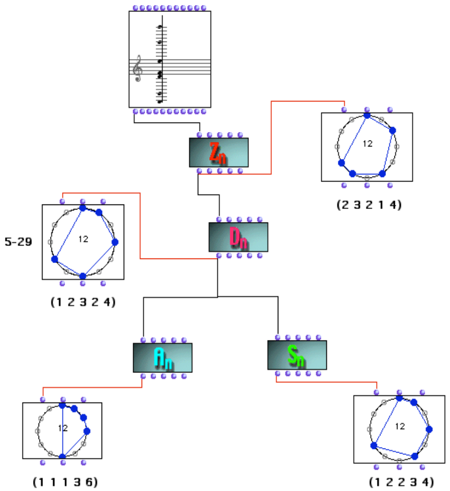
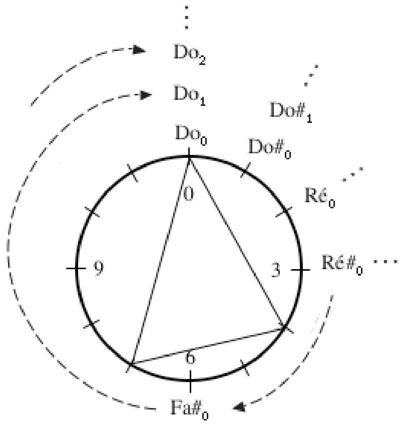


UNIVERSITÀ DI PISA



(4,3,5)

Elementi di Geometria Superiore 2

Matematica & Musica

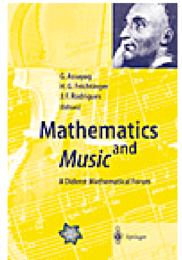
Moreno Andreatta
Equipe Représentations Musicales
IRCAM/CNRS
(In collaborazione con Carlos Agon e Emmanuel Amiot)

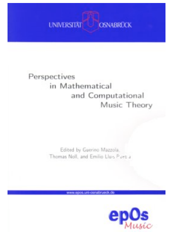
ircam Centre Pompidou

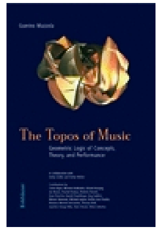
cnrs

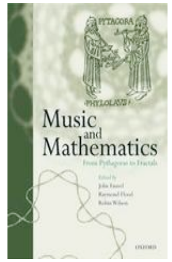
Matematica/Musica…una storia recente!

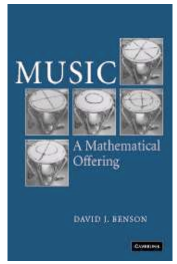
- 1999: 4e Forum Diderot (Paris, Vienne, Lisbonne), *Mathematics and Music* (Assayag et al., Springer, 2001)
- 2000-2001: Séminaire *MaMuPhi*, *Penser la musique avec les mathématiques ?* (Assayag, Mazzola, Nicolas ed., Coll. « Musique/Sciences », Ircam/Delatour, 2006)
- 2000-2003: International Seminar on *MaMuTh* (*Perspectives in Mathematical and Computational Music Theory* (Mazzola, Noll, Luis-Puebla, epOs, 2004)
- 2003: *The Topos of Music* (G. Mazzola et al.)
- 2003: *Music and Mathematics. From Pythagoras to Fractals* (J. Fauvel et al.)
- 2001 - 2006: Séminaire *MaMuX* de l'Ircam `http://recherche.ircam.fr/equipes/repmus/mamux/`
- 2004 - 2006 : Séminaire « Musique et Mathématique » (Ens/Ircam) `http://www.entretemps.asso.fr/maths`
- 2006: *Mathematical Theory of Music* (Franck Jedrzejewski), Ircam/Delatour
- 2007: *Music. A Mathematical Offering* (Dave Benson), Cambridge University Press
- 2007: *Les mathématiques naturelles* (Marc Chemillier), Odile Jacob
- 2008: *Music Theory and Mathematics* (Jack Douthett et al.), Univ. of Rochester Press
- 2007: Journal of Mathematics and Music (Taylor & Francis)
- 2008: Society of Mathematics and Computation in Music

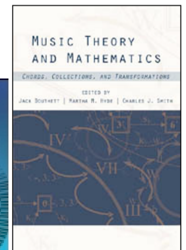



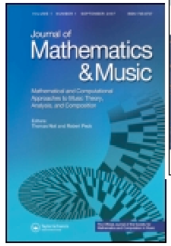










Programma del corso

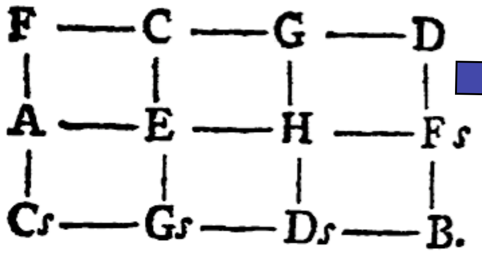
1.) Rappresentazione e formalizzazione delle strutture musicali
2.) Enumerazione e classificazione delle strutture musicali
3.) Teorie trasformazionali, diatoniche e neo-riemanniane
4.) Tessellazioni musicali: la costruzione dei canoni ritmici a mosaico
5.) Sequenze periodiche e calcolo delle differenze finite a valori in gruppi ciclici
6.) Ramificazioni filosofiche e cognitive dell'approccio algebrico in musica

Rappresentazione e formalizzazione delle strutture musicali

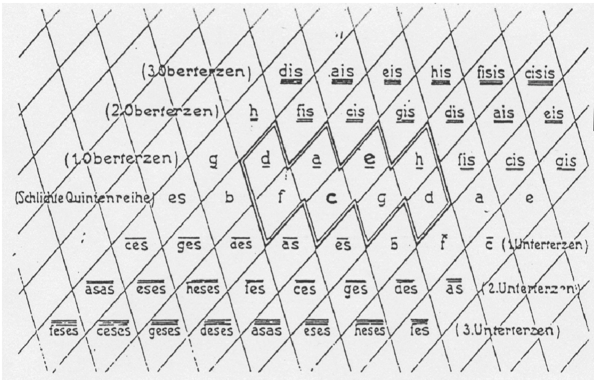
- Rappresentazioni geometriche e formalizzazioni algebriche
  - Il *Tonnetz* di Eulero
  - Rappresentazioni circolari e toroidali
  - Teoria degli *orbifolds*
  - Cenni dell'approccio categoriale

- Strumenti informatici e rappresentazioni simboliche
  - Teoria della calcolabilità
  - Teoria della complessità
  - Calcolo informatico
  - Lambda-calcolo
  - Programmazione logica e calcolo concorrente
  - Analisi musicale assistita su calcolatore

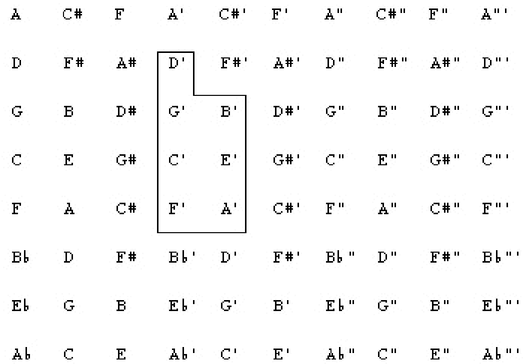
Rappresentazioni geometriche delle strutture musicali



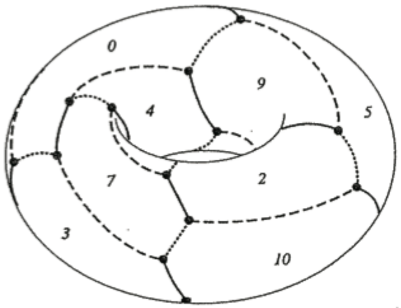
Euler : *Speculum musicum*, 1773



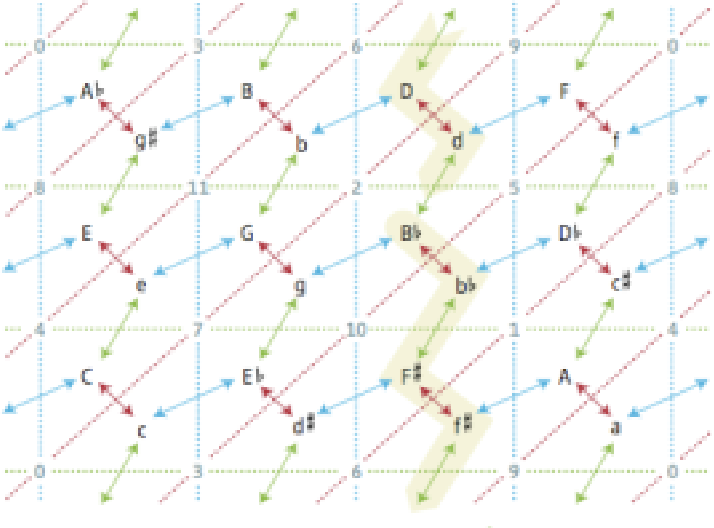
Hugo Riemann : « Ideen zu einer Lehre von den Tonvorstellung », 1914



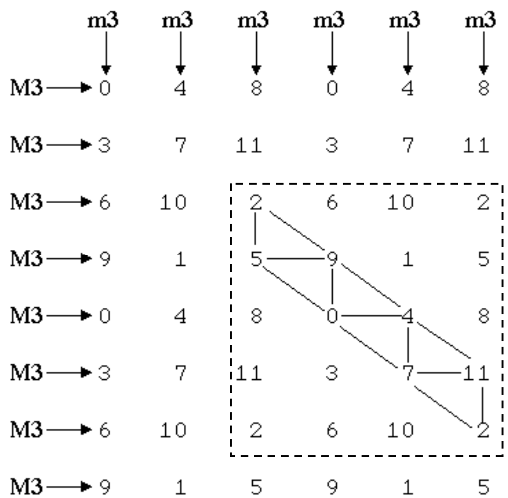
Longuet-Higgins (1962)



Douthett & Steinbach, *JMT*, 1998

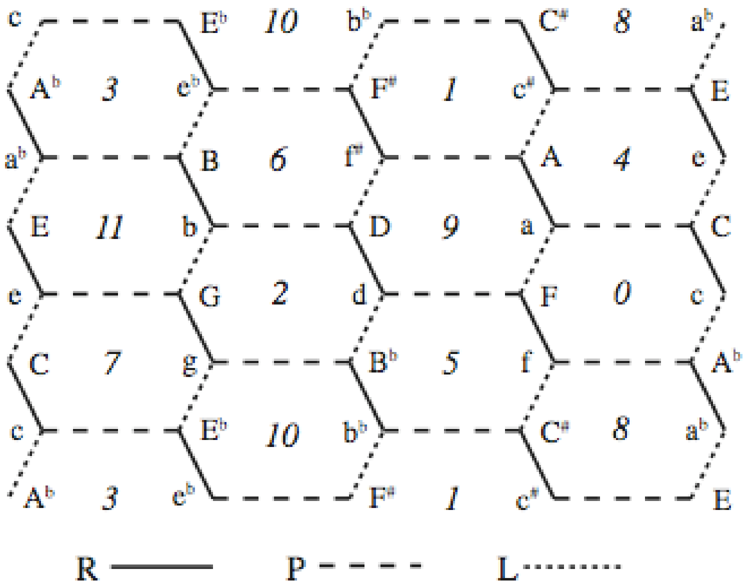


J. Hook, « Exploring Musical Space », *Science, 2006*

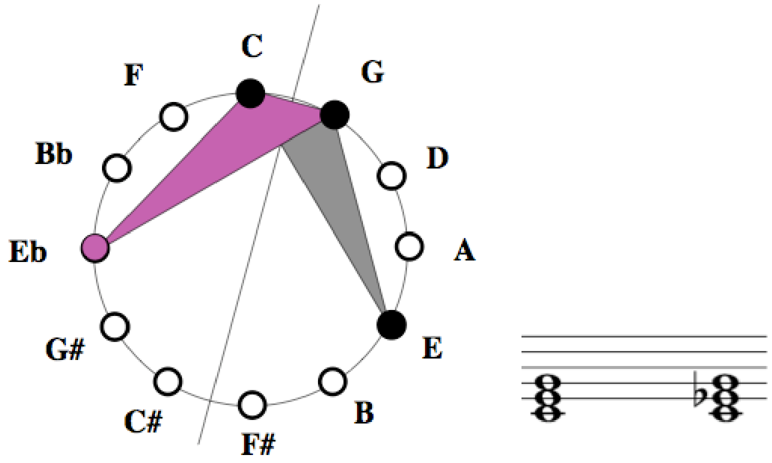


Balzano (1980)

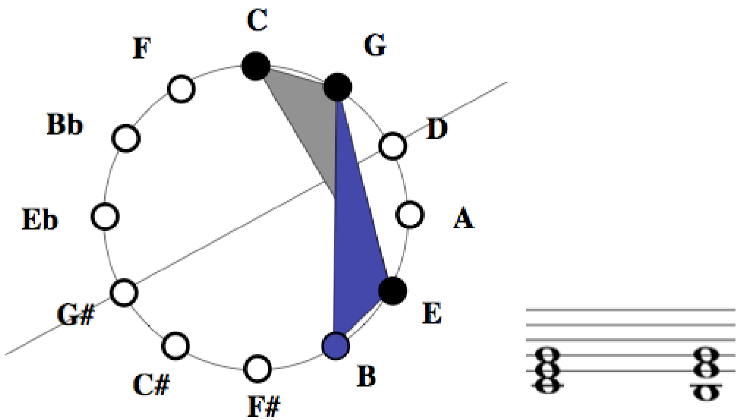
Teorie neo-Riemanniane



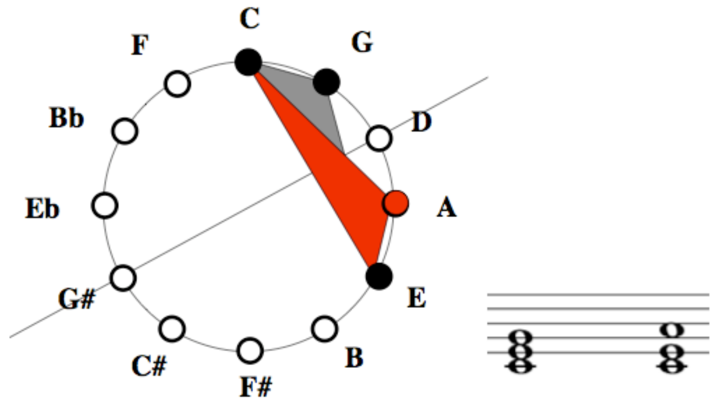
(Neo-)Riemannian Operation **P** = **„Parallel“**



(Neo-)Riemannian Operation **L** = **„Leading-Tone“**

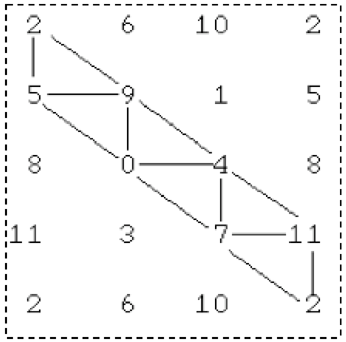


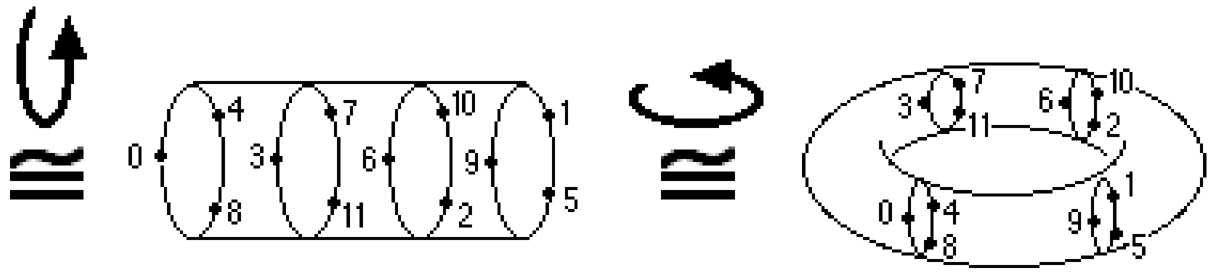
(Neo-)Riemannian Operation **R** = **„Relative“**



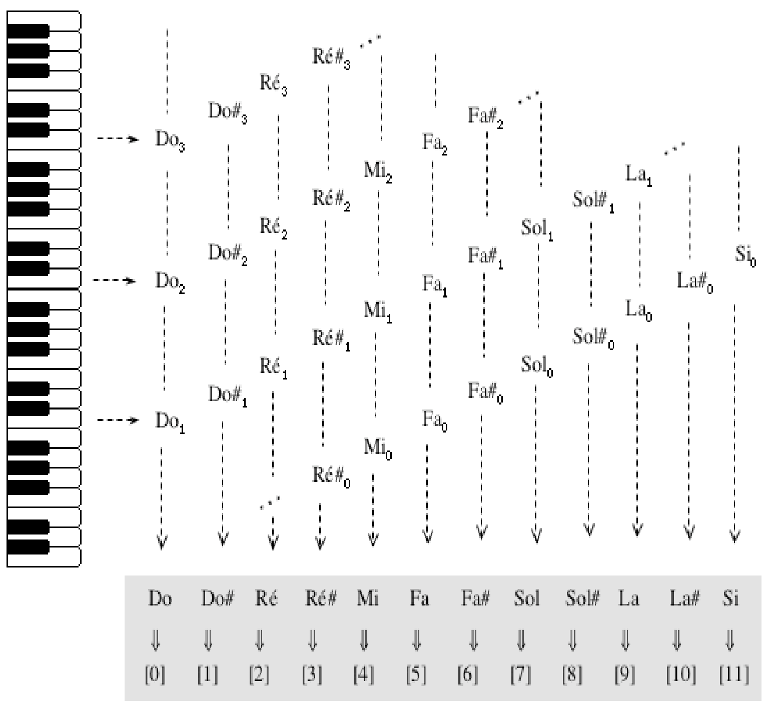
Cf. Th. Noll: The Z12 Story, MaMuX, Dec. 18 2004

Equivalenza algebrica fra rappresentazioni geometriche

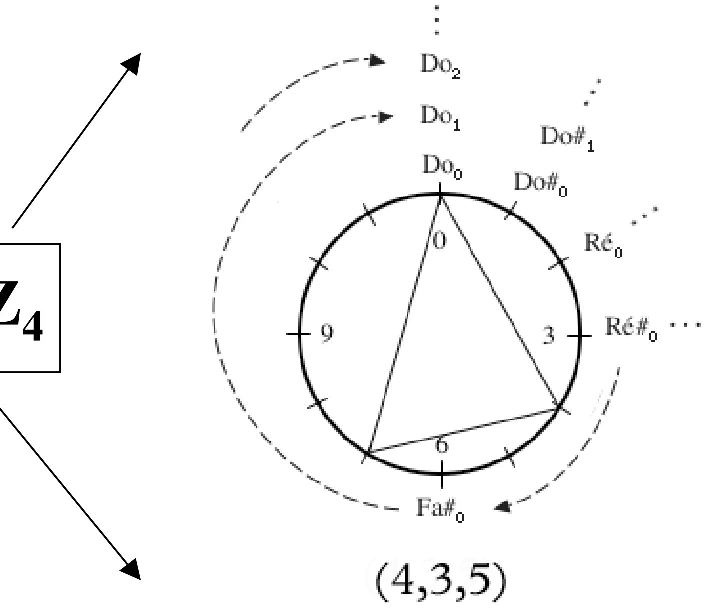




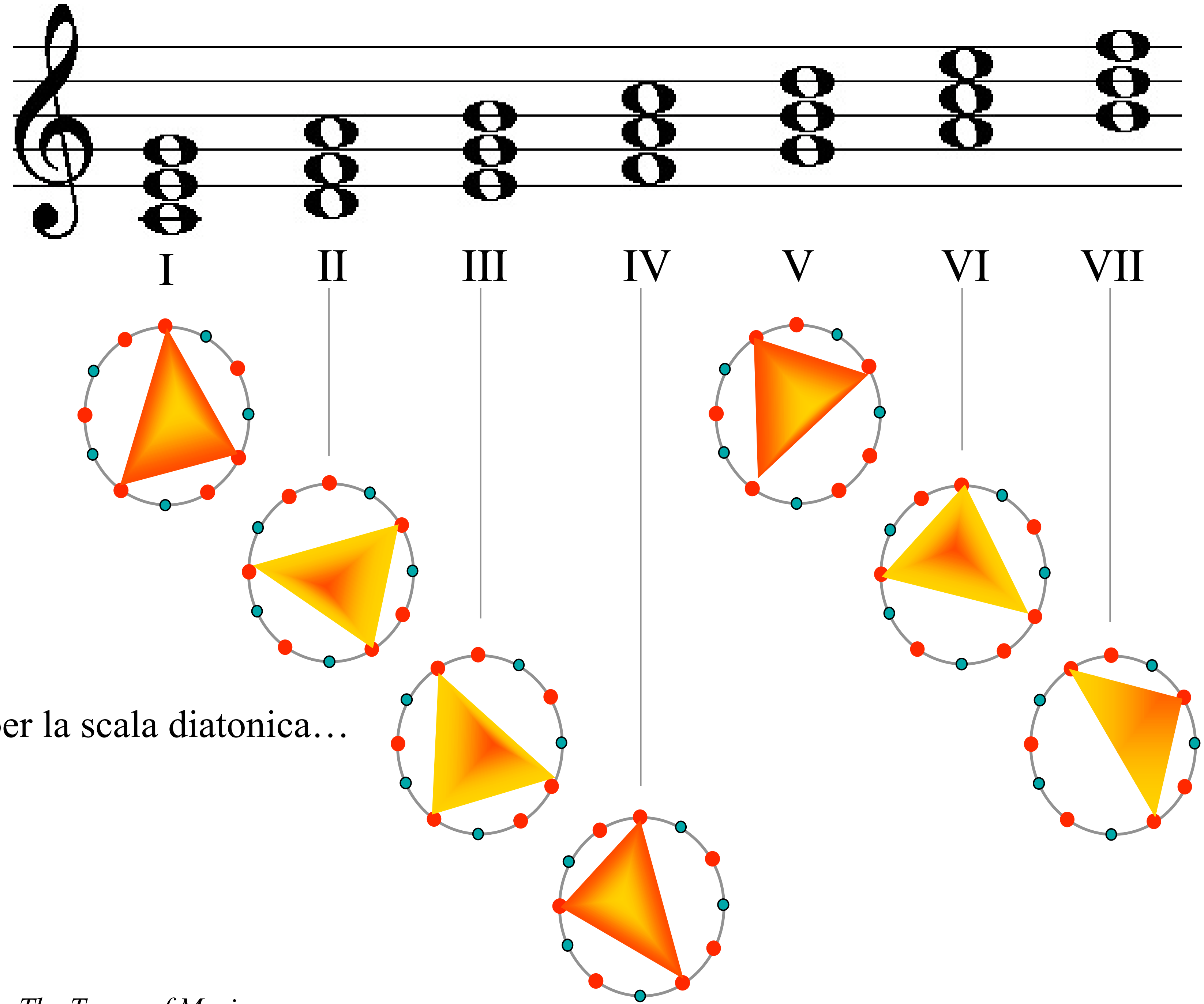
Rappresentazione toroidale



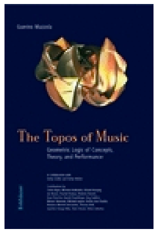
$$Z_{12} = Z_3 \times Z_4$$



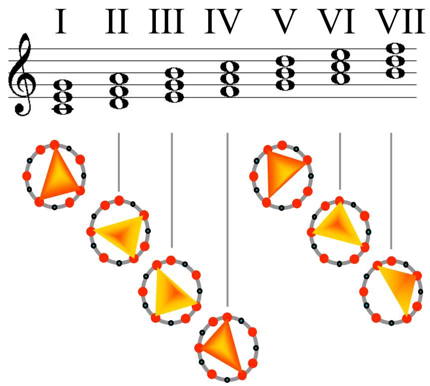
Rappresentazione circolare



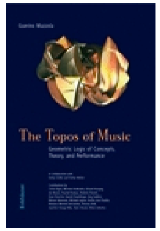
Un atlas per la scala diatonica…

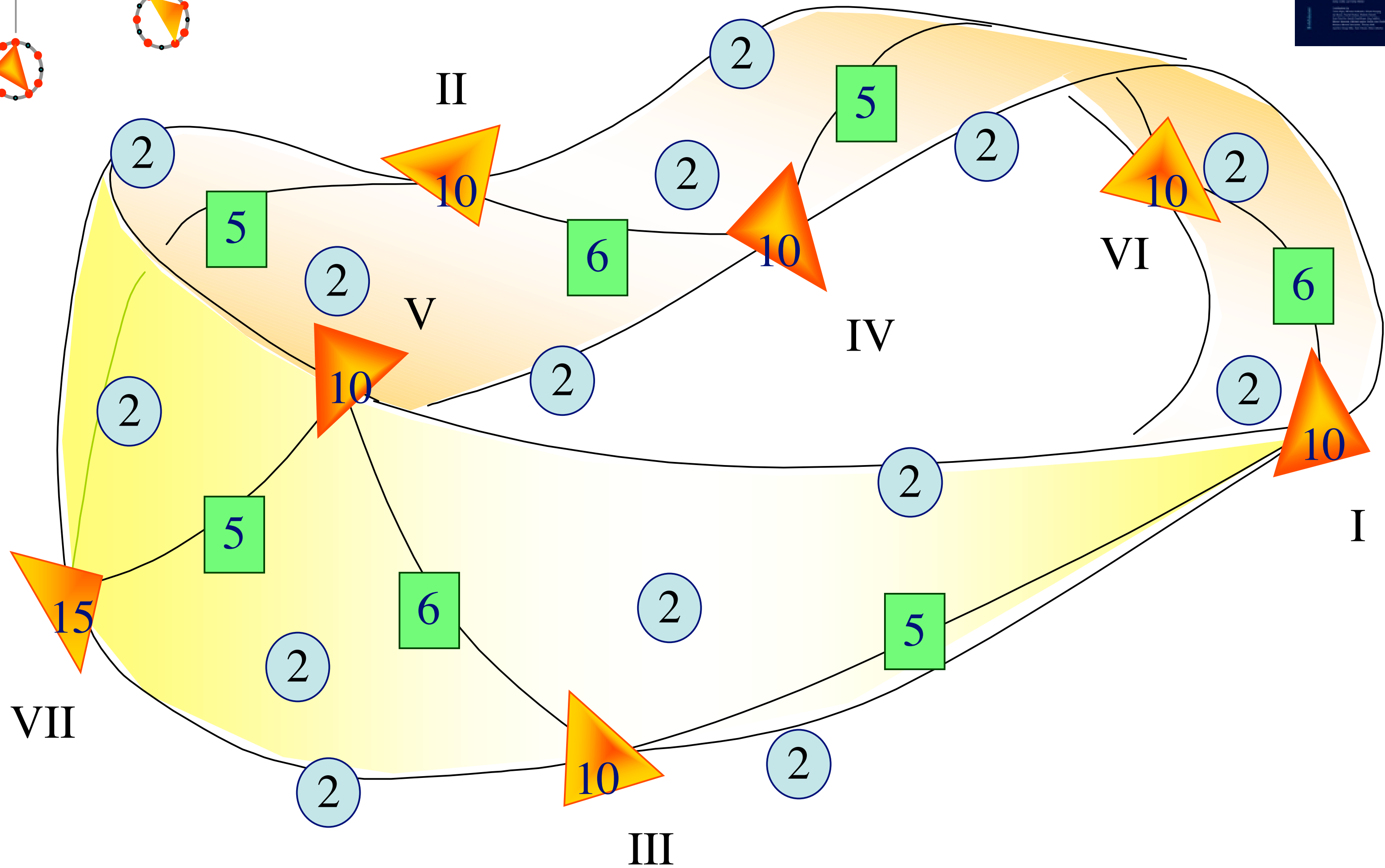


G. Mazzola, *The Topos of Music*



G. Mazzola, *The Topos of Music*



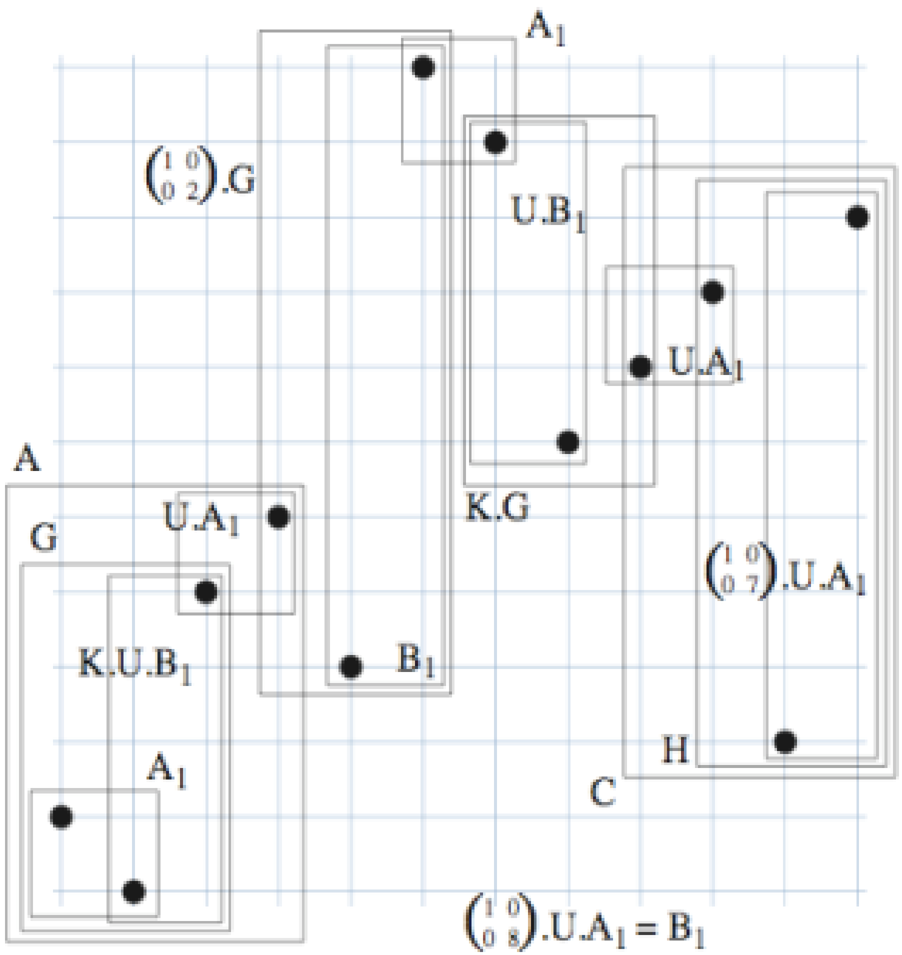


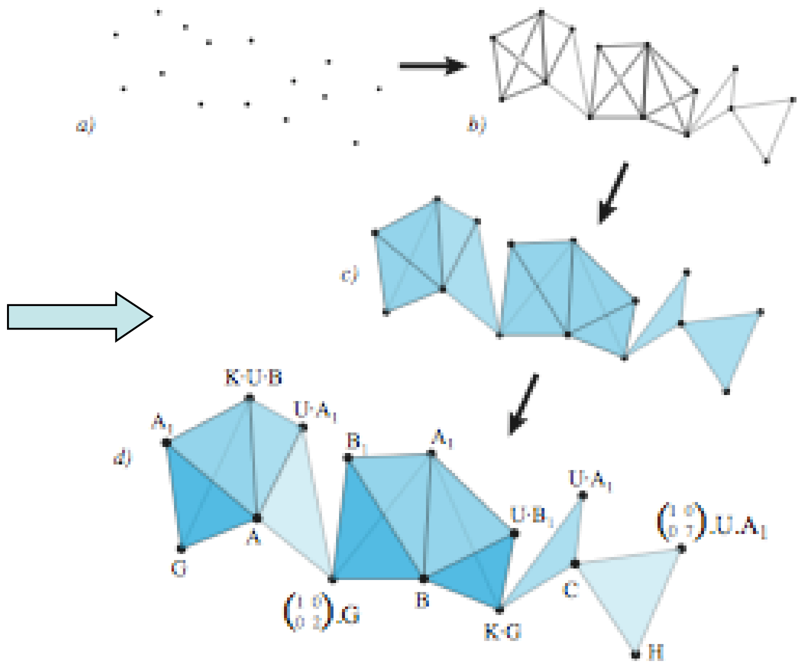
…e il nervo topologico associato

Nervo topologico e analisi musicale

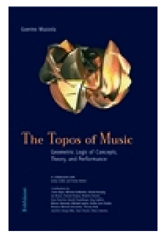
a)

b)

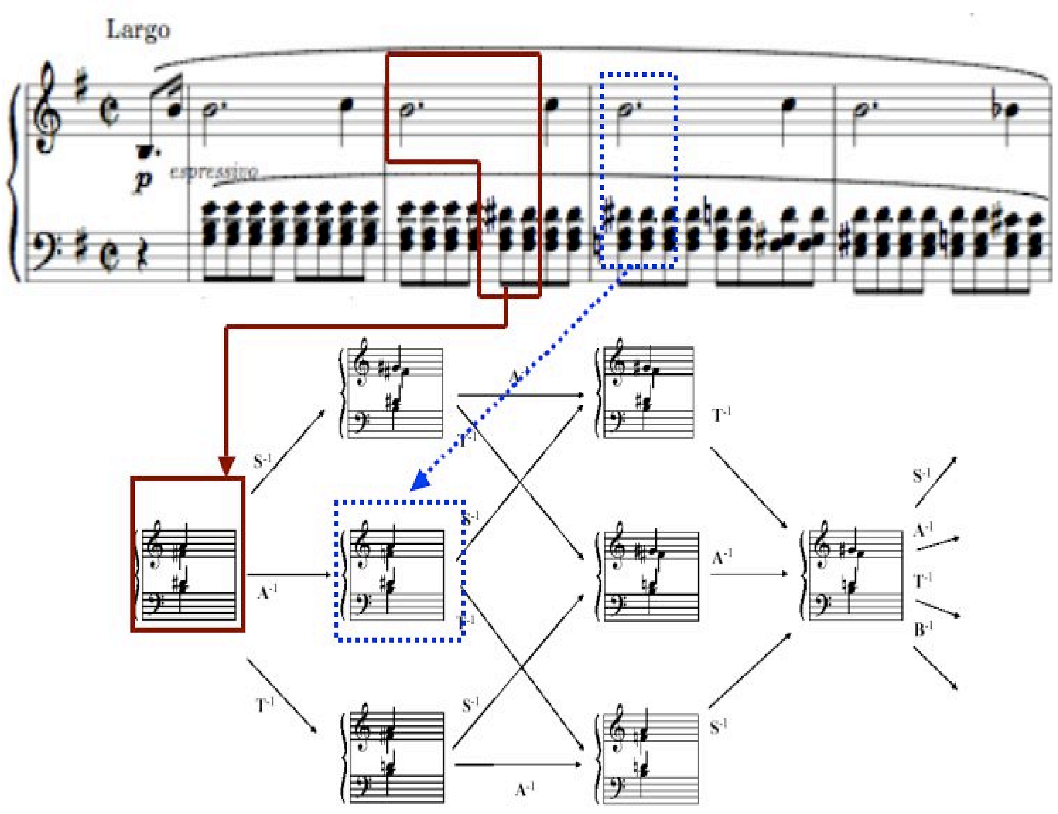


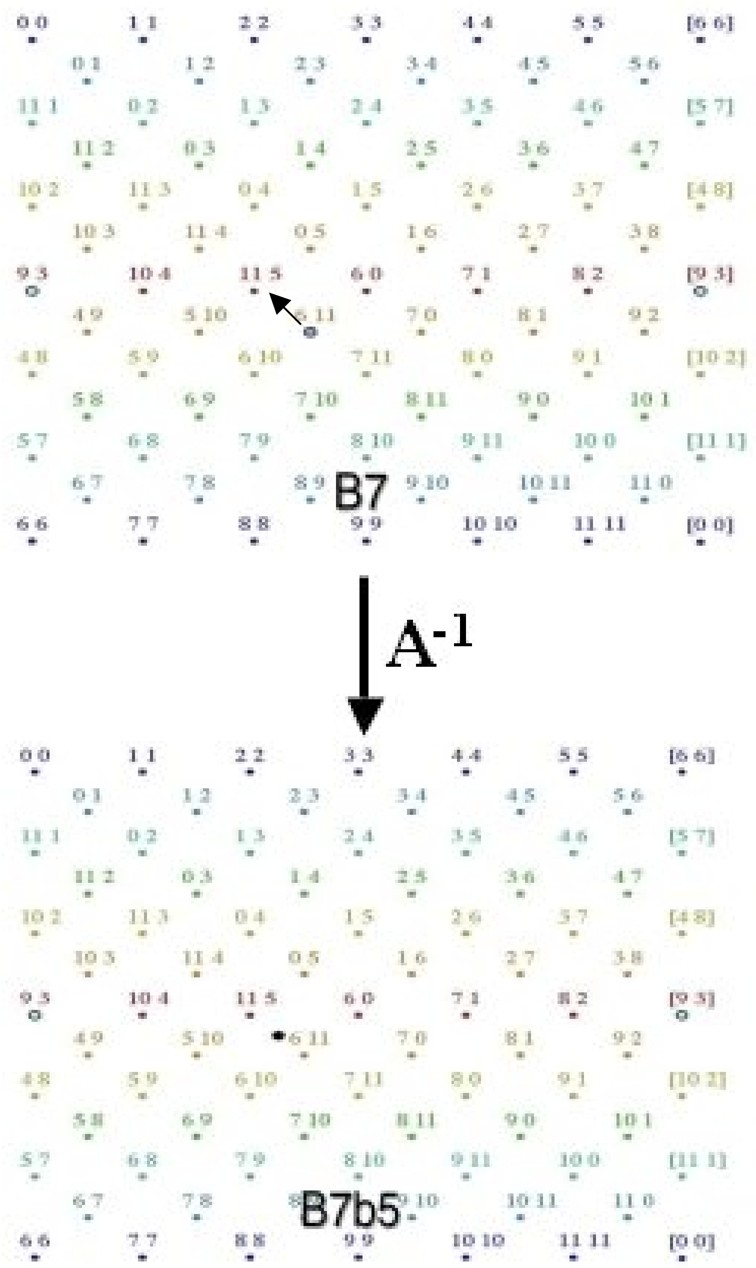


G. Mazzola : *The Topos of Music*, ch. 13 - "What are global compositions ?"



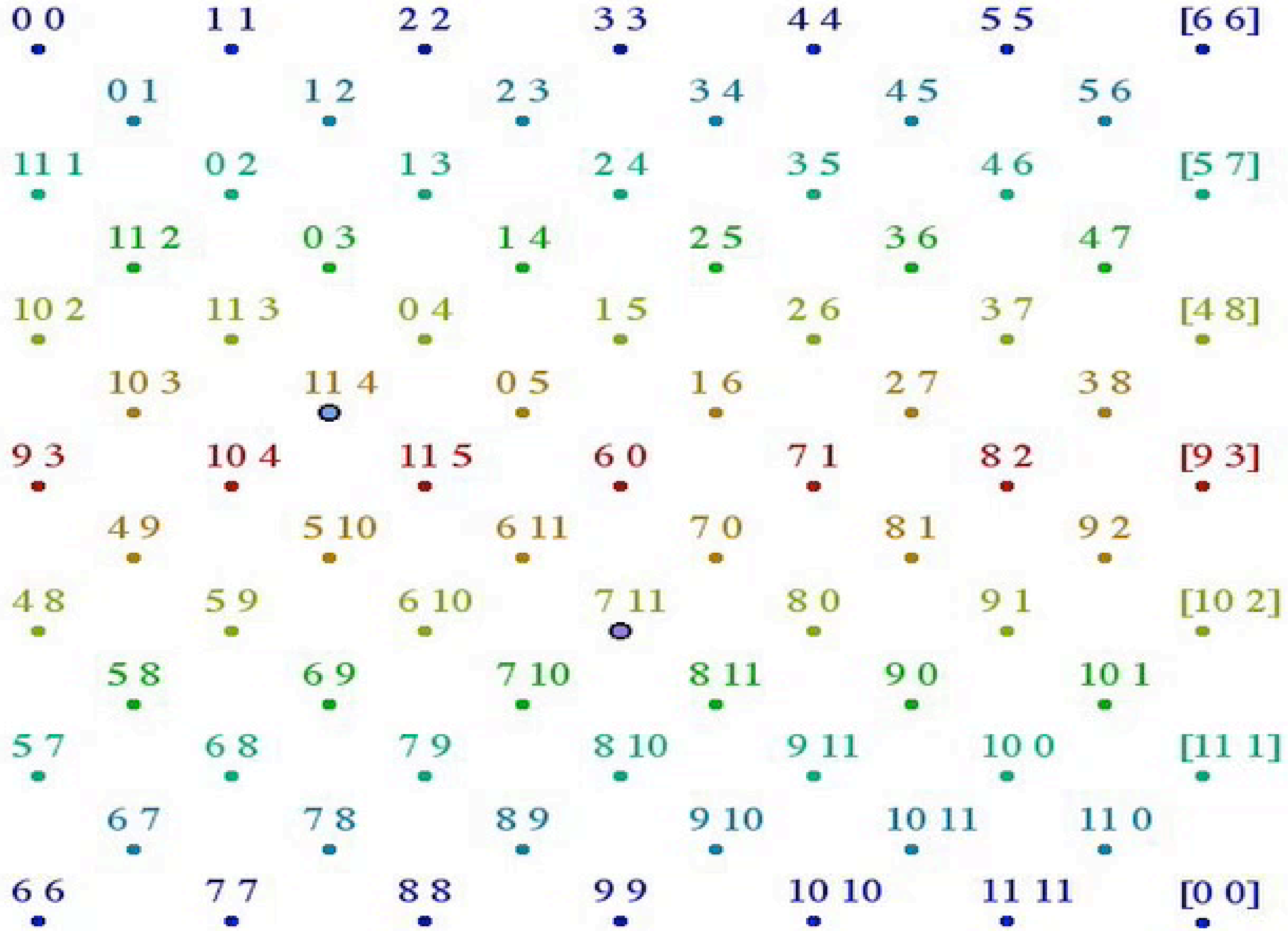
Rappresentazione non–orientabile dello spazio musicale



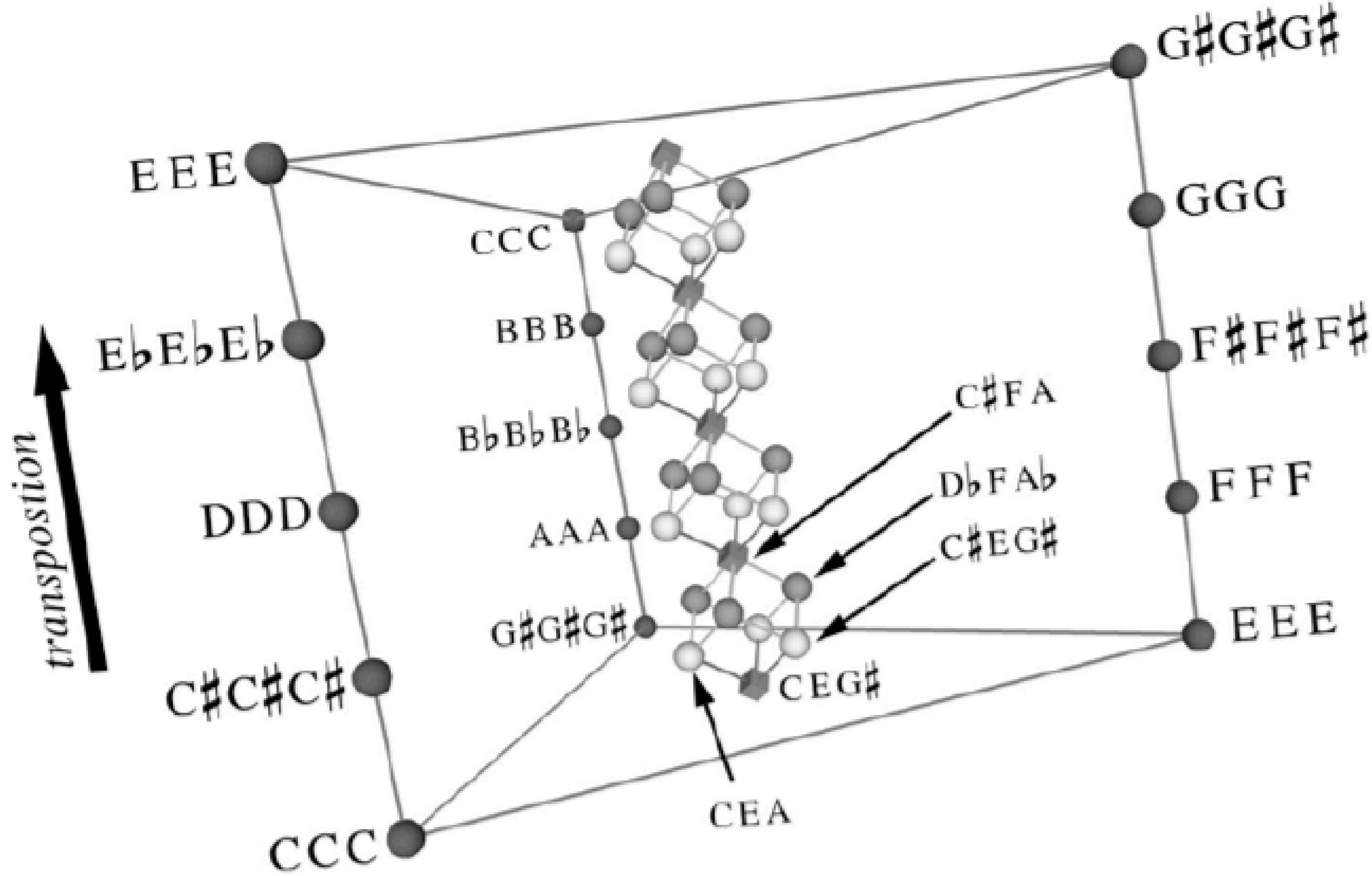


Dmitri Tymoczko :
« The Geometry of Musical Chords »,
Science, 313, 2006

$T^2 = \mathbf{R}/12\mathbf{Z} \times \mathbf{R}/12\mathbf{Z} \longrightarrow T^2/S_2$

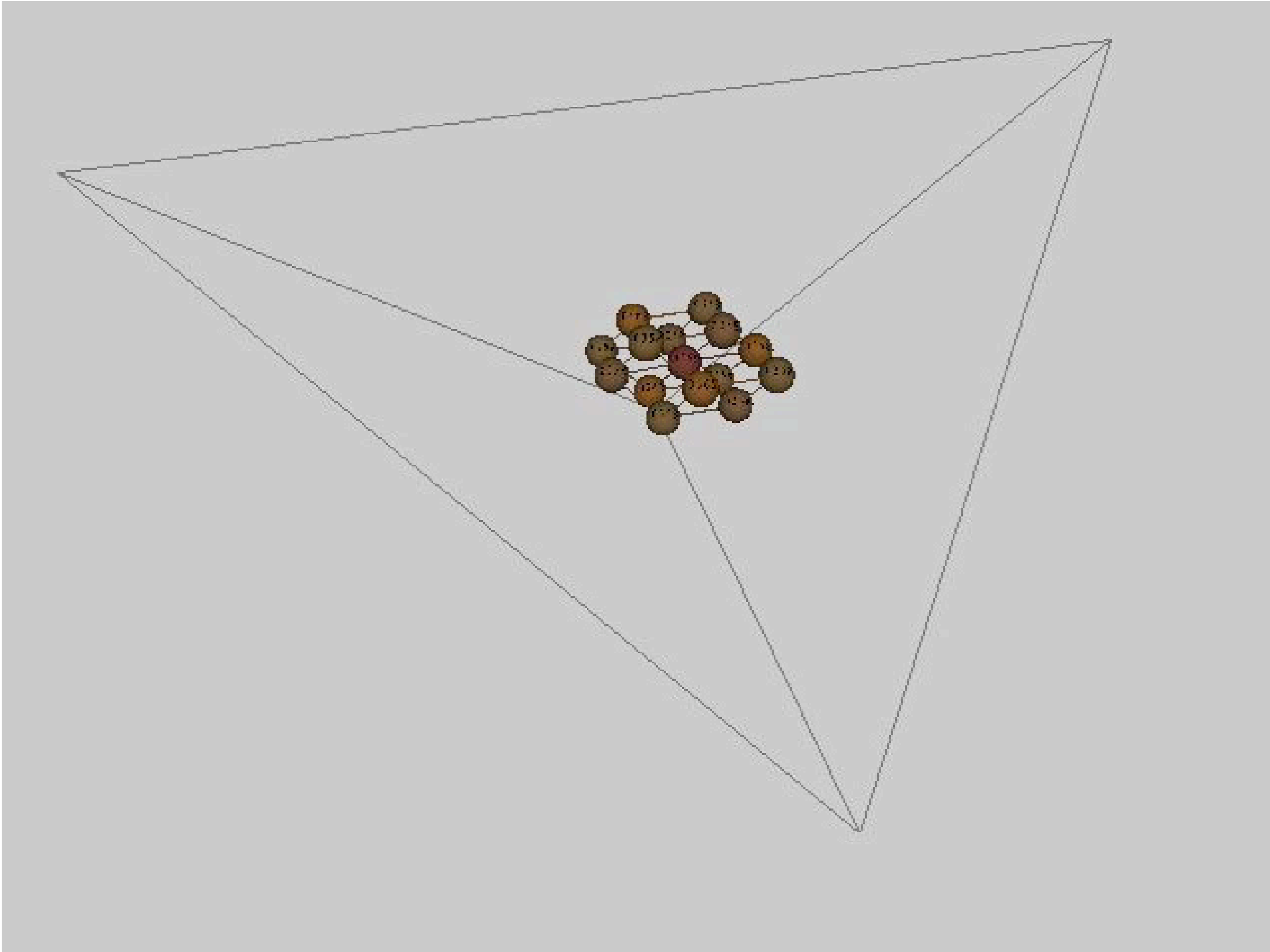


$T^3 = (\mathbf{R}/12\mathbf{Z})^3 \longrightarrow T^3/S_3$

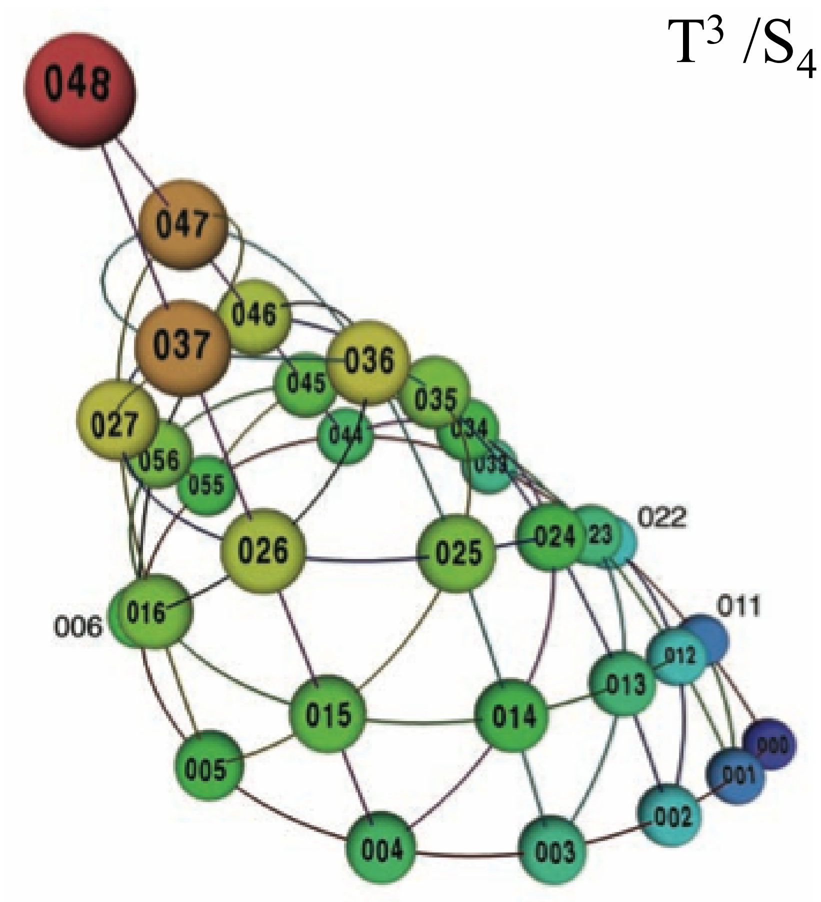


Dmitri Tymoczko, « The Geometry of Musical Chords », *Science*, 313, 2006

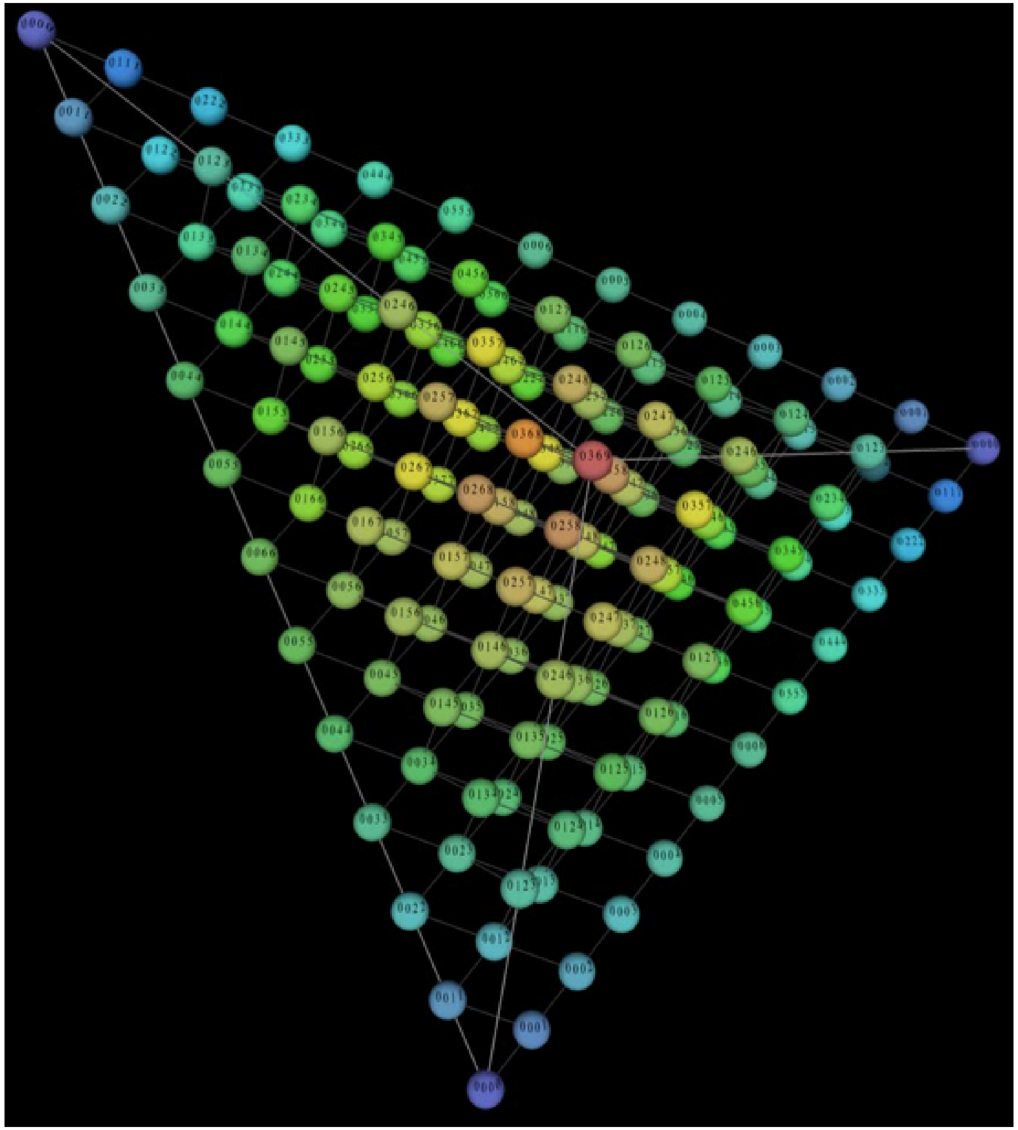
$T^4 = (\mathbf{R}/12\mathbf{Z})^4 \longrightarrow T^4/S_4$



Dmitri Tymoczko, « The Geometry of Musical Chords », *Science*, 313, 2006

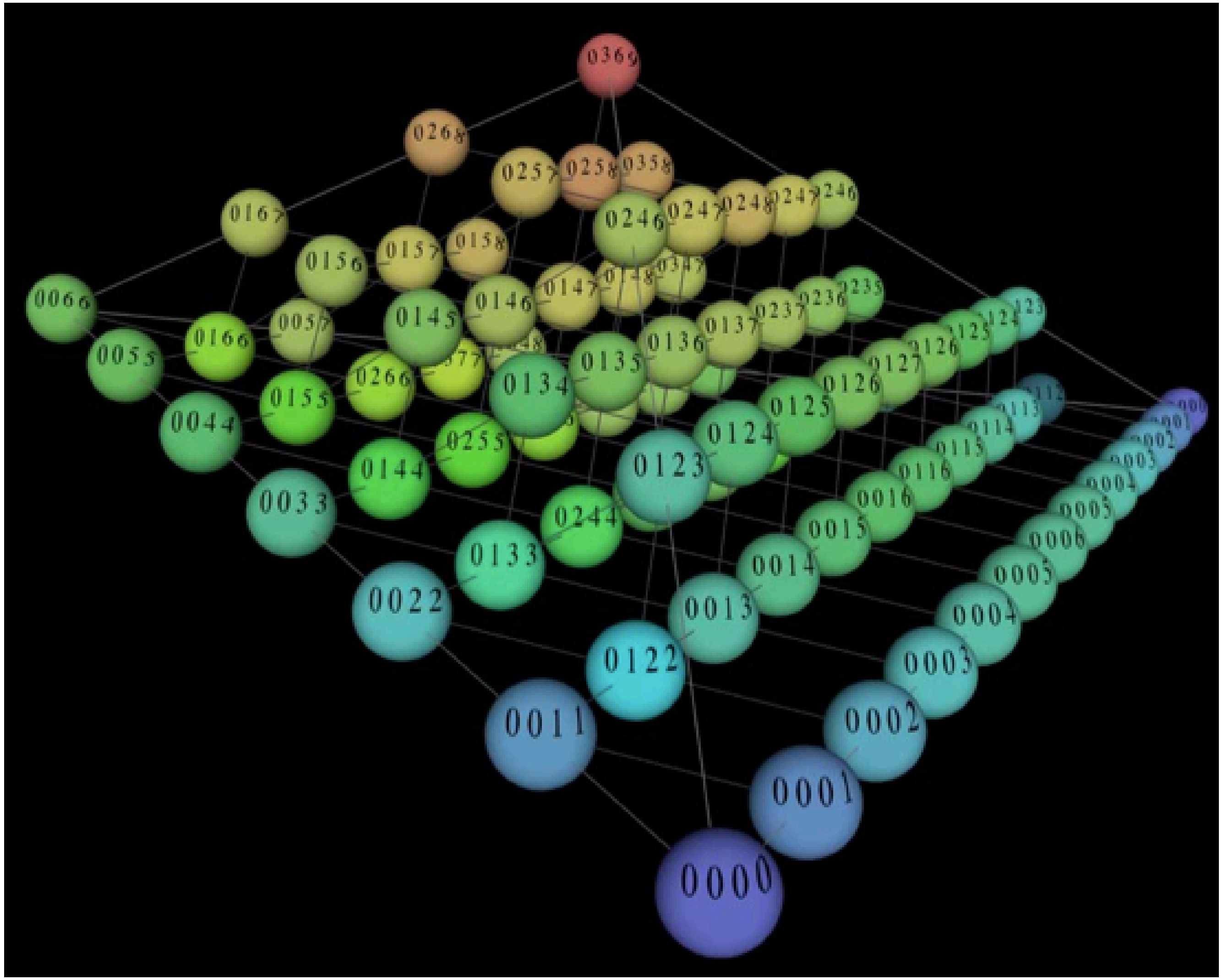


T^3/S_4



C. Callender, I. Quinn & D. Tymoczko, « Generalized Voice-Leading Spaces », *Science*, 320, 2008

$T^3/(S_4 \times Z_2)$



C. Callender, I. Quinn & D. Tymoczko, « Generalized Voice-Leading Spaces », *Science*, 320, 2008

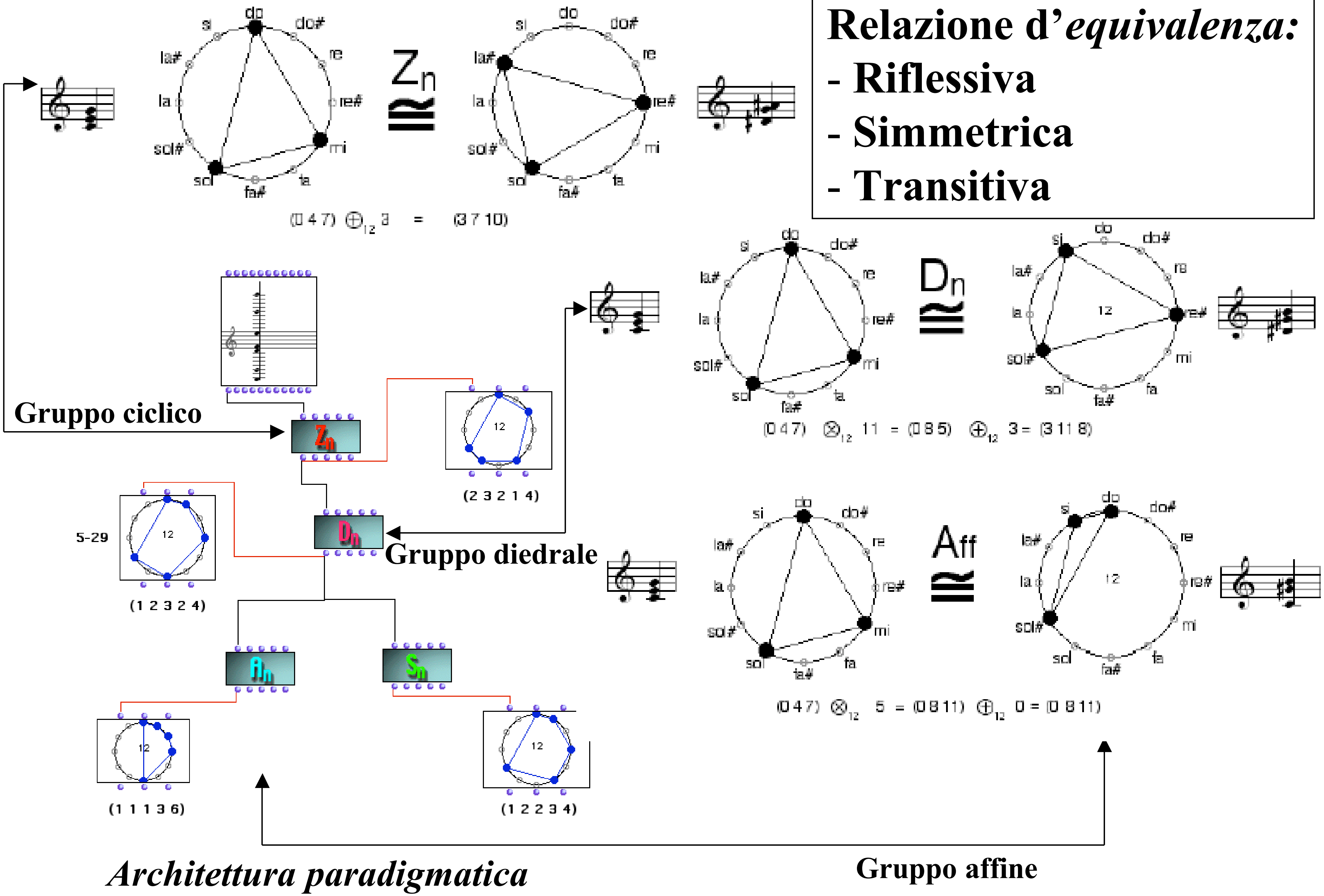
Enumerazione e classificazione delle strutture musicali

- Lemma di Burnside e teoria dell'enumerazione di Polya
  - Classificazione paradigmatica degli accordi musicali (azioni del gruppo ciclico, diedrale e affine sul sistema temperato tradizionale)
  - Modi di Messiaen a trasposizione limitata
  - Serie dodecafoniche e serie omni-intervallari
  - Asimmetria ritmica
  - Spazi microtonali

- La *Set Theory* d'Allen Forte
  - Il vettore intervallare
  - Teorema dell'esacordo (Milton Babbitt)
  - La relazione Z e gli insiemi omometrici

I gruppi come “paradigmi” per l’equivalenza fra accordi

Relazione d’*equivalenza:*
- **Riflessiva**
- **Simmetrica**
- **Transitiva**



Architettura paradigmatica e strutture algebriche in musica

« [C'est la notion de groupe qui] donne un sens précis à l'idée de structure d'un ensemble [et] permet de déterminer les éléments efficaces des transformations en réduisant en quelque sorte à son schéma opératoire le domaine envisagé. […]L'objet véritable de la science est le ***système des relations*** *et non pas les termes supposés qu'il relie. […] Intégrer les résultats - symbolisés - d'une* ***expérience*** *nouvelle revient […] à créer un canevas nouveau, un* ***groupe de transformations*** *plus complexe et plus compréhensif »*

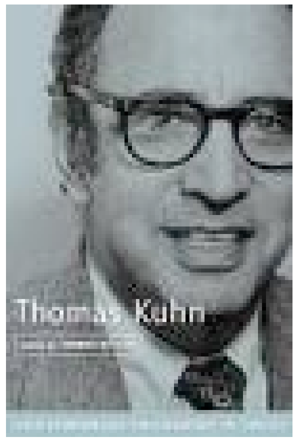
G.-G. Granger : « Pygmalion. Réflexions sur la pensée formelle », 1947

Felix Klein

Ernst Cassirer

Gilles-Gaston Granger

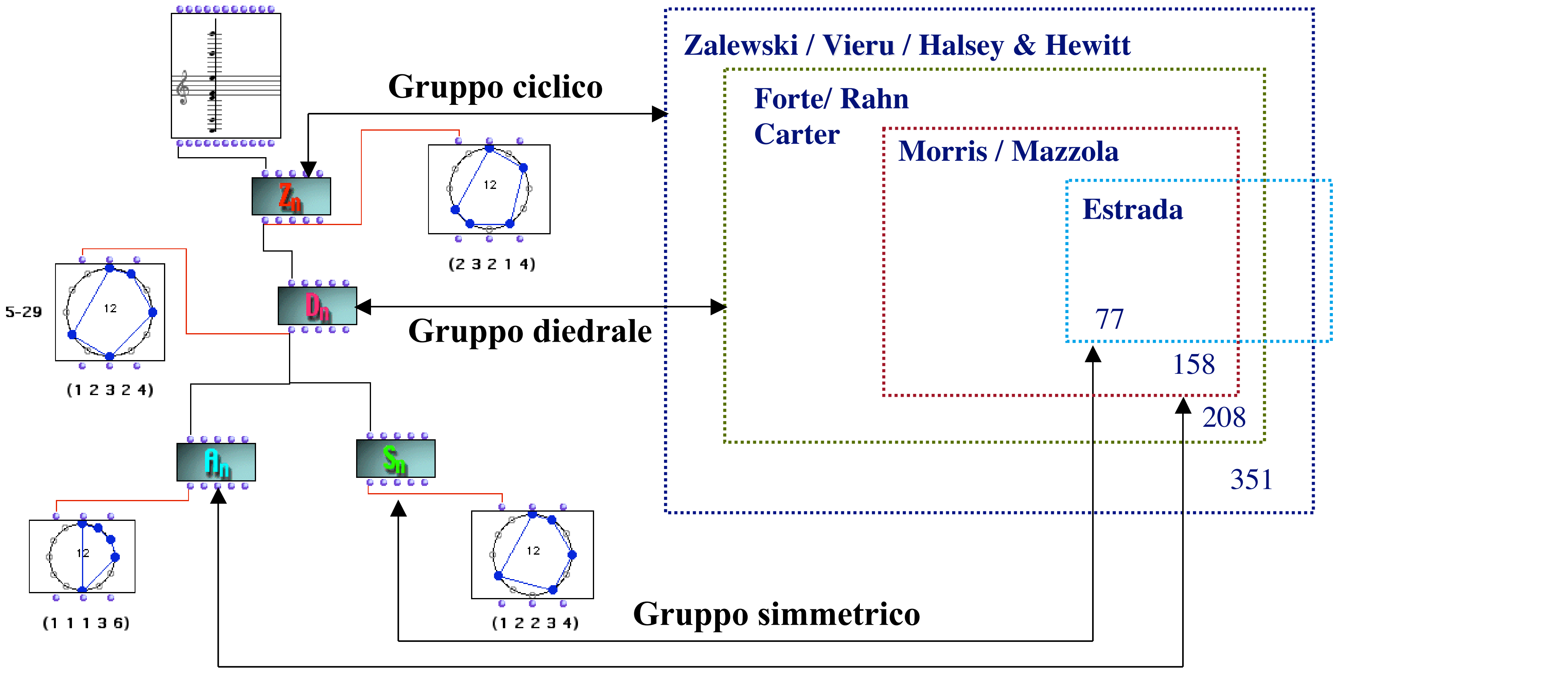
Jean Piaget



Thomas Kuhn

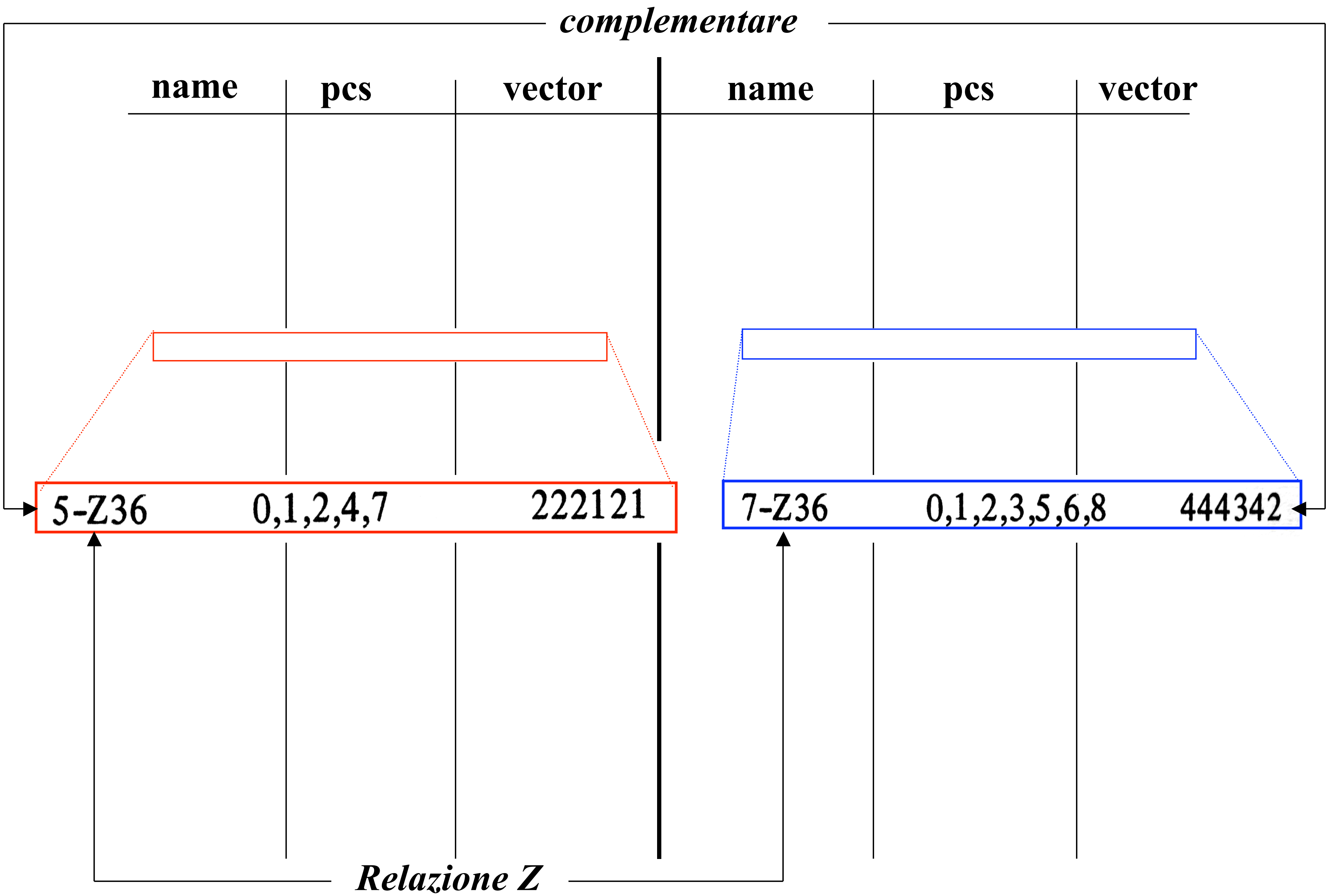
Classificazione paradigmatica delle strutture musicali

$G \backslash k$	1	2	3	4	5	6	7	8	9	10	11	12	
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1	
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1	**Set Theory**
$\mathrm{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1	

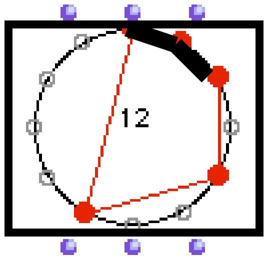


Architettura paradigmatica

La *Set Theory* d'Allen Forte: catalogo dei *pitch-class sets*



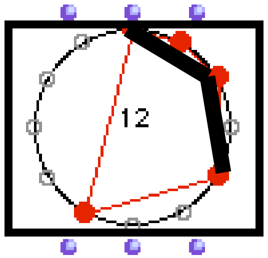
Vettore intervallare e relazione Z



Il vettore intervallare (Forte) esprime la frequenza di apparizione di ogni intervallo (modulo il suo complementare)

5-Z36 0,1,2,4,7 222121

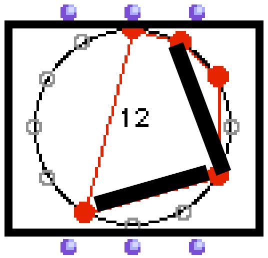
Vettore intervallare e relazione Z



Il vettore intervallare (Forte) esprime la frequenza di apparizione di ogni intervallo (modulo il suo complementare)

5-Z36	0,1,2,4,7	222121

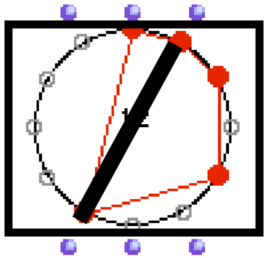
Vettore intervallare e relazione Z



Il vettore intervallare (Forte) esprime la frequenza di apparizione di ogni intervallo (modulo il suo complementare)

5-Z36	0,1,2,4,7	222121

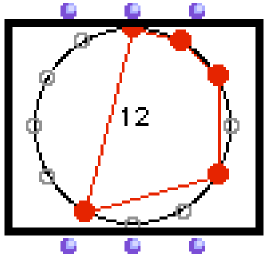
Vettore intervallare e relazione Z



Il vettore intervallare (Forte) esprime la frequenza di apparizione di ogni intervallo (modulo il suo complementare)

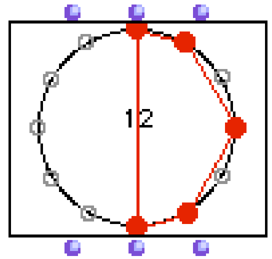
5-Z36	0,1,2,4,7	222121

Vettore intervallare e relazione Z



Il vettore intervallare (Forte) esprime la frequenza di apparizione di ogni intervallo (modulo il suo complementare)

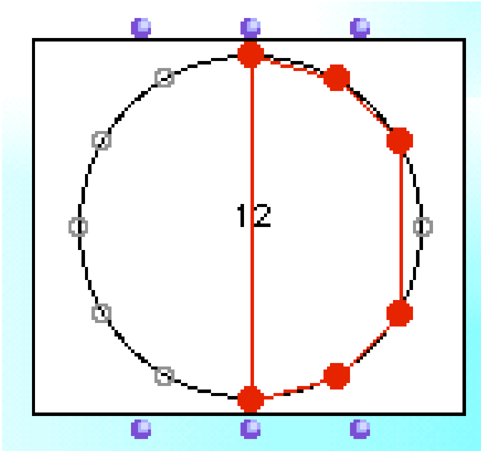
5-Z36	0,1,2,4,7	222121



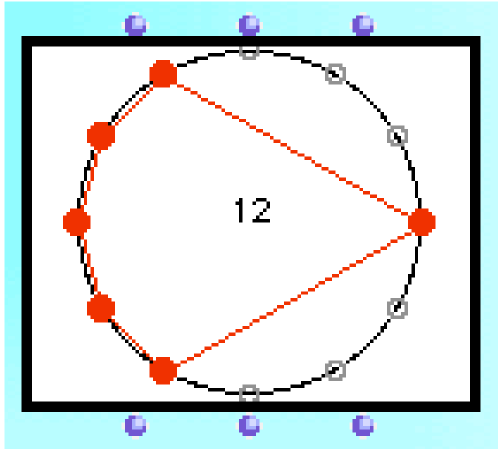
5-Z12

Teorema dell'esacordo (o teorema di Babbitt)

(Wilcox, Ralph Fox (?), Chemillier, Lewin, Mazzola, Schaub, …, Amiot [2006])



A



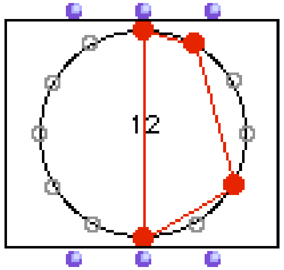
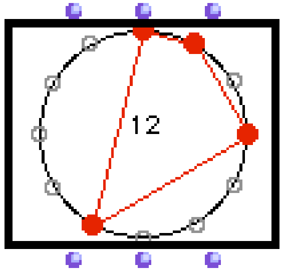
A'

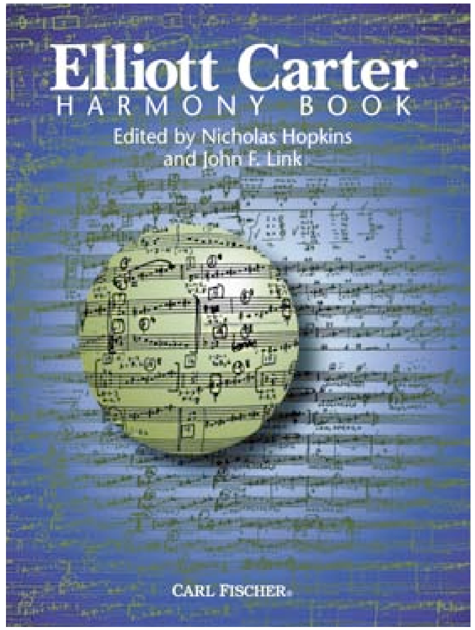
$$IV(A) = [4, 3, 2, 3, 2, 1] = [4, 3, 2, 3, 2, 1] = IV(A')$$

Un esacordo e il suo complementare hanno lo stesso vettore intervallare

=> *OpenMusic*

Elliott Carter’s *Harmony Book* (2002)

		pcset	*vecteur*	*Forte*
18		(0 1 4 6)	[111111]	4-Z15
23		(0 1 3 7)	[111111]	4-Z29



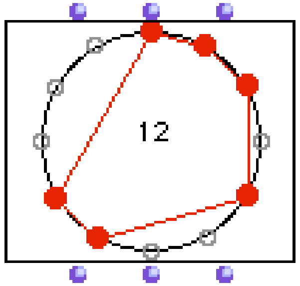
Utilizzazione (implicita) della *Z-relation*

- *Quartetto n°1* (1951)
- *Night Fantasies* (1980)
- *90+* (1994)
- …

Elliott Carter: 90+ (1994)

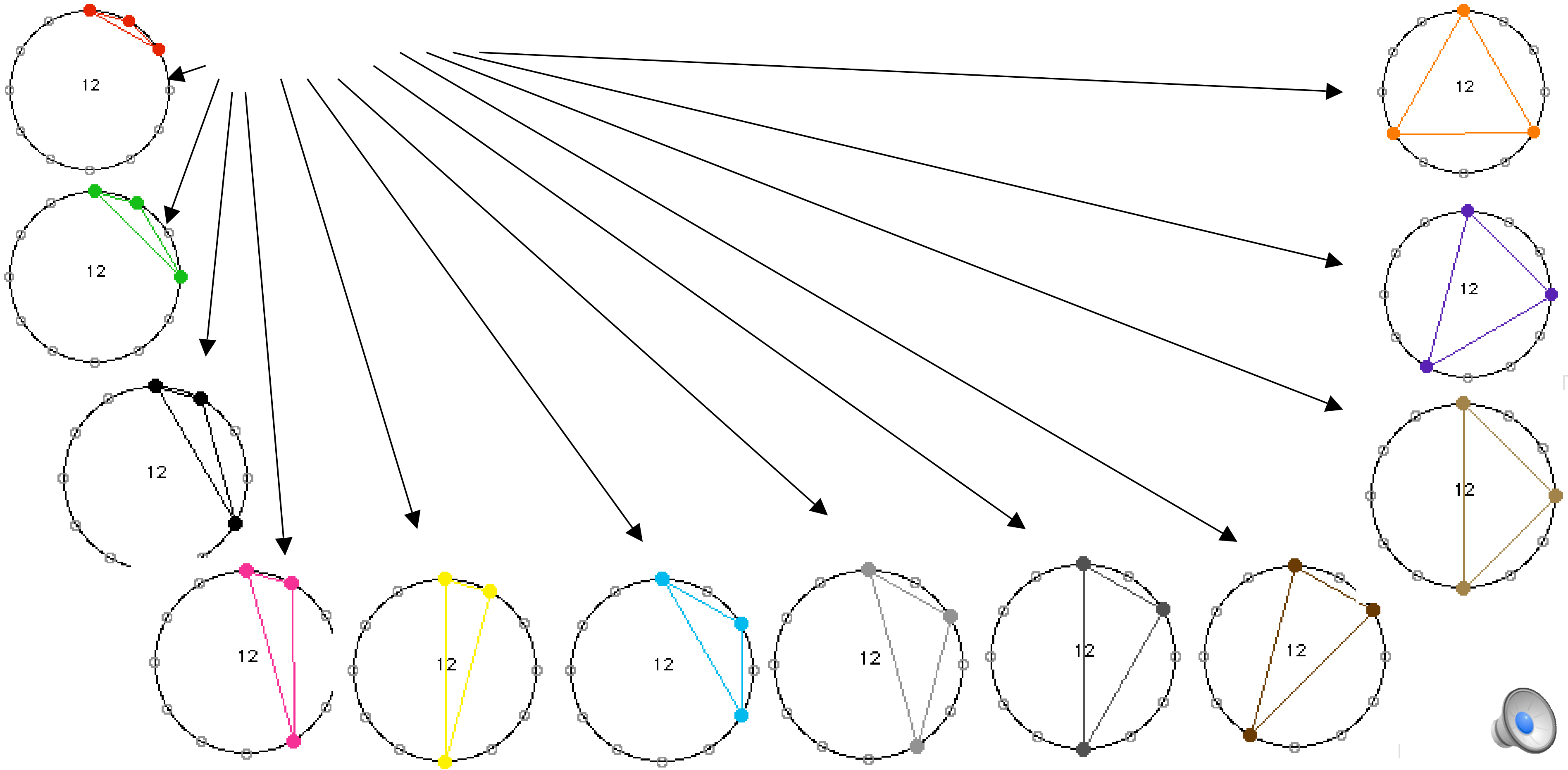
$G\backslash k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1
$\mathrm{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1

35

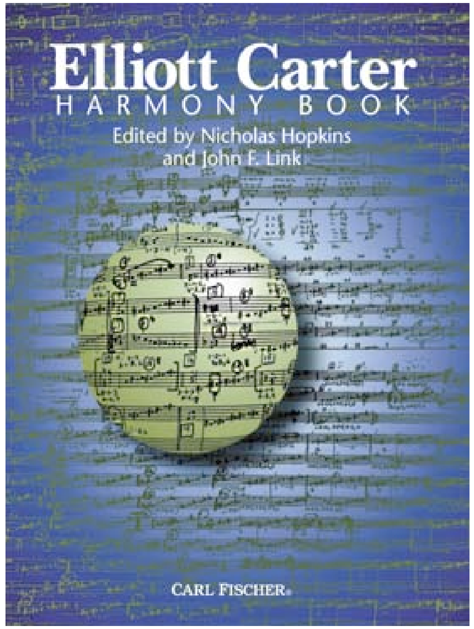


pcset	*vector*	*name*
(0 1 2 4 7 8)	[322332]	6-Z17

All-triad hexachord



Hommage à Elliott Carter

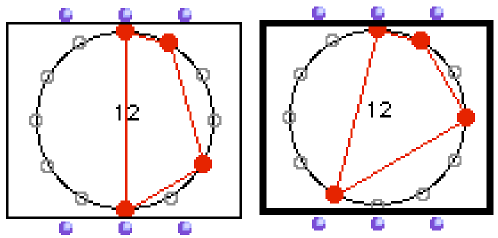


Colloque international
Organisé par le **Centre de Recherche sur les Arts et le Langage (EHESS-CNRS)**
et l'**Ircam-Centre Pompidou** avec le soutien de la Fondation Paul Sacher

Sous la direction de Max Noubel (Université de Bourgogne / CRAL, équipe Musique)
en collaboration avec Moreno Andreatta (équipe Représentations musicales, Ircam)
et Nicolas Donin (équipe Analyse des pratiques musicales, Ircam)

***1ère journée :* Jeudi 11 décembre 2008** **10h-18h**
***2ème journée :* Vendredi 12 décembre 2008** **9h30-18h**
Salle Igor-Stravinsky, Ircam, Paris
Entrée libre dans la limite des places disponibles

http://recherche.ircam.fr/carter/



Teorie trasformazionali, diatoniche e neo–riemanniane

- Il sistema d'intervalli generalizzati (GIS) di David Lewin
  - La funzione intervallare e la trasformata di Fourier discreta
  - Teorema generale dell'esacordo

- Reticoli di Klumpenhouwer (*K-nets)*
  - Isografie forti
  - Isografie positive
  - Isografie negative

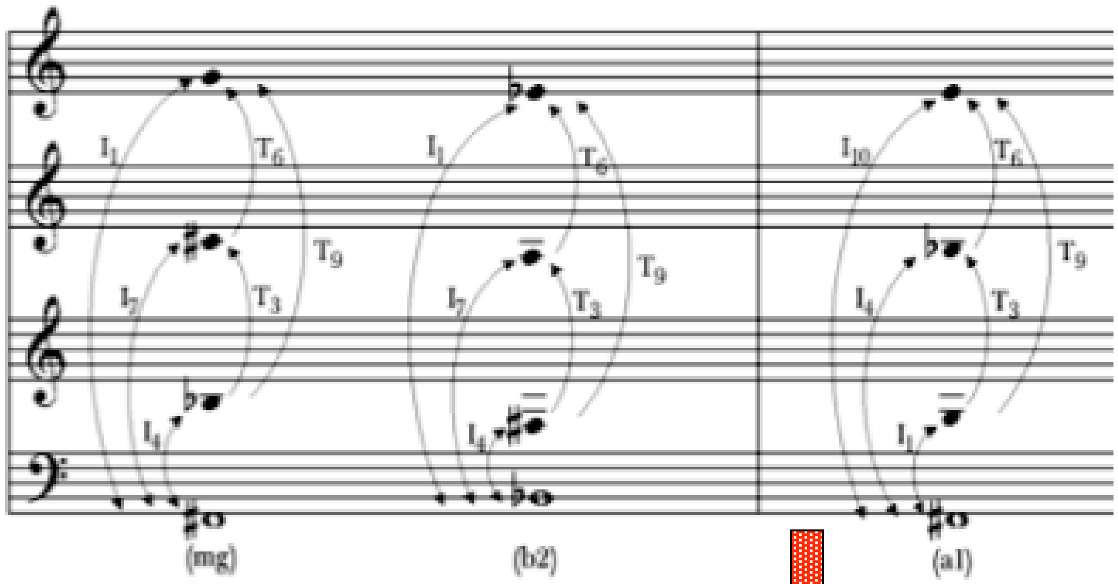
- Teorie diatoniche
  - Unicità della scala diatonica
  - Insiemi ripartiti in maniera massimale (*Maximally Even Sets*)
  - Scale ben formate (*Well-formed scales*)
  - Diatonismo *vs* cromatismo

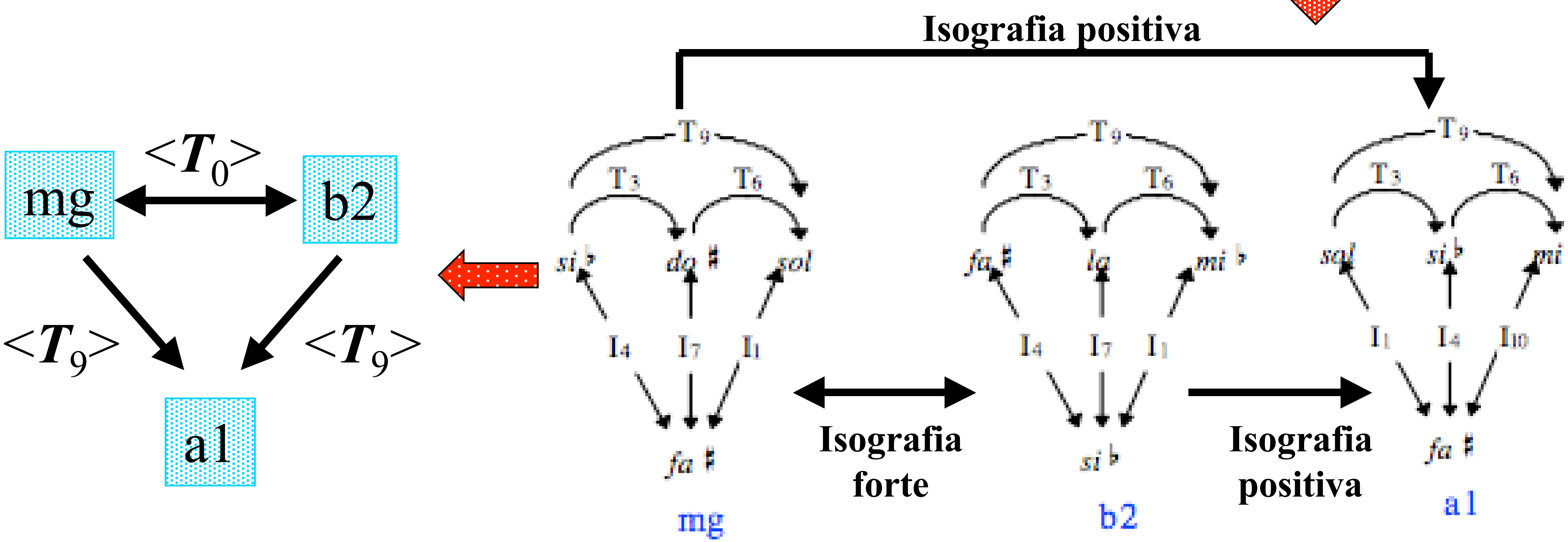
- Teorie neo-riemanniane
  - Dualità trasposizione / inversione
  - Cenni di grammatiche formali (*Christoffel words*)

Klumpenhouwer Networks (K-nets)

Xavier Hascher: « Liszt et les sources de la notion d'agrégat », Analyse Musicale, 43, 2002

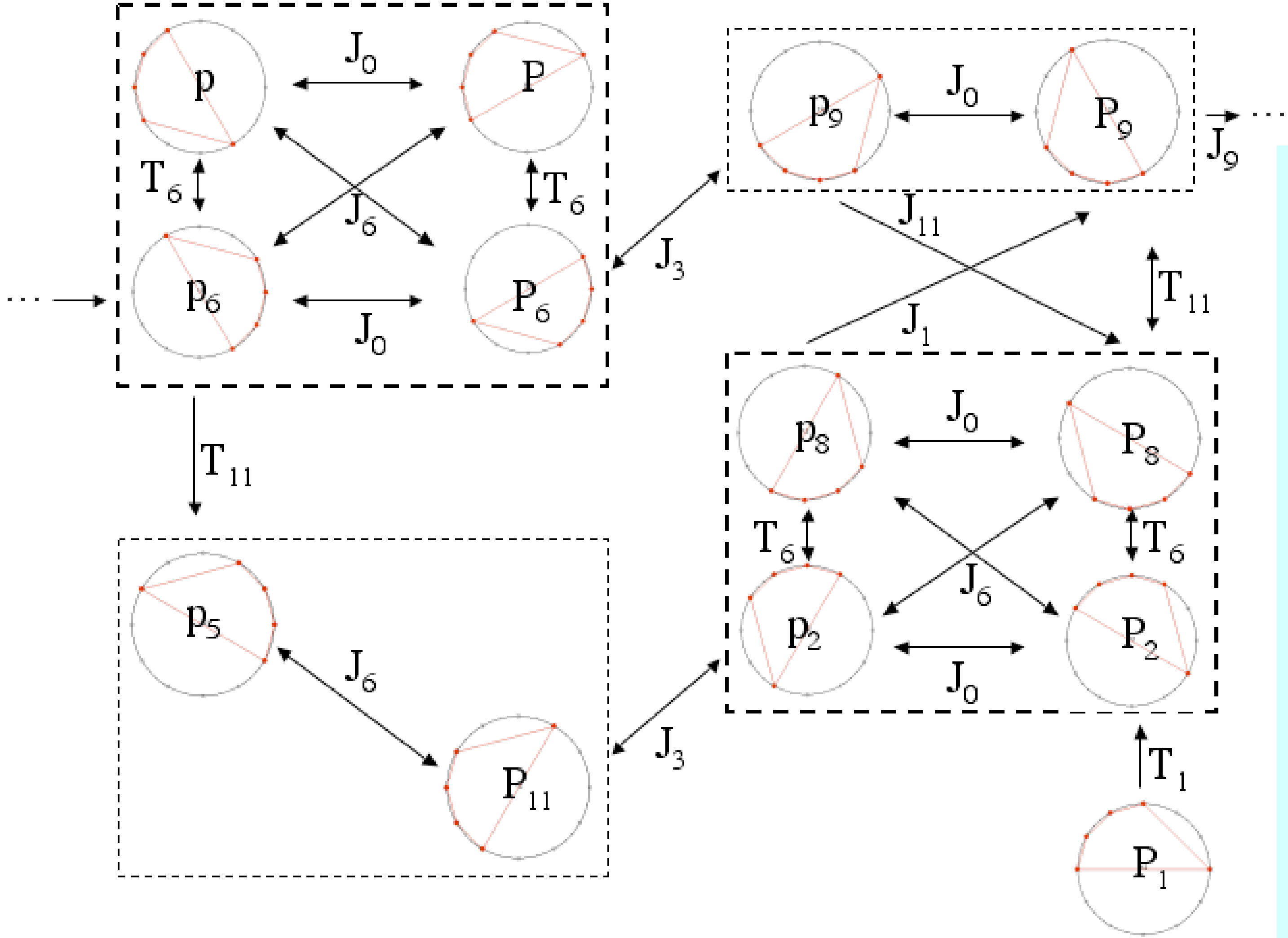
Ex. 1 - « Ladislaus Teleki » (*Historische ungarische Bildnisse* n° 4), mes. 1-7
Les agrégats dans la classification de Forte





Analisi trasformazionale: reticolo

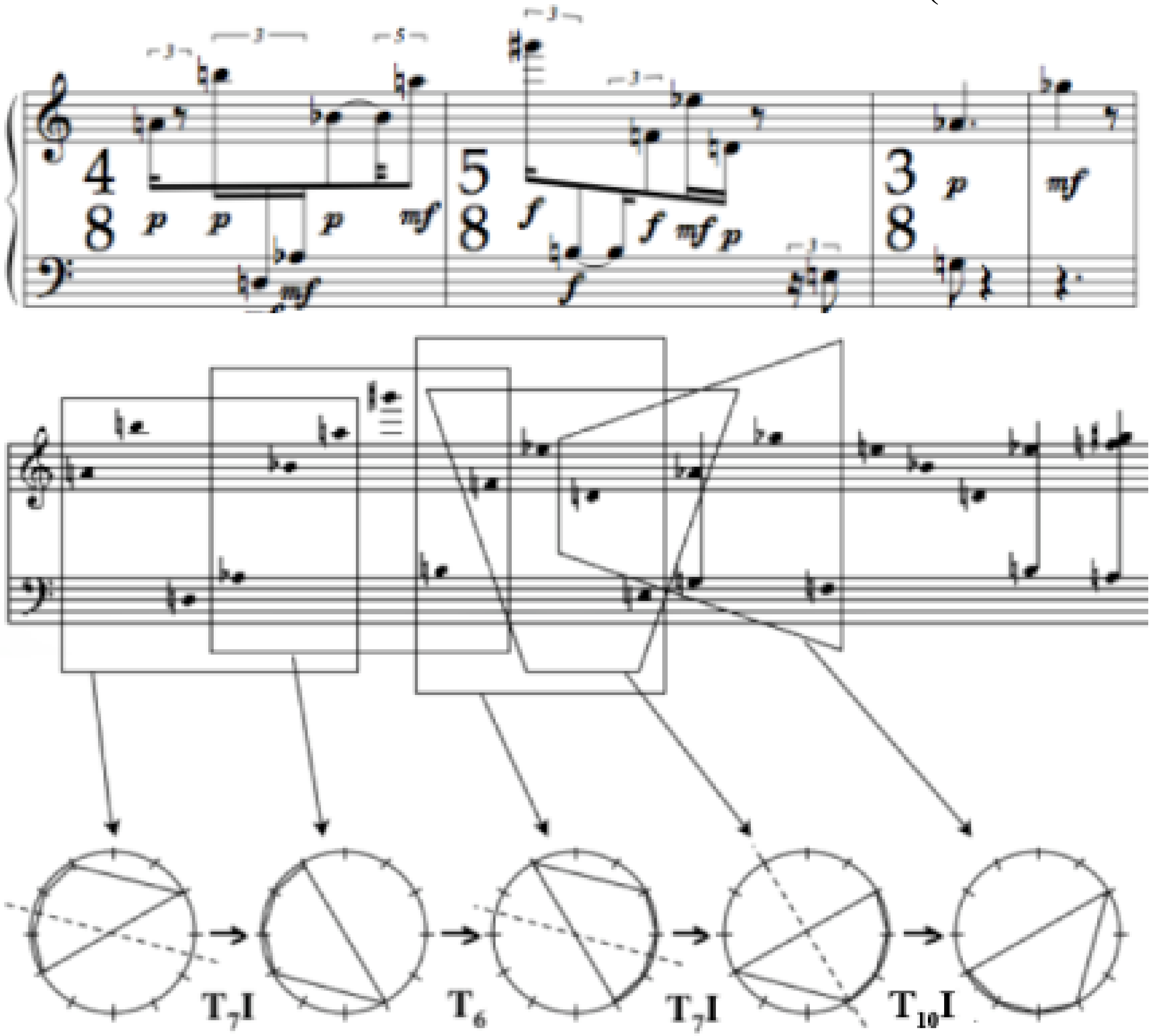
Stockhausen: *Klavierstück III* (Analisi di D. Lewin)



« [...] the sequence of events moves within a clearly defined world of possible relationships, and because - in so moving - it makes the abstract space of such a world accessible to our sensibilities. That is to say that the story projects what one would traditionally call form. »

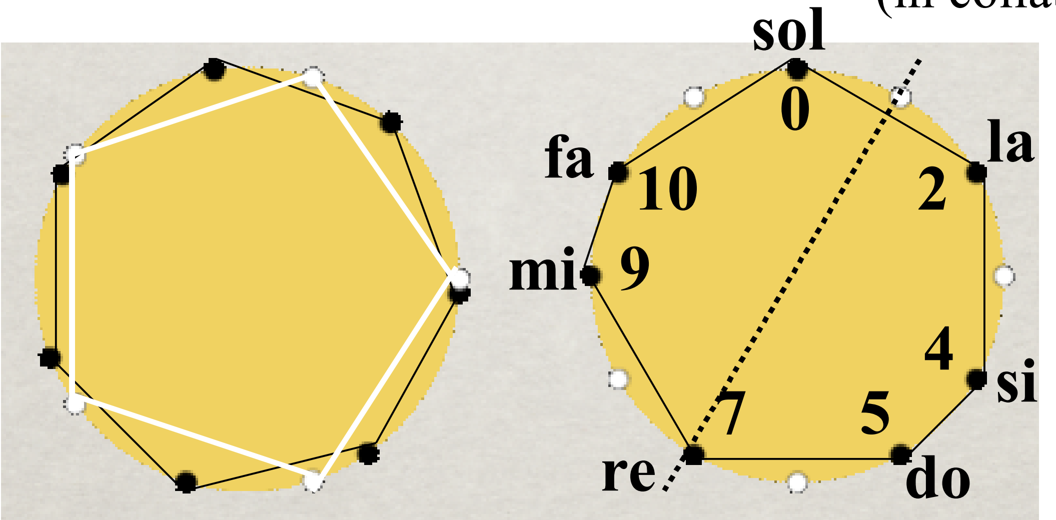
Segmentazione per « imbricazione »: progressione trasformazionale

Stockhausen: *Klavierstück III* (Analisi di D. Lewin)



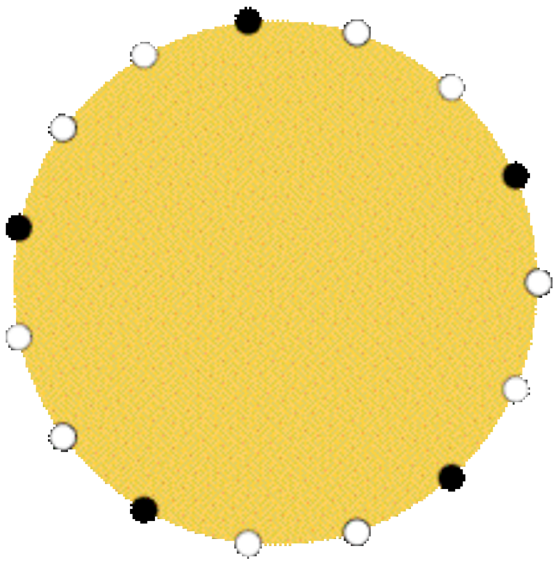
Maximally-Even Sets (Me–sets)

(in collaborazione con Emmanuel Amiot)



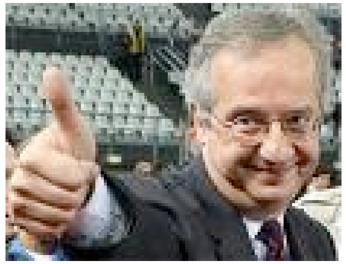
Scala diatonica:
{0, 2, 4, 5, 7, 9, 10}

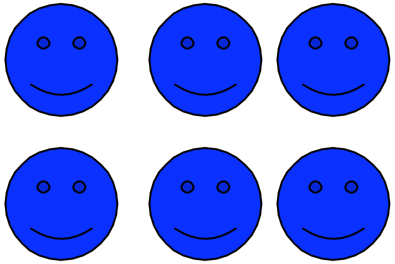
Scala pentatonica:
{1, 3, 6, 8, 11}

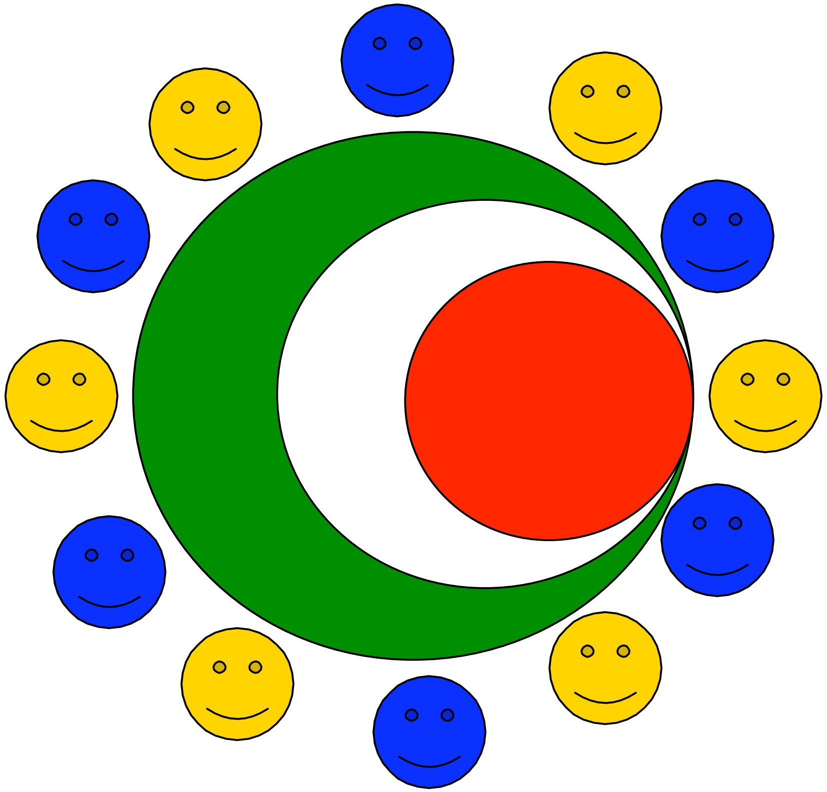


La scala diatonica come *ME-set*

<u>The dinner table problem</u> (Italian version)

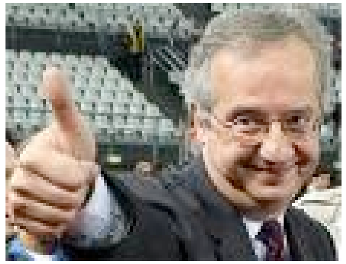


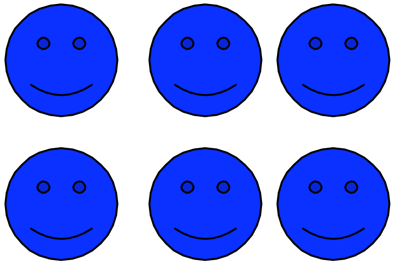


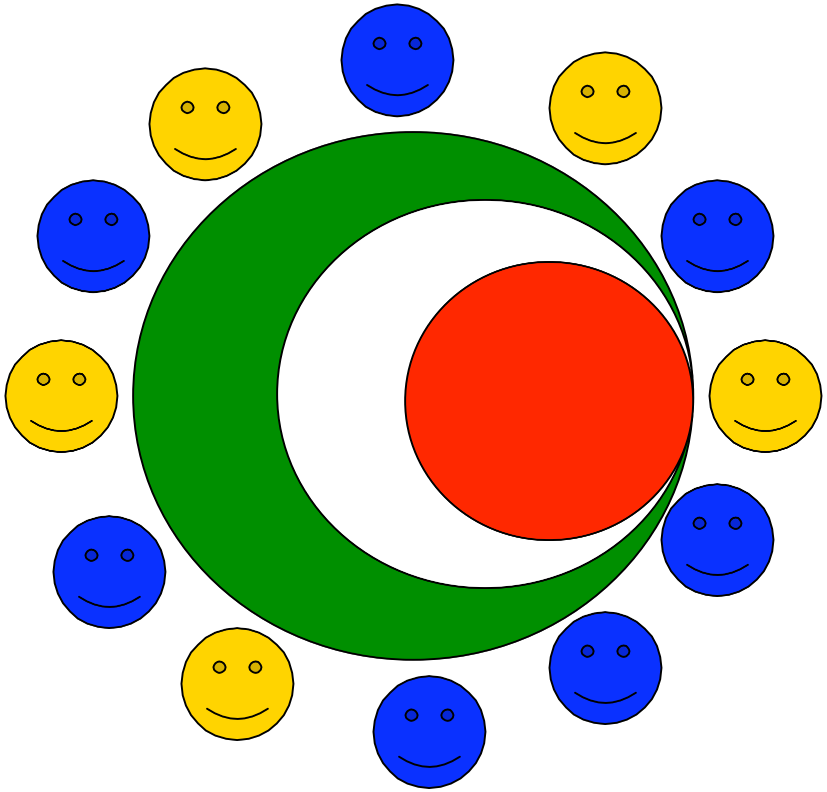


La scala diatonica come *ME-set*

<u>The dinner table problem</u> (Italian version)

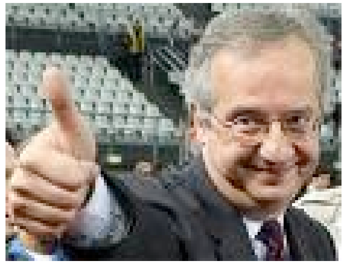


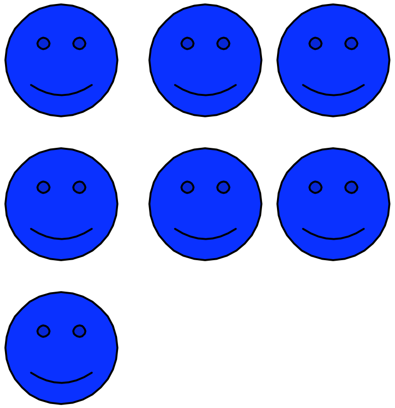


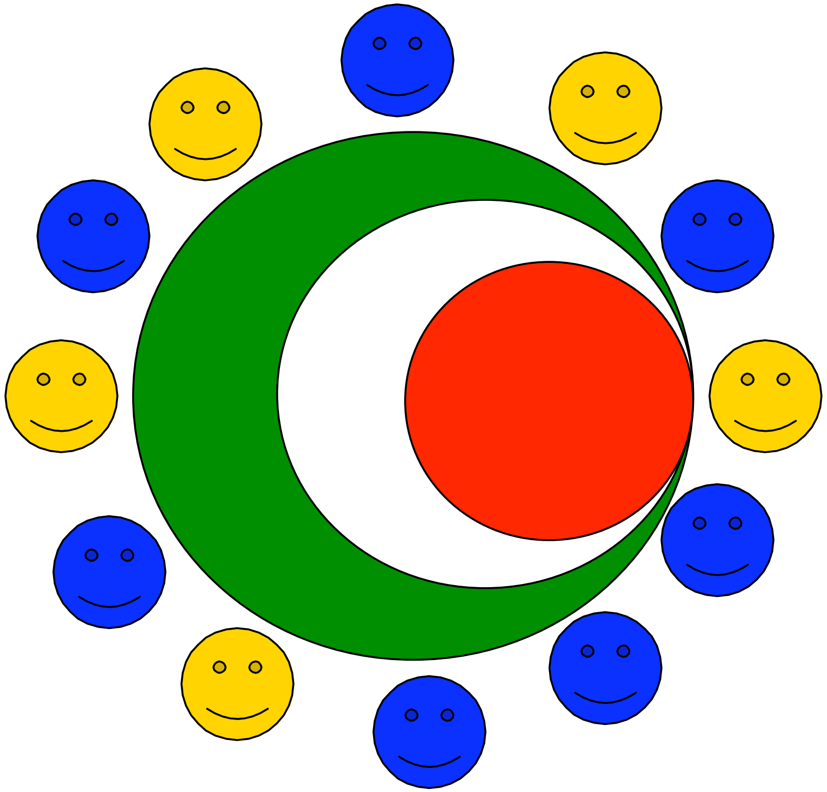


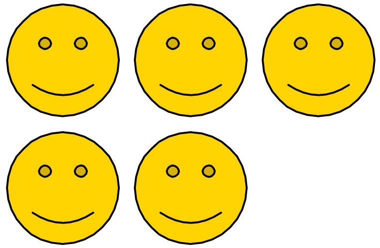
La scala diatonica come *ME-set*

<u>The dinner table problem</u> (Italian version)



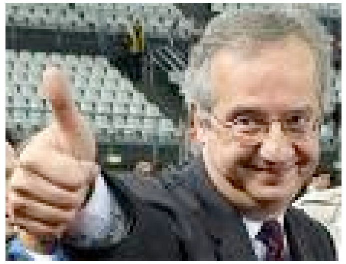


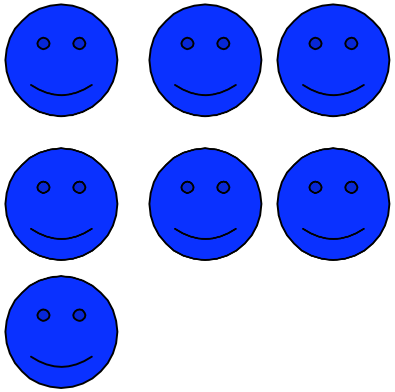


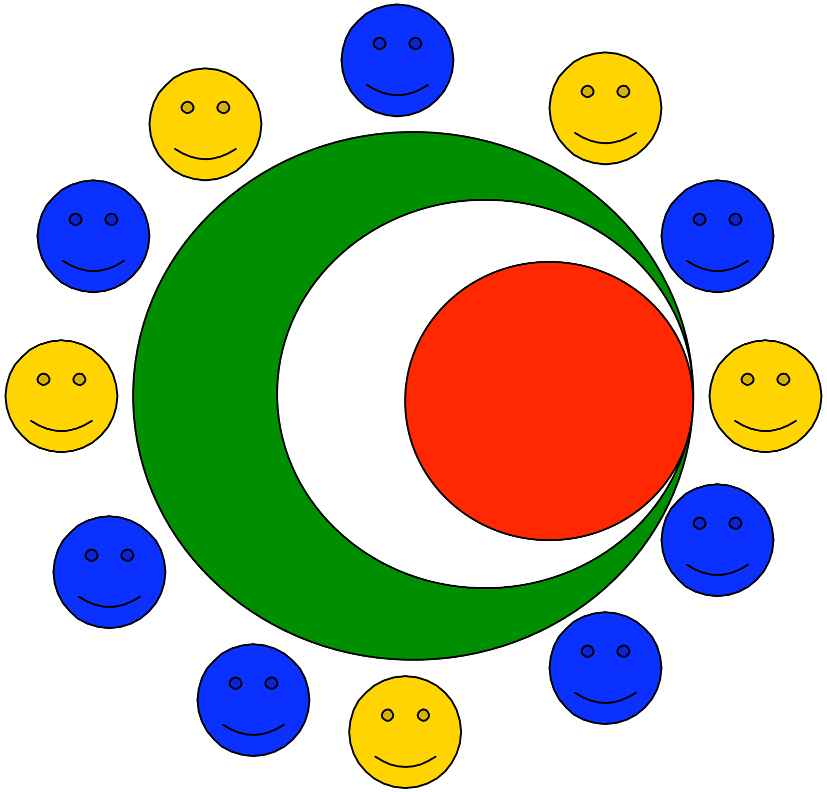


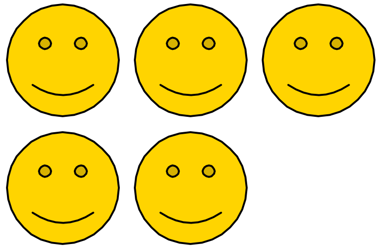
La scala diatonica come *ME-set*

The dinner table problem (Italian version)



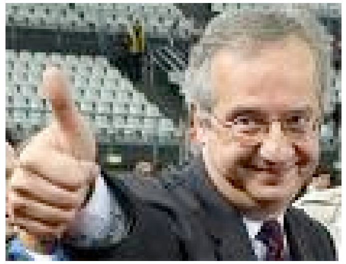


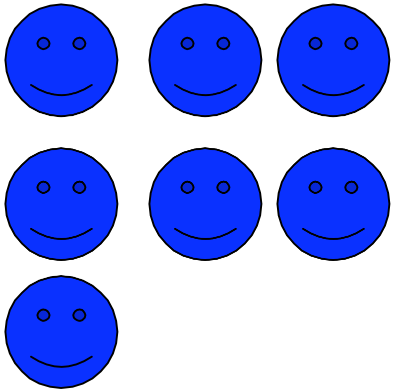


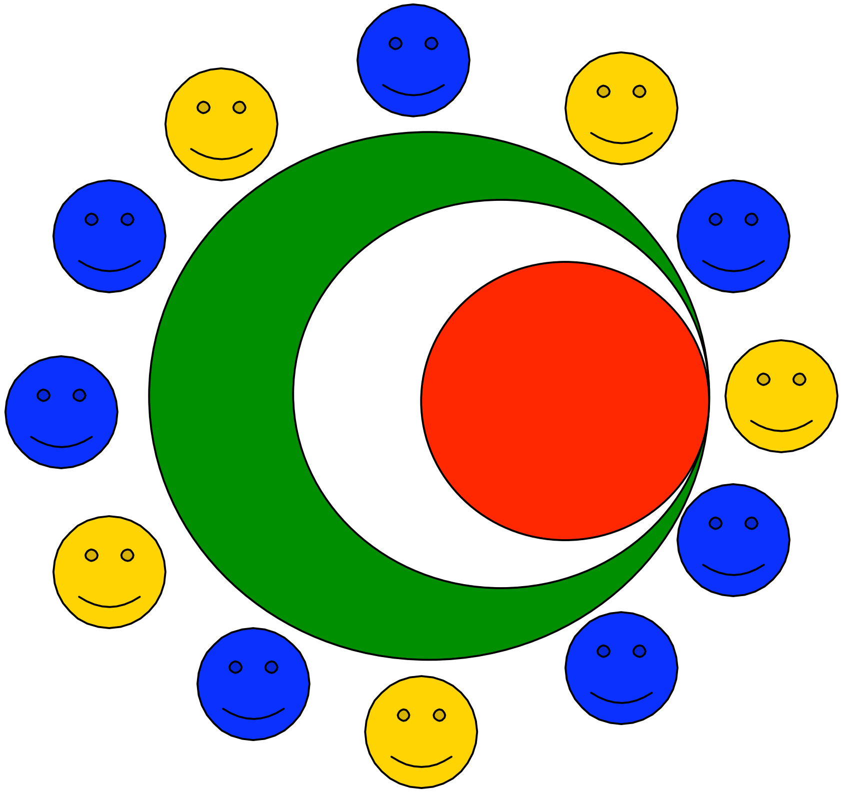


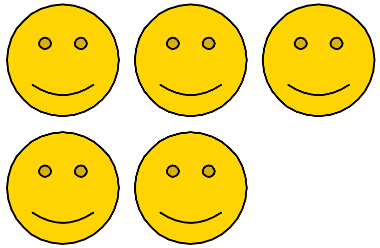
La scala diatonica come *ME-set*

<u>The dinner table problem</u> (Italian version)



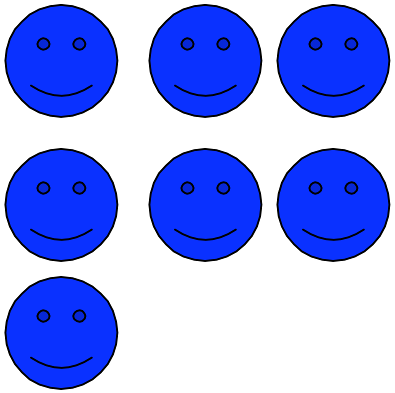


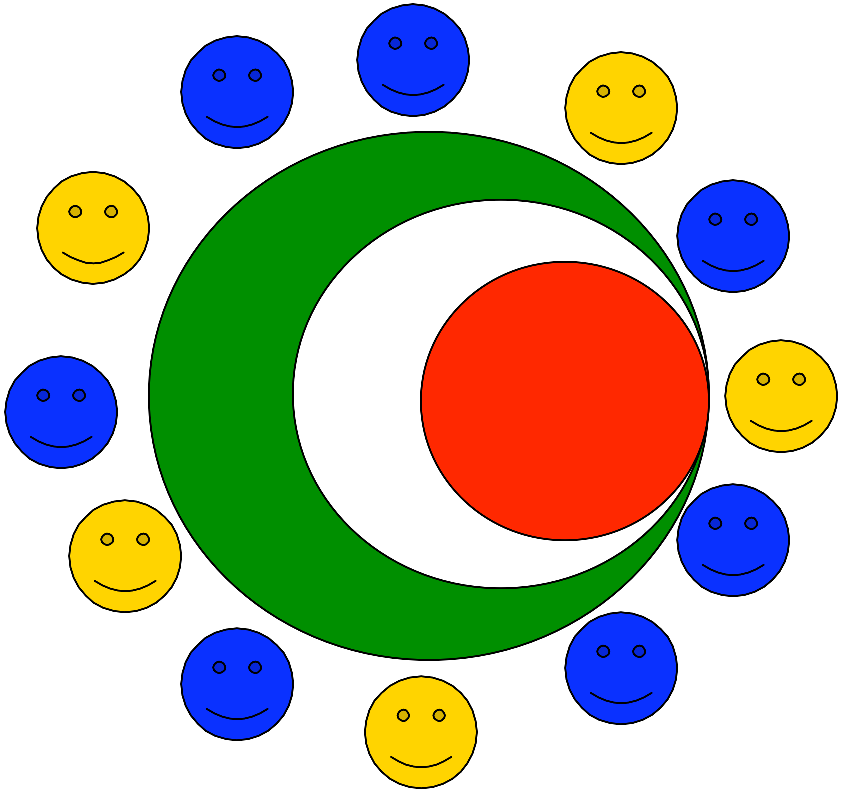


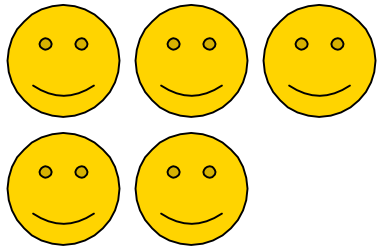


La scala diatonica come *ME-set*

The dinner table problem (Italian version)

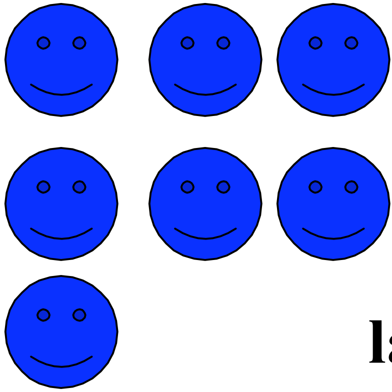


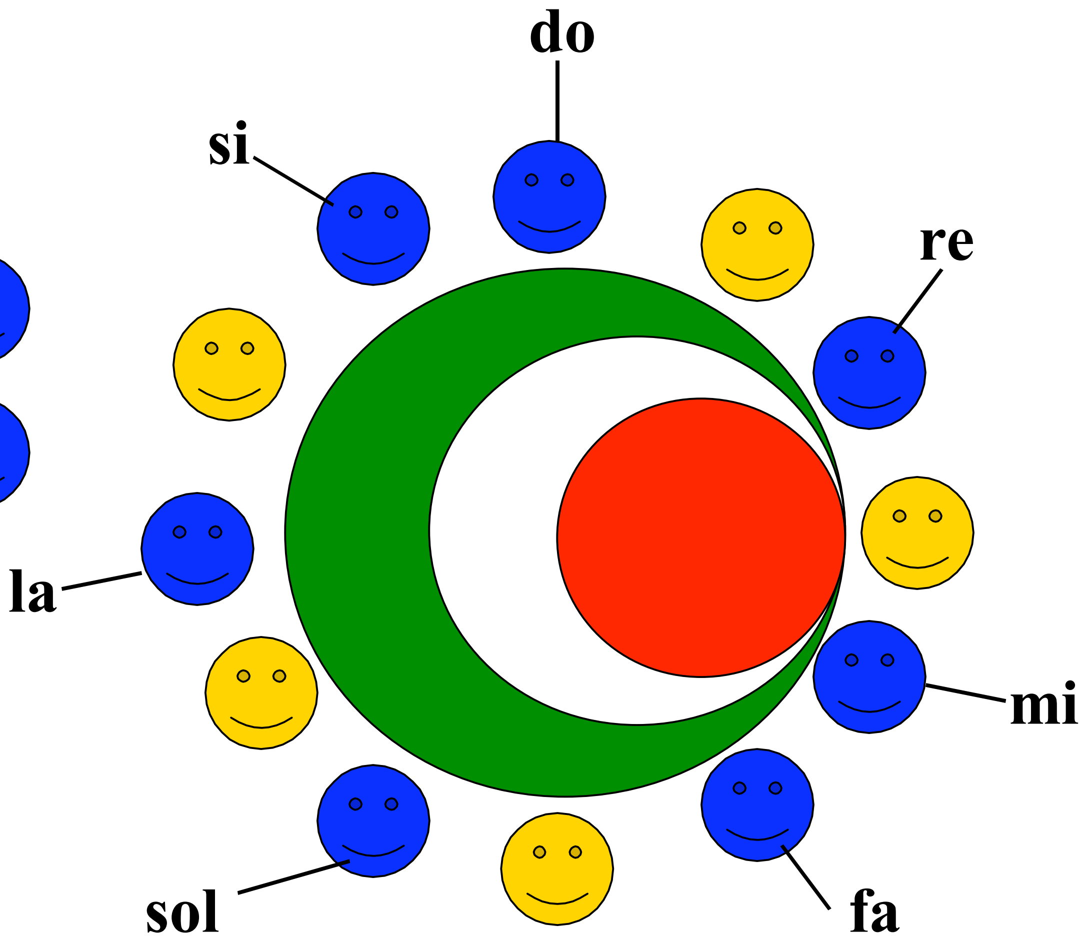


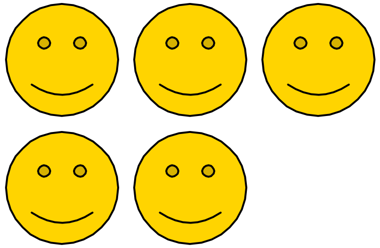


La scala diatonica come *ME-set*

The dinner table problem (Italian version)







=> *OpenMusic*

Jack Douthett &Richard Krantz, “Energy extremes and spin configurations for the one-dimensional antiferromagnetic Ising model with arbitrary-range interaction”, *J. Math. Phys.* 37 (7), July 1996

Tassellazioni musicali: la costruzione dei canoni a mosaico

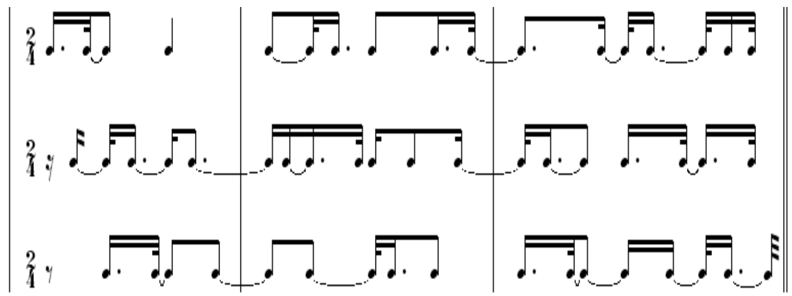
- Fattorizzazione di gruppi ciclici
  - Gruppi di Hajos e gruppi non-Hajos
  - Teorema di Hajos
  - Teorema di Redei

- Fattorizzazioni polinomiali (polinomi ciclotomici)
  - Condizioni di Coven-Meyerowitz

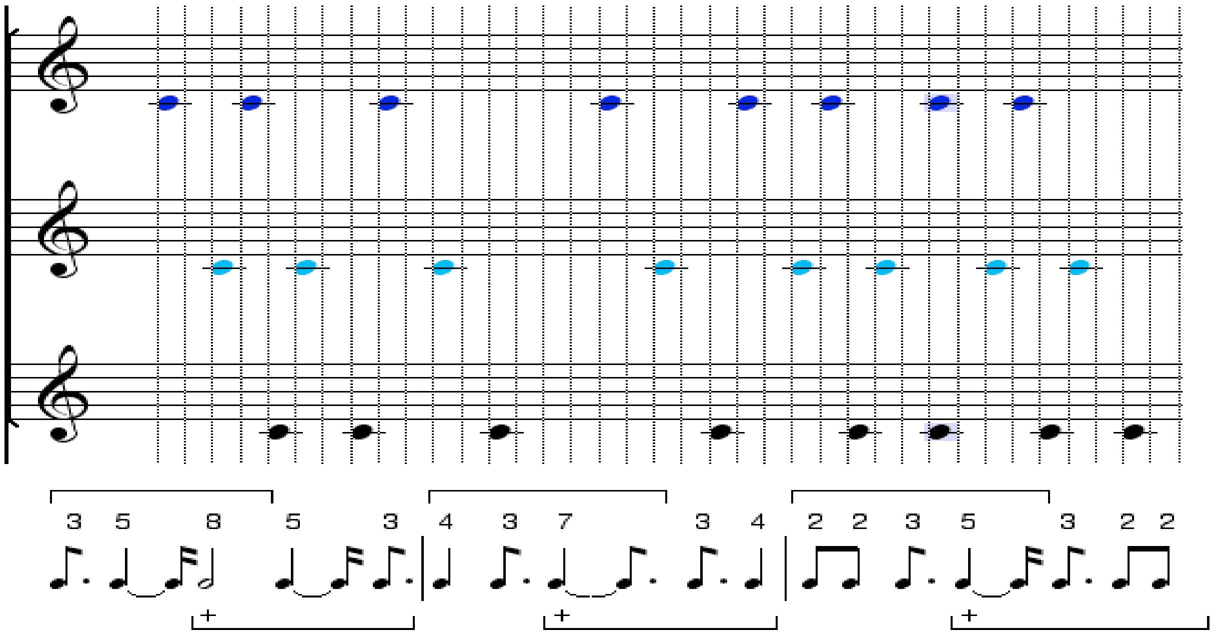
- Congetture geometrico-algebriche
  - Congettura di Minkowski
  - Congettura di Keller
  - Congettura di Fuglede (congettura spettrale)

Olivier Messiaen e i canoni ritmici

Harawi (1945)



Visions de l'Amen (1943)

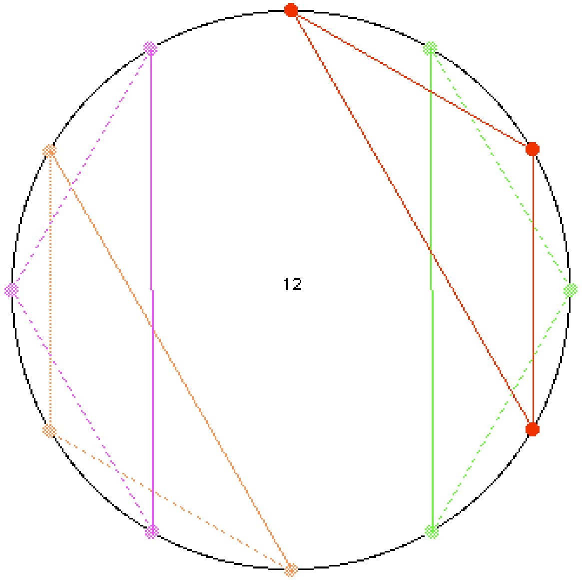


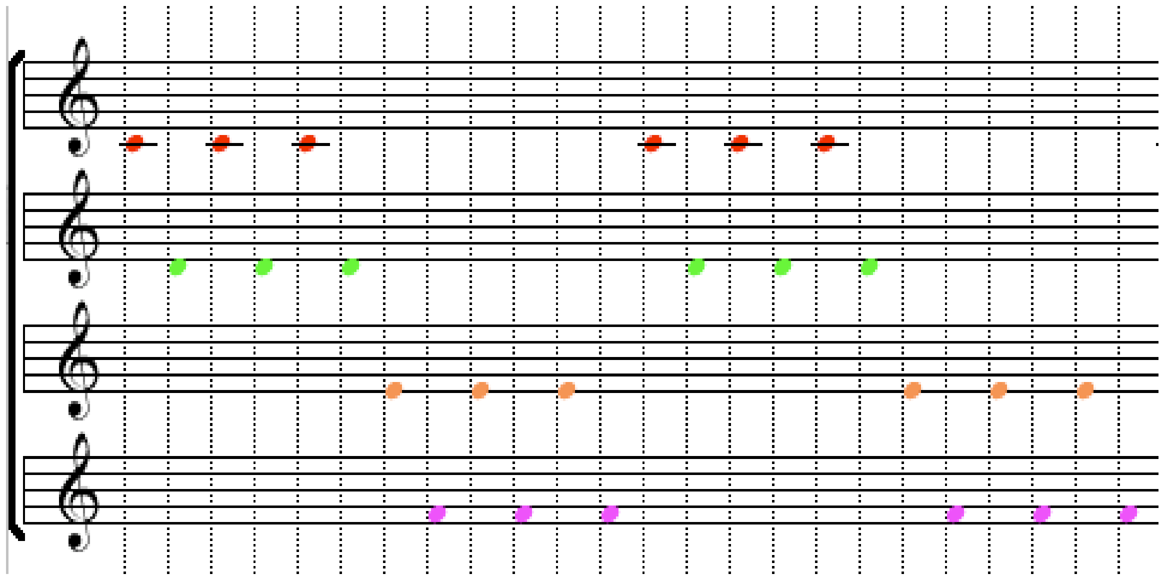
Modèle
rythmique

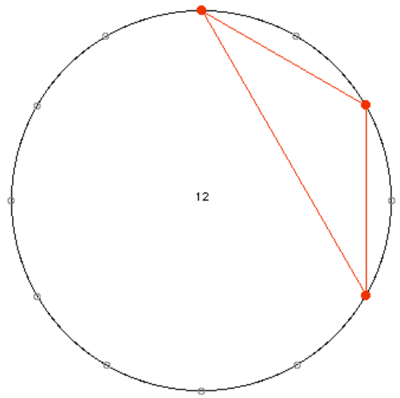
« …il résulte de tout cela que les différentes sonorités se mélangent ou s'opposent de manières très diverses, **jamais au même moment ni au même endroit […]. C'est du désordre organisé** »

O. Messiaen : *Traité de Rythme, de Couleur et d'Ornithologie,* tome 2, Alphonse Leduc, Paris, 1992

Canoni a mosaico a simmetria trasposizionale



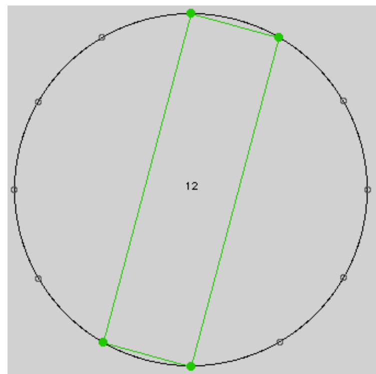




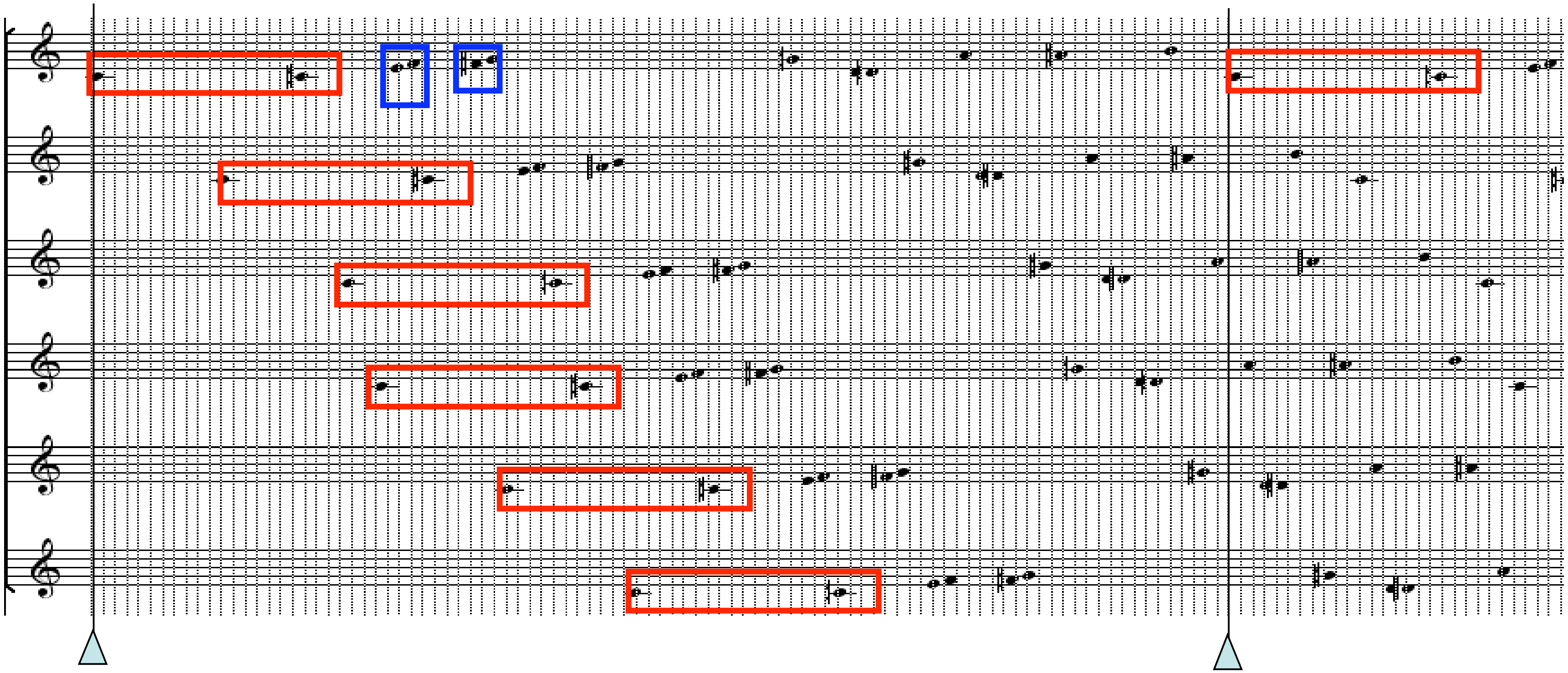
$\mathbf{Z}_{12} = A \oplus B$

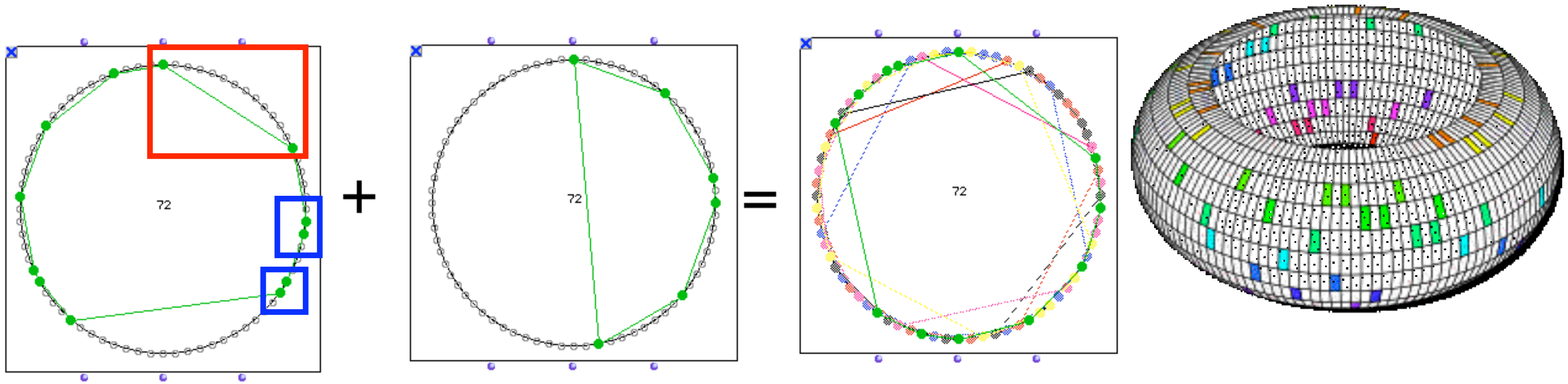
$A = \{0,2,4\}$

$B = \{0,1,6,7\}$



Vuza Canons : canoni a mosaico senza periodicità interne

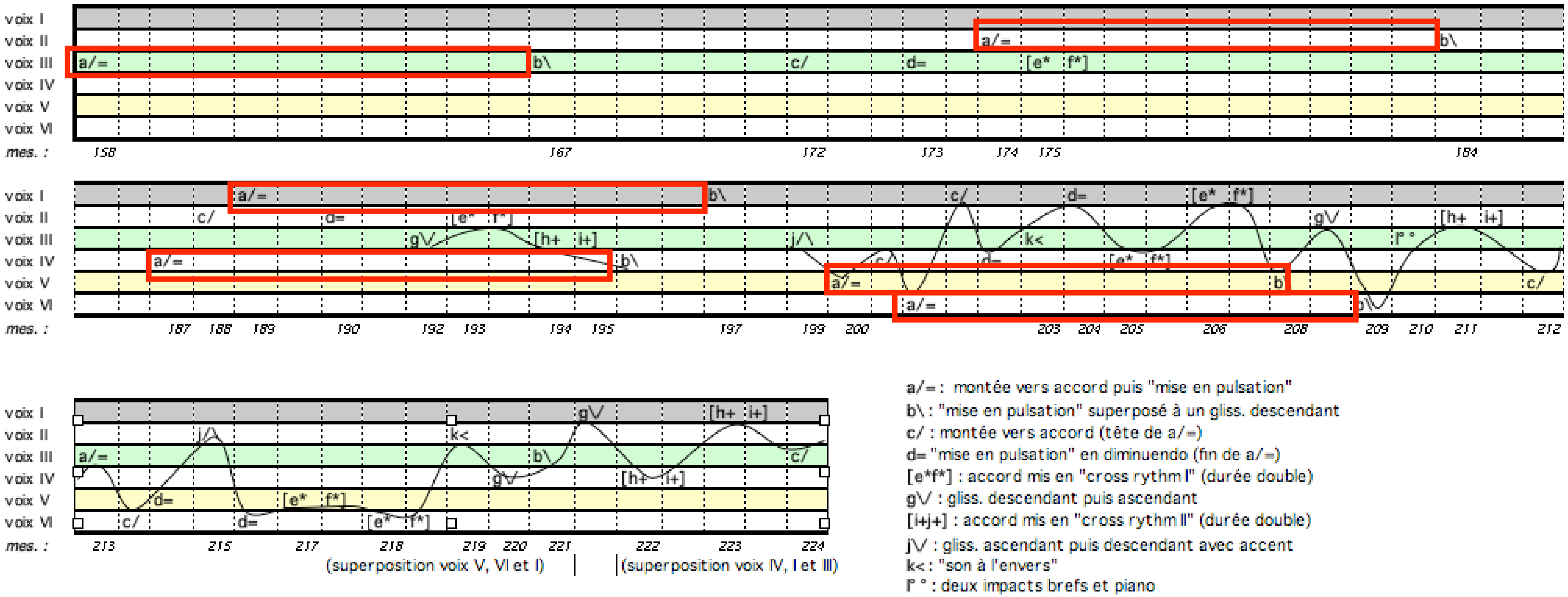




Fabien Lévy

Canoni di Vuza su gesti strumentali complessi

- *Coïncidences* (pour 33 musiciens, 1999-2007)



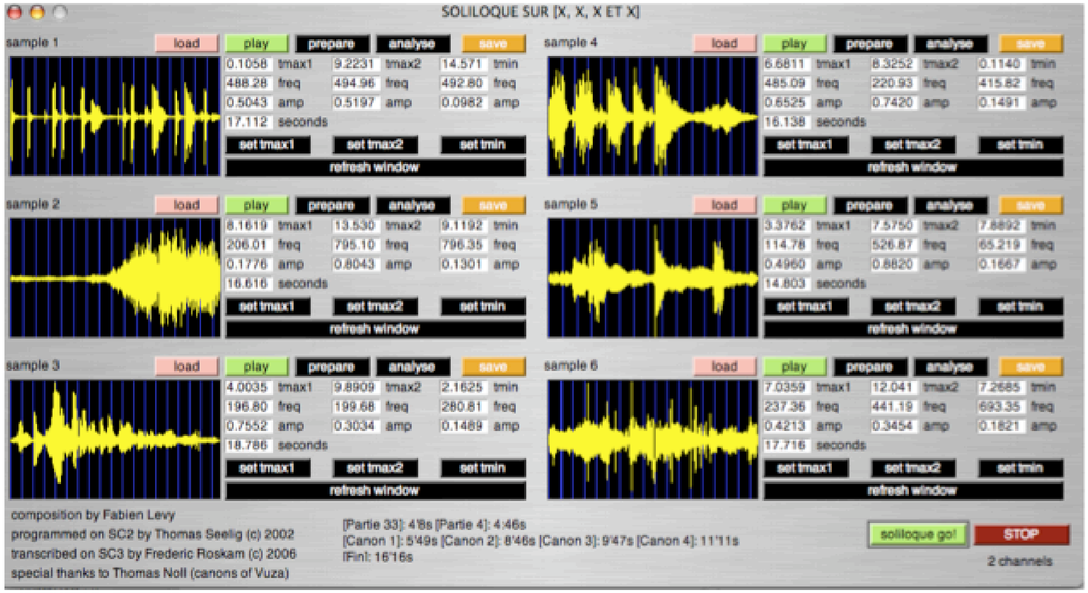
Coïncidences - Fabien Levy : déroulement du canon (mes. 158 à 226)
(chaque impact fait 3 temps)

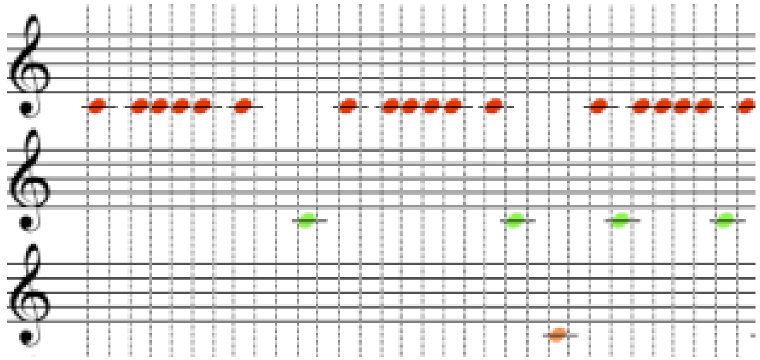
Interprètes : Tokyo Symphony Orchestra, Dir.: Kazuyoshi Akiyama, 05/09/2007, Suntory Hall, Tokyo, Japon

Fabien Lévy

Canoni « aumentati » e CAO

• *Soliloque sur [X, X, et X], commentaire par un ordinateur d'un concert mal compris de lui*

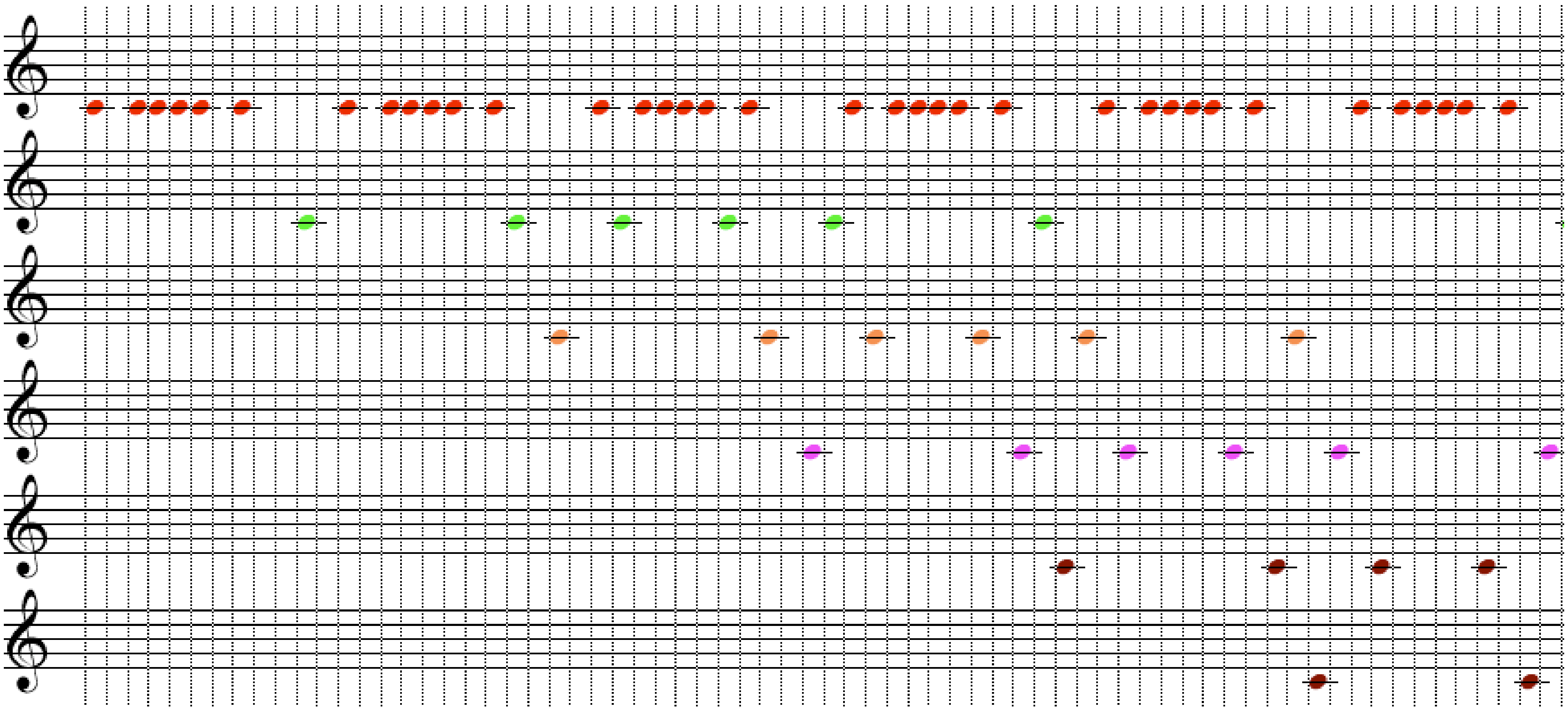




(in collaborazione con Thomas Noll)

Augmented Tiling Canons o l'azione del gruppo affine

(in collaborazione con Thomas Noll)



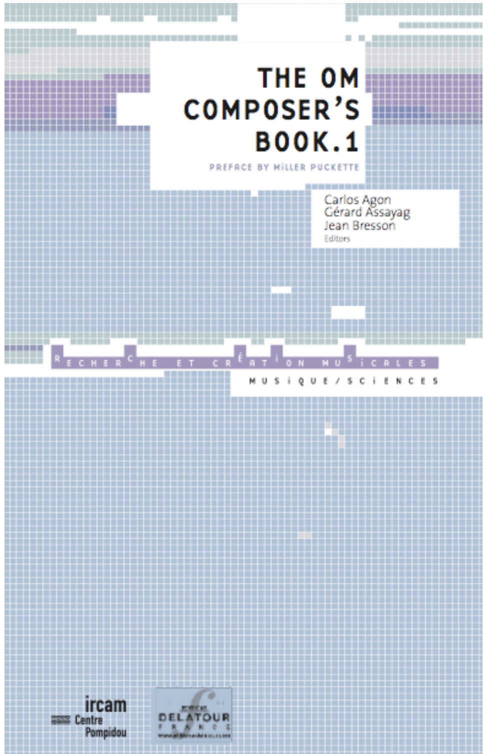
=> *OpenMusic*

Georges Bloch

Strategie compositive a partire da un modello formale

- Organisation métrique d’un canon mosaïque
- Réduction d’un canon par auto-similarité
- Modulation métrique entre canons
- Transformation d’un canon dans une texture
- Canons mosaïques et IAO (*OMax*)

- ***Projet Beyeler*** (2001)
- *Projet Hitchcock*
- *Visite des tours de la cathédrale de Reims*
- *Noël des Chasseurs*
- *Canons à marcher*
- *Canon à eau*
- *Harawun* (2004)
- *L’Homme du champ* (2005)
- *A piece based on Monk* (2007)
- *Peking Duck Soup* (2008)



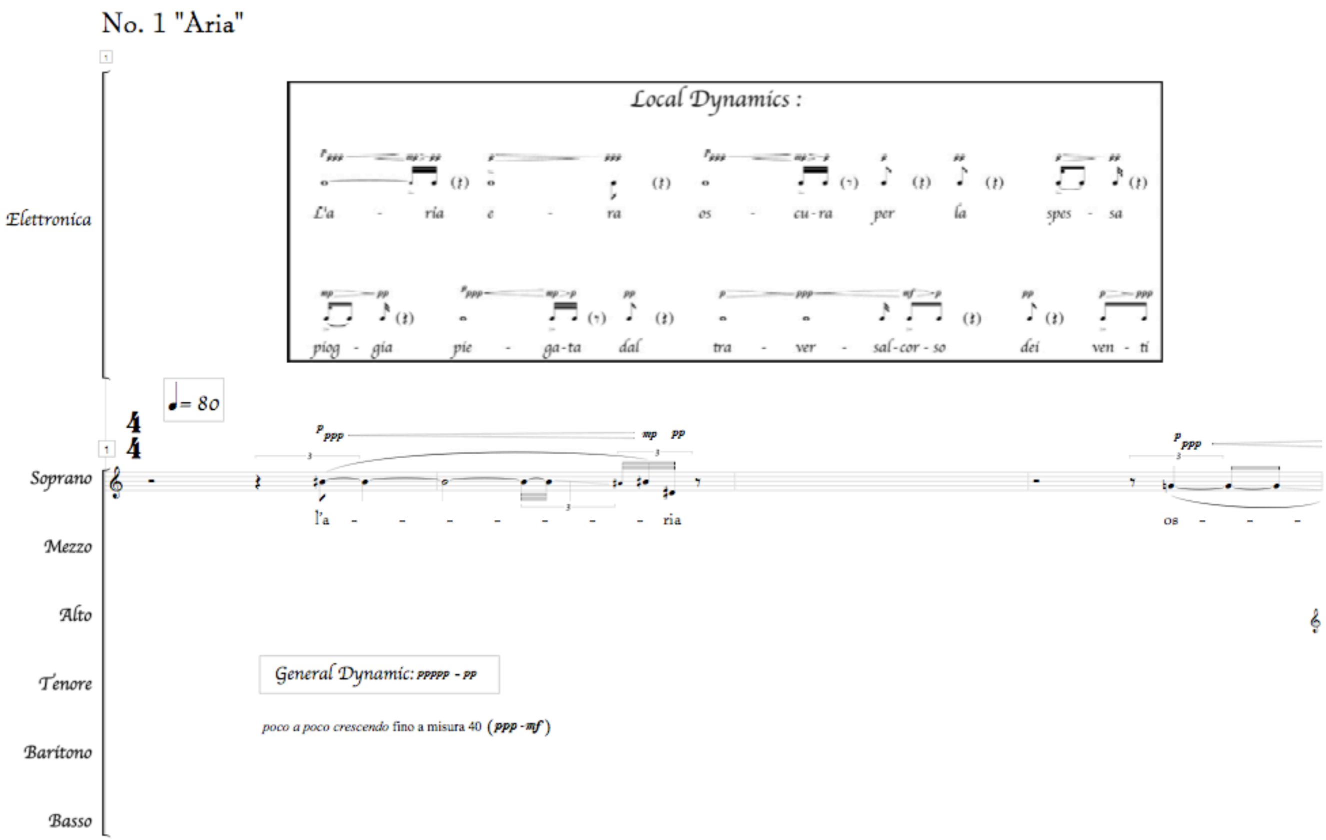
- *A piece based on Monk* (2007) (« Well You Need'nt »)

Mauro Lanza

Canoni di Vuza e periodicità locali

- *La descrizione del diluvio* (Ricordi, 2007-2008)

Canon à 14 voix sur le pattern rythmique :
(1 3 25 27 1 3 11 14 27 1 3 25 27 4 25 27 1 3 25 14 13 1 3 25 27 1 3 52)



6 voix sont en live et 8 dans l'électronique. L'unité est la double-croche de triolet. Le choix des notes et des durées est fait en cherchant à souligner certaines ***quasi-périodicités*** *du canon de Vuza, et cela donne à chaque voix un caractère beaucoup plus "redondant".*

Sequenze periodiche e calcolo delle differenze finite

- Sequenze riducibili, riproducibili e teorema di fattorizzazione

- Applicazione alle grammatiche formali e alla teoria dell’imparità ritmica

Sequenze periodiche e calcolo delle differenze finite

$Df(x)=f(x)-f(x-1)$

f = 7 11 10 11 7 2 7 11 10 11 7 2 7 11…

\ / \ /

Df = 4 11 1 8 7 5 4 11 1 8 7 5 4 11…

\ /

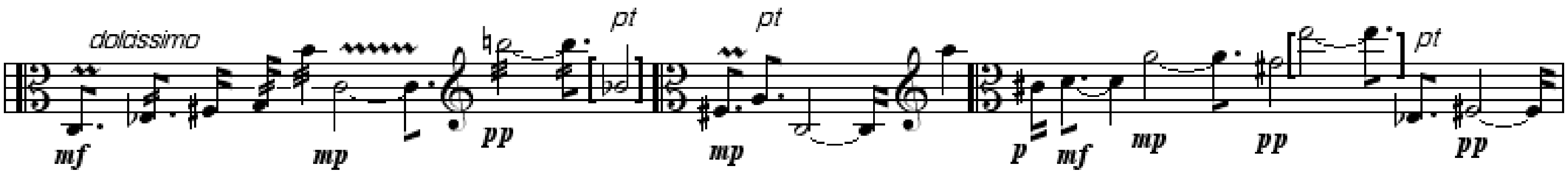
D^2f = 11 7 2 7 11 0 11 7 2 7 11 0…

\ /

D^3f = 1 8 7 5 4 11 1 8 7 5 4 11…

\ /

D^kf = …



V	0	3	8	7	11	0	11	10	6	9	0	9	1	2	9	8	4	3	6
VIII	0	0	0	0	3	3	7	2	0	0	0	6	3	3	3	4	8	0	0
IV	3	3	4	4	1	11	11	8	3	3	9	4	1	7	11	8	11	3	9
IX	0	0	0	0	0	3	6	[1]	3	3	3	3	9	0	3	6	[10]	6	6
IV	0	10	3	9	10	0	9	7	0	6	7	9	6	4	9	3	4	6	3

Anatol Vieru: *Zone d'oubli* pour alto (1973)

Sequenze riducibili e riproduttibili

=> *OpenMusic*

f = 11 6 7 2 3 10 11 6 …

Df = 7 1 7 1 7 1 7 1…

D^2f = 6 6 6 6 6…

D^4f = 0 0 0

Sequenze riducibili
$\exists\, k \geq 1$ tel que $D^k f = 0$

f = 7 11 10 11 7 2 7 11 …

Df = 4 11 1 8 7 5 4 11 …

D^2f = 11 7 2 7 11 0 11 7 …

D^4f = 1 8 7 5 4 11 1 8 …

D^5f = 7 11 10 11 7 2 7 11 …

Sequenze riproducibili
$\exists\, k \geq 1$ tel que $D^k f = f$

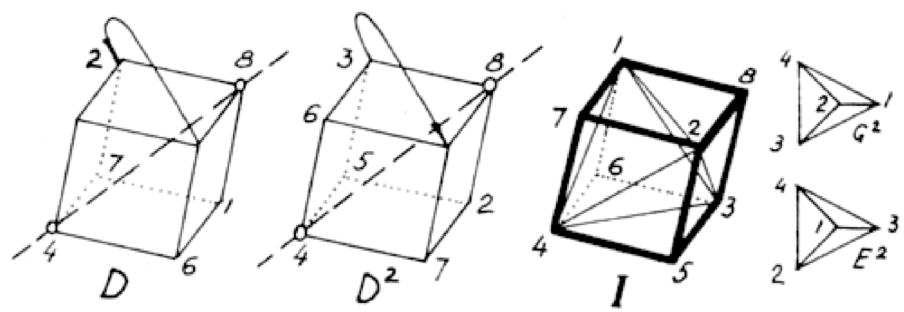
Teorema di decomposizione: Ogni sequenza periodica (a valori in un gruppo ciclico **Z/nZ**) può essere decomposta in maniera unica in una somma di una sequenza riducibile e di una sequenza riproducibile (2001)

Ramificazioni filosofiche e cognitive dell'approccio algebrico

- L'eredità del positivismo logico del circolo di Vienna nella tradizione americana
- Strutturalismo matematico e strutturalismo musicale
- Strumenti informatici e rappresentazioni simboliche
  - Teoria della calcolabilità
  - Teoria della complessità
  - Calcolo informatico
  - Lambda-calcolo
  - Programmazione logica e calcolo concorrente
  - **Analisi musicale assistita su calcolatore**

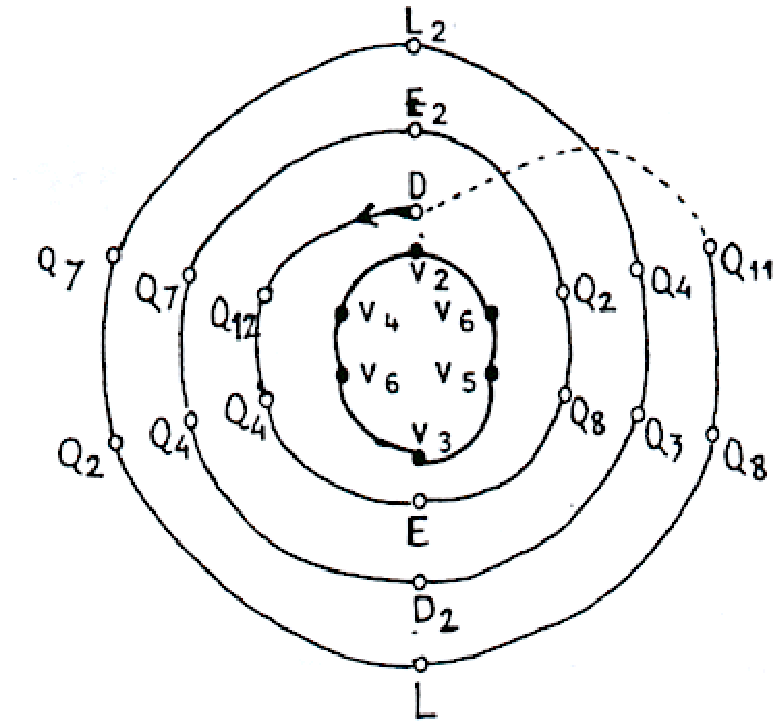
Analisi musicale computazionale: *Nomos Alpha* di I. Xenakis

La questione delle simmetrie (identità spaziali) e delle periodicità (identità nel tempo) ha un ruolo fondamentale nella musica, a tutti i livelli, da quello dei campioni sonori della sintesi del suono mediante computer, fino all'architettura di un intero brano musicale

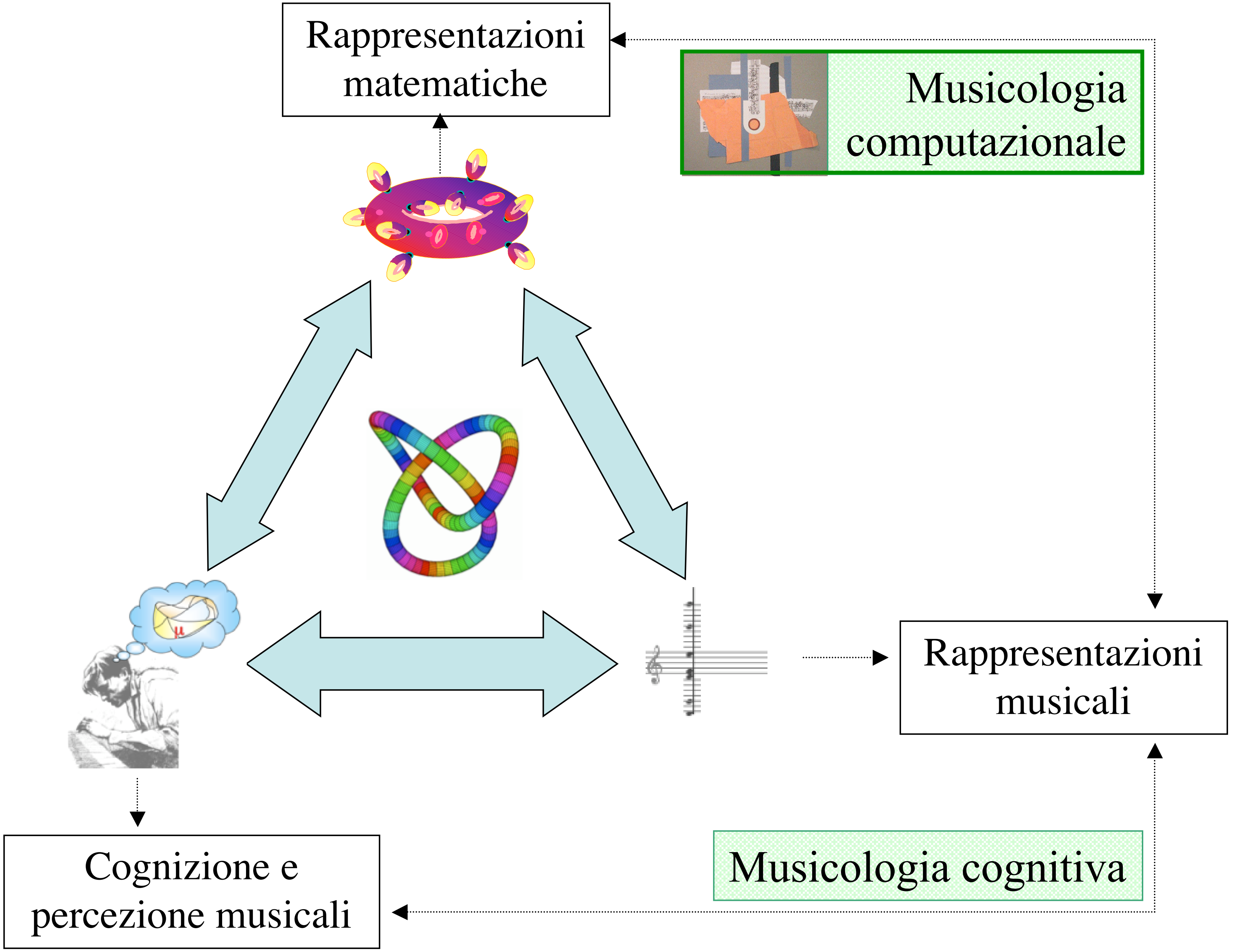


Nomos Alpha (1966)

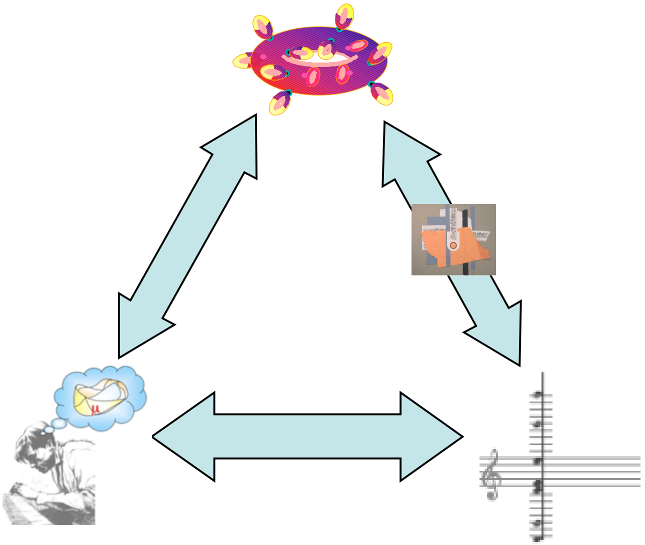
Musique symbolique pour violoncelle seul, possède une architecture "hors-temps" fondée sur la théorie des groupes de transformations.

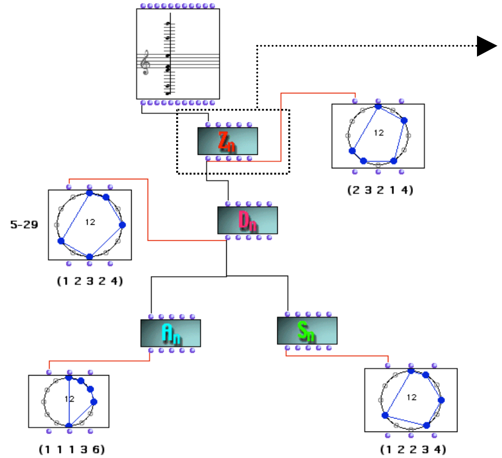


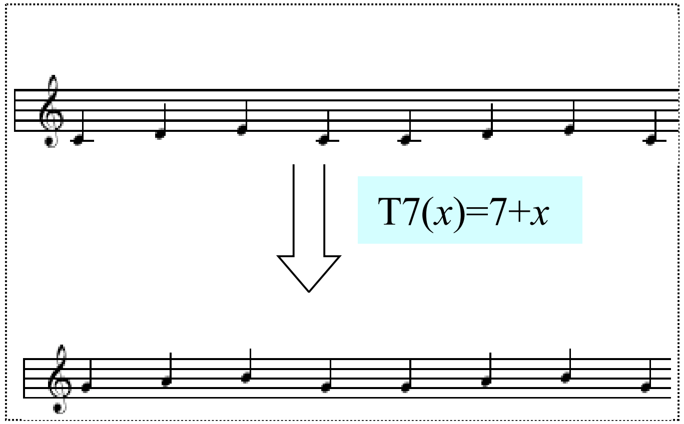
Matematica/Musica & Cognizione/Percezione



Ramificazioni percettive e cognitive dei metodi algebrici in musica



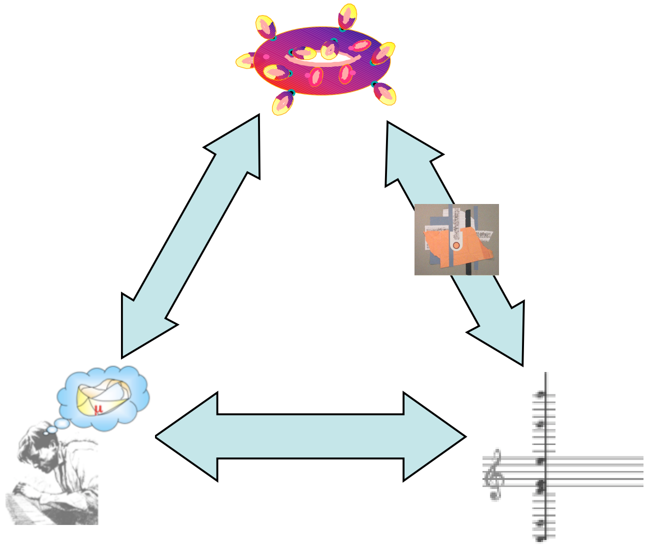


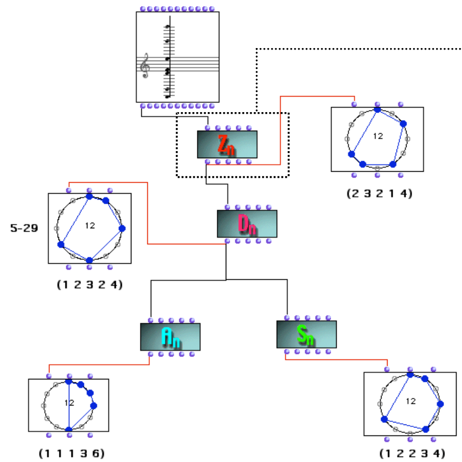


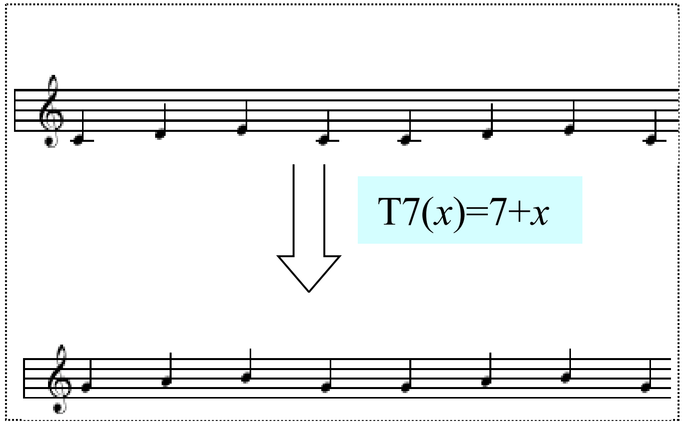
The nature of a given geometry is […] defined by the *reference* to a determinate **group** and the way in which spatial forms are related within that type of geometry. [*Cf. Felix Klein Erlangen Program - 1872*][…] We may raise the question whether there are any concepts and principles that are, although in different ways and different degrees of distinctness, necessary conditions for both the *constitution* of the **perceptual world** and the construction of the universe of geometrical thought. It seems to me that the concept of **group** and the concept of **invariance** are such principles.

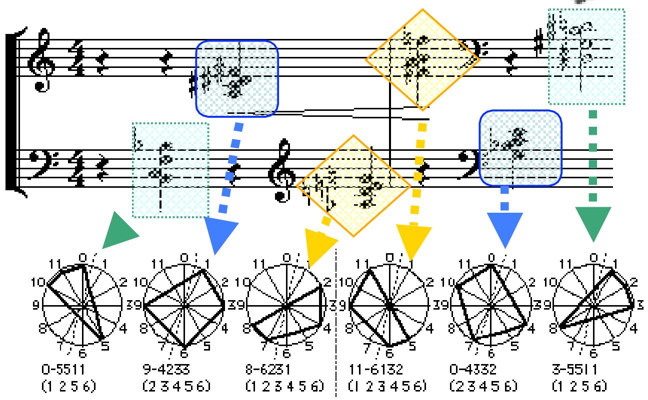
E. Cassirer, "The concept of group and the theory of perception", 1944

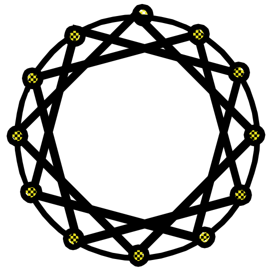
Ramificazioni percettive e cognitive dei metodi algebrici in musica

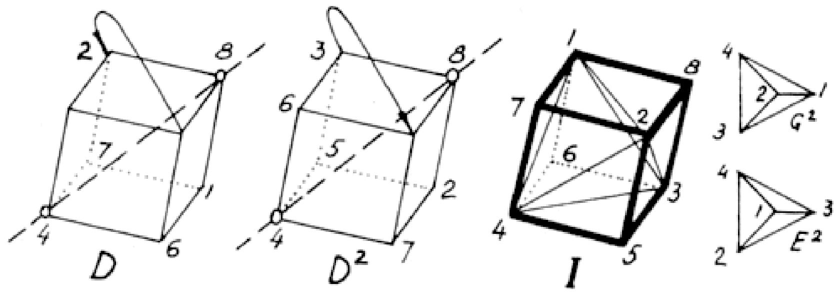












Il carattere singolare dell'esperienza musicale è dovuto in parte alle strutture particolari di ***gruppo*** *che la musica rende accessibile* [consciamente o inconsciamente] *all'ascoltatore.*

G. Balzano : « The group-theoretic description of 12-fold and microtonal pitch systems », 1980