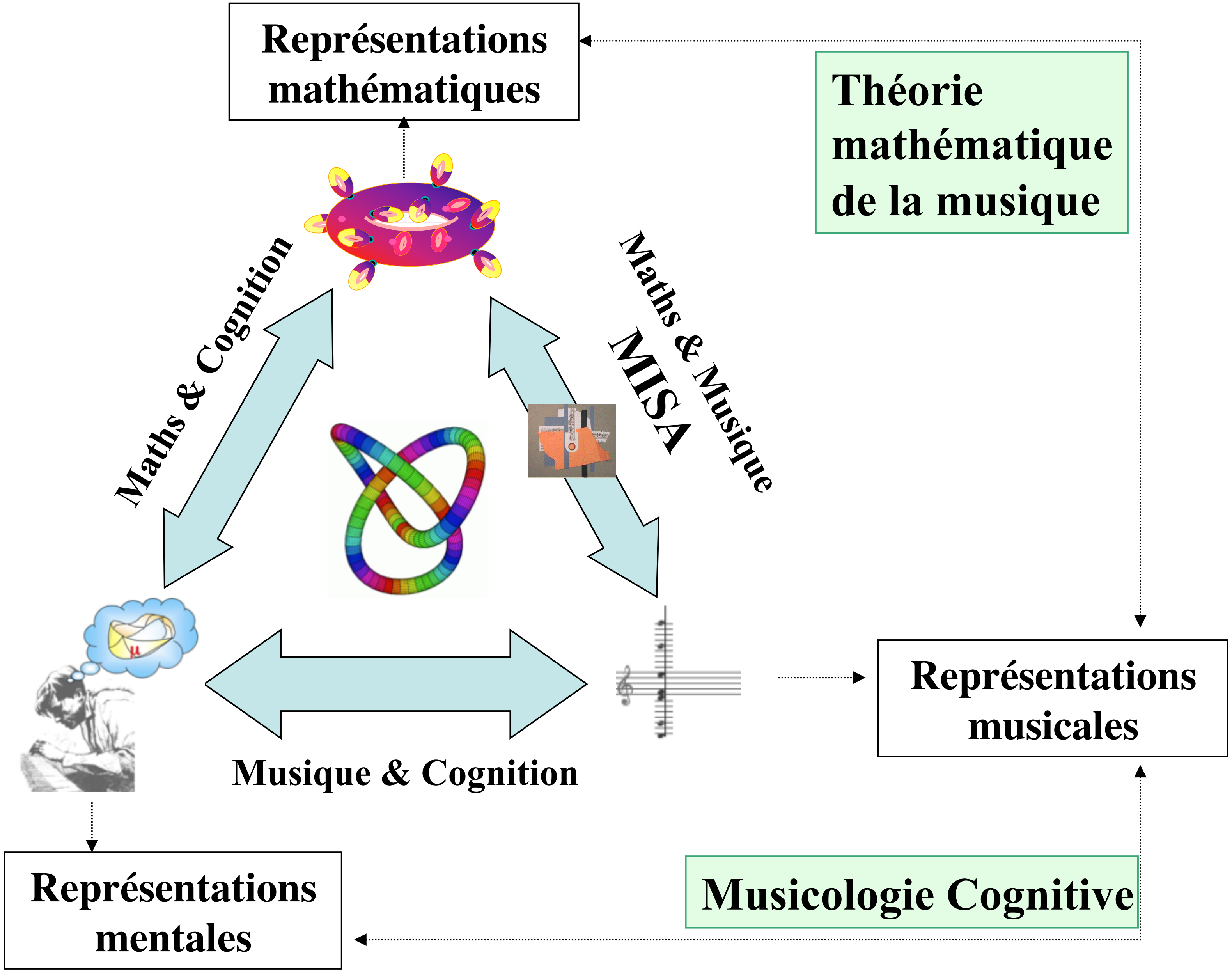


Fig. 15 Initial values equal to 4

GdT Mathématiques/Musique & Cognition

http://recherche.ircam.fr/equipes/repmus/mamux/Cognition.html



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