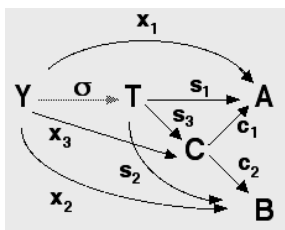
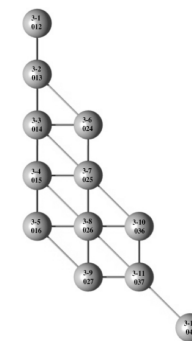
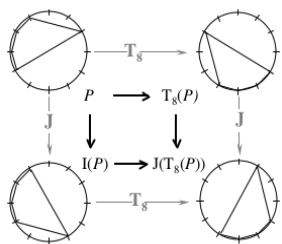
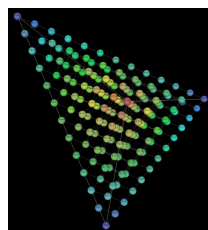


Quelques éléments pour une interprétation philosophique des approches transformationnelles en théorie et analyse musicales

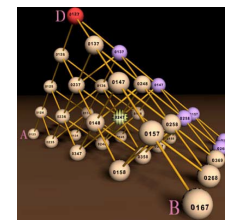
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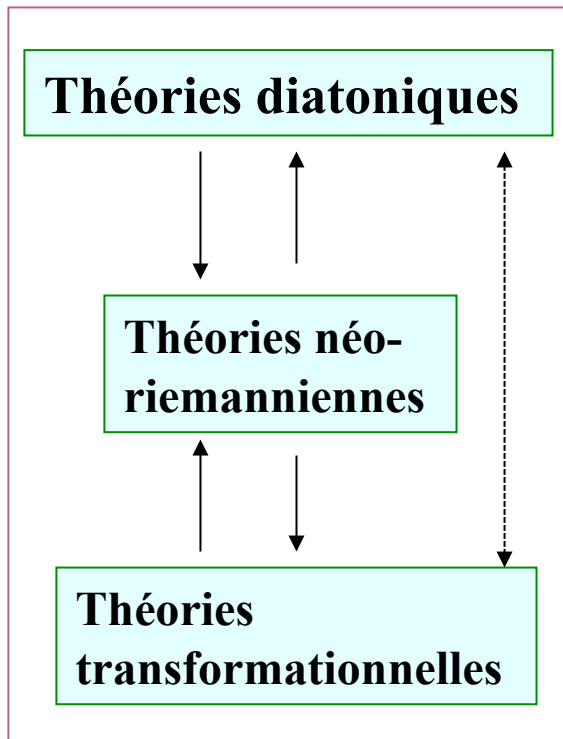
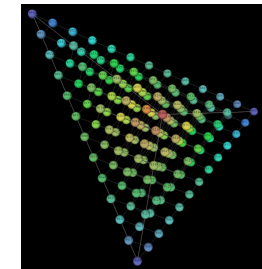
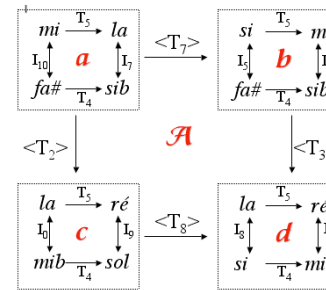
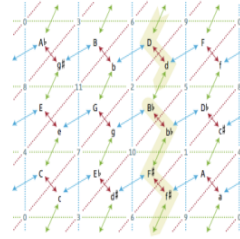
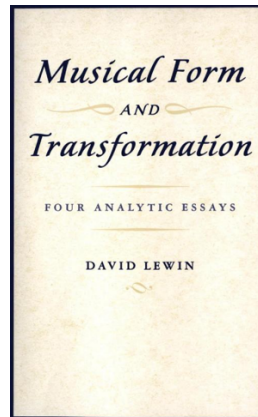
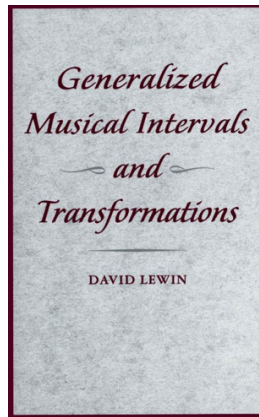
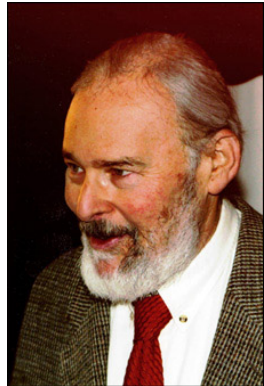
← transposition →						
00	↑ 11	22	33	44	55	[66] unison
↓ 01	12	23	34	45	56	↑ minor second
01	↑ 02	13	24	35	46	[57] major second
↓ 02	03	14	25	36	47	↑ minor third
12	↑ 03	04	15	26	37	[48] major third
↓ 13	04	05	16	27	38	↑ perfect fourth
13	↑ 04	05	16	27	38	[39] tritone
↓ 14	05	06	17	28	39	↑ perfect fourth
14	↑ 05	06	17	28	39	[20] major third
↓ 15	06	07	18	29	40	↑ minor third
15	↑ 06	07	18	29	40	[61] major second
↓ 16	07	08	19	30	41	↑ minor second
16	↑ 07	08	19	30	41	[00] unison



Moreno ANDREATTA
Equipe Représentations Musicales
IRCAM/CNRS UMR 9912



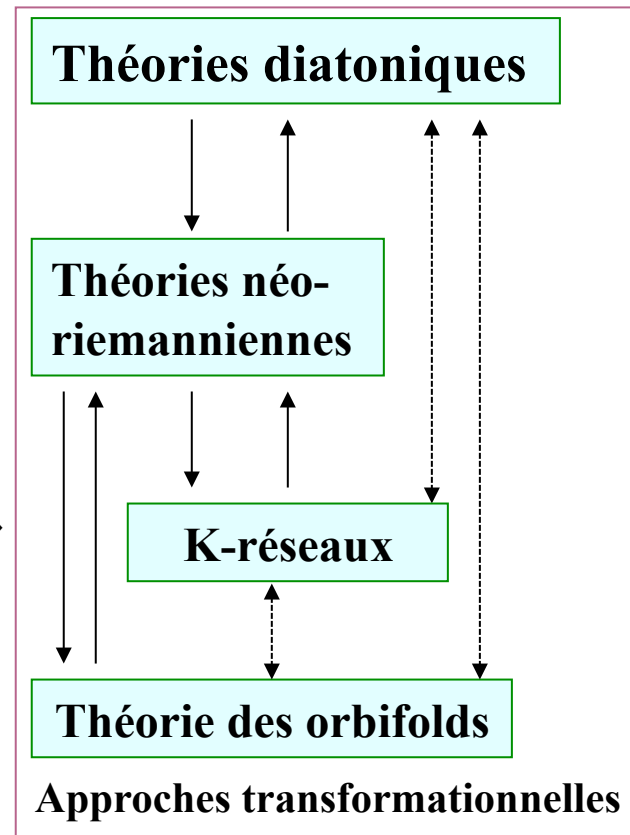
Approches transformationnelles en théorie et analyse musicales



Théories néo-schenkeriennes

[Cf. *MaMuPhi* 18 Nov. 2006]

→
GdT « Maths/Musique & Cognition »
PEPS « GdI e musique »



Approches transformationnelles
Théories néo-schenkeriennes

Plan de l'exposé :

- **Aspects théoriques de l'approche transformationnelle**

- Définition d'un GIS et premiers exemples
- Equivalence entre GIS et action simplement transitive
- Le cas du GIS non commutatif (dualité)
- Aperçu des aspects logiques (principes de « transfert »)
- Problèmes « mathémusicaux » ouverts (DFT et théorie de l'homométrie)

- **Aspects analytiques de l'approche transformationnelle**

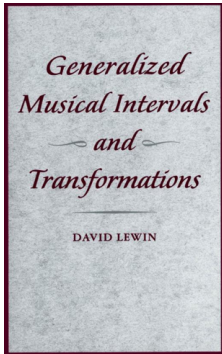
- L'analyse du *Klavierstück III* de Stockhausen par Lewin
- Les K-réseaux

- **Implications philosophiques d'une démarche transformationnelle**

- Cartésianisme vs anti-cartésianisme (Klumpenhouwer)
- Autour de la notion de « structure »
- Sur les possibilités d'un structuralisme phénoménologique en musique

- **Théorie des orbifolds en analyse musicale**

- Algèbre/géométrie
- Liens avec les K-réseaux



Système d'Intervalles Généralisés - Système Généralisé d'Intervalles

David Lewin's *Generalized Interval System* [GMIT, 1987]

$$\mathbf{GIS} = (S, G, \text{int})$$

S = ensemble

(G, \bullet) = groupe d'intervalles

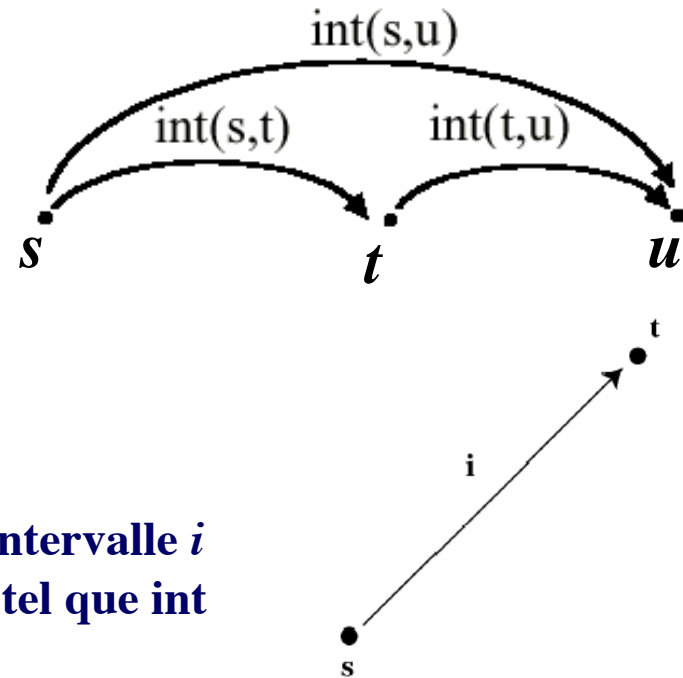
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$$S \times S \xrightarrow{\text{int}} G$$

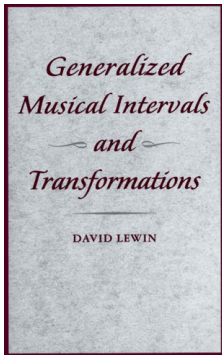
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2. Pour tout objet s dans S et tout intervalle i dans G il y a un seul objet t dans S tel que $\text{int}(s,t) = i$



- $S = \{ \dots, do, do_{\#} = ré_b, ré, \dots, si, do', \dots \}$, $G = \mathbf{Z}$, $\text{int}(do, ré) = 2$, $\text{int}(fa, do) = -5$ etc.
- $S = \{ \dots, do, ré, mi, fa, sol, la, si, do', \dots \}$, $G = \mathbf{Z}$, $\text{int}(do, ré) = 1$, $\text{int}(fa, do) = -3$ etc.
- $S = G = \mathbf{Z}_{12} = \{ do, do_{\#} = ré_b, ré, \dots, si \}$, $\text{int}(do, ré) = 2$, $\text{int}(fa, do) = 7$ etc.



Système d'Intervalles Généralisés - Système Généralisé d'Intervalles

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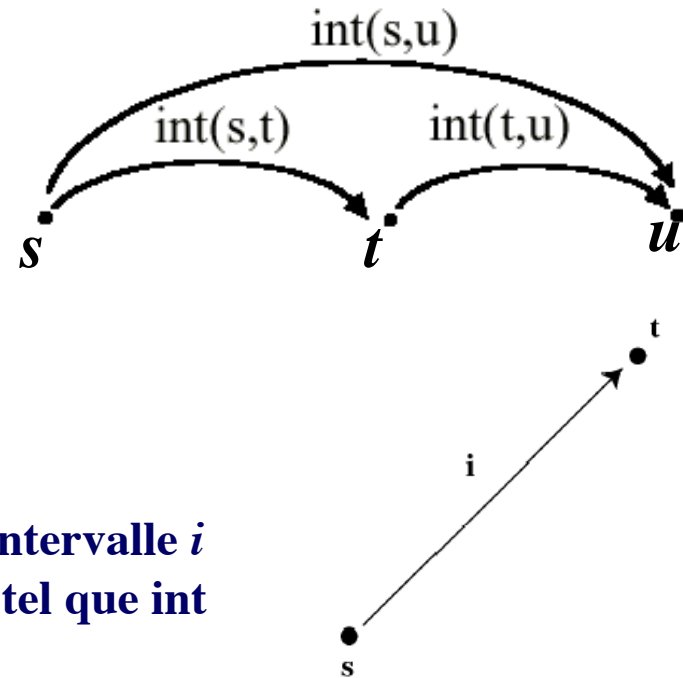
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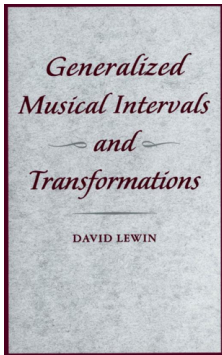
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- $S = \{(a,x) \in \mathbf{R} \times \mathbf{R}^+\}$, $G = (\mathbf{R}, +) \times (\mathbf{R}^+, \times)$, $(s, x) \bullet (t, y) = (s+t, xy)$
 $\text{int}((s,x), (t,y)) = (t-s, y/x) \rightarrow (S, G, \text{int})$ est un GIS commutatif
- $S = \{(a,x) \in \mathbf{R} \times \mathbf{R}^+\}$, $G = (\mathbf{R}, +) \times (\mathbf{R}^+, \times)$, $(s, x) \bullet (t, y) = (s+xt, xy)$
 $\text{int}((s,x), (t,y)) = ((t-s)/x, y/x) \rightarrow (S, G, \text{int})$ est un GIS non commutatif [Cf. ToM, p. 83]



Premières généralisations : transposition

$$\mathbf{GIS} = (S, G, \text{int})$$

S = ensemble

(G, \bullet) = groupe d'intervalles

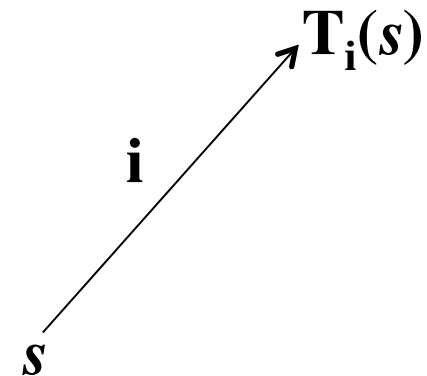
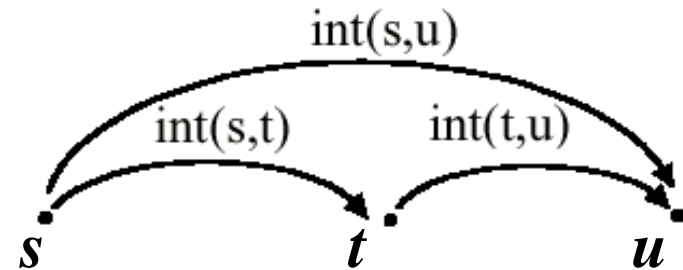
int = fonction intervallique

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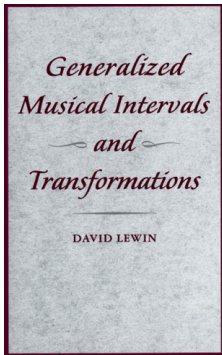
2. Pour tout objet s dans S et tout intervalle i dans G il y a un seul objet t dans S tel que $\text{int}(s,t) = i$



Généralisation de la notion de transposition (musicale)

Pour tout élément i dans G , la transposition T_i est une application

$$T_i : S \rightarrow S \quad \text{telle que} \quad \text{int}(s, T_i(s)) = i \quad \text{pour tout élément } s \text{ dans } S$$



Premières généralisations : inversion

$$\mathbf{GIS} = (S, G, \text{int})$$

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(G, \bullet) = groupe d'intervalles

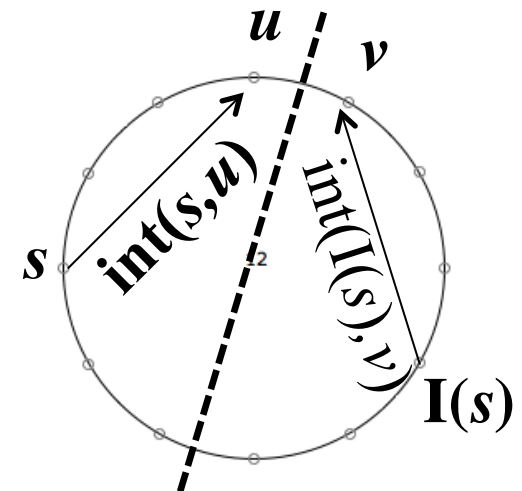
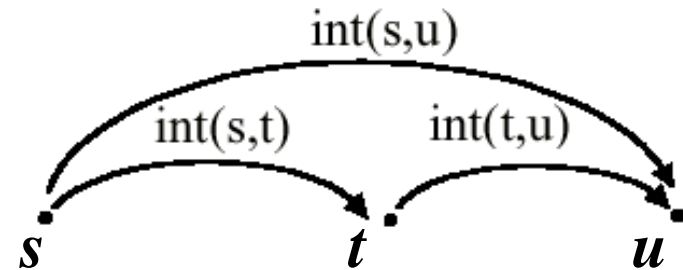
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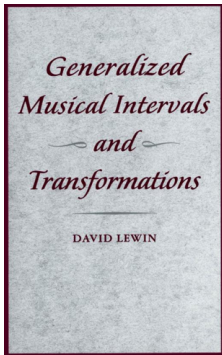
2. Pour tout objet s dans S et tout intervalle i dans G il y a un seul objet t dans S tel que $\text{int}(s,t) = i$



Généralisation de la notion d'inversion (par rapport à u et v)

Pour tous éléments u, v dans S , l'inversion I est une application

$$I_{u,v} : S \rightarrow S \quad \text{telle que} \quad \text{int}(s, u) = \text{int}(v, I_{u,v}(s))$$



Premières généralisations : inversion

$$\mathbf{GIS} = (S, G, \text{int})$$

S = ensemble

(G, \bullet) = groupe d'intervalles

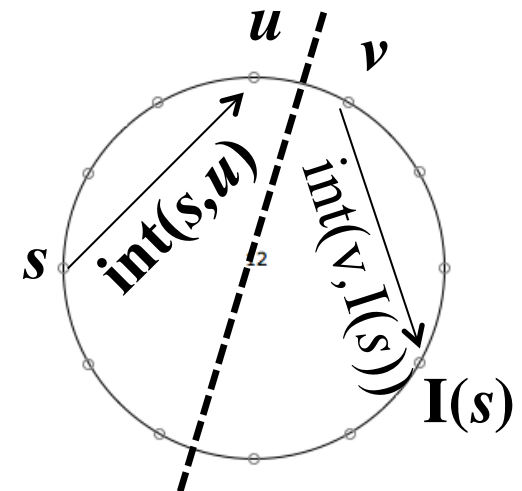
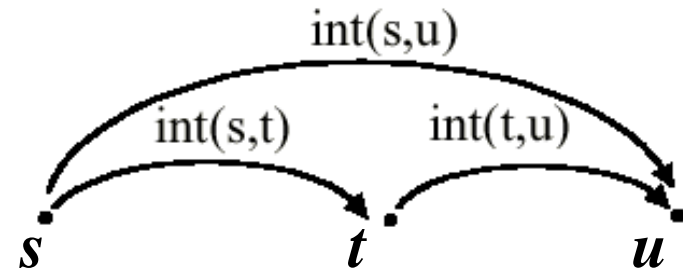
int = fonction intervallique

$$S \times S \xrightarrow{\text{int}} G$$

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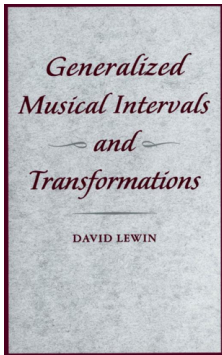
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Généralisation de la notion d'inversion (par rapport à u et v)

Pour tous éléments u, v dans S , l'inversion I est une application

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Premières propriétés intervalliques

$$\text{GIS} = (S, G, \text{int})$$

S = ensemble

(G, \bullet) = groupe d'intervalles

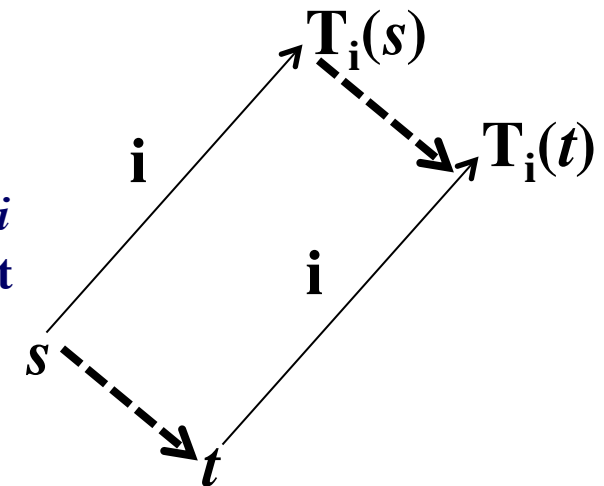
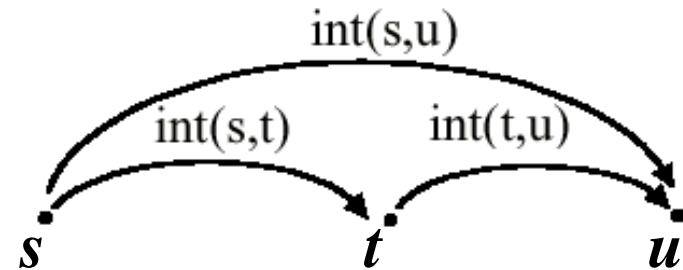
int = fonction intervallique

$$S \times S \xrightarrow{\text{int}} G$$

1. Pour tout objets s, t, u dans S :

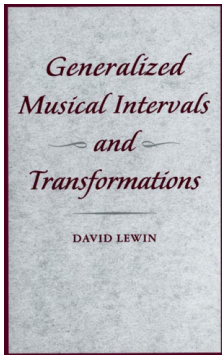
$$\text{int}(s,t) \bullet \text{int}(t,u) = \text{int}(s,u)$$

2. Pour tout objet s dans S et tout intervalle i dans G il y a un seul objet t dans S tel que $\text{int}(s,t) = i$



Dans un GIS commutatif, la transposition préserve les rapports intervalliques

$$\text{int}(s, t) = \text{int}(T_i(s), T_i(t))$$



Premières propriétés intervalliques

$$\mathbf{GIS} = (S, G, \text{int})$$

S = ensemble

(G, \bullet) = groupe d'intervalles

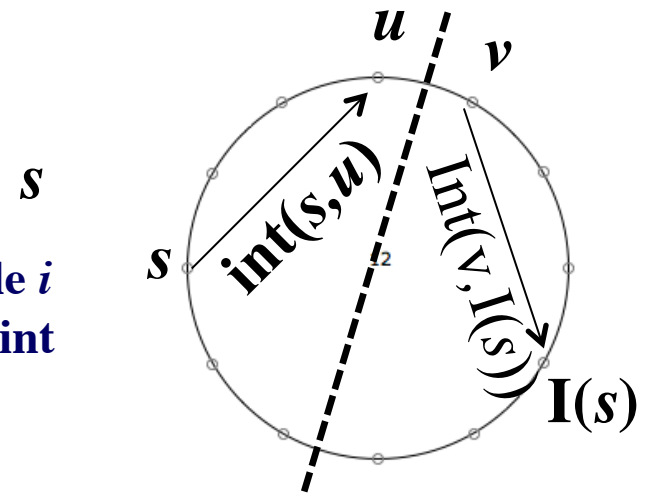
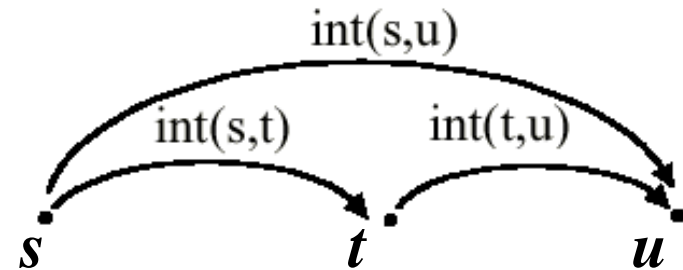
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$$S \times S \xrightarrow{\text{int}} G$$

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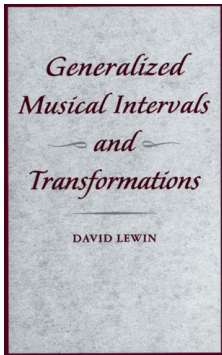
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Dans un GIS commutatif, l'inversion reverse les rapports intervallaires

$$\text{int}(s, u) = \text{int}(I(u), I(s))$$



Premières propriétés intervalliques

$$\mathbf{GIS} = (S, G, \text{int})$$

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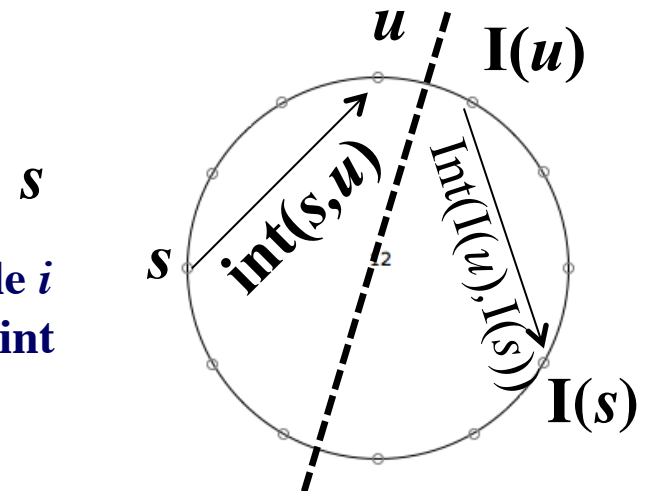
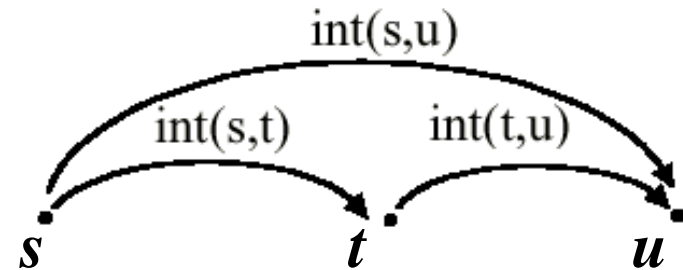
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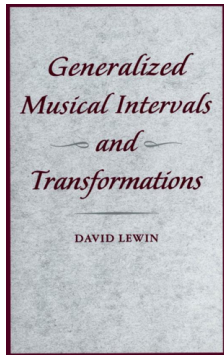
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Dans un GIS commutatif, l'inversion reverse les rapports intervallaires

$$\text{int}(s, u) = \text{int}(I(u), I(s))$$



Premières propriétés d'un GIS non commutatif

$$\text{GIS} = (S, G, \text{int})$$

S = ensemble

(G, \bullet) = groupe non commutatif

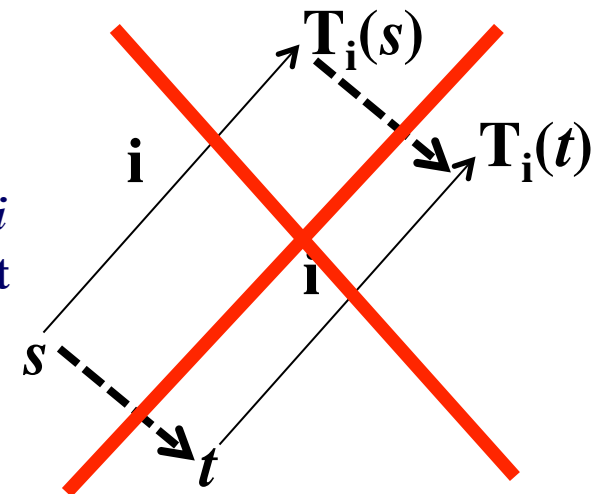
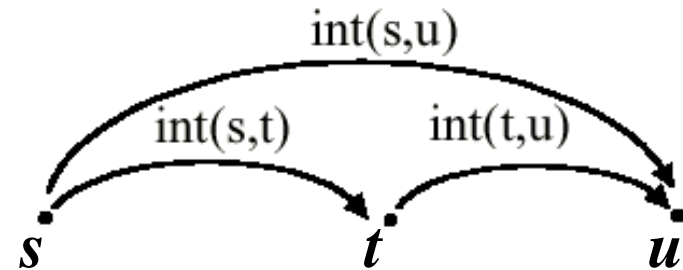
int = fonction intervallique

$$S \times S \xrightarrow{\text{int}} G$$

1. Pour tout objets s, t, u dans S :

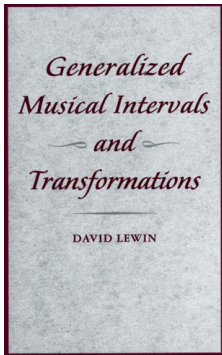
$$\text{int}(s,t) \bullet \text{int}(t,u) = \text{int}(s,u)$$

2. Pour tout objet s dans S et tout intervalle i dans G il y a un seul objet t dans S tel que $\text{int}(s,t) = i$



Il a des transpositions qui ne préservent pas les intervalles et il y a des transformations qui préservent les intervalles et qui ne sont pas des transpositions [GMIT, p. 50]

Il n'y a pas de transformations qui reversent les intervalles [GMIT, p. 59]



Equivalence entre GIS et action de groupe

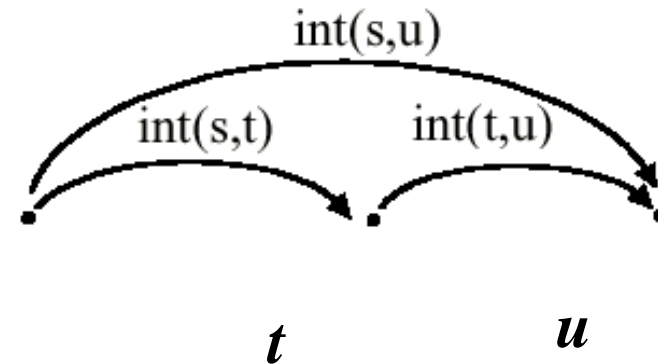
$$\text{GIS} = (S, G, \text{int})$$

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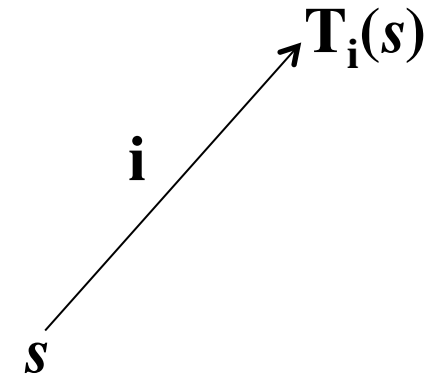
$$S \times S \xrightarrow{\text{int}} G$$



Action simplement transitive

1. Pour tout objets s, t, u dans S :
 $\text{int}(s,t) \bullet \text{int}(t,u) = \text{int}(s,u)$

2. Pour tout objet s dans S et tout intervalle i dans G il y a un seul objet t dans S tel que $\text{int}(s,t) = i$



Soit $\tau = \{T_i ; i \in G\}$ le groupe des transpositions

$$\text{GIS} = (S, G, \text{int}) \Leftrightarrow \tau \times S \rightarrow S \text{ telle que } (T_i, s) \rightarrow T_i(s)$$

Terminologies équivalentes :

- Un GIS est un G -torseur à gauche
- S est un ensemble principal homogène [Bourbaki]

Principes de transfert pour un GIS (Kolman, 2004)

$$\text{GIS}_1 = (S, G, \text{int})$$

S = ensemble

(G, \bullet) = groupe d'intervalles

int = fonction intervallique

$$S \times S \xrightarrow{\text{int}} G$$

$$\text{GIS}_2 = (S', G', \text{int}')$$

S' = ensemble

(G', \circ) = groupe d'intervalles

int' = fonction intervallique

$$S' \times S' \xrightarrow{\text{int}'} G'$$

Théorème d'équivalence :

GIS₁ et GIS₂ sont isomorphes ssi G et G' sont isomorphes en tant que groupes

→ Classification des GIS (à isomorphisme près)

- 5 GIS d'ordre 12
- 2 GIS octatoniques non-commutatifs (D_8 et Q_8)

→ Extension de la théorie transformationnelle aux GIS topologiques

→ Etude des propriétés logiques d'un GIS

Aspects logiques de la théorie transformationnelle

The class of commutative GIS has the same complexity as the class of commutative groups: the theory of each is decidable. This contrasts strongly with the class of finite (non commutative) GIS, which is not even recursively axiomatizable

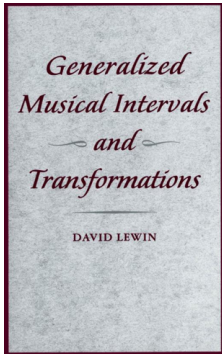
[Kolman 2004]

→ GIS can be considered as models of a theory in first-order language

The application of logic that I believe is potentially interesting for finite GIS theory concerns the minimal axiomatic strength of formal systems [...] necessary to prove certain finite combinatorial principles of GIS theory. How strong must an axiomatic system be in order to answer (i.e. prove or refute) all the combinatorial problems of finite GIS theory? Can every combinatorial question of finite GIS theory be answered using just finitistic methods? How much mathematics does one need to do music theory?

[Kolman 1999]

→ Reverse Mathematical Music Theory?



Limitations de la théorie des GIS (Rahn, Tymoczko, ...)

$$\text{GIS} = (S, G, \text{int})$$

S = ensemble

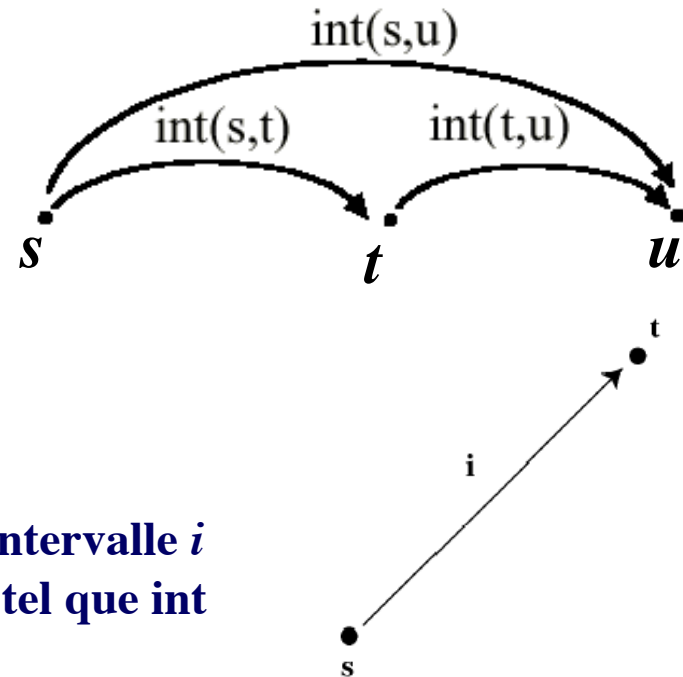
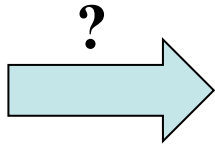
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2. Pour tout objet s dans S et tout intervalle i dans G il y a un seul objet t dans S tel que $\text{int}(s,t) = i$



« [...] Other problems associated with Lewin's approach to musical intervals -problems that arise even when we can transport arrows from point to point. These result from Lewin's requirement that intervals always be defined at every point in the space, and that they be represented by functions whose identity is entirely determined by their inputs and outputs. As we will see, these requirements further constrain the range of applicability of Lewin's theory by prohibiting spaces with boundaries and by eliding the distinct paths that might connect the same pairs of points.

Aspects logiques de la théorie transformationnelle

The class of commutative GIS has the same complexity as the class of commutative groups: the theory of each is decidable. This contrasts strongly with the class of finite (non commutative) GIS, which is not even recursively axiomatizable

→ GIS can be considered as models of a theory in first-order language

[Kolman 2004]

The application of logic that I believe is potentially interesting for finite GIS theory concerns the minimal axiomatic strength of formal systems [...] necessary to prove certain finite combinatorial principles of GIS theory. How strong must an axiomatic system be in order to answer (i.e. prove or refute) all the combinatorial problems of finite GIS theory? Can every combinatorial question of finite GIS theory be answered using just finitistic methods? How much mathematics does one need to do music theory?

[Reverse Mathematical Music Theory ->]

→ Trouver de problèmes « finitistically unprovable » qui peuvent être exprimé comme des énoncé [statements] dans la théorie des GIS finis

[Kolman 1999]

Fonction Intervallique IFUNC dans un GIS

$\text{GIS} = (S, G, \text{int})$

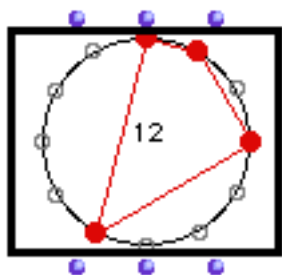
S ensemble

H et H' dans S

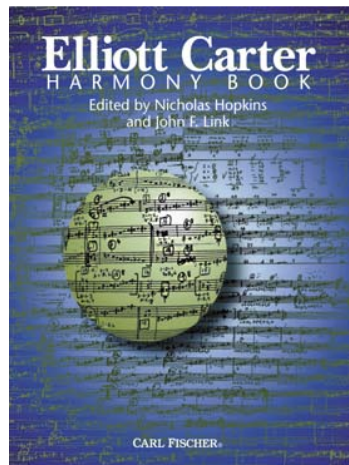
$$\text{IFUNC}(H, H')(i) = \#\{(a, b) \in H \times H' \mid \text{int}(a, b) = i\}$$

$$\text{IFUNC}(H, H')(2) = 4$$

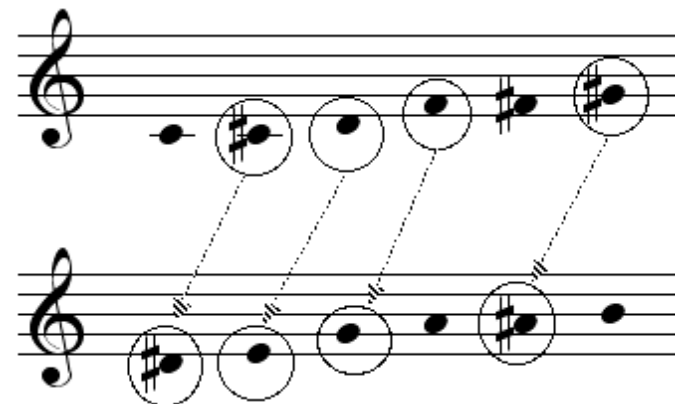
23



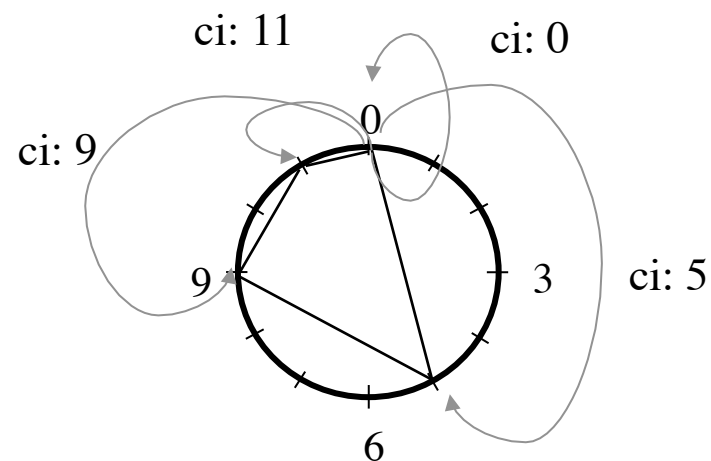
$$VI = [111111]$$



$$H = \{0, 1, 2, 5, 6, 8\}$$



$$H' = \{3, 4, 7, 9, 10, 11\}$$

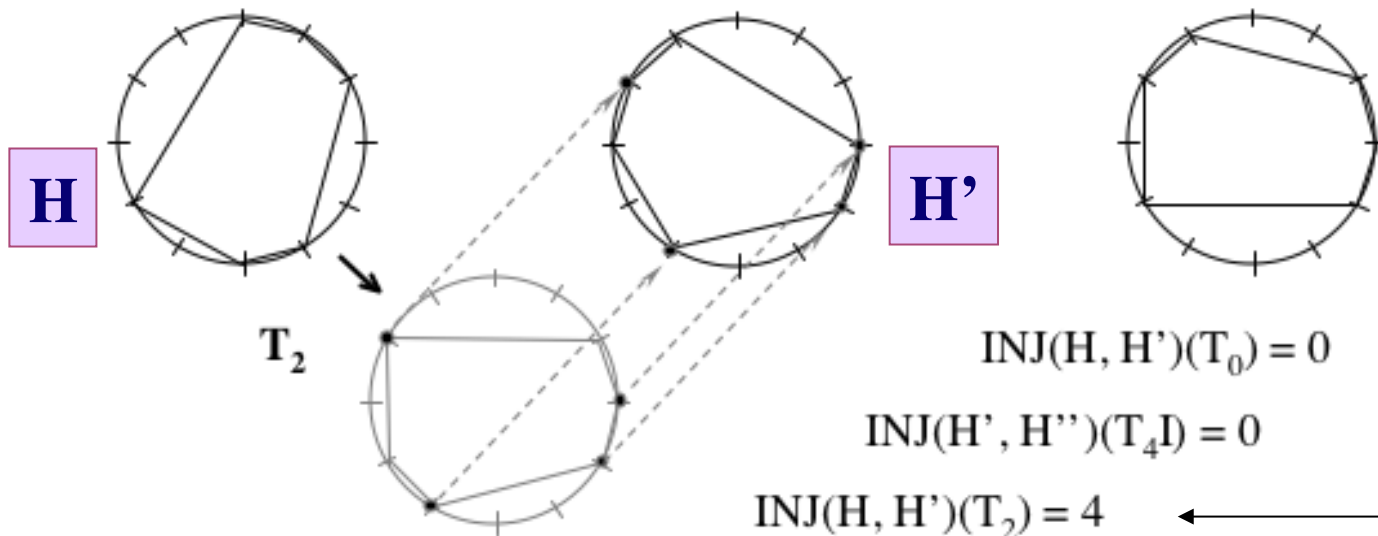


$$A = \{0, 5, 9, 11\} \Rightarrow IC_A(k) = 1 \forall k$$

Fonction d'Injection comme généralisation de relation d'inclusion



$$H = \{0, 1, 2, 5, 6, 8\} \quad H' = \{3, 4, 7, 9, 10, 11\} \quad H'' = T_4 I(H) = \{2, 3, 4, 8, 10, 11\}$$

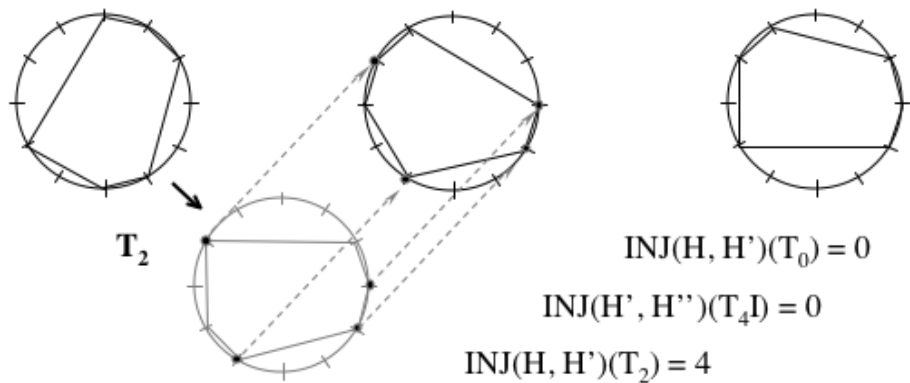


$$\text{INJ}(H, H')(T_i) = \#\{a \in H \mid T_i(a) \in H'\}$$

$i=2$

Fonction d'Injection (INJ) et fonction intervallaire (IFUNC)

$H = \{0, 1, 2, 5, 6, 8\}$ $H' = \{3, 4, 7, 9, 10, 11\}$ $H'' = T_4 I(H) = \{2, 3, 4, 8, 10, 11\}$



$INJ(H, H')(T_i) =$
 nombre d'éléments a
 de H tels que $T_i(a) \in H'$

||

H

H'

$IFUNC(H, H')(i) =$
 nombre d'éléments
 (a, b) dans $H \times H'$ tels
 que $int(a, b) = i$

« *Here the basic hierarchical scope of the (twelve-tone) system is contained essentially in the simple theorem that:*

Given a collection of pitches (pitch classes), the multiplicity of occurrence of any interval (...) determines the number of common pitches between the original collection and the transposition by the interval »

(Milton Babbitt, Past and Present Concepts, 1961)

« *...on peut remplacer entièrement le concept d'intervalle dans un GIS avec le concept de transposition dans un espace »*

« ...on peut remplacer le concept même de GIS avec l'idée d'un espace S sur lequel on a un groupe d'opérations qui opère »

(David Lewin, Generalized Musical Intervals and Transformations, 1987)

Babbitt

↕ $GIS = Z_{12}$

↕ GIS

Lewin

$$\mathbf{INJ}(A, B)(T_i) = \mathbf{IFUNC}(A, B)(i)$$

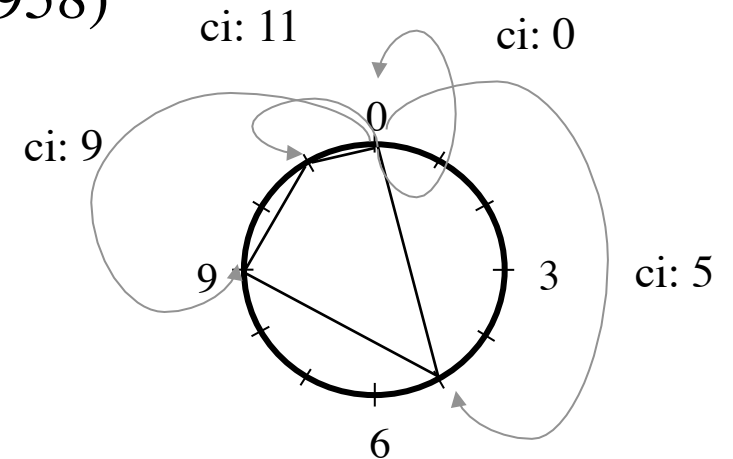
Approche transformationnelle et théorie de l'homométrie

$$\text{INJ}(A, B)(T_k) = \text{IFUNC}(A, B)(k)$$

- Dans le cas du GIS Z_{12} si $A = B$ on obtient la définition de “contenu intervallique” d'un accord
- Le contenu intervallique est équivalent à un produit de convolution de fonctions caractéristiques (Lewin, *JMT*, 1958)

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

Diagram illustrating musical sets B and C on a staff and their corresponding chromatic circle diagrams. Set B is represented by the interval set $(0, 1, 4, 6)$ and Set C by $(0, 1, 3, 7)$. The chromatic circle diagrams show the 12 notes of the circle with red dots indicating the members of sets B and C.



$$A = \{0, 5, 9, 11\} \Rightarrow IC_A(k) = 1 \forall k$$

$$B = \{0, 1, 4, 6\}$$

$$C = \{0, 1, 3, 7\}$$

$$\Rightarrow IC_B(k) = IC_C(k) = IC_A(k) = 1 \forall k$$

Approche transformationnelle et théorie de l'homométrie

$$\text{INJ}(A, B)(T_k) = \text{IFUNC}(A, B)(k)$$

- Dans le cas du GIS Z_{12} si $A = B$ on obtient la définition de “contenu intervallique” d'un accord
- Le contenu intervallique est équivalent à un produit de convolution de fonctions caractéristiques (Lewin, *JMT*, 1958)

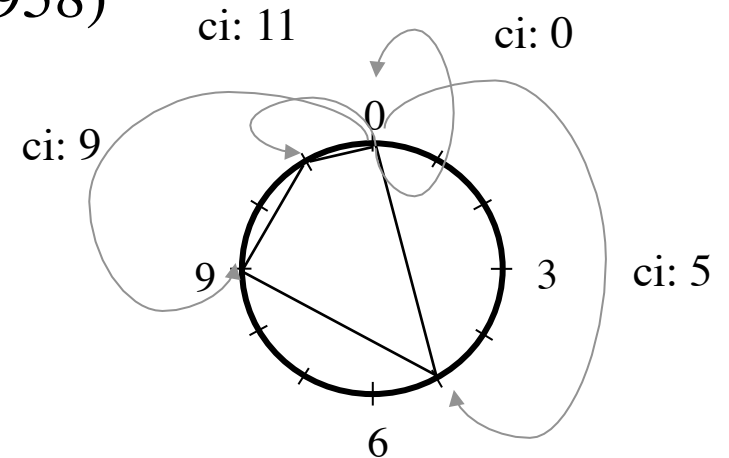
$$IC_A(k) = (1_A \star 1_{-A})(k)$$

$$1_A \star \tilde{1}_B(k) = \sum_i 1_A(i) \times 1_B(i - k) = \sum_{\substack{i \in A \\ i - k \in B}} 1$$

$$\mathcal{F}(1_A \star \tilde{1}_B) = \mathcal{F}(1_A) \times \mathcal{F}(\tilde{1}_B)$$

$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$

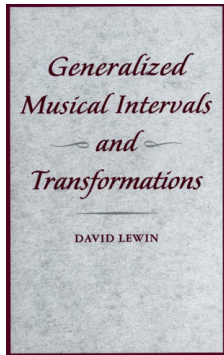
$$\mathcal{F}(IC_A) = \mathcal{F}_A \times \mathcal{F}_{-A} = |\mathcal{F}_A|^2$$



$$A = \{0, 5, 9, 11\} \Rightarrow IC_A(k) = 1 \forall k$$

➔ **Reconstruction de la phase**

Cf. D. Ghisi, J. Mandereau, E. Amiot, C. Agon, M. Andreatta, “Generalized Z-relation and Homometric Theory”, paper in progress, to be submitted to the *Journal of Mathematics and Music*



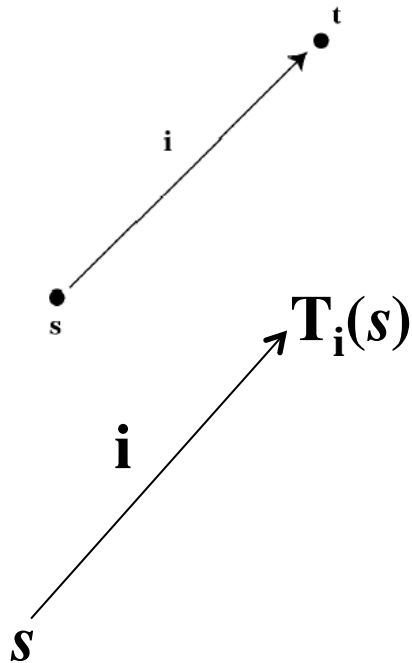
Premières implications philosophiques de l'équivalence

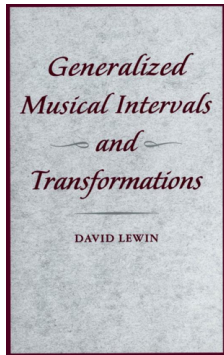
$$\text{GIS} = (S, G, \text{int}) \quad \Leftrightarrow$$

Action simplement transitive

Cartésianisme vs anti-cartésianisme

« To some extent for cultural-historical reasons, it is easier for us to hear “intervals” between individual objects than to hear transpositional relations between them; we are more used to conceiving transpositions as affecting Gestalts built up from individual objects. As this way of talking suggests, we are very much under the influence of Cartesian thinking in such matters. We tend to conceive the primary objects in our musical spaces as atomic individual “elements” rather than contextually articulated phenomena like sets, musical series, and the like. And we tend to imagine ourselves in the position of observers when we theorize about musical space; the space is “out there,” away from our dancing bodies or singing voices. “The interval from s to t ” is thereby conceived as modeling a relation of extension, observed in that space external to ourselves; we “see” it out there just as we see distances between holes in a flute or points along a stretched string. . . . » [GMIT, p. 158–59].





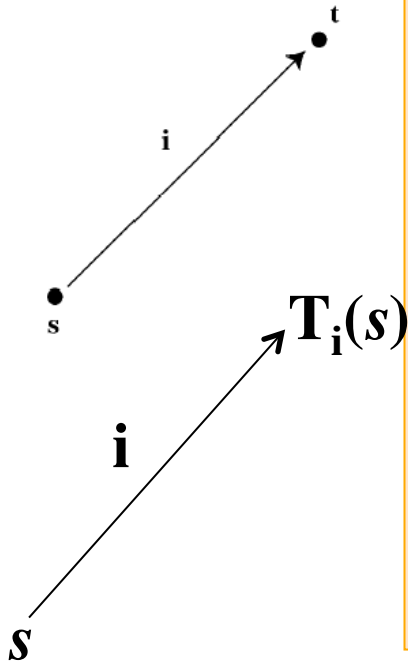
Premières implications philosophiques de l'équivalence

$GIS = (S, G, \text{int}) \iff$

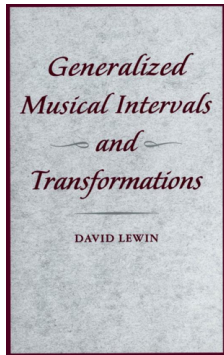
Action simplement transitive

Cartésianisme vs anti-cartésianisme

« In contrast, the transformational attitude is much less Cartesian. Given locations s and t in our space, this attitude does not ask for some observed measure of extension between reified 'points'; rather it asks: 'If I am at s and wish to get to t , what characteristic gesture (e.g. member of STRANS) should I perform in order to arrive there?' The question generalizes in several important respects: 'If I want to change Gestalt 1 into Gestalt 2 (as regards to content, or location, or anything else), what sorts of admissible transformations in my space (members of STRANS or otherwise) will do the best job?' Perhaps none will work completely, but 'if only . . . ,' etc. This attitude is by and large the attitude of someone inside the music, as idealized dancer and/or singer. No external observer (analyst/listener) is needed » [GMIT, p. 158–59].



« ...instead of regarding the i -arrow [...] as a measurement of extension between points s and t observed passively « out there » in a Cartesian *res extensa*, one can regard the situation actively, like a singer, player or composer, thinking « I am at s ; what characteristic transformation do I perform in order to arrive at t ? » [GMIT, p. xiii].

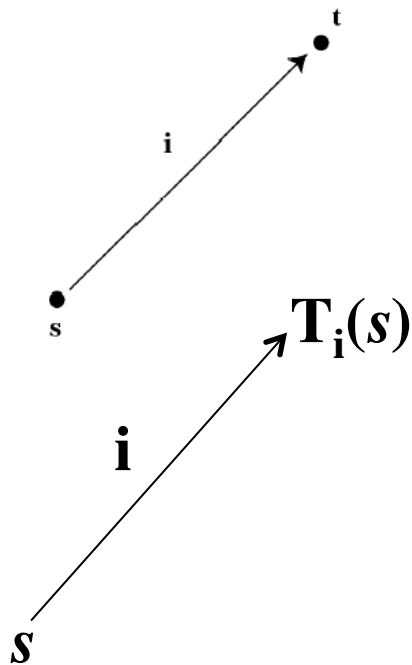


Premières implications philosophiques de l'équivalence

$$\text{GIS} = (S, G, \text{int}) \iff$$

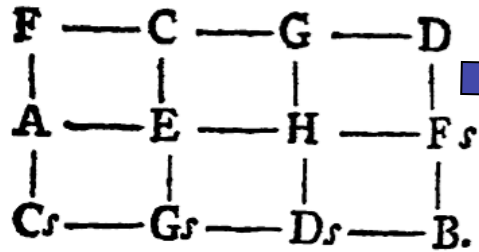
Action simplement transitive

Cartésianisme vs anti-cartésianisme

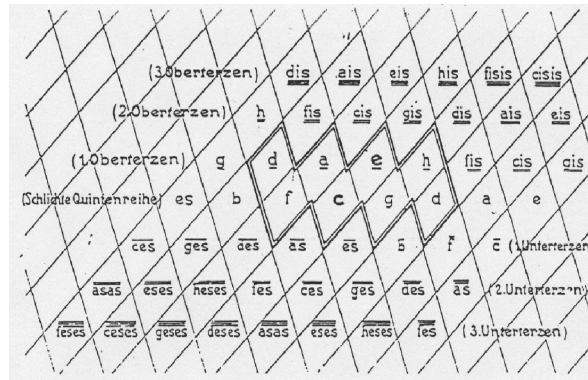


« [A]bove we sketched a mathematical dichotomy between intervals in a GIS and transposition-operations on a space: Either can be generated formally from the characteristic properties of the other. More significant than this dichotomy, I believe, is the *generalizing* power of the transformational attitude. It enables us to *subsume* the theory of GIS structure, along with the theory of simply transitive groups, into a broader theory of transformations. This enables us to consider intervals-between-things and transpositional-relations-between-Gestalts not as alternatives, but as the *same* phenomenon manifested in different ways » [GMIT, p. 159].

Approches transformationnelles diatoniques et néo-riemanniennes



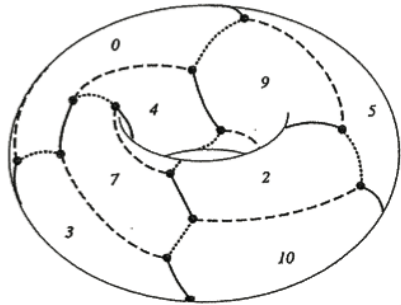
Euler : *Speculum musicum*, 1773



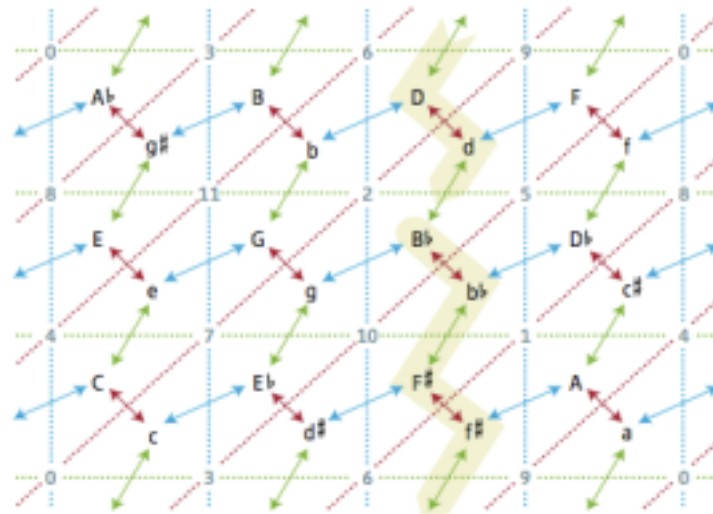
Hugo Riemann : « Ideen zu einer Lehre von den Tonvorstellung », 1914

A	C#	F	A'	C#'	F'	A''	C#''	F''	A'''
D	F#	A#	D'	F#'	A#'	D''	F#''	A#''	D'''
G	B	D#	G'	B'	D#'	G''	B''	D#''	G'''
C	E	G#	C'	E'	G#'	C''	E''	G#''	C'''
F	A	C#	F'	A'	C#'	F''	A''	C#''	F'''
Bb	D	F#	Bb'	D'	F#'	Bb''	D''	F#''	Bb'''
Eb	G	B	Eb'	G'	B'	Eb''	G''	B''	Eb'''
Ab	C	E	Ab'	C'	E'	Ab''	C''	E''	Ab'''

Longuet-Higgins (1962)



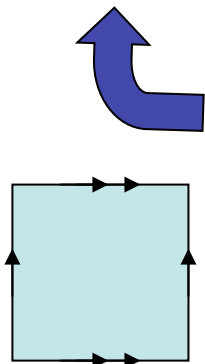
Douthett & Steinbach, *JMT*, 1998



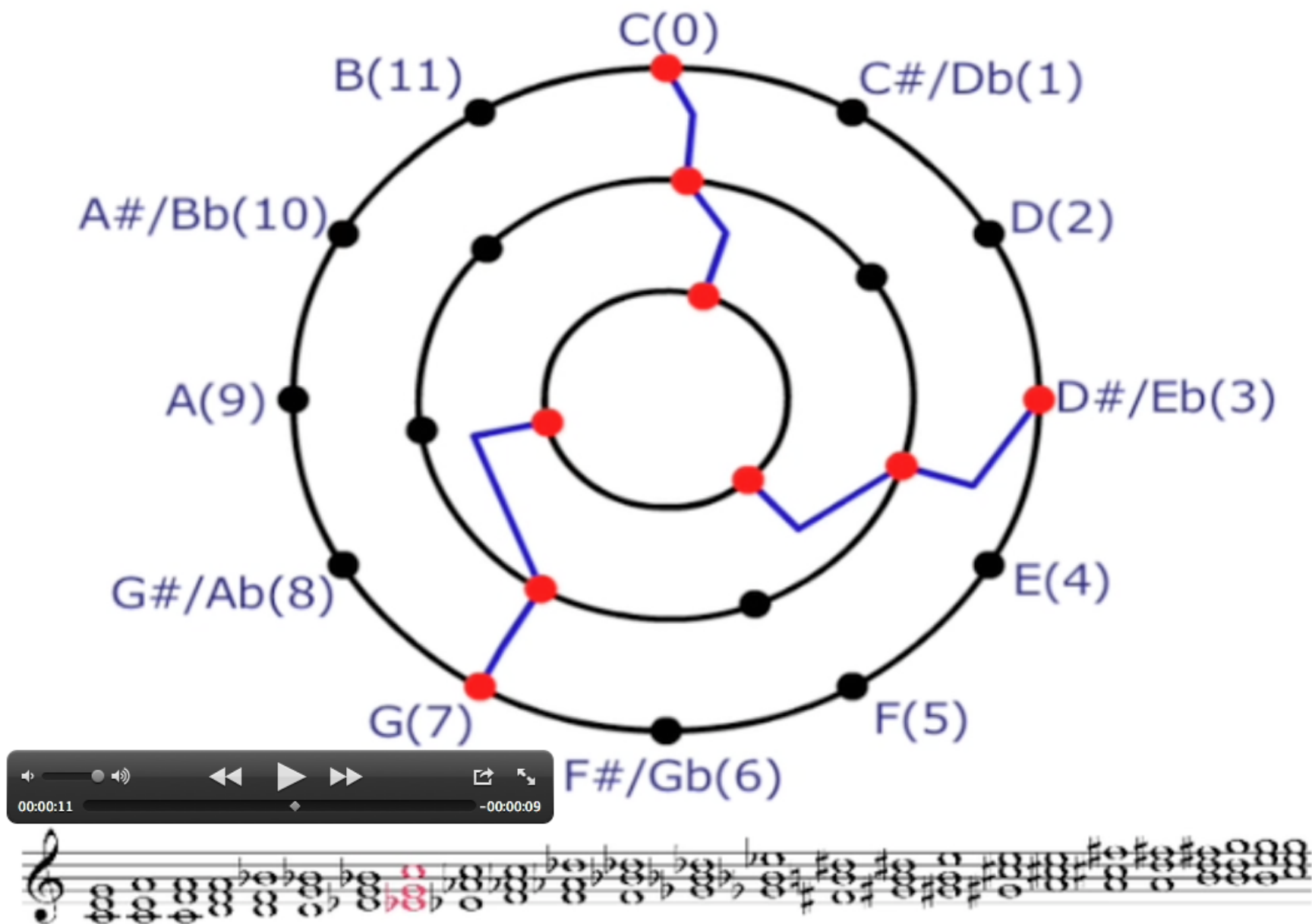
J. Hook, « Exploring Musical Space », *Science*, 2006

	m3	m3	m3	m3	m3	m3
M3 → 0	4	8	0	4	8	
M3 → 3	7	11	3	7	11	
M3 → 6	10	2	6	10	2	
M3 → 9	1	5	9	1	5	
M3 → 0	4	8	0	4	8	
M3 → 3	7	11	3	7	11	
M3 → 6	10	2	6	10	2	
M3 → 9	1	5	9	1	5	

Balzano (1980)

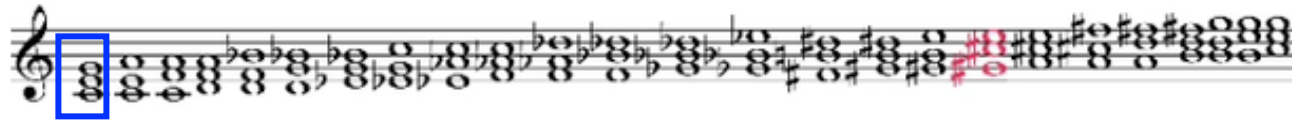


Beethoven's 9th Symphony
Mvmt. 2, mm. 143-176

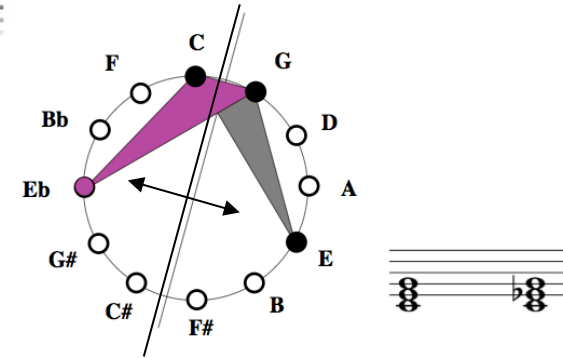


Cf. Jack Douthett, “Filtered Point-Symmetry and Dynamical Voice-Leading”, dans *Music Theory and Mathematics. Chords, Collections, and Transformations*, edited by J. Douthett, M. M. Hyde, C. J. Smith, URP, 2008.

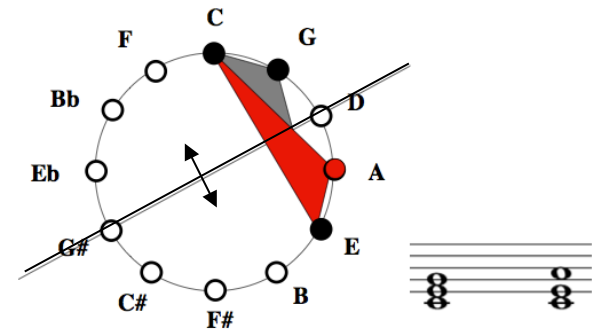
Le Tonnetz d'Oettingen/Riemann



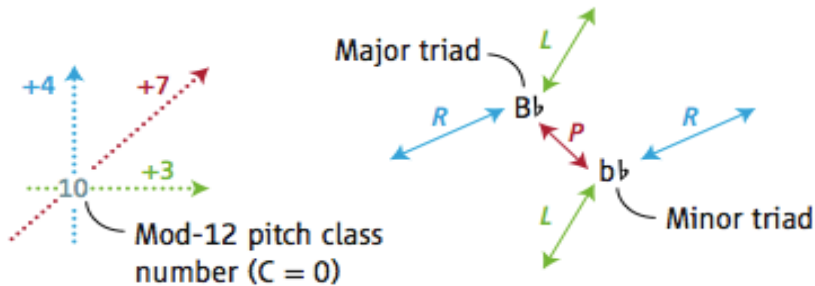
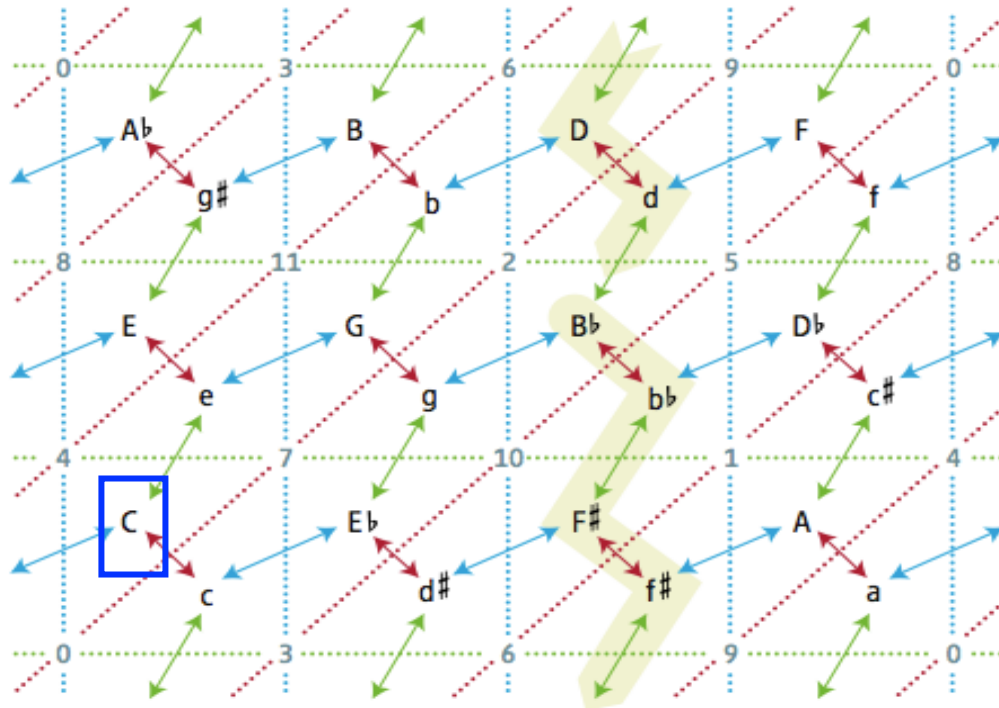
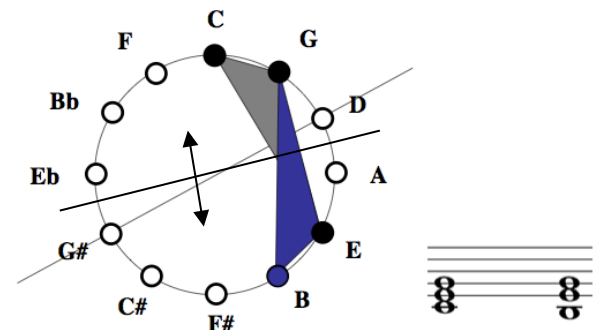
(Neo-)Riemannian Operation P = „Parallel“ [Noll04]



(Neo-)Riemannian Operation R = „Relative“

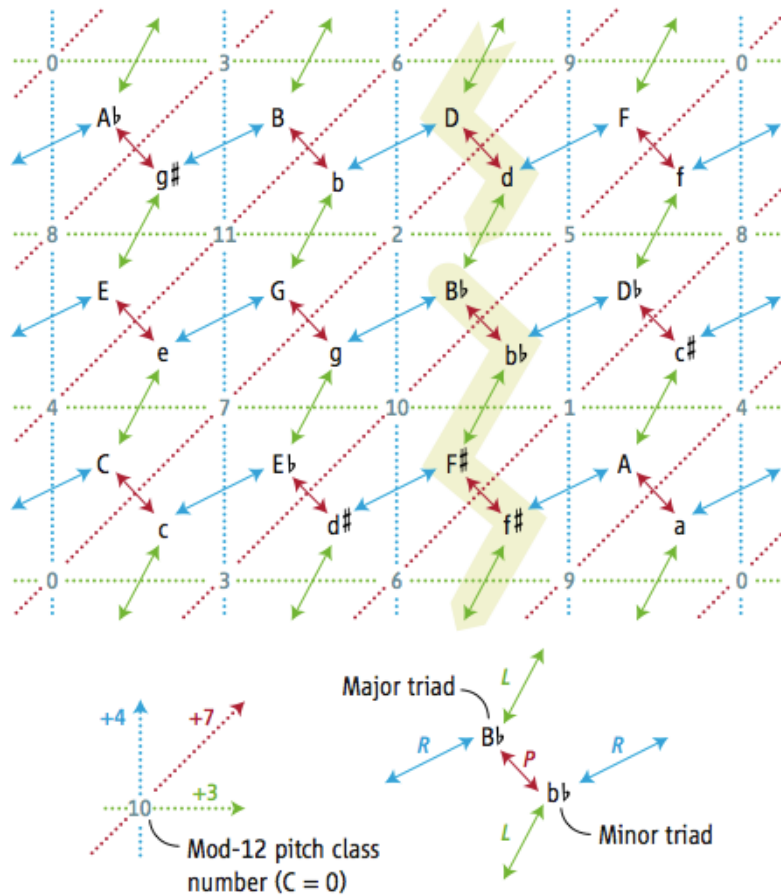


(Neo-)Riemannian Operation L = „Leading-Tone“



[J. Hook 06]

Le Tonnetz en tant que GIS



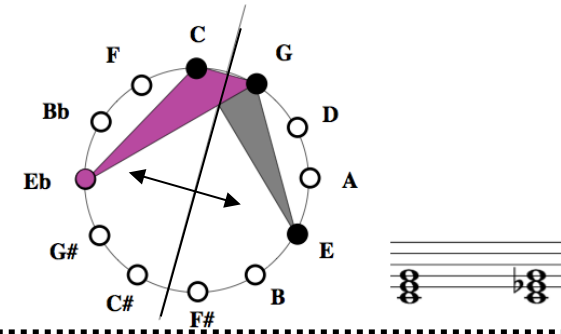
$$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 ; LRL = L(LR)^{-1} \rangle$$

• ρ opère de façon simplement transitive sur l'ensemble S des 24 triades consonantes

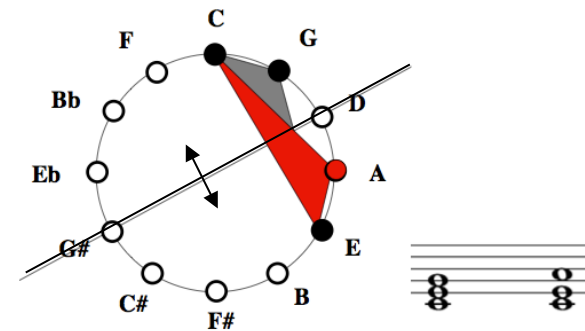
$\Rightarrow (S, \rho, \text{int})$ est un GIS

(Neo-)Riemannian Operation P = „Parallel“

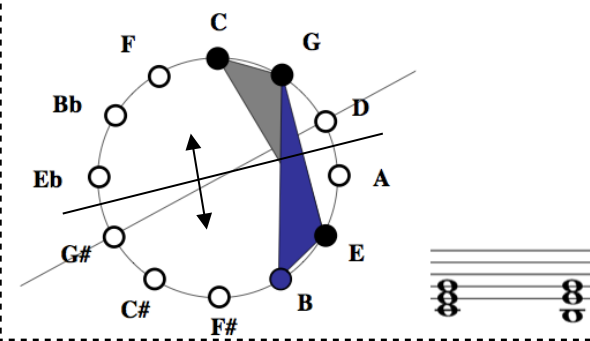
[Noll04]



(Neo-)Riemannian Operation R = „Relative“



(Neo-)Riemannian Operation L = „Leading-Tone“



Une autre structure de GIS sur l'espace S

L	R	RL	I_{11}
C → e	C → a	C → G	C → e
c → A \flat	c → E \flat	c → f	c → E
D \flat → f	D \flat → B \flat	D \flat → A \flat	D \flat → e \flat
c \sharp → A	c \sharp → E	c \sharp → f \sharp	c \sharp → E \flat
D → f \sharp	D → b	D → A	D → d
d → B \flat	d → F	d → g	d → D
E \flat → g	E \flat → c	E \flat → B \flat	E \flat → c \sharp
d \sharp → B	d \sharp → F \sharp	d \sharp → g \sharp	d \sharp → C \sharp
E → g \sharp	E → c \sharp	E → B	E → c
e → C	e → G	e → a	e → C
F → a	F → d	F → C	F → b
f → D \flat	f → A \flat	f → B \flat	f → B
F \sharp → a \sharp	F \sharp → d \sharp	F \sharp → C \sharp	F \sharp → B \flat
f \sharp → D	f \sharp → A	f \sharp → b	f \sharp → B \flat
G → b	G → e	G → D	G → a
g → E \flat	g → B \flat	g → c	g → A
A \flat → c	A \flat → f	A \flat → E \flat	A \flat → a \flat
g \sharp → E	g \sharp → B	g \sharp → c \sharp	g \sharp → G \sharp
A → c \sharp	A → f \sharp	A → E	A → g
a → F	a → C	a → d	a → G
B \flat → d	B \flat → g	B \flat → F	B \flat → f \sharp
a \sharp → F \sharp	a \sharp → C \sharp	a \sharp → d \sharp	a \sharp → F \sharp
B → e \flat	B → g \sharp	B → F \sharp	B → f
b → G	b → D	b → e	b → F

[Satyendra 2004]

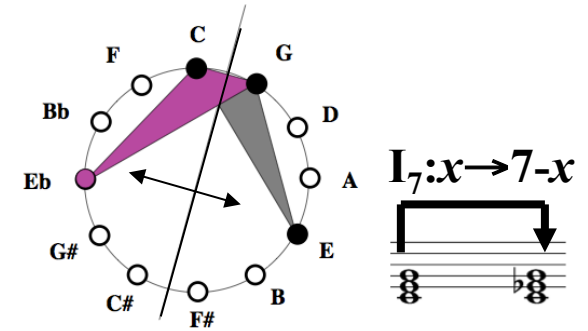
$$D_{12} = \langle I, T \mid I^2 = T^{12} = 1 ; ITI = I(IT)^{-1} \rangle$$

- D_{12} opère de façon simplement transitive sur l'ensemble S des 24 triades consonantes

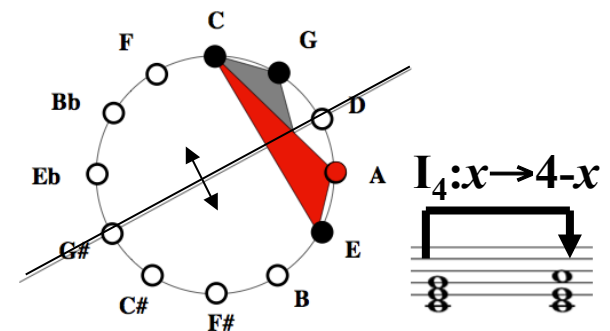
$\Rightarrow (S, D_{12}, \text{int})$ est un GIS

(Neo-)Riemannian Operation P = „Parallel“

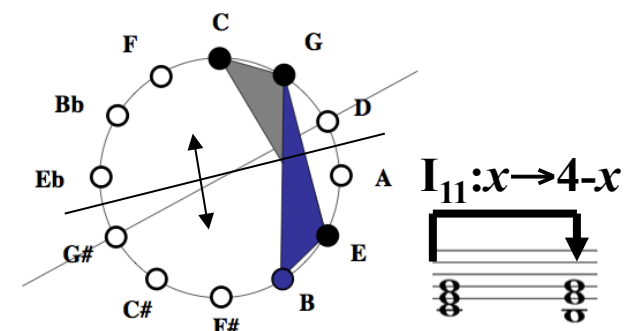
[Noll04]



(Neo-)Riemannian Operation R = „Relative“



(Neo-)Riemannian Operation L = „Leading-Tone“



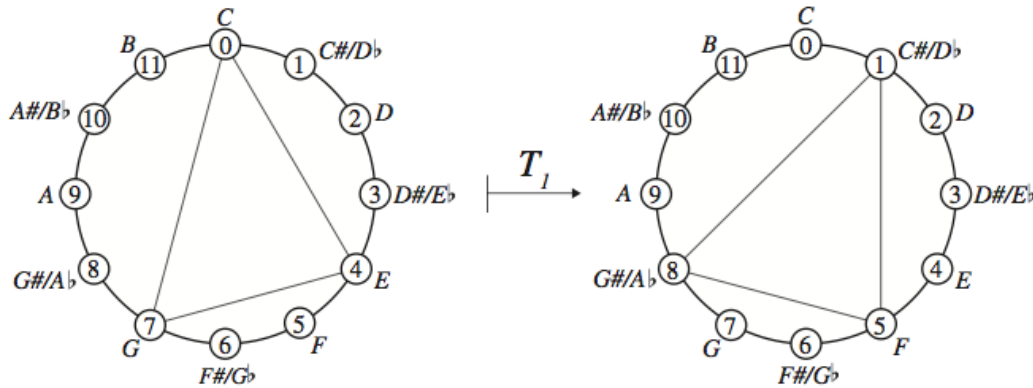
Dualité entre (S, ρ, int) et (S, D_{12}, int)

$$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 ; LRL = L(LR)^{-1} \rangle$$

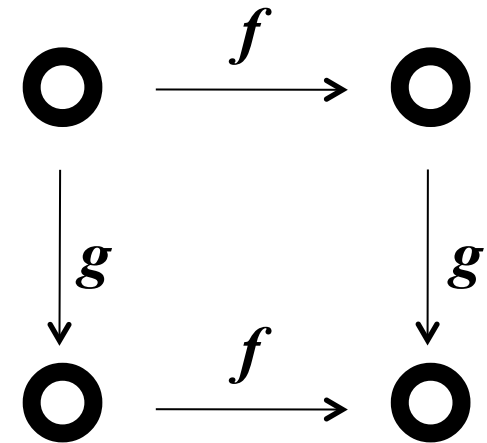
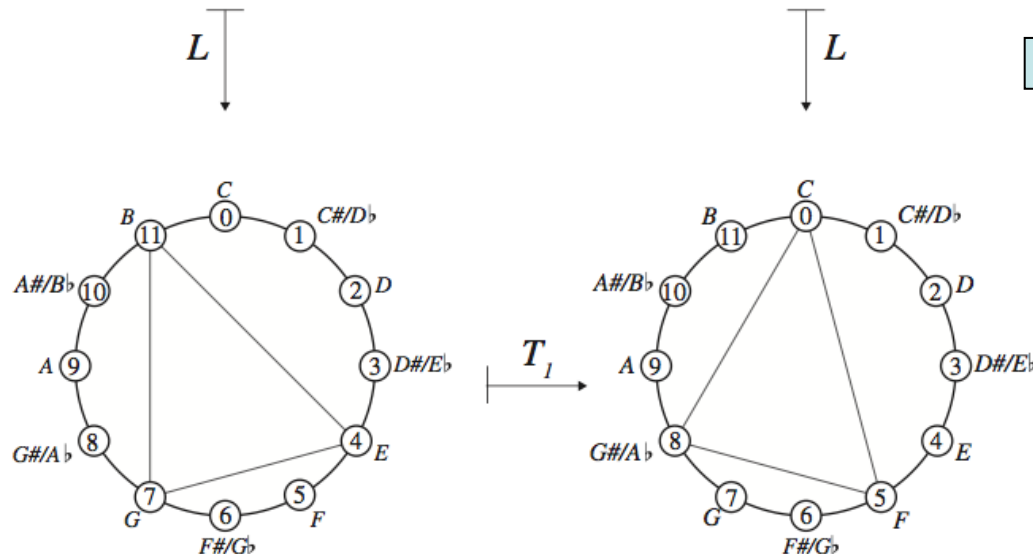
\Leftrightarrow

$$D_{12} = \langle I, T \mid I^2 = T^{12} = 1 ; ITI = I(IT)^{-1} \rangle$$

$\Rightarrow \rho$ et D_{12} sont l'un le *centralisateur* de l'autre (dans le groupe symétrique $Sym(S)$)



$(S, \rho, \text{int}) \neq (S, D_{12}, \text{int})$
[cf. équivalence entre GIS]



Tout diagramme commute

$$\forall f \in D_{12}$$

$$\forall g \in \rho$$

« Making and Using a Pcset Network for Stockhausen's Klavierstück III »

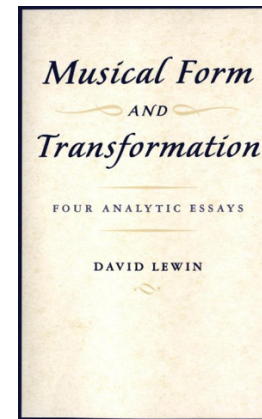
Measures 1-4 of the score. The top staff is in treble clef, and the bottom staff is in bass clef. The key signature has one flat (B-flat). Measure 1: Treble clef has a quarter note G4, quarter note A4, quarter note Bb4, quarter note C5, quarter note Bb4, quarter note A4, quarter note G4. Bass clef has a quarter note G2, quarter note F2, quarter note E2, quarter note D2, quarter note C2, quarter note B1, quarter note A1. Measure 2: Treble clef has a quarter note G4, quarter note A4, quarter note Bb4, quarter note C5, quarter note Bb4, quarter note A4, quarter note G4. Bass clef has a quarter note G2, quarter note F2, quarter note E2, quarter note D2, quarter note C2, quarter note B1, quarter note A1. Measure 3: Treble clef has a quarter note G4, quarter note A4, quarter note Bb4, quarter note C5, quarter note Bb4, quarter note A4, quarter note G4. Bass clef has a quarter note G2, quarter note F2, quarter note E2, quarter note D2, quarter note C2, quarter note B1, quarter note A1. Measure 4: Treble clef has a quarter note G4, quarter note A4, quarter note Bb4, quarter note C5, quarter note Bb4, quarter note A4, quarter note G4. Bass clef has a quarter note G2, quarter note F2, quarter note E2, quarter note D2, quarter note C2, quarter note B1, quarter note A1.

Trois interprétations :

 Henck

 Kontarsky

 Tudor

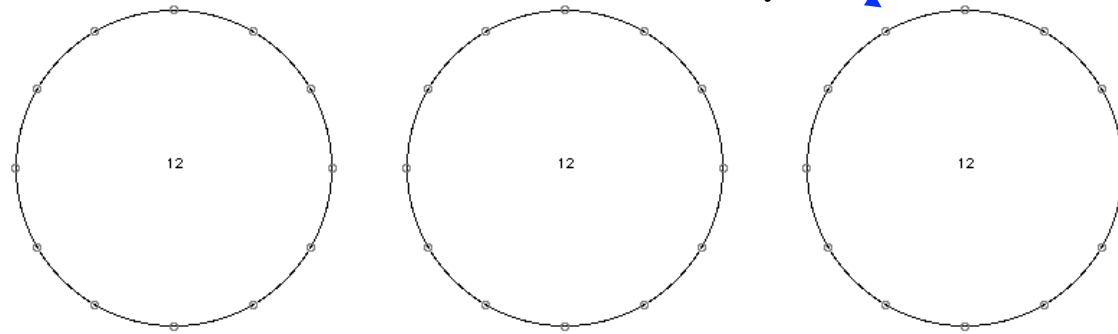
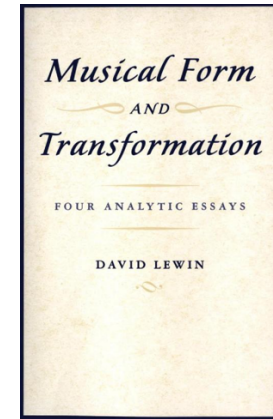


Measures 5-8 of the score. Measure 5: Treble clef has a quarter note G4, quarter note A4, quarter note Bb4, quarter note C5, quarter note Bb4, quarter note A4, quarter note G4. Bass clef has a quarter note G2, quarter note F2, quarter note E2, quarter note D2, quarter note C2, quarter note B1, quarter note A1. Measure 6: Treble clef has a quarter note G4, quarter note A4, quarter note Bb4, quarter note C5, quarter note Bb4, quarter note A4, quarter note G4. Bass clef has a quarter note G2, quarter note F2, quarter note E2, quarter note D2, quarter note C2, quarter note B1, quarter note A1. Measure 7: Treble clef has a quarter note G4, quarter note A4, quarter note Bb4, quarter note C5, quarter note Bb4, quarter note A4, quarter note G4. Bass clef has a quarter note G2, quarter note F2, quarter note E2, quarter note D2, quarter note C2, quarter note B1, quarter note A1. Measure 8: Treble clef has a quarter note G4, quarter note A4, quarter note Bb4, quarter note C5, quarter note Bb4, quarter note A4, quarter note G4. Bass clef has a quarter note G2, quarter note F2, quarter note E2, quarter note D2, quarter note C2, quarter note B1, quarter note A1.

Measures 9-12 of the score. Measure 9: Treble clef has a quarter note G4, quarter note A4, quarter note Bb4, quarter note C5, quarter note Bb4, quarter note A4, quarter note G4. Bass clef has a quarter note G2, quarter note F2, quarter note E2, quarter note D2, quarter note C2, quarter note B1, quarter note A1. Measure 10: Treble clef has a quarter note G4, quarter note A4, quarter note Bb4, quarter note C5, quarter note Bb4, quarter note A4, quarter note G4. Bass clef has a quarter note G2, quarter note F2, quarter note E2, quarter note D2, quarter note C2, quarter note B1, quarter note A1. Measure 11: Treble clef has a quarter note G4, quarter note A4, quarter note Bb4, quarter note C5, quarter note Bb4, quarter note A4, quarter note G4. Bass clef has a quarter note G2, quarter note F2, quarter note E2, quarter note D2, quarter note C2, quarter note B1, quarter note A1. Measure 12: Treble clef has a quarter note G4, quarter note A4, quarter note Bb4, quarter note C5, quarter note Bb4, quarter note A4, quarter note G4. Bass clef has a quarter note G2, quarter note F2, quarter note E2, quarter note D2, quarter note C2, quarter note B1, quarter note A1.

« Making and Using a Pcset Network for Stockhausen's Klavierstück III »

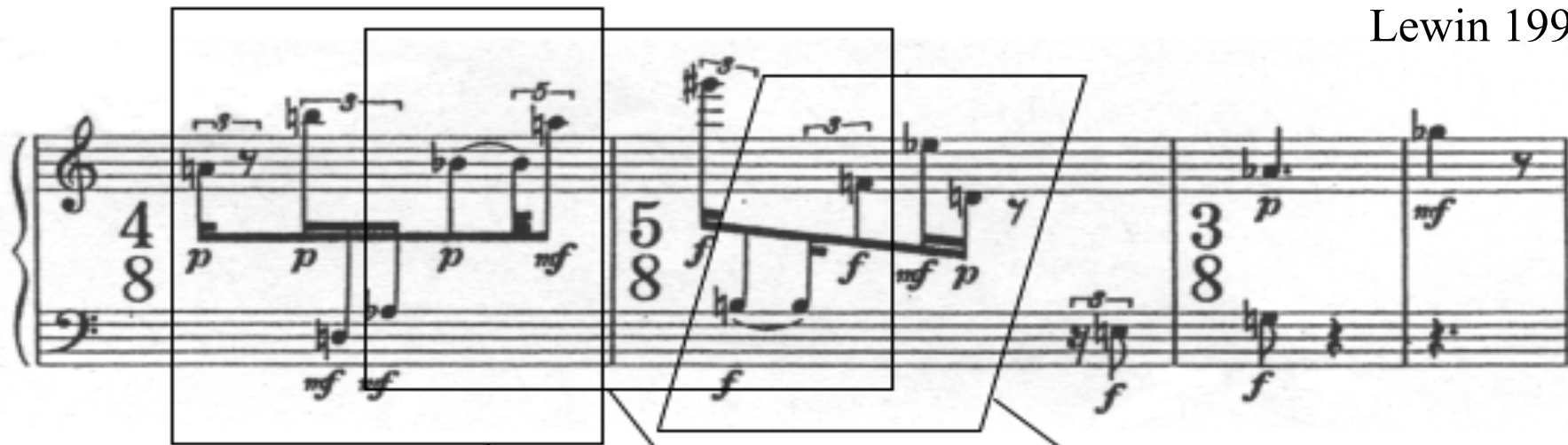
The image shows a musical score for Stockhausen's Klavierstück III. The score is in 4/8, 5/8, and 3/8 time signatures. It features various dynamics such as *p*, *mf*, *f*, and *mf*. Three colored boxes (red, green, and blue) highlight specific passages in the score. Red arrows point from these boxes to three circular diagrams below, each containing the number 12.



« The most ‘theoretical’ of the four essays, it focuses on the forms of one pentachord reasonably ubiquitous in the piece. A special **group of transformations** is developed, one suggested by the musical interrelations of the pentachord forms. Using that group, the essay arranges **all pentachord forms** of the music into a **spatial configuration** that illustrates network structure, for this particular phenomenon, over the entire piece. »

« *Making and Using a Pcset Network for Stockhausen's Klavierstück III* »

Lewin 1993



SI: (1, 1, 1, 3, 6)

(6, 3, 1, 1, 1)

(6, 3, 1, 1, 1)

IFUNC: [5 3 2 2 1 1 1 1 2 2 3]

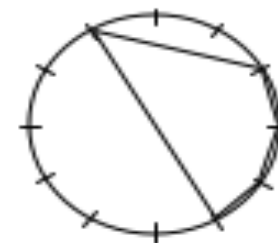
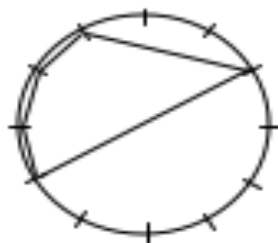
[5 3 2 2 1 1 1 1 2 2 3]

[5 3 2 2 1 1 1 1 2 2 3]

VI: [3 2 2 1 1 1]

[3 2 2 1 1 1]

[3 2 2 1 1 1]



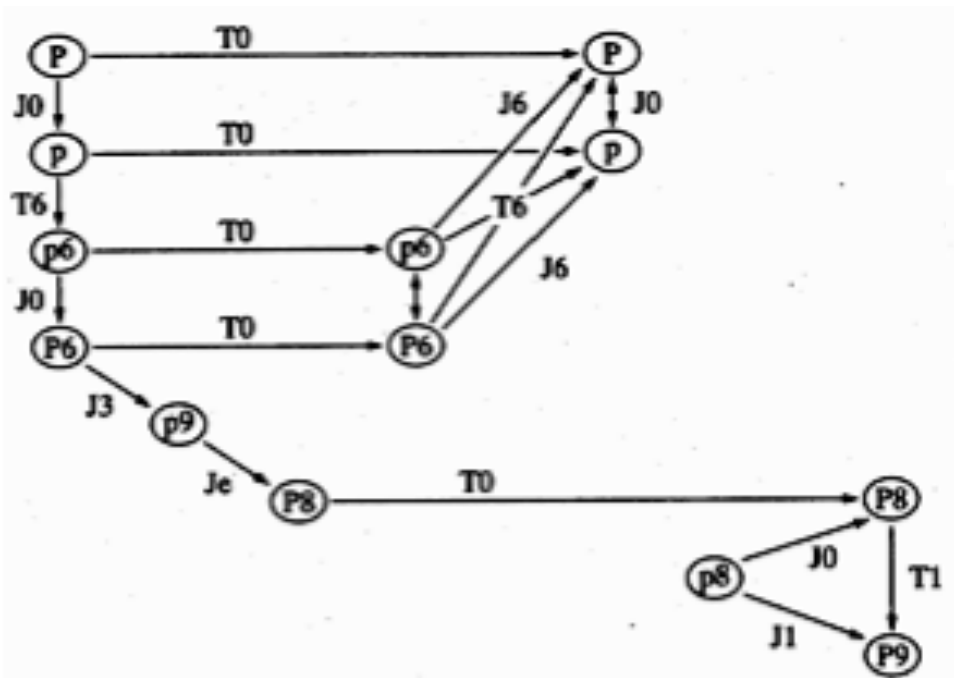
Segmentation par « imbrication »: progression transformationnelle

Stockhausen: *Klavierstück III* (Analisi di D. Lewin)

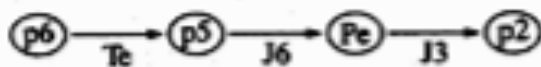
Musical score for Klavierstück III, measures 1-3. The score is in 4/8, 5/8, and 3/8 time signatures. The first measure is in 4/8 with dynamics *p p p mf*. The second measure is in 5/8 with dynamics *f f mf p*. The third measure is in 3/8 with dynamics *p mf*. The score includes various ornaments and articulations.

Diagram illustrating transformational progression through imbrication. The top part shows a musical score with overlapping boxes highlighting specific segments. Arrows point from these boxes to a sequence of five circular diagrams below, each representing a transformational stage. The diagrams are labeled T_7I , T_6 , T_7I , $T_{10}I$, and $T_{10}I$. The diagrams show the evolution of a set of points (notes) on a circle, with arrows indicating the progression from one stage to the next.

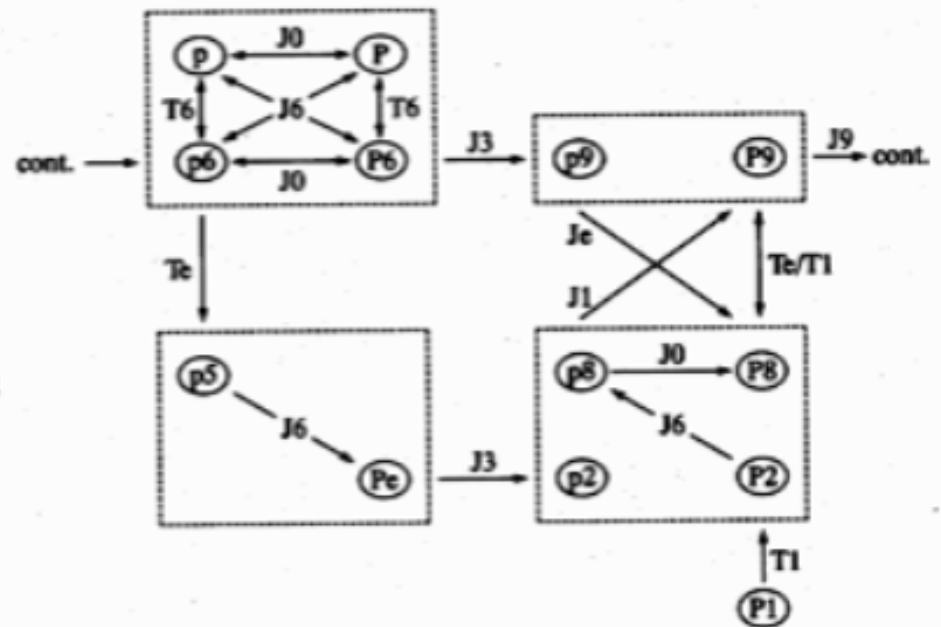
Progression transformationnelle vs réseau transformationnel



...and so on, ending with



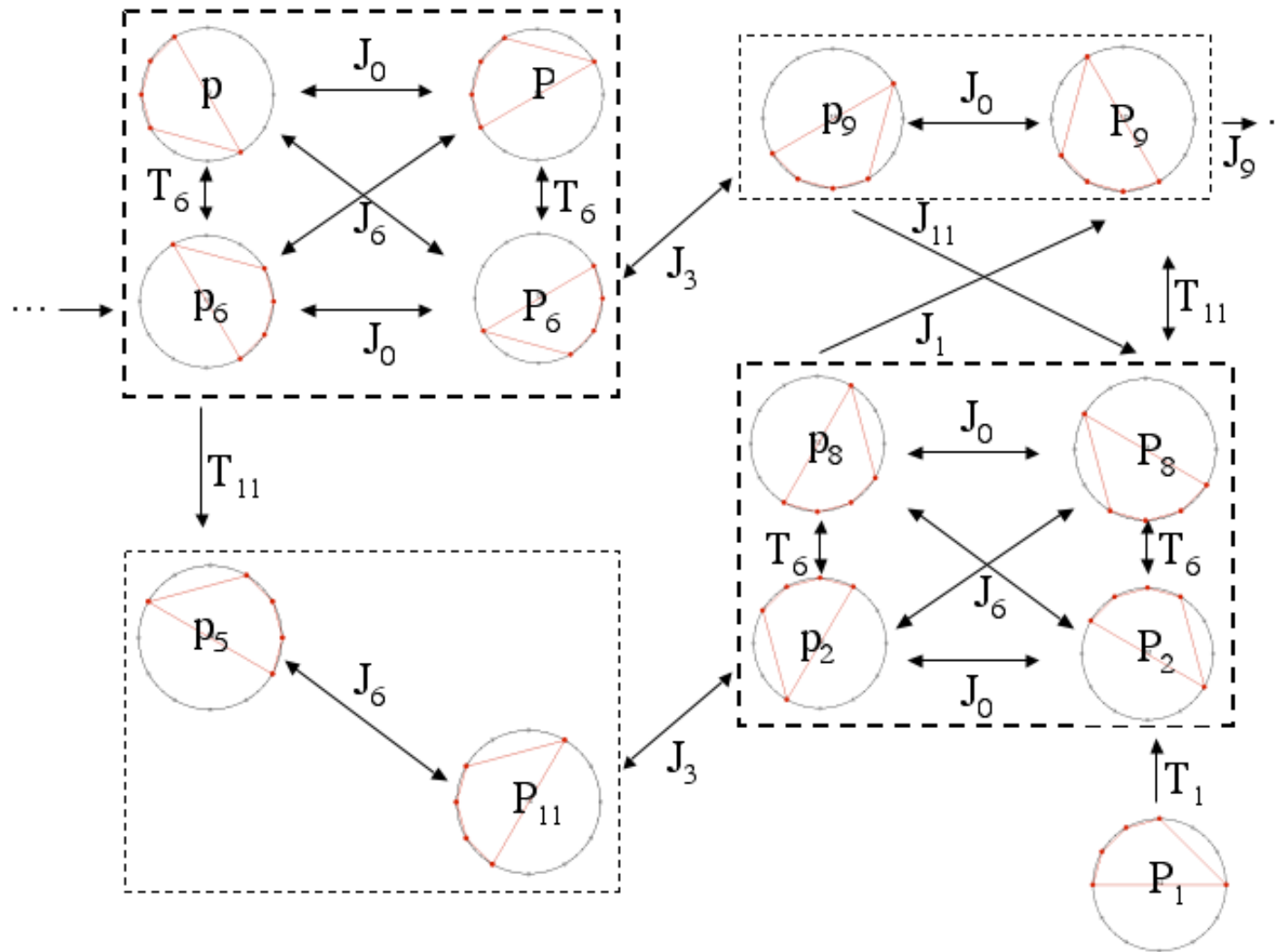
Example 2.4. A network whose left-to-right layout reflects the chronological progress of the piece through P/p forms.



« Rather than asserting a network that follows pentachord relations one at a time, according to the chronology of the piece, I shall assert instead a network that displays all the pentachord forms used and all their potentially functional interrelationships, in a very compactly organized little spatial configuration. »

Reseau transformationnel

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

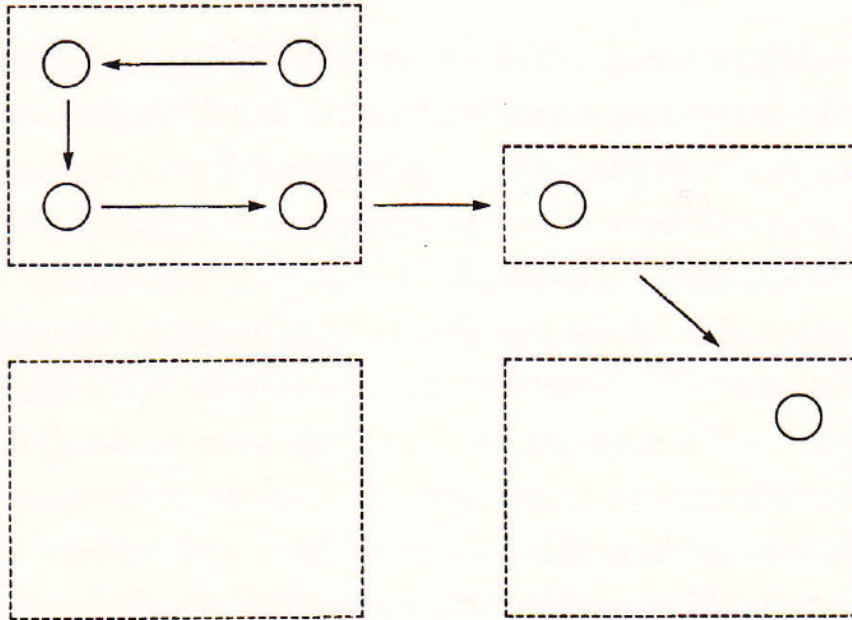


« [...] the sequence of events moves within a clearly defined world of possible relationships, and because - in so moving - it makes the abstract space of such a world accessible to our sensibilities. That is to say that the story projects what one would traditionally call form. »

Parcours multiples d'écoute dans un réseau transformationnel

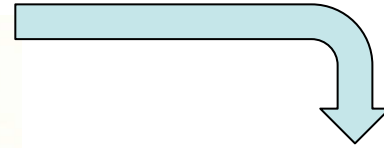
Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

Pass 1 (mm. 1-5).

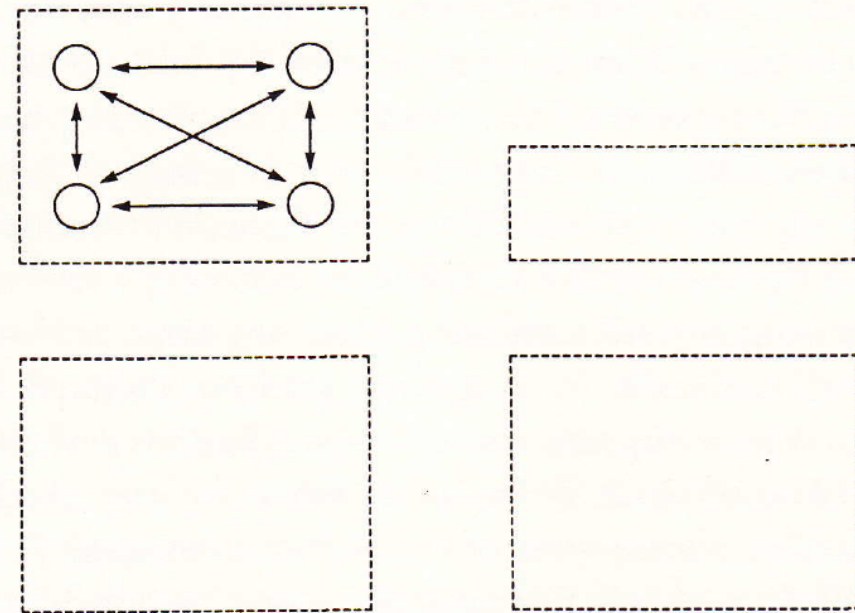


a

horizontal arrows within boxes = J0; between boxes = J3 or J9
 vertical arrows within boxes = T6; between boxes = Te or T1
 diagonal arrows within boxes = J6; between boxes = Je or J1

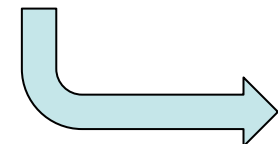


Pass 2 (mm. 5-8) goes back and elaborates the beginning area of pass 1.



b

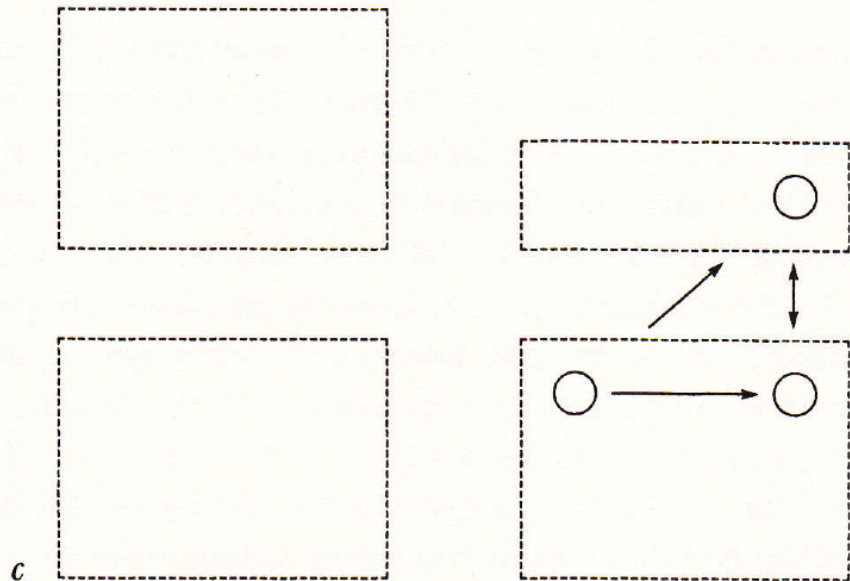
horizontal arrows within boxes = J0; between boxes = J3 or J9
 vertical arrows within boxes = T6; between boxes = Te or T1
 diagonal arrows within boxes = J6; between boxes = Je or J1



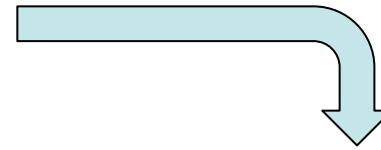
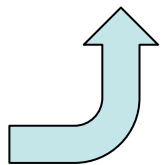
Parcours multiples d'écoute dans un réseau transformationnel

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

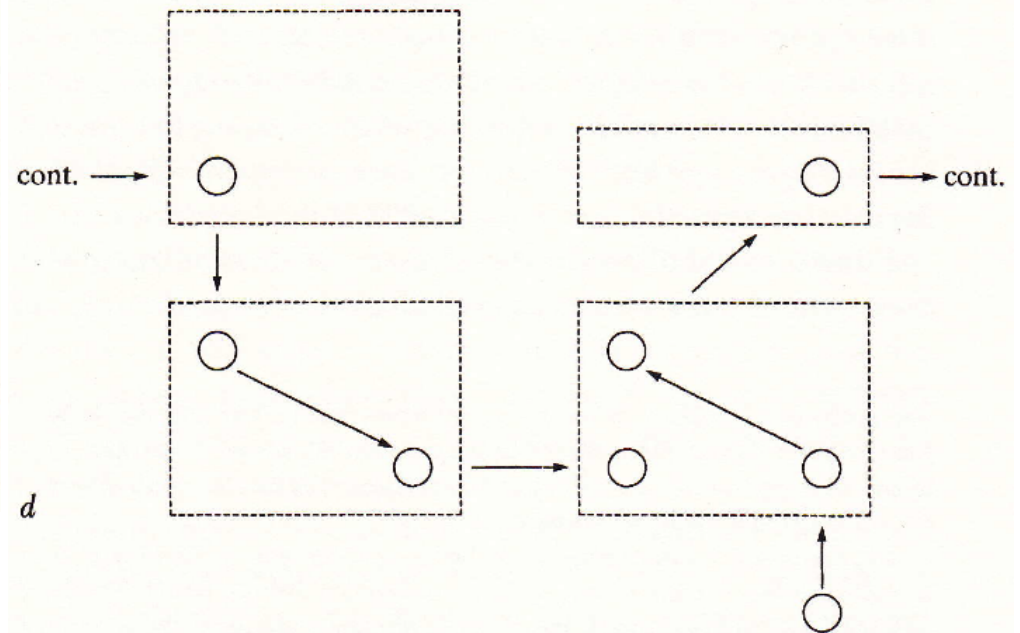
Pass 3 (mm. 8-10) picks up and elaborates the ending area of pass 1.



horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1



Pass 4 (mm. 9-16) expands the p8 + P8 area of pass 3 to activate P2 and p2 as well. P2 is the "essential" incipit of pass 4; p2 is the end of the pass, and of the piece.



horizontal arrows within boxes = J0; between boxes = J3 or J9
vertical arrows within boxes = T6; between boxes = Te or T1
diagonal arrows within boxes = J6; between boxes = Je or J1

Exercices d'écoute

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

m. 1 1-2 2 2-3 2-5 2-5

P0 p0 p6 P6 p9 P8

m. 5-7 5-7 5-7 5-7 8-10 8-10 8-10

P6 p6 P0 p0 p8 P8 P9

m. 9-11 10-11 11-12 11-12 11-13 12-13 13-14 13-15

P1 P2 p8 P9 p6 p5 Pe p2

Example 2.7. An ear-training aid for listening to P/p forms and their inter-relations.

« I doubt that [Nicholas] Cook [cf. *A Guide to Music Analysis*, 1987] would have much patience with my network analysis; I suspect he would read it as yet one more exercise in what he calls “cracking the code.” Let me be the first to say emphatically that the network analysis is very far from an analysis of the piece, that I find it problematical, and that it took some effort for me to develop the aural agenda of [the ear-training exercises in] example 2.7. »

Exercices d'écoute

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

The image displays three systems of musical notation for Stockhausen's *Klavierstück III*. Each system consists of a treble and bass staff with a figured bass line below. The first system (measures 1-6) has figures: P0, p0, p6, P6, p9, P8. The second system (measures 7-13) has figures: P6, p6, P0, p0, p8, P8, P9. The third system (measures 14-20) has figures: P1, P2, p8, P9, p6, p5, Pe, p2. Measure numbers are indicated above the staves.

Example 2.7. An ear-training aid for listening to P/p forms and their inter-relations.

« However, I must say that I enjoyed developing that agenda, which of course I did gradually as my work developed, and not in so neatly packaged a way as in this essay. I felt I was getting at something in the piece that very much involved “what the music did to me,” if only in one of its aspects. I felt I was responding in some measure to a strong sense of challenge I felt about the piece. No matter to what degree I am deluding myself, I miss in Cook the sense of having to extend my ear in response to a sense of challenge ».

Computer-Aided Transformational Analysis in OpenMusic

mm. 1-2

mm. 1-2

$J = T7I$
 $p \implies p$

Musical notation for measures 1-2, showing a treble clef staff with a key signature of one sharp (F#) and a common time signature. The notes are: m. 1: F#4, G4, A4, B4; m. 2: C5, B4, A4, G4.

mm. 2-3

mm. 2-3

$T6$
 $p6 \implies P6$

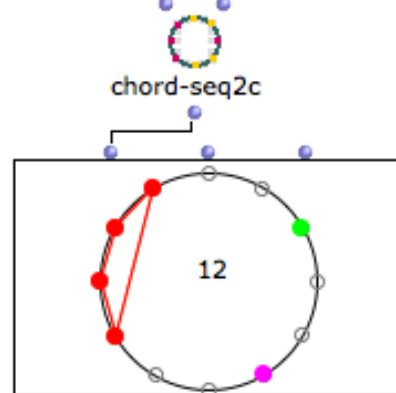
Musical notation for measures 2-3, showing a treble clef staff with a key signature of one sharp (F#) and a common time signature. The notes are: m. 2: F#4, G4, A4, B4; m. 3: C5, B4, A4, G4.

mm. 5-7

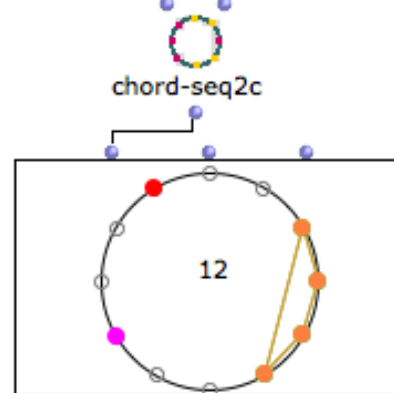
mm. 5-7

J
 $P6 \implies p6$

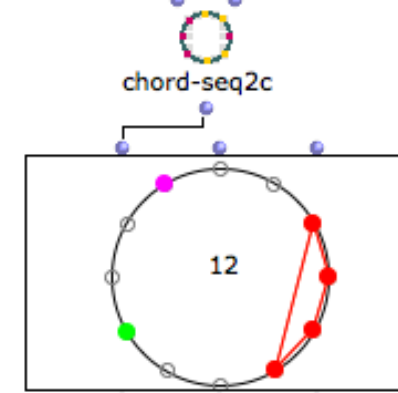
Musical notation for measures 5-7, showing a treble clef staff with a key signature of one sharp (F#) and a common time signature. The notes are: m. 5: F#4, G4, A4, B4; m. 6: C5, B4, A4, G4; m. 7: F#4, G4, A4, B4.



Calcule



Calcule



Calcule

➔ <http://recherche.ircam.fr/equipes/repmus/OpenMusic/>

Visualisations multimédia de l'analyse transformationnelle

R. Attas : Metaphors in Motion: Agents and Representation in Transformational Analysis, *MTO*, 15(1), 2009
<http://mto.societymusictheory.org/issues/mto.09.15.1/mto.09.15.1.attas.html>

Animation 1. Klavierstück III

MAP

p/589te P/89te2 p9/25678 P9/5678t
p6/e2345 P6/23458

p5/25678 Pe/789t1 p8/14567 P8/4567t
p2/7te01 P2/te014

day

2

Klavierstück III

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Visualisations multimédia de l'analyse transformationnelle

R. Attas : Metaphors in Motion: Agents and Representation in Transformational Analysis, *MTO*, 15(1), 2009
<http://mto.societymusictheory.org/issues/mto.09.15.1/mto.09.15.1.attas.html>

Animation 2. Grow Your Own Pentachord

The interface features a central 3D illustration of a brown pot containing soil and several white flowers with yellow centers. To the left, there are two rows of glowing wands: 'T WANDS' (yellow) labeled T1, T6, Te and 'J WANDS' (blue) labeled J0, J1, J3, J6, J9, Je. In the top left, a 'PLAYBACK SPEED' control is shown with a progress bar. In the top right, a 'PENTACHORD MAP' displays a grid with nodes (p, P, T6, J6, J0, P6, J3, p9, p5, p8) and arrows, with a red dot indicating 'You are here'. Below the map is a large 'P' symbol with '89te2' and 'CURRENT PENTACHORD' text. At the bottom right, there are buttons for 'INSTRUCTIONS', 'POT LAYOUT', and 'GUIDE'. The bottom of the screen has a black banner with the text 'grow your own pentachord!' and a footer: 'This website is best viewed in 1024 x 768 resolution. GROW YOUR OWN PENTACHORD! Copyright © 2007-2008 UBC School of Music. All Rights Reserved.'

Vers une modélisation informatique de l'analyse transformationnelle

YunKang Ahn, L'analyse musicale computationnelle, thèse, Université de Paris VI / Ircam, déc 2009

The diagram illustrates the process of transforming musical analysis into a computational model. It features piano score excerpts on the left and right, a central geometric diagram with labels (a, b, c, d), and two portraits of YunKang Ahn and a man with a chalkboard.

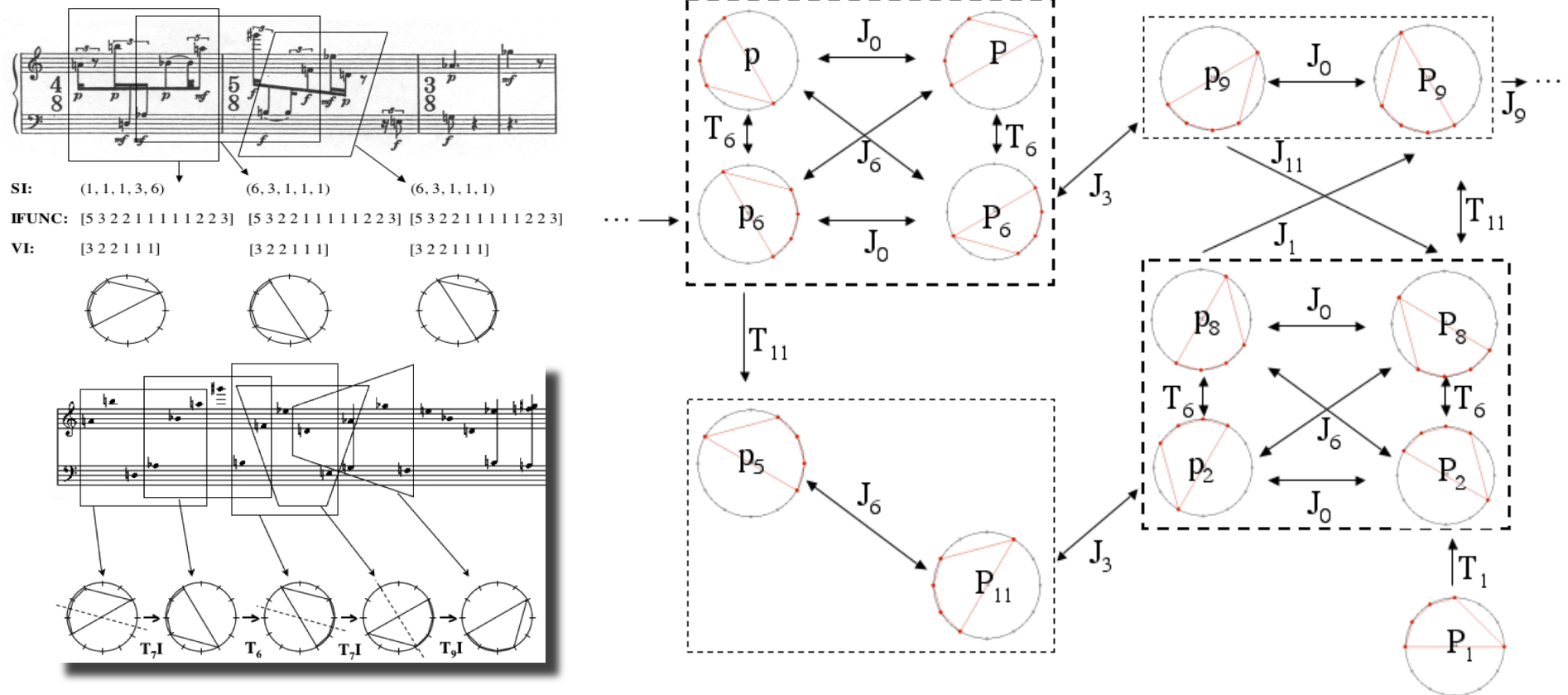
The piano score excerpts show musical notation with various annotations, including dynamic markings (p, f, ff) and time signatures (4/8, 3/8). The left side shows the original score with some annotations, while the right side shows the score with more extensive annotations, including red and blue boxes highlighting specific sections.

The central diagram consists of several parts:

- a) A set of points representing musical data.
- b) A network of interconnected nodes representing relationships between musical elements.
- c) A series of blue triangles representing a geometric model of the musical structure.
- d) A detailed geometric model with vertices labeled A_1, B, C, H and edges labeled $K-U-B, U-A_1, U-B_1, U-A_1, G, B, C, K-G, H$.

Two portraits are included: the top one is YunKang Ahn, and the bottom one is a man with a beard and a red tie, likely a professor or researcher involved in the project. The bottom portrait is positioned in front of a chalkboard with mathematical notations, including E , ED , $\text{Simple } (\mathbb{R}^2)$, \mathbb{Z}_n , and \mathbb{R}^3 .

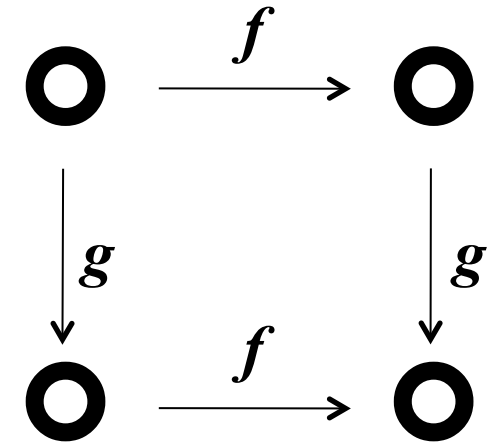
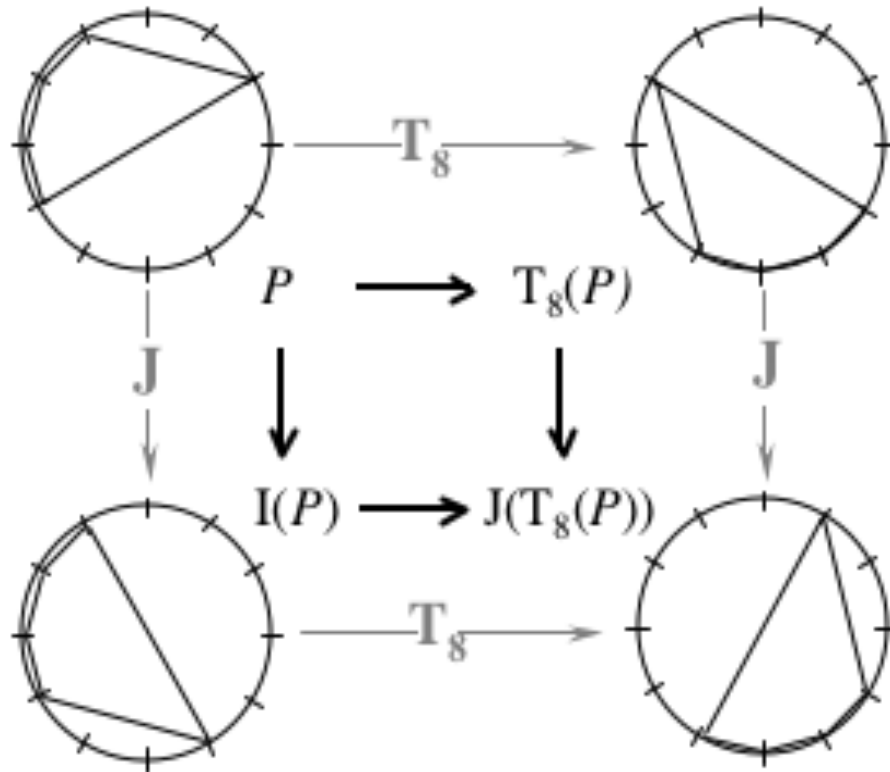
Progression transformationnelle vs réseau transformationnel



« A rational reconstruction of a work or works, which is a theory of the work or works, is an explanation not, assuredly, of the 'actual' process of construction, but of how the work or works may be construed by a hearer, how the 'given' may be 'taken' »

M. Babbitt : « Contemporary Music Composition and Music Theory as Contemporary Intellectual History », 1972

Inversions « contextuelles » et commutativité



Tout diagramme commute

$$\forall f, g \in \langle T, J \rangle$$

Le groupe des 24 transformations $\sigma = \{T_0, T_1, \dots, T_{11}, T_0J, T_1J, \dots, T_{11}J\}$ est commutatif et opère de manière simplement transitive sur l'espace S des 24 formes du pentacorde de base (i.e. l'ensemble de ses 12 transpositions et de ses 12 inversions)

$\Rightarrow (S, \sigma, \text{int})$ est un GIS

Autres implications philosophiques de l'approche transformationnelle

1. Les groupes comme structures sous-jacentes des GIS

The nature of a given geometry is [...] defined by the *reference* to a determinate **group** and the way in which spatial forms are related within that type of geometry. [Cf. *Felix Klein Erlangen Program - 1872*][...] We may raise the question whether there are any concepts and principles that are, although in different ways and different degrees of distinctness, necessary conditions for both the *constitution* of the **perceptual world** and the construction of the universe of geometrical thought. It seems to me that the concept of **group** and the concept of **invariance** are such principles.

E. Cassirer, "The concept of group and the theory of perception", 1944



Felix Klein



Ernst Cassirer

Autres implications philosophiques de l'approche transformationnelle

2. L'action de groupe comme « paradigme » de la dualité objectale/opératoire

« [C'est la notion de groupe qui] donne un sens précis à l'idée de structure d'un ensemble [et] permet de déterminer les éléments efficaces des transformations en réduisant en quelque sorte à son schéma opératoire le domaine envisagé. [...] L'objet véritable de la science est le **système des relations** et non pas les termes supposés qu'il relie. [...] Intégrer les résultats - symbolisés - d'une **expérience** nouvelle revient [...] à créer un canevas nouveau, un **groupe de transformations plus complexe et plus compréhensif** »

G.-G. Granger : « Pygmalion. Réflexions sur la pensée formelle », 1947



Felix Klein



Ernst Cassirer



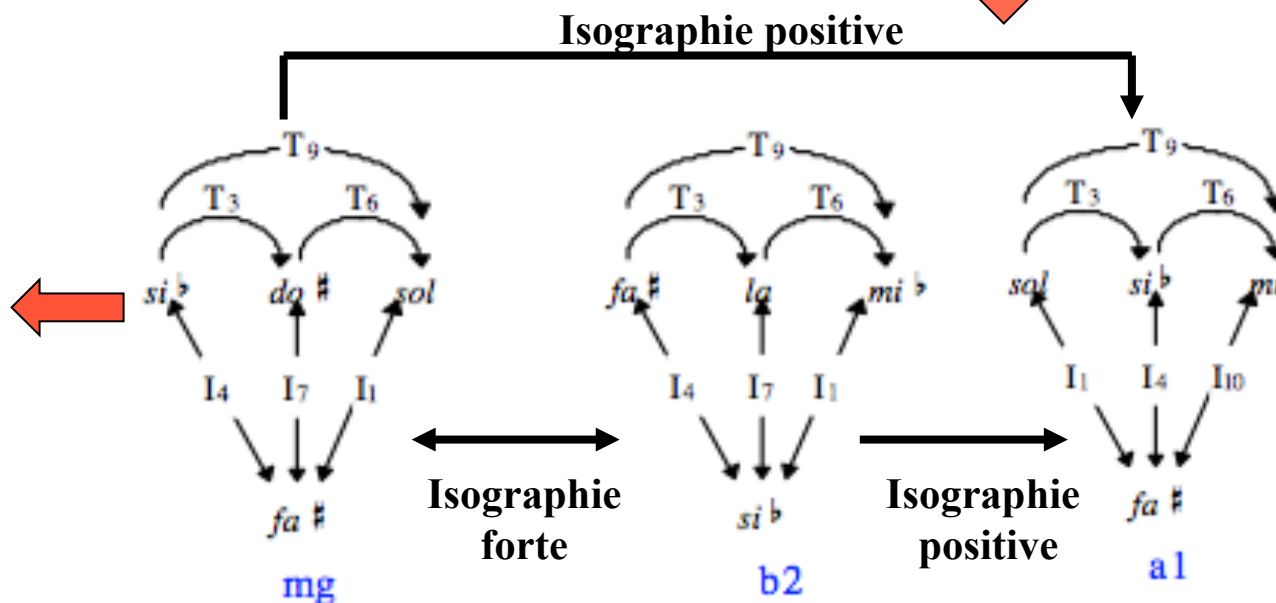
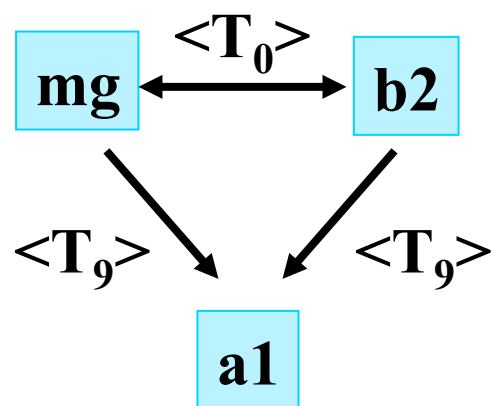
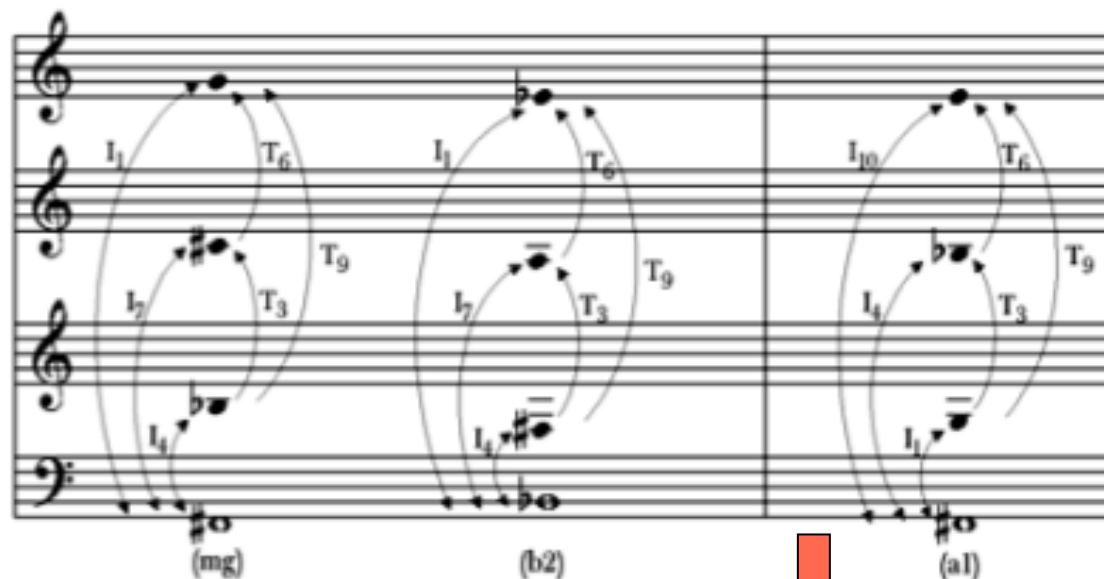
Gilles-Gaston Granger

Klumpenhouver Networks (K-réseaux)

Xavier Hascher: « Liszt et les sources de la notion d'agrégat », *Analyse Musicale*, 43, 2002



Ex. 1 - « Ladislaus Teleki » (*Historische ungarische Bildnisse* n° 4), mes. 1-7
Les agrégats dans la classification de Forte

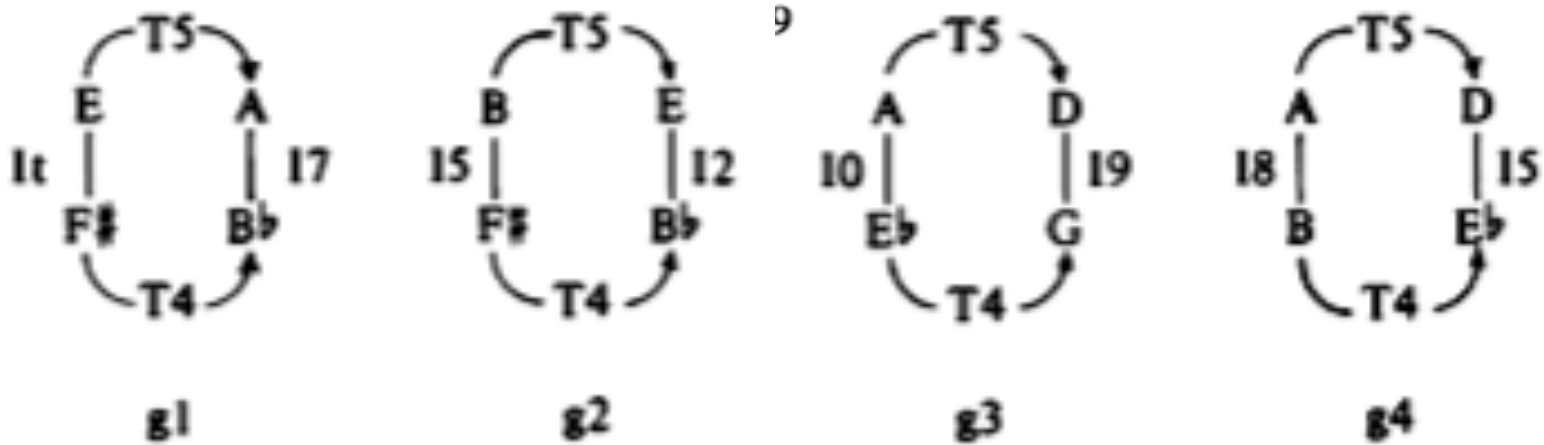


Clumpenhouwer Networks (K-réseaux)

$$\langle T_k \rangle : T_m \rightarrow T_m$$

$$I_m \rightarrow I_{k+m}$$

David Lewin: «A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », JMT, 1994



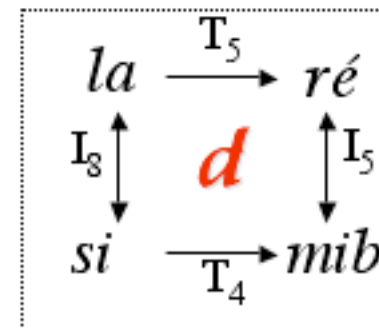
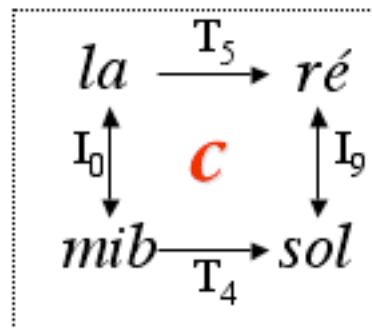
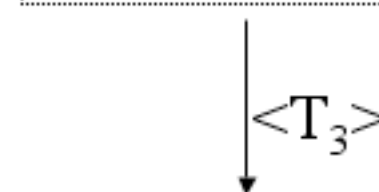
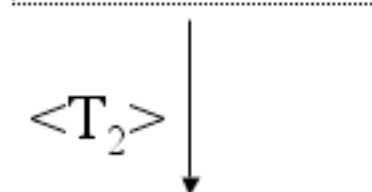
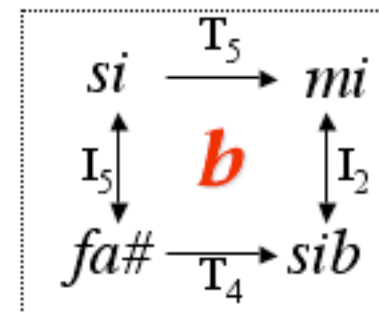
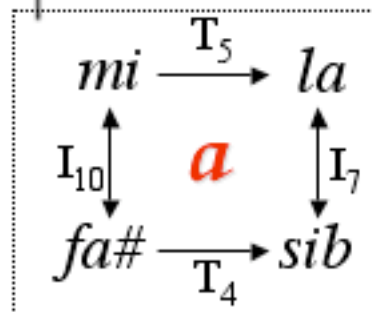
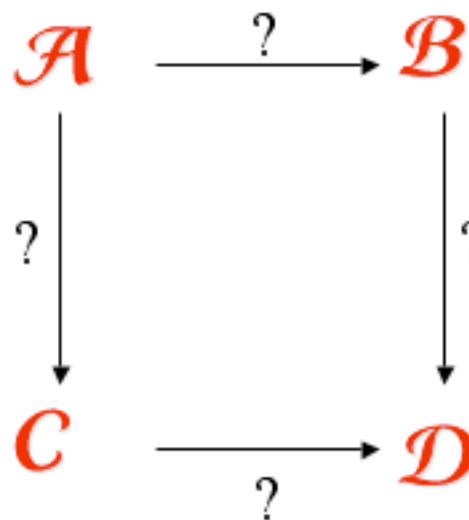
Isographie positive



Isographie positive

Klumpenhower Networks (K-réseaux) : recursivité

David Lewin: «A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », JMT, 1994



\mathcal{A}

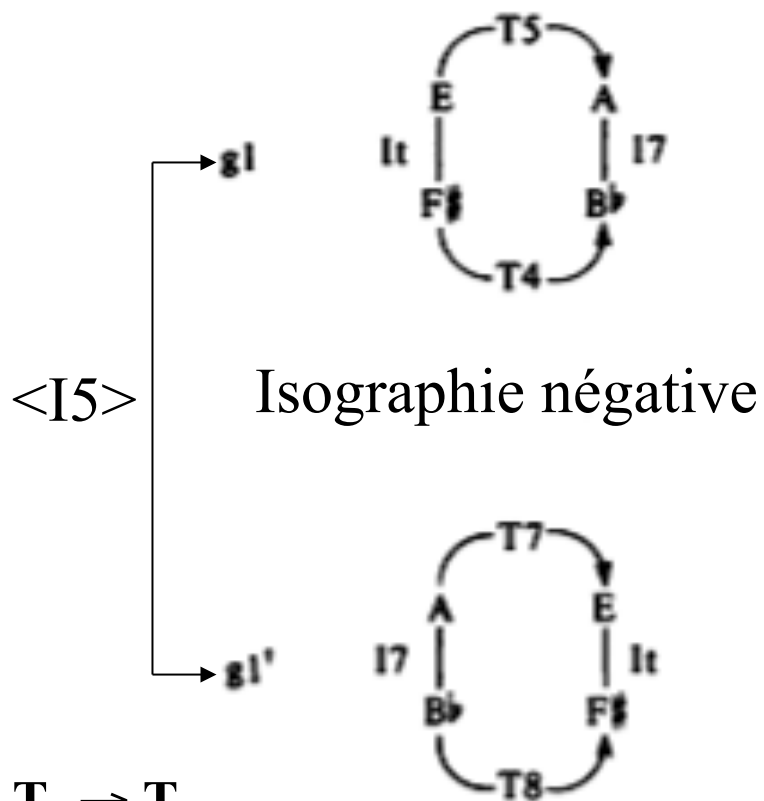


Clumpenhower Networks (K-nets)

David Lewin: «A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », JMT, 1994

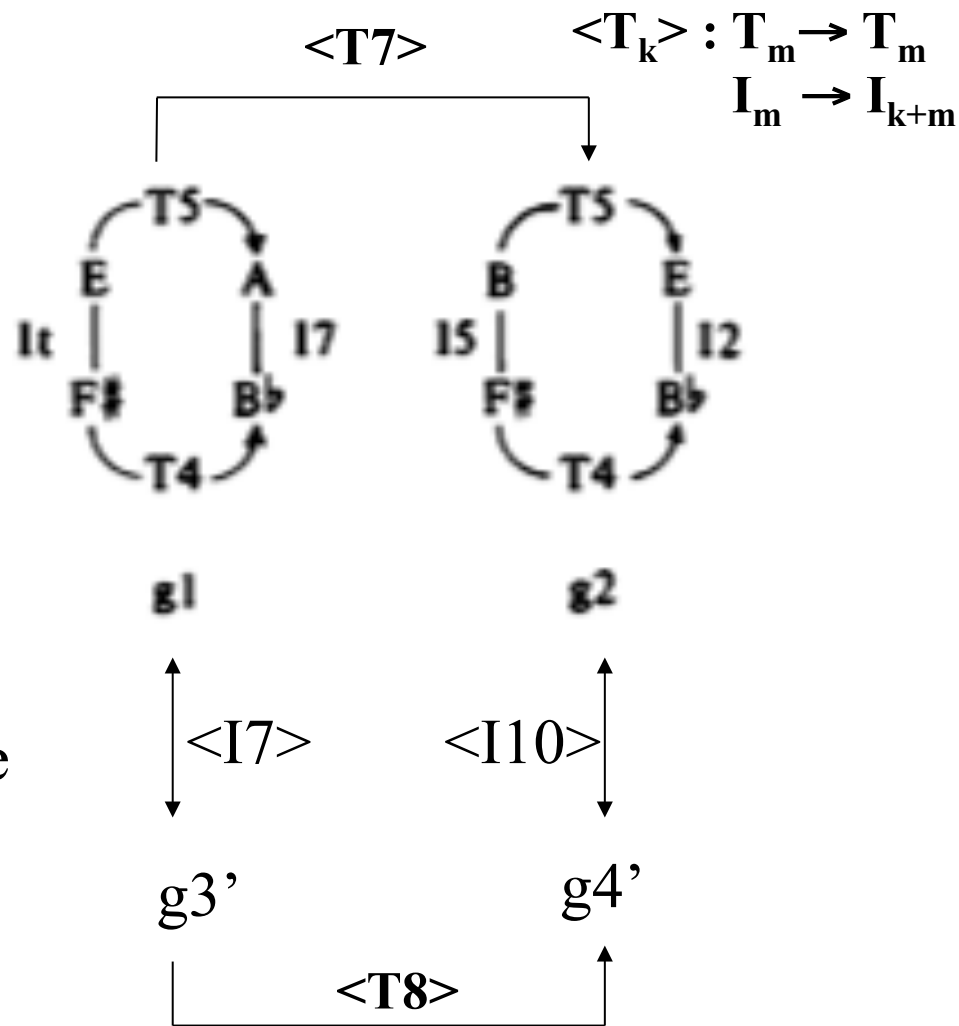


Example 9

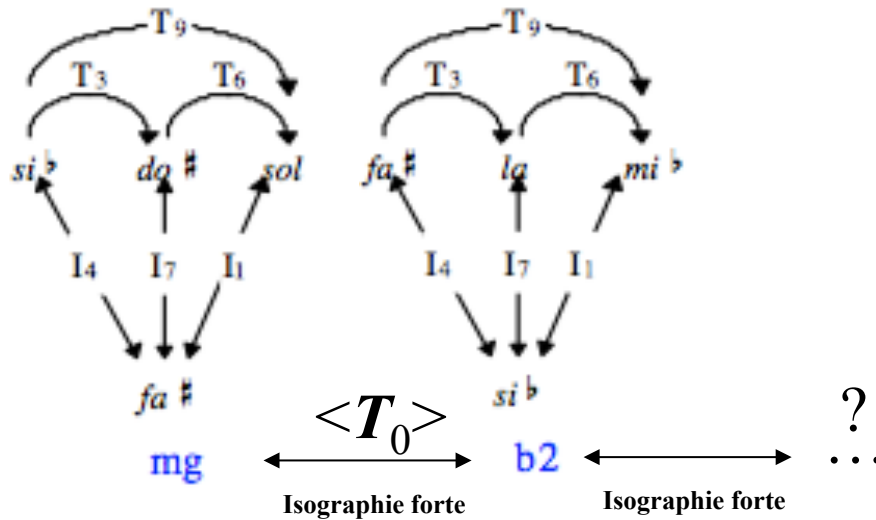


$$\langle I_k \rangle : T_m \rightarrow T_{-m}$$

$$I_m \rightarrow I_{k-m}$$



Énumération des K-nets en relation d'isographie forte



$$\begin{array}{ccccc}
 x & \xrightarrow{T_3} & x+3 & \xrightarrow{T_6} & x+9 \\
 \swarrow I_4 & & \uparrow I_7 & & \searrow I_1 \\
 & & 4-x & = & 7-(x+3) = 1-(x+9)
 \end{array}$$

\Rightarrow 12 solutions

$$\begin{array}{ccc}
 re \xrightarrow{T_4} fa^\# & & x \xrightarrow{T_4} x+4 \\
 M_5 \downarrow & \longleftrightarrow \text{Isographie forte} & M_5 \downarrow \\
 sib \xrightarrow{T_6 I} sol^\# & & 5x \xrightarrow{T_6 I} 6-5x=2-(x+4) \implies 8=4x \implies x=2, 5, 8, 11 \\
 & & \Rightarrow 4 \text{ solutions}
 \end{array}$$

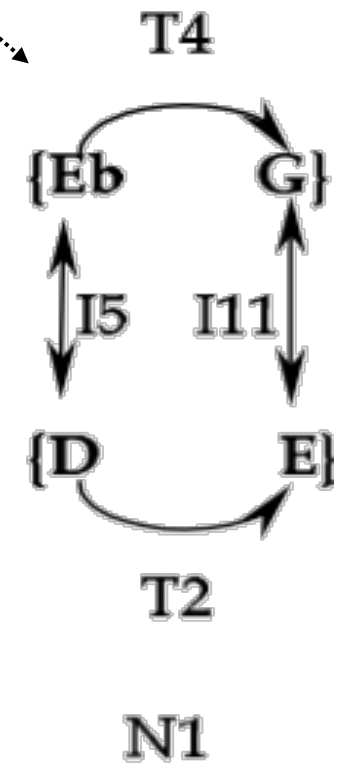
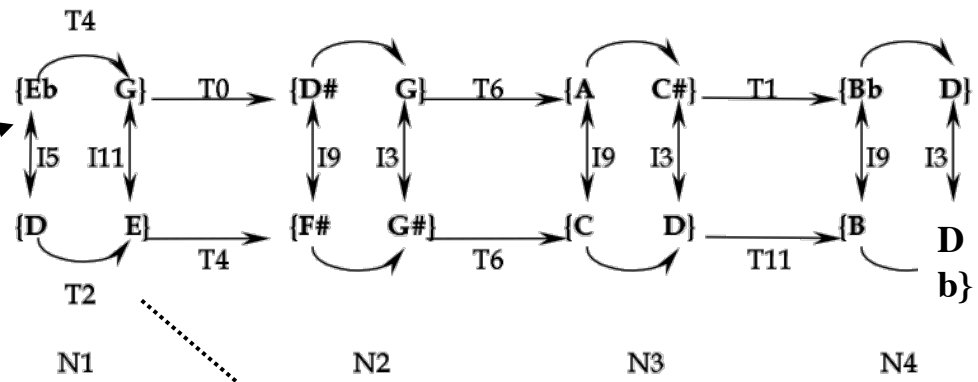
$$\begin{array}{ccc}
 re^\# \xrightarrow{M_3} la & & x \xrightarrow{M_1} x \\
 M_1 \downarrow & \longleftrightarrow \text{Isographie forte} & M_1 \downarrow \\
 re^\# \xrightarrow{M_{11}} la & & x \xrightarrow{M_{11}} 11x=7x \implies 4x=0 \implies x=0, 3, 6, 9 \\
 & & \Rightarrow 4 \text{ solutions}
 \end{array}$$

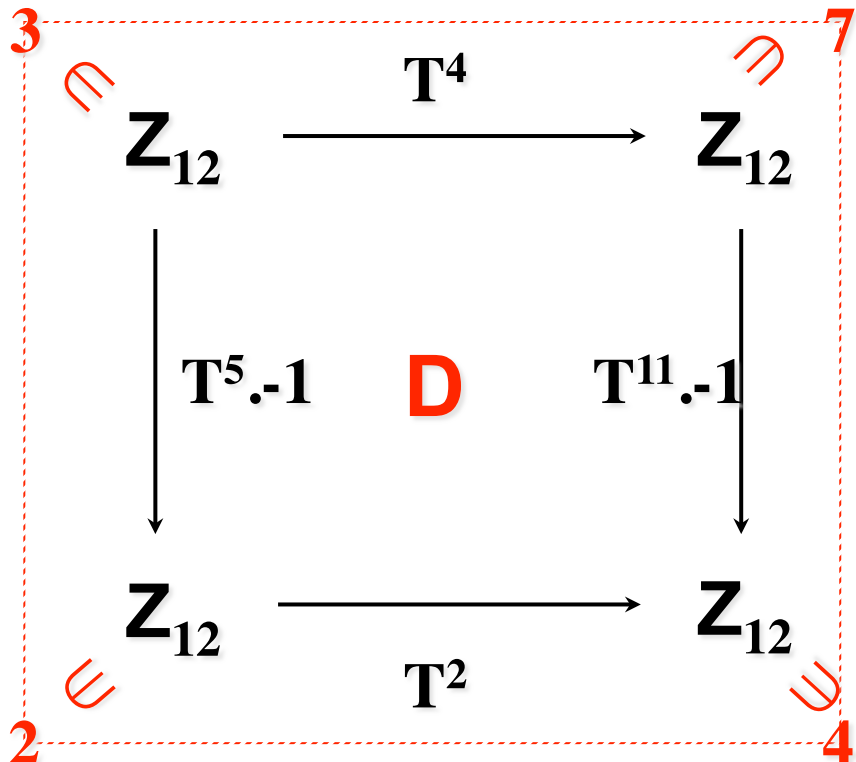
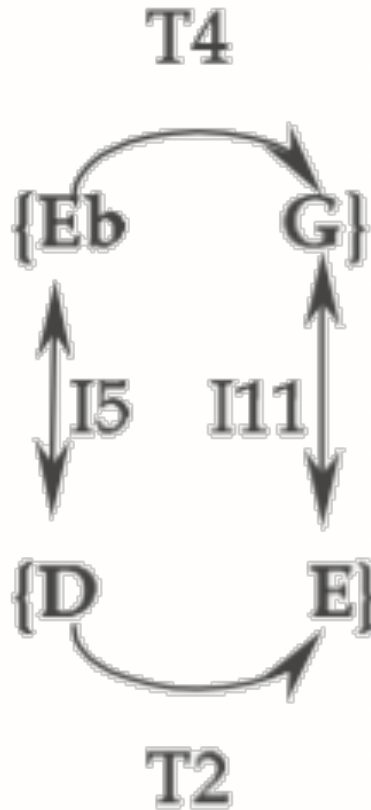
Henry Klumpenhouwer: *Deep Structure in K-net Analysis with Special Reference to Webern (Opus 16, n°4)*

J₁ [0125] J₂ [0125] J₃ [0125]
 Sehr lebhaft (♩ = ca 112) rit.
 1 2
Gesang
 As - ter - ges me, No -
Baß-Klarinette
fp

J₄ [0134] J₅ [0145] J₆ [0347] J₇ [0134]
 langsamer (♩ = ca 84) *sehr zart* tempo I. (♩ = ca 112)
 3 4
 mi - ne, hys - so - po, et mun -
sehr zart *pp* *pp* *p*
Bkl.

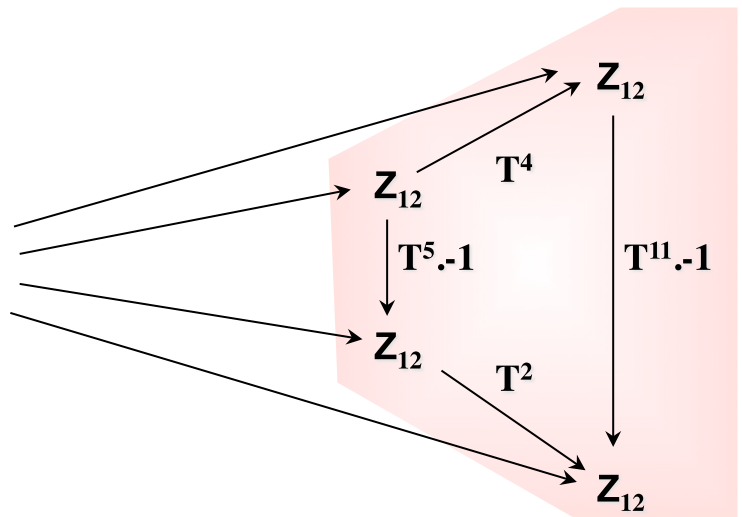
J₈ [0125]
 5 6
 da - bor: la - - va - bis me, et
fp *p*
Bkl.





N1

$(3, 7, 2, 4) \in \mathbf{lim}(\mathbf{D})$

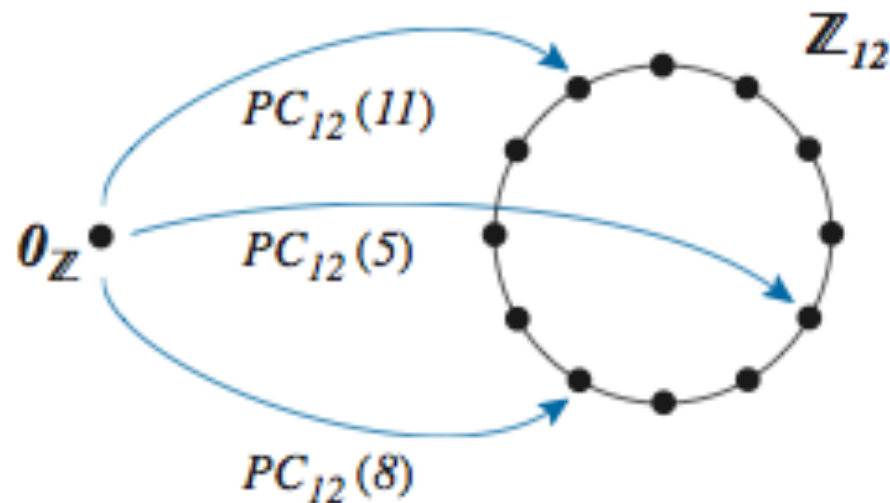


$$\mathbf{Z}_i = \mathbf{Z}_{12}$$

$$f_{ij}^t \in \mathbf{Z}_i @ \mathbf{Z}_j$$

$$0_{\mathbf{Z}} @ \mathbf{Z}_{12}$$

$$Cr = \{PC_{12}(5), PC_{12}(8), PC_{12}(11)\}$$



$$\mathbf{Z}_{12} @ \mathbf{Z}_{12}$$

$$Cr = \{[10,11], [6,1], [1,1]\}$$

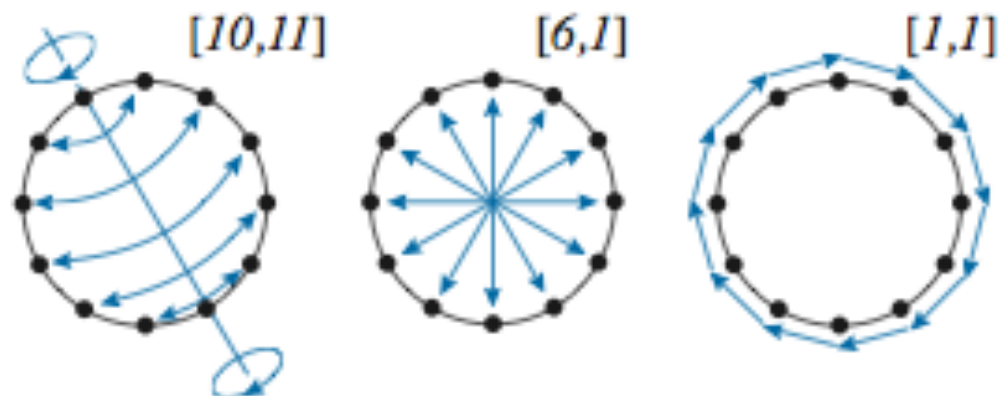


Figure 7.2: Above, a zero-addressed 12-tempered class 3-chord, below a self-addressed 12-tempered class 3-chord [ToM, p. 111].

$$\mathbf{Z}_i = \mathbf{Z}_{12}$$

$$f_{ij}^t \in \mathbf{Z}_i @ \mathbf{Z}_j$$

$\text{lim}(\mathbf{D}) =$ family of strongly-isographic networks

Z

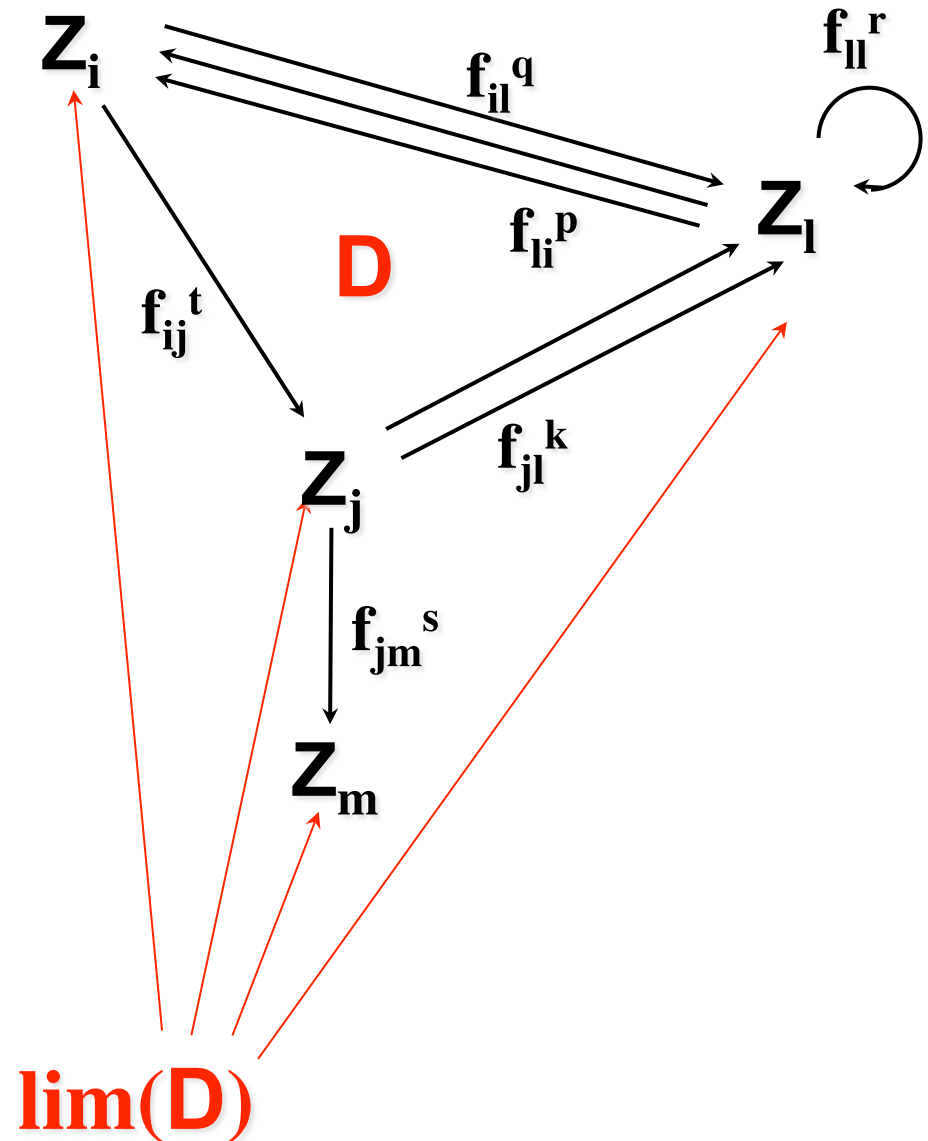
Fact:

$$\text{lim}(\mathbf{D}) \approx \mathbf{U}$$

$\mathbf{U} =$ (empty or) subgroup of $(\mathbf{Z}_{12})^n$

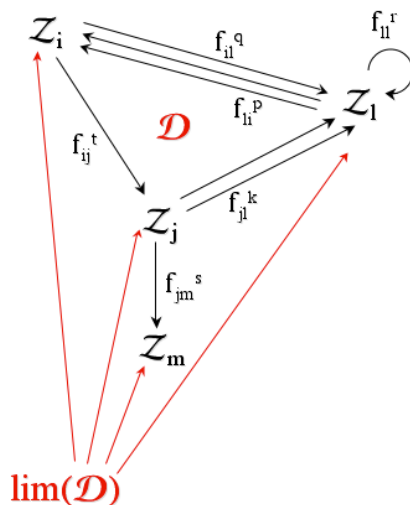
If $f_{**}^* =$ isomorphisms
 $\text{card}(\mathbf{U})$ (= 0 or) divides 12

G. Mazzola & M. Andreatta: From a Categorical Point of View: K-nets as Limit Denotators, *PNM*, 2006



$$\mathbf{Z}_i = \mathbf{Z}_{12}$$

$$\mathbf{f}_{ij}^t \in \mathbf{Z}_i @ \mathbf{Z}_j$$

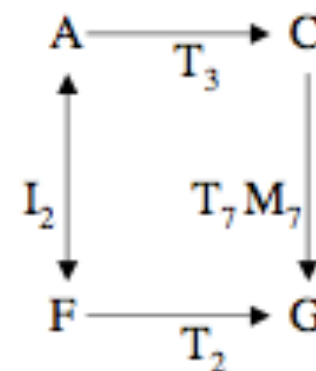
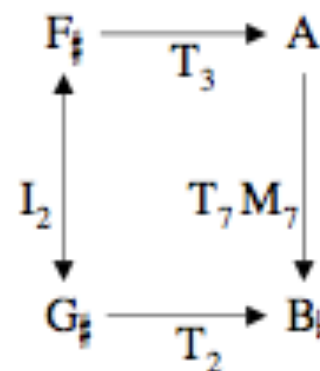
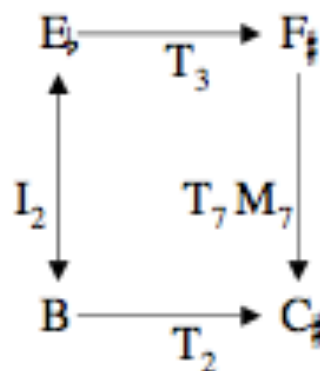
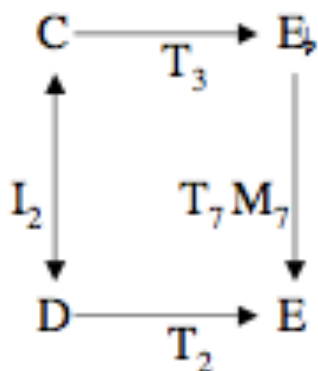


Fact:
 $\lim(\mathcal{D}) \approx U$



$U =$ (empty or)
 subgroup of $(\mathbf{Z}_{12})^n$

If $f_{**}^* =$ isomorphisms
 card (U) (= 0 or)
 divides 12



EXAMPLE 6: THE FOUR SOLUTIONS (STRONGLY ISOGRAPHIC K-NETS) OF THIS DIAGRAM ILLUSTRATE THAT THE CARDINALITY OF THE SOLUTION SET IS A DIVISOR OF 12. HERE, THE OPERATOR M_7 DENOTES THE MULTIPLICATION BY 7

Autres implications philosophiques de l'approche transformationnelle

3. Aspects cognitifs de la formalisation catégorielle

« *La théorie des catégories est une théorie des constructions mathématiques, qui est macroscopique, et procède d'étage en étage. Elle est un bel exemple d'abstraction réfléchissante, cette dernière reprenant elle-même un principe constructeur présent dès le stade sensori-moteur. Le style catégoriel qui est ainsi à l'image d'un aspect important de la genèse des facultés cognitives, est un style adéquat à la description de cette genèse* »

Jean Piaget, Gil Henriques et Edgar Ascher, *Morphismes et Catégories. Comparer et transformer*, 1990

- G. S. Halford & W. H. Wilson, "A Category Theory Approach to Cognitive Development", *Cognitive Psychology*, 12, 1980
- J. Macnamara & G. E. Reyes, *The Logical Foundation of Cognition*, OUP, 1994
- ...
- A. Ehresmann, J.-P. Vanbremerch, *Memory Evolutive Systems, Hierarchy, Emergence, Cognition*, 2007



Felix Klein



Ernst Cassirer



Gilles-Gaston Granger



Jean Piaget

Autres implications philosophiques de l'approche transformationnelle

4. Dépassement du cadre positivistico-logique

*« Because a compelling interpretation of musical perceptions is needed to turn a theoretically true statement into a meaningful statement, analytical judgment plays a central role in meaningfully reducing the scope of the theory. So although it may appear that the mathematics of Lewin's work is a language of scientific positivism, the emphasis on perceptual context and interpretation actually distances GMIT's theory from scientific theory—at least the kind of “covering law” theory often cited in connection with scientific research. A music theory for communicating perceptions and intuitions locates music in **experience** and not in **nature**. »*

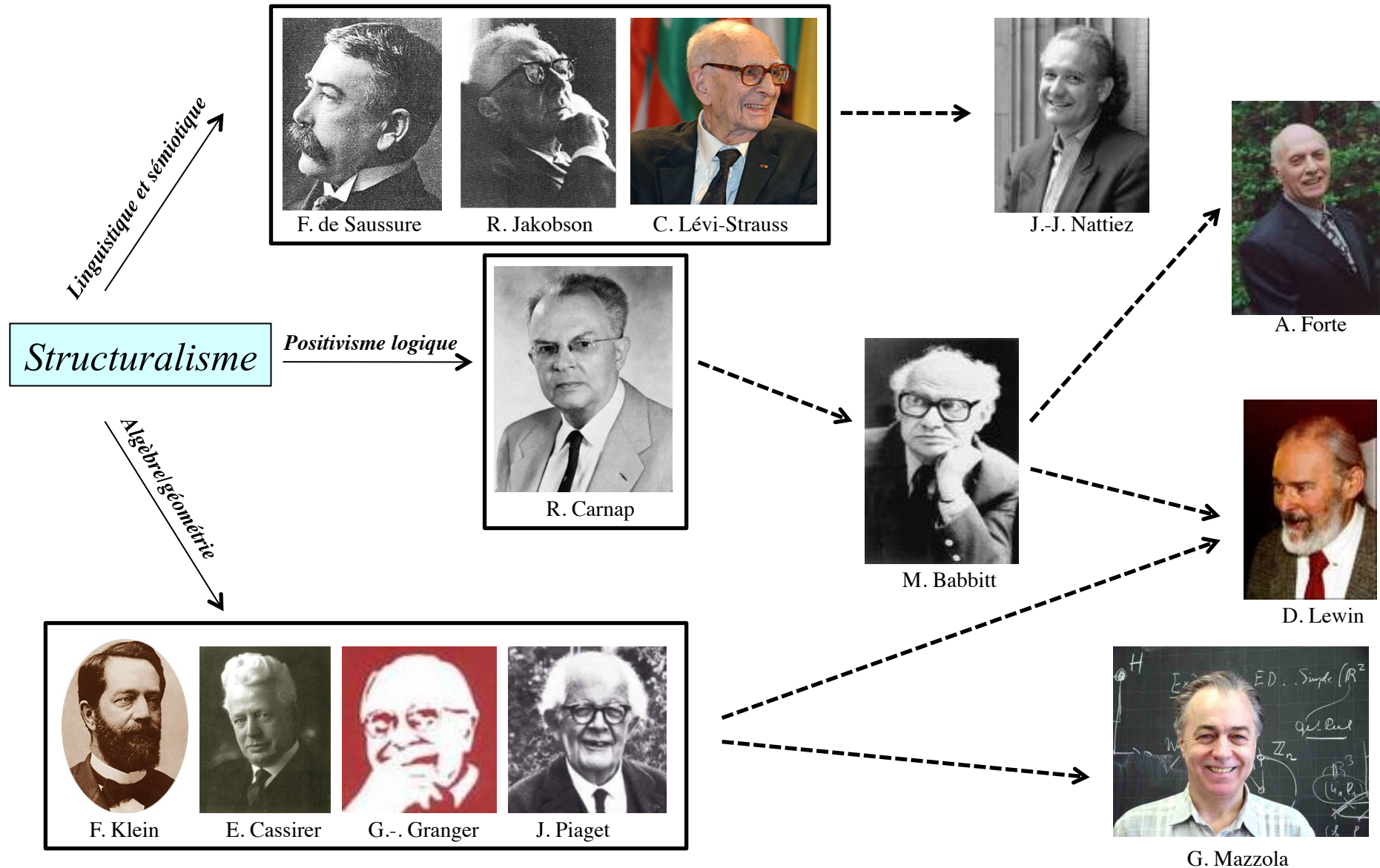
R. Satyendra, « An Informal Introduction to Some Formal Concepts from Lewin's Transformational Theory », *Journal of Music Theory*, 48, p. 99-141.



Quelle philosophie pour l'analyse transformationnelle ?

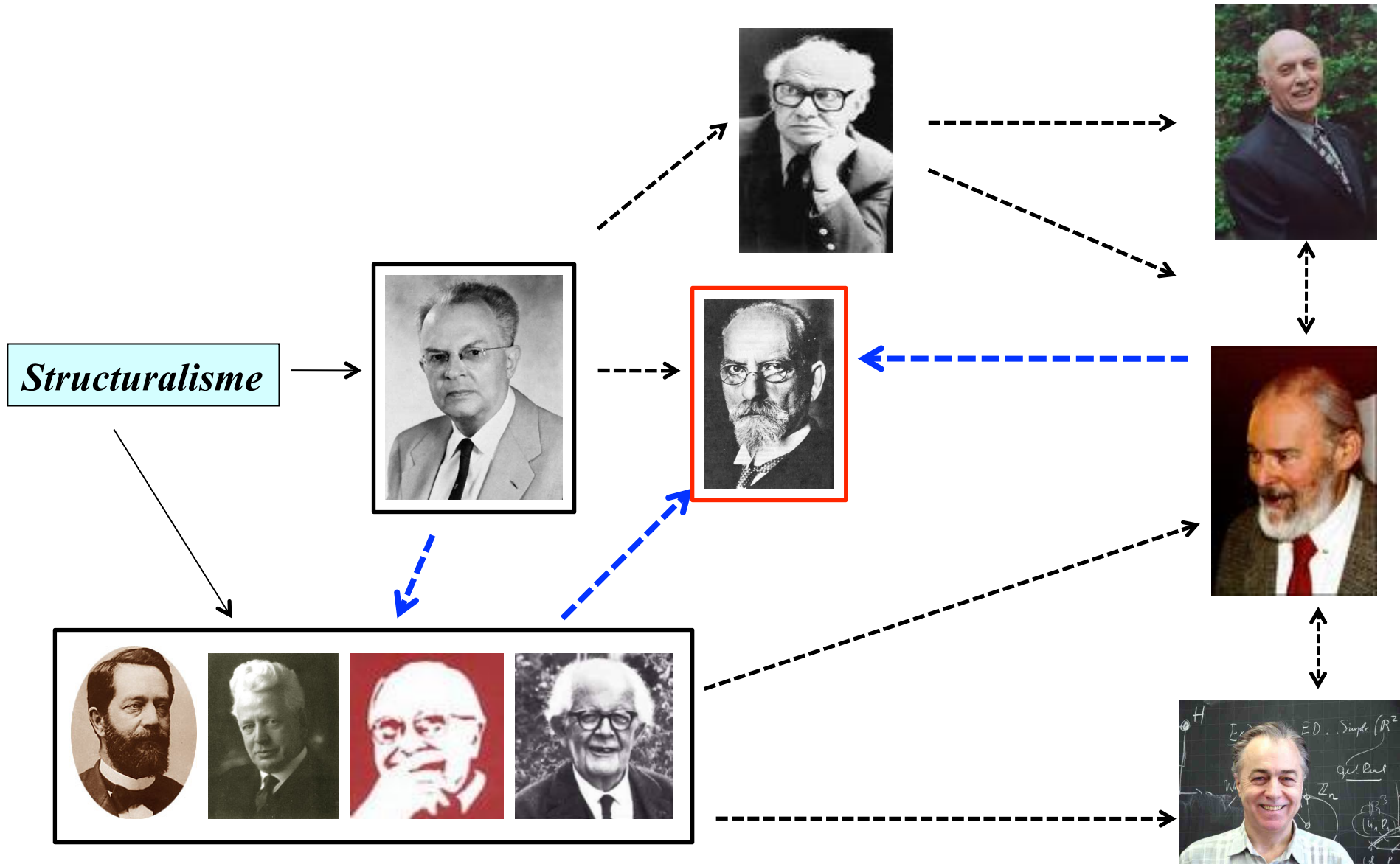
Autres implications philosophiques de l'approche transformationnelle

5. Trois orientations pour une démarche structurale en musique



Autres implications philosophiques de l'approche transformationnelle

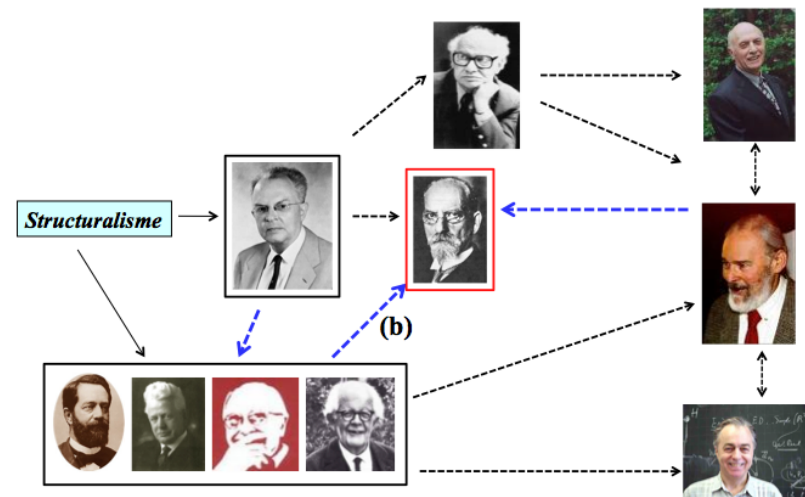
6. Vers une démarche structurale-phénoménologique en musique



Vers une démarche structurale-phénoménologique en musique

...et les mettre en mouvement (b)

« *La phénoménologie husserlienne des mathématiques est structurale en ce qu'elle se fixe sur les invariances (donc ce qui apparaît par variation), dont elle fait le cœur de l'objectivité mathématique, en tant qu'objectivité formelle. Elle est aussi structurale, dans un seul et même engagement, par le fait de privilégier l'opération sur l'objet.* »



« [...] *La pensée catégoriale n'est pas du tout étrangère, dans ses fondements, au type de « structuralisme » qui est celui de la phénoménologie, simplement en en faisant, avec des moyens que la phénoménologie ne pouvait pas soupçonner pour des raisons tenant à l'avancement du savoir mathématique autour de 1900, un structuralisme dynamique qui, par certains côtés, est beaucoup plus phénoménologique [...] que celui-là même que la phénoménologie pouvait proposer. En un certain sens, la pointe du structuralisme, ce n'est pas la structure, mais ce qu'on fait de la structure* »

« [...] *Il nous semble que, aujourd'hui, la théorie mathématique des catégories fournit, peut-être pour la première fois, un cadre théorique à ce que pourrait être une véritable épistémologie phénoménologique des mathématiques, ainsi que, du point de vue philosophique en général, un extraordinaire champ d'application à la phénoménologie. Elle nous donne enfin les moyens de remplir ce qui a toujours été le programme de la phénoménologie, à savoir ne jamais séparer le concept de l'intuition* »

J. Benoist, « Mettre les structures en mouvement: la phénoménologie et la dynamique de l'intuition conceptuelle. Sur la pertinence phénoménologique de la théorie des catégories », dans L. Boi, P. Kerszberg, F. Patras (éd.), *Rediscovering Phenomenology*. Springer, 2007

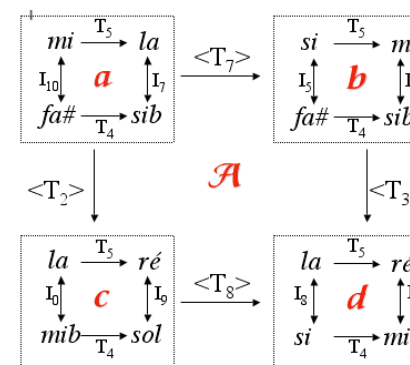
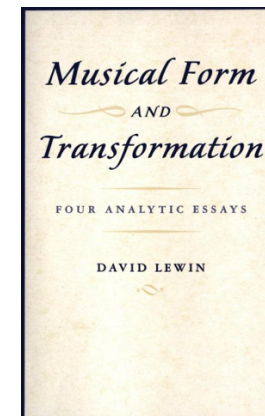
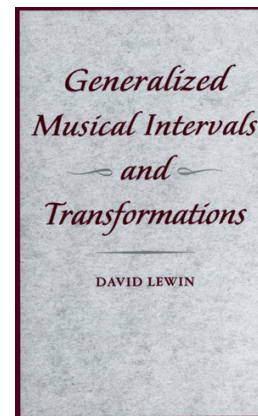
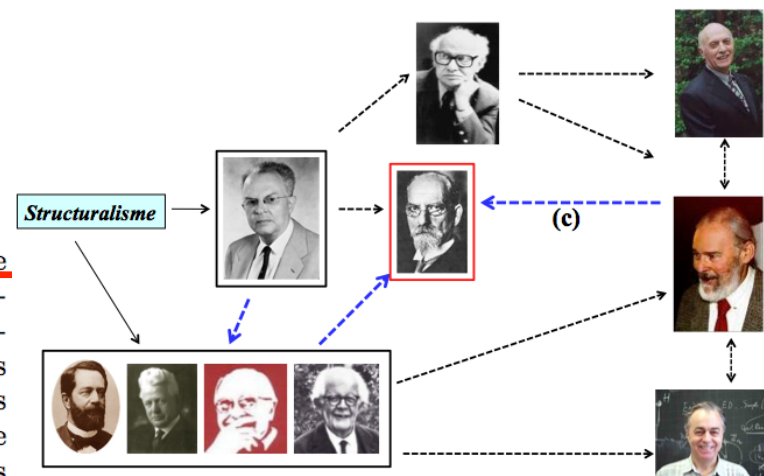
Vers une démarche structurale-phénoménologique en musique

(c) Analyse transformationnelle et phénoménologie husserlienne

La théorie de Lewin des réseaux transformationnels construit un méta-modèle de structure sur des bases phénoménologiques. Lewin développe méticuleusement le formalisme adéquat à ses réseaux : des systèmes de points/flèches aux graphes de transformation jusqu'aux réseaux de transformation. Lewin définit aussi des isomorphismes entre des systèmes de points/flèches, entre des graphes de transformations, ainsi que des « isographies » entre des réseaux de transformation (*si et seulement si* leurs graphes de transformation sont isomorphes); et de la même façon, il définit les homomorphismes entre les objets précédents, ainsi que les concepts usuels de relation, et ainsi de suite. L'ouvrage *GMIT* illustre brillamment chaque distinction mathématique par une foultitude d'exemples analytiques musicaux dont certains, comme la théorie néo-riemannienne, sont devenus eux-même des nouveaux secteurs de la théorie musicale. Lewin a convaincu la communauté américaine de la théorie musicale de considérer avec attention ce cadre sous-jacent des réseaux de transformation; et ce, jusqu'à ce que cette conception soit élevée au rang de nouveau paradigme d'« expression » de la pensée musicale.

Les réseaux transformationnels de Lewin sont formellement des graphes dont les flèches sont « étiquetées » (*labeled*) dans un demi-groupe, et les nœuds, dans un ensemble quelconque, sur lequel opère un demi-groupe, de telle manière que le diagramme résultant « commute » au sens de la théorie des catégories. Bien entendu, si le demi-groupe dont les éléments étiquettent les flèches possède une identité, le graphe-monoïde se trouvera ainsi lié à une catégorie¹⁷.

J. Rahn, « La déclinaison et le flot », in *Autour de la Set Theory. Rencontre musicologique franco-américain 15-16 octobre 2003* (dir. M. Andreatta, J.-M. Bardez et J. Rahn), Collection « Musique/Sciences », Ircam/Delatour, 2008



Séminaire **MaMuX**
Mathématiques, musique et relations avec d'autres disciplines

Autour de la géométrisation dans la logique et l'informatique musicale

Trois usages de notions spatiales en programmation et en représentation des connaissances

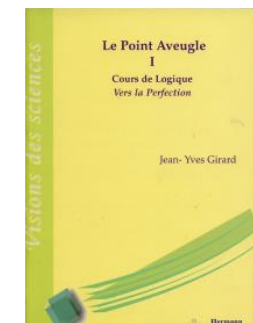
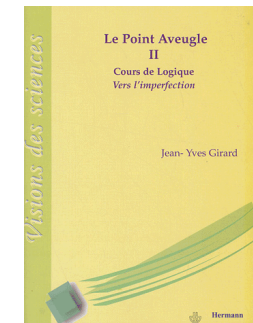
Carte blanche à l'équipe « Informatique, Biologie Intégrative et Systèmes Complexes » (IBISC, FRE 3190),
Université d'Evry Val d'Essonne, Genopole

Vendredi 13 novembre 2009

de 14h30 à 18h30

Ircam, Salle I. Stravinsky
1, place I. Stravinsky 75004 Paris

(Entrée libre dans la mesure des places disponibles)

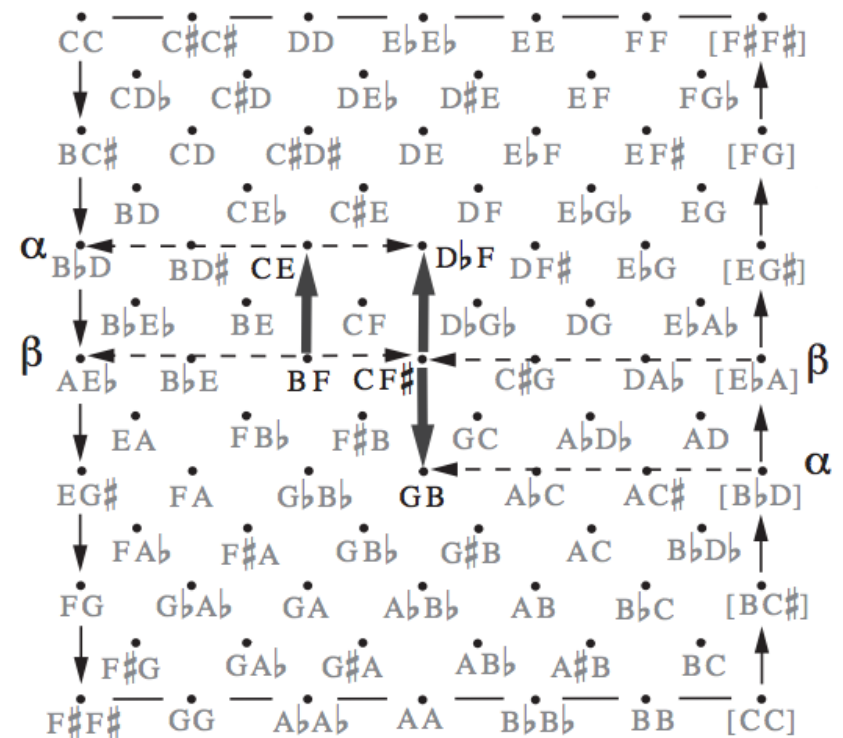
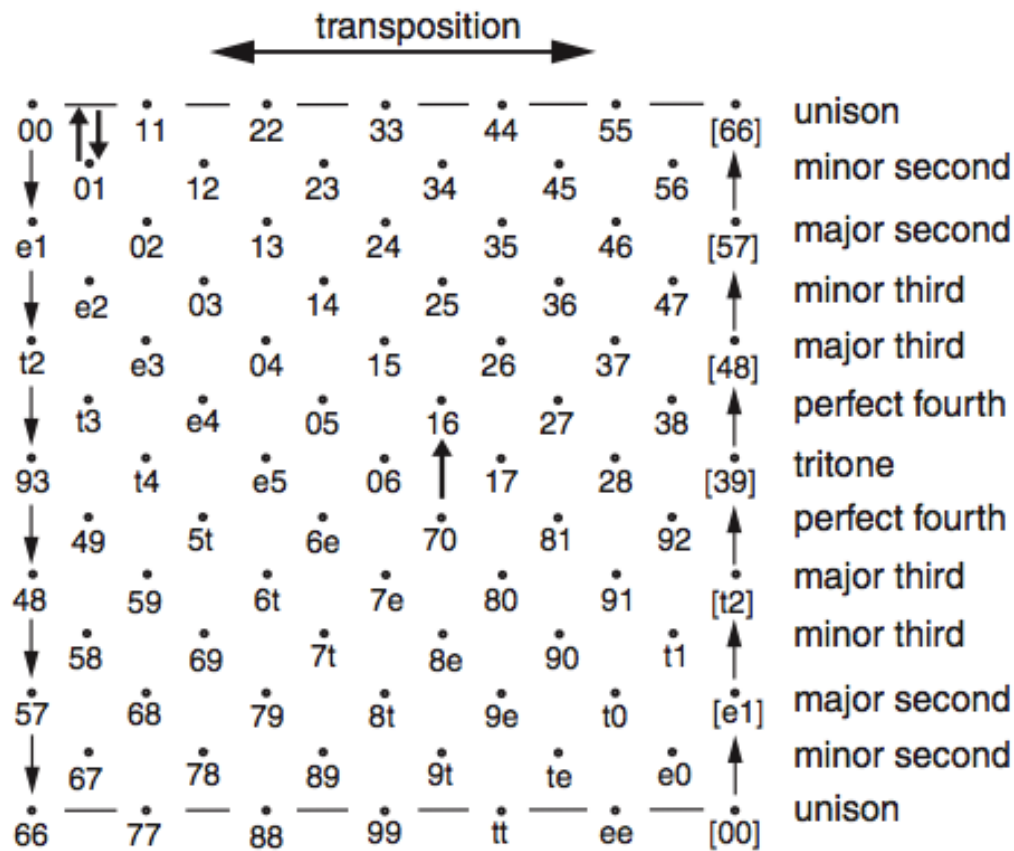
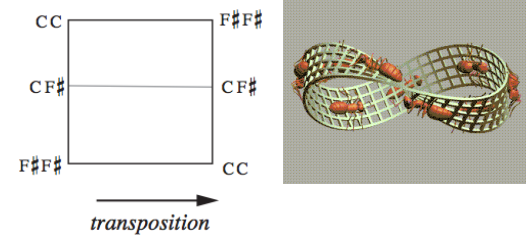


Programme :

- Moreno Andreatta : Un survol sur les approches géométriques et topo(s)-logiques en théorie mathématique de la musique
- Olivier Michel : Une brève présentation du langage Huit-et-demi
- Antoine Spicher : Une brève présentation du langage MGS
- Jean-Louis Giavitto : Résoudre des analogies en se déplaçant dans un espace
- Discussion finale

Se (dé)placer dans un orbifold

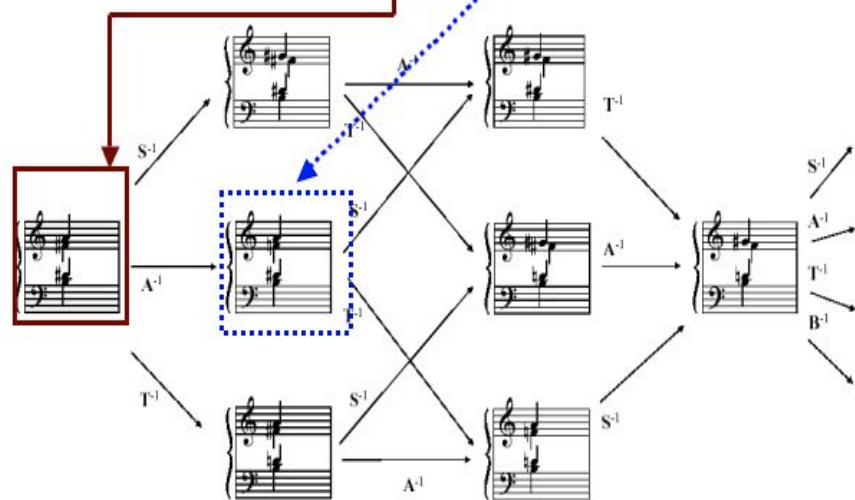
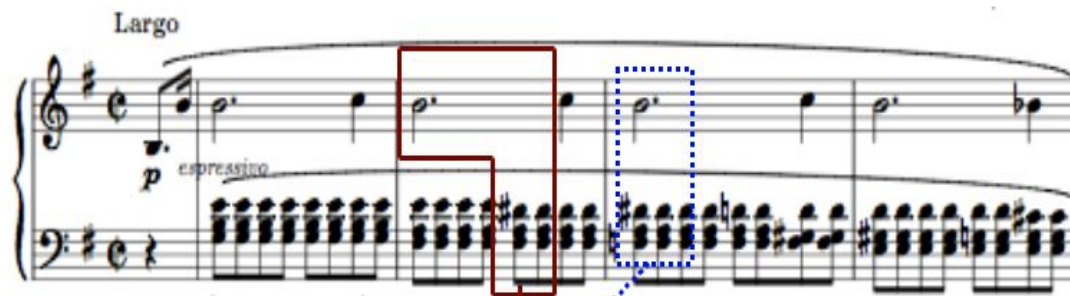
$$T^2 = \mathbf{R}/12\mathbf{Z} \times \mathbf{R}/12\mathbf{Z} \longrightarrow T^2 / S_2$$



Dmitri Tymoczko :
 « The Geometry of Musical Chords »,
Science, 313, 2006

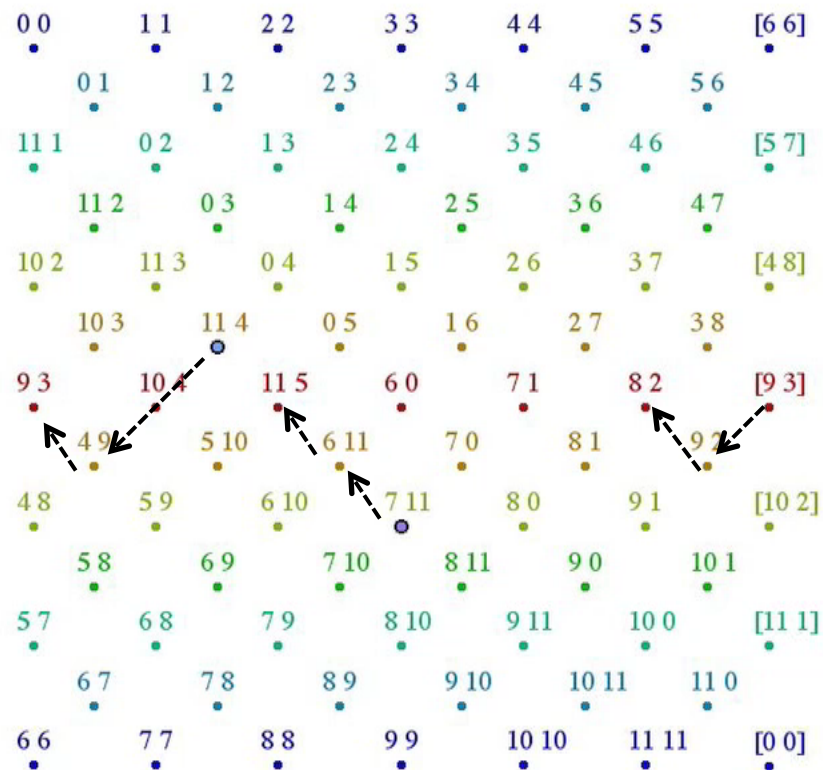
[Tymoczko 2010]

$$T^2 = R/12Z \times R/12Z \longrightarrow T^2 / S_2$$



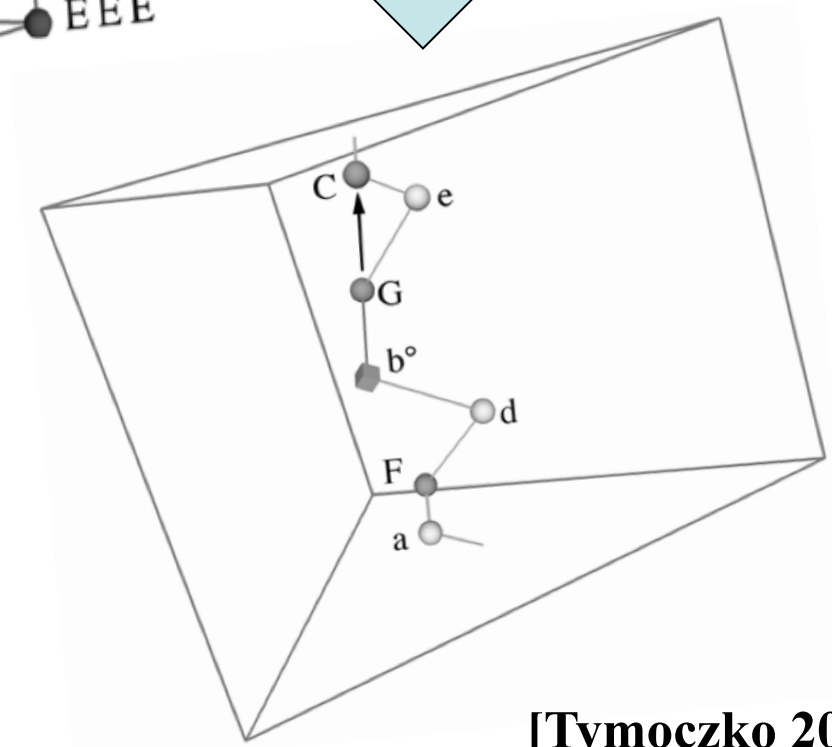
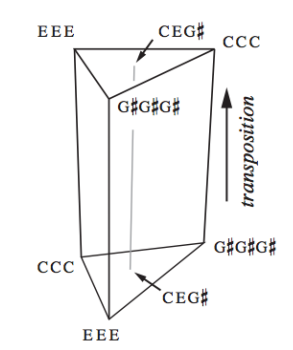
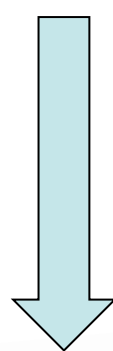
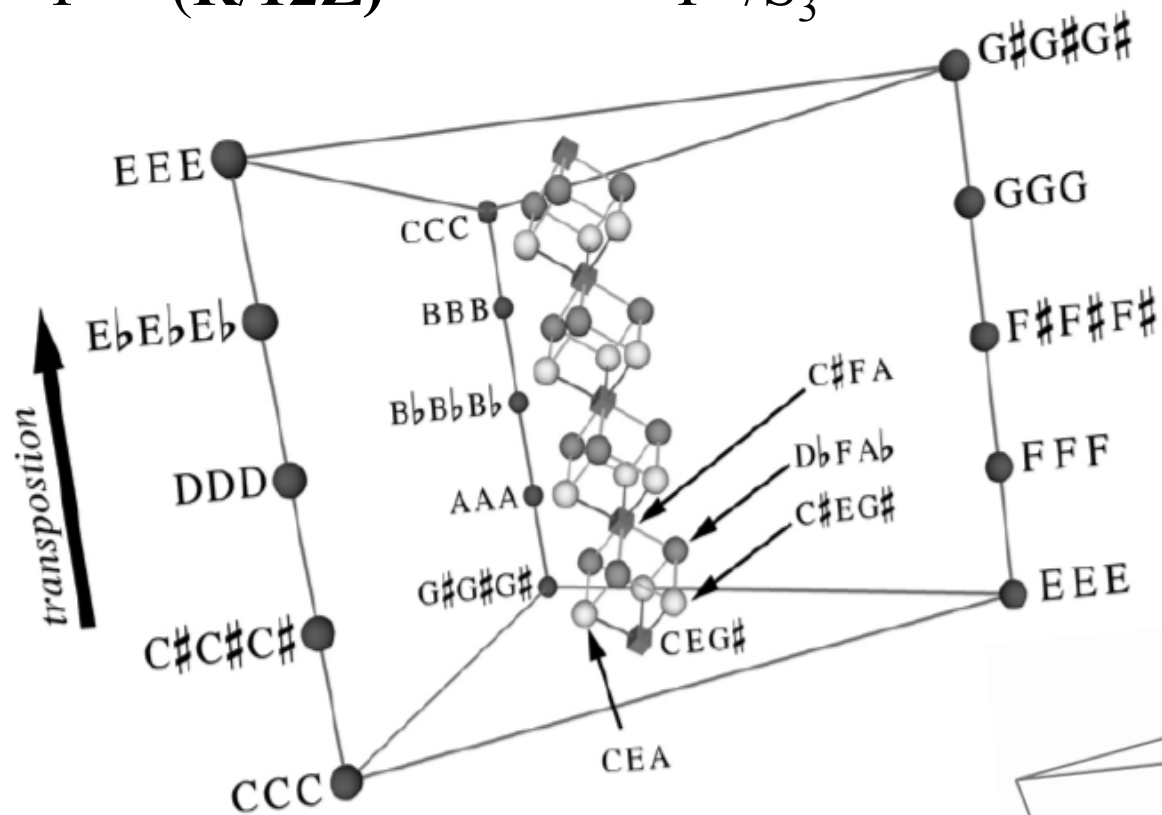
← transposition →

00	11	22	33	44	55	[66]	unison
01	12	23	34	45	56	[57]	minor second
e1	02	13	24	35	46	[57]	major second
e2	03	14	25	36	47	[48]	minor third
t2	e3	04	15	26	37	[48]	major third
t3	e4	05	16	27	38	[39]	perfect fourth
93	14	e5	06	17	28	[39]	tritone
48	59	6t	7e	80	91	[t2]	perfect fourth
49	5t	6e	70	81	92	[t2]	major third
58	69	7t	8e	90	t1	[e1]	minor third
57	68	79	8t	9e	t0	[e1]	major second
67	78	89	9t	te	e0	[00]	minor second
66	77	88	99	tt	ee	[00]	unison



Dmitri Tymoczko :
 « The Geometry of Musical Chords »,
Science, 313, 2006

$$T^3 = (\mathbb{R}/12\mathbb{Z})^3 \longrightarrow T^3 / S_3$$

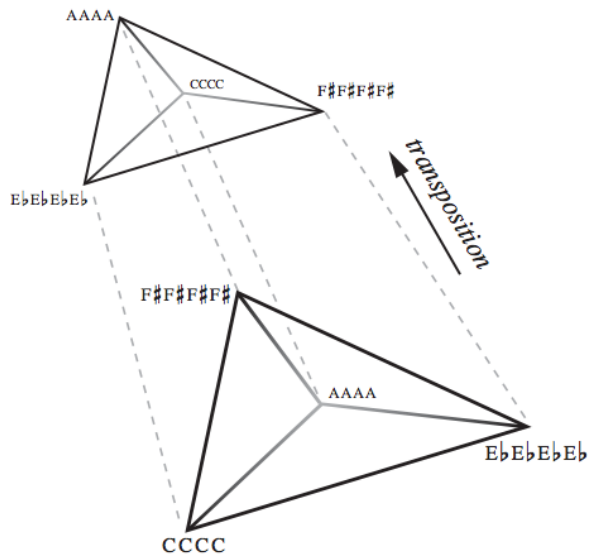


Dmitri Tymoczko :
 « The Geometry of Musical Chords »,
Science, 313, 2006

[Tymoczko 2010]

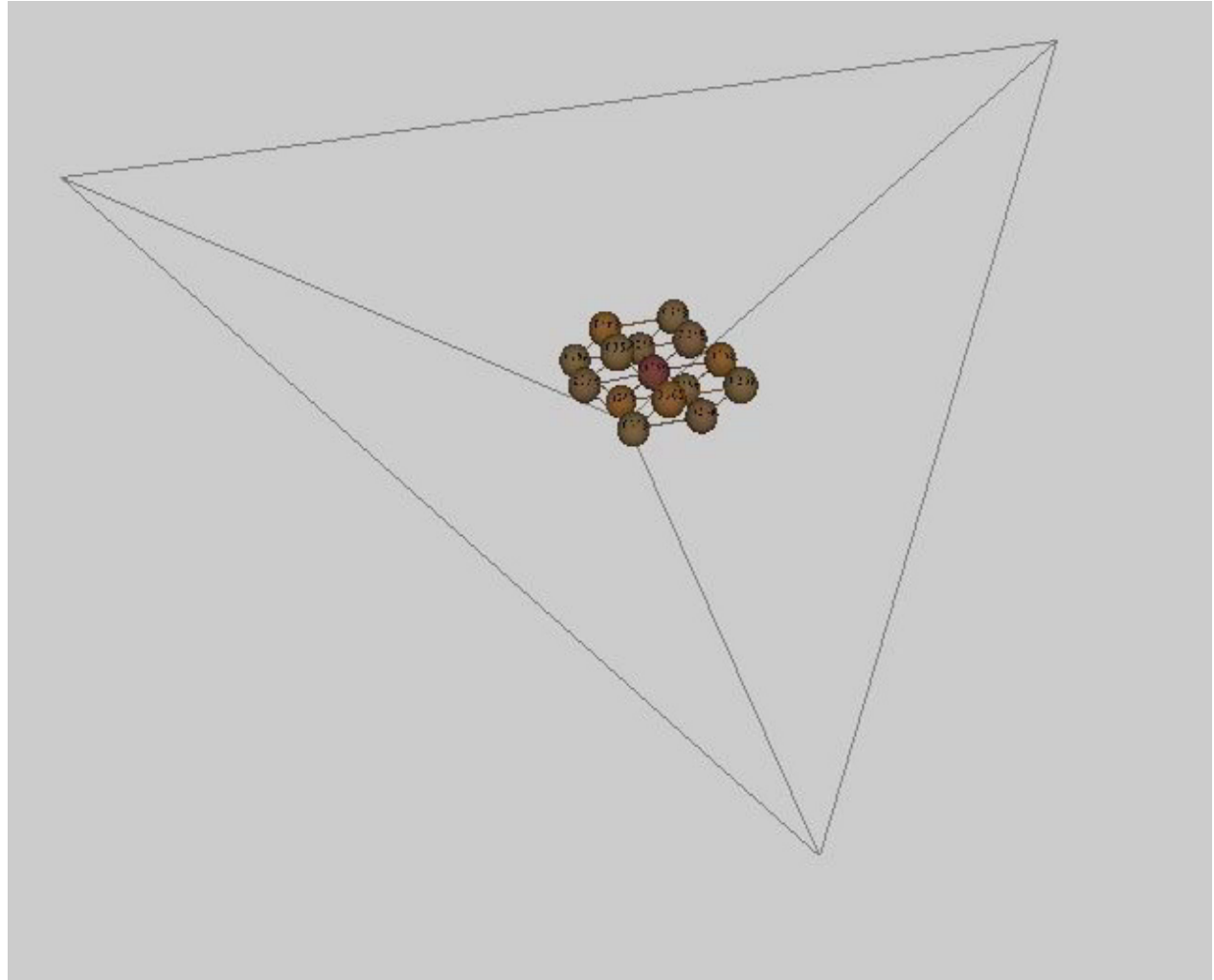
Se (dé)placer dans un espace de dim. 4

$$T^4 = (\mathbf{R}/12\mathbf{Z})^4 \longrightarrow T^4 / S_4$$



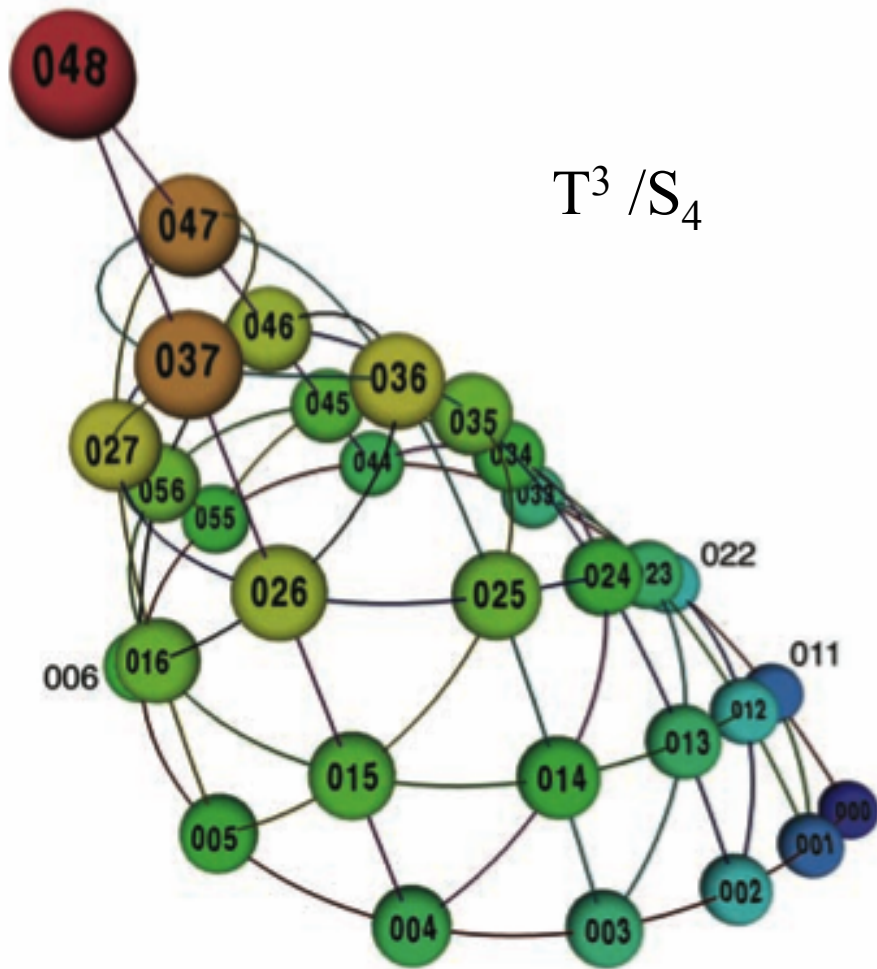
Largo

A musical score in G major, 4/4 time, marked "Largo" and "p" (piano). The score shows a sequence of chords. A red box highlights a specific chord, and a blue dashed box highlights another. Below the score is a chord progression diagram showing the relationships between chords. The diagram uses labels S¹, A¹, and T¹ to denote different types of chords or transformations. Arrows indicate the progression from one chord to the next.

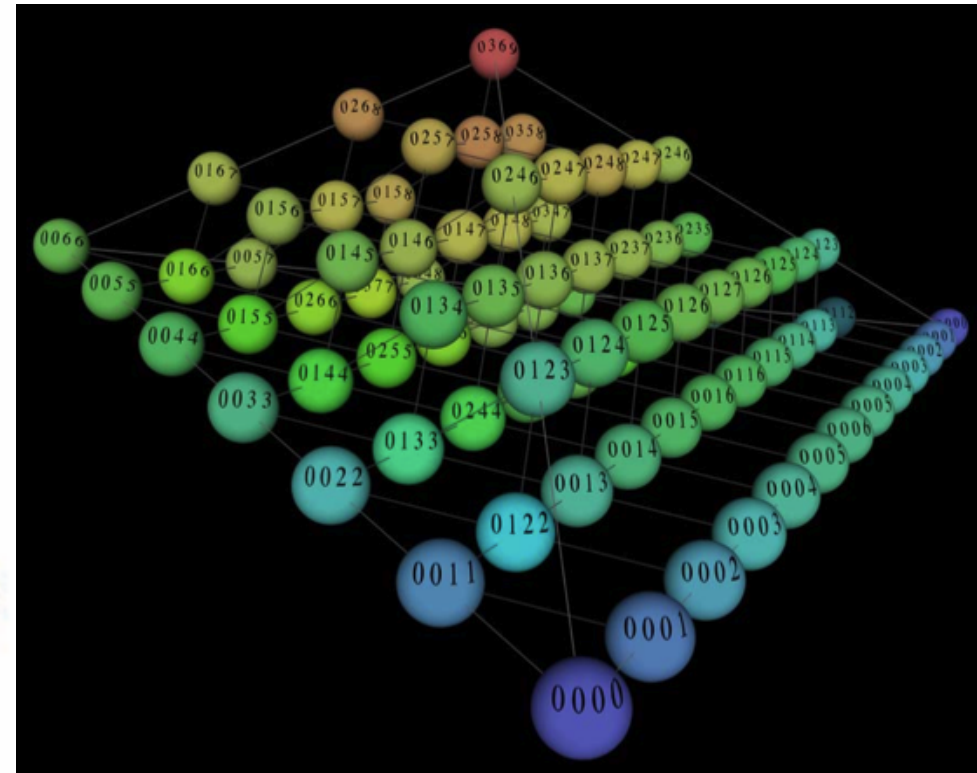


Dmitri Tymoczko, « The Geometry of Musical Chords », *Science*, 313, 2006

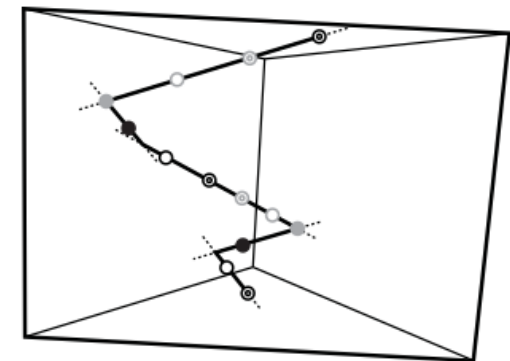
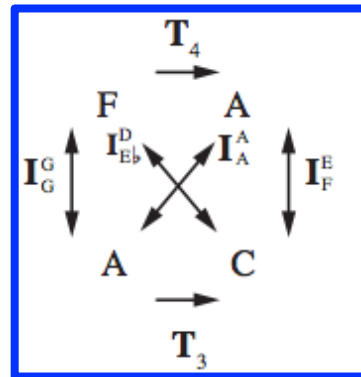
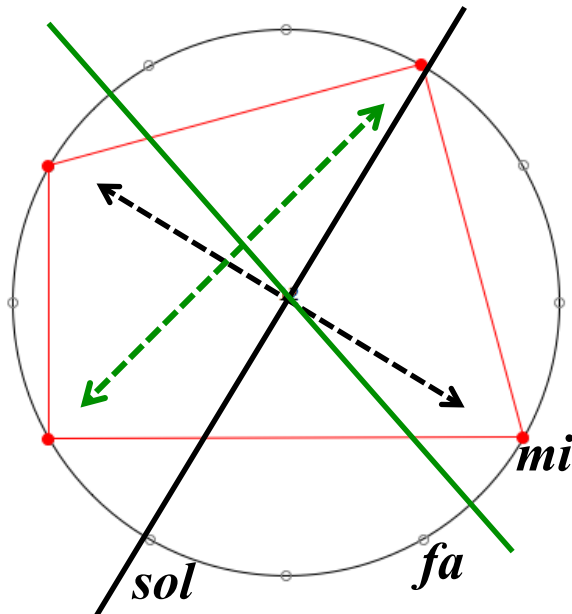
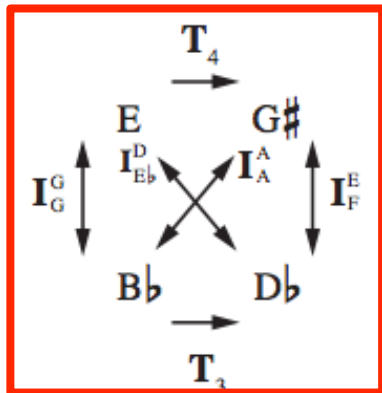
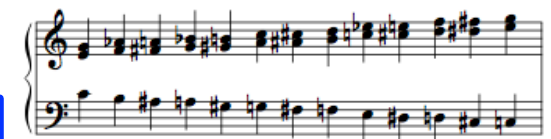
Autres exemples d'orbifolds

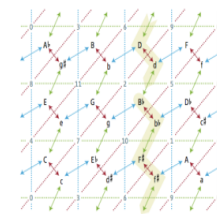
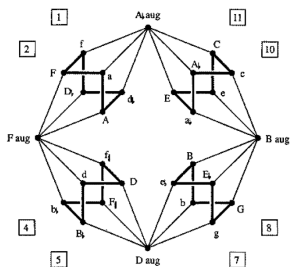


$T^3 / (S_4 \times Z_2)$



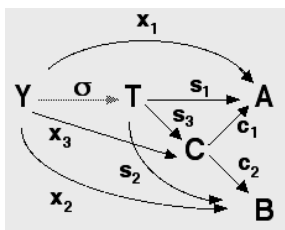
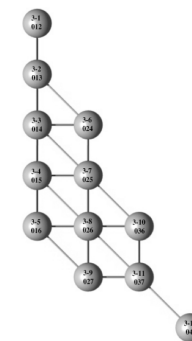
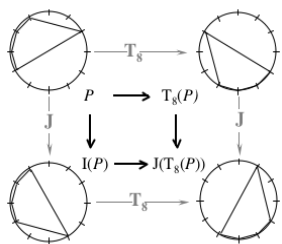
Orbifolds et analyse transformationnelle



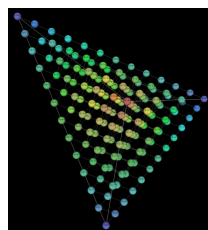


Quelques éléments pour une interprétation philosophique des approches transformationnelles en théorie et analyse musicales

samedi 16 janvier 2009



← transposition →							
00	11	22	33	44	55	66	unison
01	12	23	34	45	56	67	minor second
02	13	24	35	46	57	68	major second
03	14	25	36	47	58	69	minor third
04	15	26	37	48	59	60	major third
05	16	27	38	49	50	61	perfect fourth
06	17	28	39	40	51	62	tritone
07	18	29	30	41	52	63	perfect fourth
08	19	30	41	52	63	74	major third
09	20	31	42	53	64	75	minor third
10	21	32	43	54	65	76	major second
11	22	33	44	55	66	77	minor second
12	23	34	45	56	67	78	unison



Moreno ANDREATTA
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