AROUND SET THEORY

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PROCEEDINGS OF THE SYMPOSIUM AROUND SET THEORY

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Edited by
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In memoriam David Lewin (1933-2003)

Introduction

In the contemporary panorama of analytical disciplines, Set Theory is with no doubt the best illustration of the deep differences between North-American and European musicological traditions. Introduced by Milton Babbitt's thoughts, one of the most original composers/theorists of the 20^{th} century, Set Theory gradually imposed itself in the United States as being the main paradigm regarding the musical analysis of a certain contemporary music, i.e. atonal music from the Second Vienna School (Schoenberg, Webern and Berg). The development of this discipline these last twenty years, particularly thanks to the emergence of the transformational approaches offered by David Lewin, generalizing Allen Forte's traditional approach, do seem to suggest an extension of the application domain of these analytical tools to other dimensions than pitch (for instance the ones of rhythm and tones) as well as to other repertoires than the one of atonal music. Despite an abundance of publications on Set Theory, French musicology and, more generally, European musicology, have retained a certain scepticism regarding this approach, which basic principles, however, are often poorly understood. Yet, if we trace back the history of mathematical tools used in 20^{th} century musical analysis and theory, European research has given many examples of theoretical constructions very close to the ones developed in the United States. One of the objectives of this Conference was to approach American Set Theory from a larger perspective, incorporating the works on formalizing musical structures proposed by several composers and theorists of the last century (from Costère to Xenakis) as well as recent applications of some mathematical tools within analysis and computer-assisted composition.

The Conference "Around Set Theory" which brought together at IRCAM a number of authorities on this subject, was actually the exceptional occasion of evoking its history as well as opening the debate on the present situation from an analytical point of view and for which the compositional applications in addition to the mathematical ramifications are no way from being exhausted. Music theorists, musicologists, analysts, composers and mathematicians, sometimes coming from quite different cultural and geographic backgrounds, had the opportunity to confront their views on that occasion and which also enabled putting forth the question of the reach, stakes and limits of a computational-type musicological conception, and for which American Set Theory represents one of the most striking examples of the past 20^{th} century.

A *Babbitt Forum* gave also the opportunity to examine the works of Milton Babbitt, works both theoretical and musical and that has had, even though in margin of European artistic life, a great influence on American contemporary music. A concert, focusing on a few works composed by the ones that actually influenced Set Theory or that were influenced, took place, in close relationship with the structure of the Conference. As to draw this symbolic bridge between Europe and the United States, we had invited two pianists representing both traditions. Marilyn Nonken, recognized in New York as being one of the best specialists for the contemporary repertoire and who was very close to

Milton Babbitt, as well as to young composers such as Paul Nauert and Jason Eckardt. Dimitri Vassilakis, soloist for the Ensemble Intercontemporain, is more familiar with the European repertoire. Olivier Messiaen's "Mode de valeurs et d'intensités" marking a turning point, was played by both of them.[1] Should I mention here the considerable influence this piece had on all a generation of composers as the germ of a thought in Europe about the generalized series? By offering these two pianists to share this piece, we put forth a hypothesis, directly influenced by Milton Babbitt on the very conception of this third piece from the *Quatre études de rythme*, and for which the boldness, as Célestin Deliège asserts, seems to be at variance with the level-headed nature of the French composer.[2] This hypothesis is corroborated by Milton Babbitt that gave us the confirmation that he participated to the composition classes given by Olivier Messiaen in Tanglewood in 1948, that is, one year prior the creation of the piece "Mode de valeurs et d'intensités". Yes, the first two *Etudes de rythmes* were composed in the United States, as the frontispiece of the partition indicates. The reference "Darmstadt – 1949" on the score of "Mode de valeurs et d'intensités" seems therefore to confirm that the piece was indeed composed after Messiaen and Babbitt met.[3]

The conference "Around Set Theory" laid the foundations for a new dialogue between an American school always on the threshold and part of the French and European musicology falling within a computational approach. This exchange goes on today, five years after the Conference, following an always more institutional path, as shows the quite recent launch of the first peer review journal on mathematics on the relationship between mathematics and music (*Journal of Mathematics and Music*, Taylor & Francis) as well as the creation of a scholar society on the subject: the *Society for Mathematics and Computation in Music*. The articles gathered in the present volume include most of the interventions of the Conference, and have been organized according to the main axes we used to elaborate these conferences.

A first half-day was dedicated to historical and theoretical introductions to American Set Theory, with special attention given to theoretical concepts that developed themselves in parallel in Europe. Within this very vast domain of relationships between mathematics and music, several problems concern questions regarding a "set-theoretical"[4] approach, as stresses John Rahn's article called "The swerve and the flow: Music's Relationship to Mathematics". By placing the problem of the relationship of music with mathematics within a philosophical perspective, the author suggests the existence of deep-structural relations between the transformational approach developed in the United States starting at the end of the 80's and the birth, almost at the same time, in Europe, of a mathematical theory of music based on a categorial approach. This intuition appeared to be quite right, as many articles have shown later on.[5] The relationships among the American set-theoretical tradition and Music mathematical theory as it has developed itself in Europe around the 50's have been treated by Luigi Verdi's historical introduction who, quite clearly, expresses one of the major ambitions of this conference, i.e. the postulate that *around* Set Theory, as traditionally put forth in music analysis textbooks, a movement with common ideas developed itself independently from cultural traditions.

The second half-day was dedicated to the relationships between "classic"[6] set-theoretical approach and David Lewin's transformational conception. As to put into relief their similarities and differences, we chose a piece from the atonal repertoire lending itself ideally to analysis under one or the other of these perspectives: Arnold Schoenberg's

Klavierstücke Opus 19, no. 4. The reader will therefore be able to choose to start with Allen Forte's analysis that dissects the piece by offering segmentations coming out onto a catalogue of very small pitch-class sets, or immerse himself *ex abrupto* into Xavier Hascher's transformational reading using Klumpenhouwer's networks (or K-nets). These papers perfectly express the dialectic relationship of these two visions. From an epistemological point of view, by making use of the categories put forth by Gilles-Gaston Granger,[7] one could qualify the first approach of "objectal", by opposition to the second, i.e. "operative". However, one should not imagine the objectal/operative duality to be an exclusively analytical category. This duality can be found in compositional approaches we could consider as "set-theoretical", either because they use in composition the basic principles of Set Theory, or because the underlying theoretical tools to the compositional processes perfectly fit into a set-theoretical approach.

Although not referring to Granger, Milton Babbitt is with no doubt one of the first composers to be aware of the theoretic reach of this duality and to make use of it in his composition, influencing directly in this regard the development of Set Theory as a subject when he asserts that the dodecaphonic system is a "system" mathematically speaking, that is to say a "set of elements, relationships among those elements as well as operations on these elements".[8] The will of this Forum was to pay a tribute to this emblematic composer/theorist and for which Andrew Mead and Joseph Dubiel presented a few analytical aspects. Babbitt's compositional approach raises the question of the relationship between Set Theory and dodecaphonic composition, as discussed in detail by Robert Morris in his contribution entitled "Compositional Theory, Musical Spaces and compositional Designs". This article gives another point of view, compositional this time, regarding what can be *around* Set Theory. In using the notion of "compositional theory", as being a "genre of musical research that is distinguished from other forms of music theory, such as that involved in pedagogy and musical analysis", Morris gives us a precious help in understanding the approach of some composers, from Boulez to Xenakis, for whom having a natural affinity with Set Theory was never really highlighted. It is certainly the case for André Riotte, French composer and theorist who has lead formal researches on some of the properties of the dodecaphonic system, particularly on all-interval series ("balanced cycles" in his own terminology) that precede sometimes, historically, the ones of his American colleagues that are maybe, more broadly known in the musicological community. In "Formalisms and freedom of the imagination", André Riotte puts forth his compositional techniques that generalize Iannis Xenakis' Sieve theory, and for which the implementation in software to help analyzing music with a computer constitutes a remarkable feature of the European tradition in comparison with American Set Theory. This is an approach that is at the heart of the researches lead at IRCAM, particularly regarding the programming language concerning theory, computer-assisted analysis and composition (OpenMusic); this approach having, this time, been taken up again and having integrated the theoretic tools offered by André Riotte and Marcel Mesnage at the start of the 80's within a coherent algebraic approach. These tools are sometimes being developed directly by young composers for whom the techniques do not fit anymore in the set-theoretical tradition born from dodecaphonism, as Paul Nauert and Jason Eckardt prove, being two composers and theorists using intensively set-theoretical methods in computer-assisted composition. Nauert's compositional strategies, emerging from a theoretical research going beyond the very parameters of pitch, stress the limited character of Set Theory traditional descriptions, often criticized precisely for its one-way

applications to the domain of pitch as well as to the atonal/dodecaphonic repertoire. Another example is given by Jason Eckardt, a composer whose techniques have been inspired by Set Theory and which also apply to microtonal systems. We do wish to stress this aspect here, as it represents quite a very important research orientation in the domain of relationships among mathematics and music, and for which we have already evoked a few aspects. The study of microtonal systems from a set-theoretical approach belongs to what is called in the American tradition "diatonic theory". It's a subject for which both traditions, American and European, have recently found several theoretic points of convergence, particularly regarding the use of formal grammars and Fourier's discrete transformed one as to formalize certain musical properties of the diatonic theory (pairwise well-formed scales, well-formed scales, etc.).[9]

The ambition of the Conference "Around Set Theory" was not to establish a critical assessment of Set Theory twenty-five years after the publication of *The Structure of Atonal Music*, Allen Forte's book that gives a birth date to Set Theory as an analytical subject.[10] However, we could not resist trying to give a few elements for a critical assessment. A first critical evaluation of Allen Forte' Set Theory was submitted by Jean-Jacques Nattiez, starting with the tripartition model elaborated by the author in collaboration with Jean Molino. The author's critic concerns more specifically the delimitation criteria of basic units, that is to say segmentation, an aspect raising epistemological problems, especially when regarding their pertinence when confronted to poietic strategies. Other critical elements have been put forth by Célestin Deliège in his essay on Set Theory's cohabitation and atonal harmony. It is about a critical discussion of some of the basic principles of Allen Forte's Set Theory and, mostly, about a new theoretic proposition aiming at establishing a hierarchical figuring that could allow a "set-theoretical" analysis from the acoustic properties of the spectrum. As to offer a "meta-perspective" on these two critical evaluations of Set Theory, we have asked Marcel Mesnage to try and have a critical mind regarding the work of Jean-Jacques Nattiez and Célestin Deliège. It's obviously a very delicate thing to do as well as much presumptuous, as the author himself did mention. Let us add, as far as we are concerned, that it is mathematically quite dangerous, as this almighty critic should be followed by an other ever almighty critic, and so forth. But hopefully we managed to find a way of concluding this book that would have eventually found some room in Jorge Luis Borges' library, and this, thanks to John Rahn's commentaries and Jean-Michel Bardez's postscript...

We would now like to thank all the ones that have made this event possible. First, the members of the scientific and organizing committee: Jean-Marc Chouvel (Reims University), Guerino Mazzola (Zürich University and Minnesota University), André Riotte (SFAM), Hugues Vinet (Scientific Director of IRCAM/CNRS), Gérard Assayag (Head of IRCAM's Music Representation Team) and Stéphan Schaub (doctoral student in computational musicology at IRCAM/Sorbonne Paris IV University). We would like to particularly thank Stéphan as he played an essential part in the search for partnerships, such as the American Embassy in France (cultural department). The other partners were IRCAM, the SFAM, the University of Washington at Seattle, Princeton University, the Multimedia Laboratory of the University of Zürich (Switzerland) and the CNRS. We would like to thank the journal *Perspectives of New Music* that gave us the authorizations to use the articles that in the meantime had already been published. We also thank Sylvie Benoit for her help in organizing logistically the conference. The project

of organizing an international Conference on Set Theory at IRCAM had been welcomed at the time, especially by some composers, with little greetings, under the motives, do we think, of prejudice more than because of a true deep knowledge on the matter. We do want to thank Bernard Stiegler, IRCAM Director at the time, for having supported this initiative as well as the artistic directors Eric De Visscher and Jean-Michel Lejeune for their help in the conception of the concert programme that has managed to draw a landscape around Set Theory for which many aspects remain to be discovered.

Moreno Andreatta

Notes

1. Besides "Mode de valeurs et d'intensités" by Messiaen, the following pieces were interpreted: *Allegro Penseroso*, *Partitions* and *Post-Partitions* by Milton Babbitt, *A Collection of Caprices* by Paul Nauert, *Echoes' White Veil* by Jason Eckardt, *Douze notations* by Pierre Boulez, *Mists* by Iannis Xenakis and eight *Inventions* by André Riotte (no. 1, 6, 7, 10, 11, 12, 13 and 17).

2. Cf. Célestin Deliège, *Cinquante ans de modernité musicale : de Darmstadt à l'IRCAM. Contribution historiographique à une musicologie critique*, Brussels: Mardaga, 2003.

3. I would like to thank Georges Bloch and Jean-Claude Risset for having brought to my attention this littly known fact of the history of integral serialism.

4. We shall be using all through this book this neologism which seems to having been fairly well accepted by the "mathemusical" community.

5. See, in particular, Guerino Mazzola and Moreno Andreatta, "From a Categorical Point of View : K-Nets as Limit Denotators", *Perspectives of New Music*, 44, no. 2, p. 88-113 and John Rahn, "Cool tools", *Journal of Mathematics and Music*, Vol. 1, no. 1, March 2007, p. 7-22. Let's also keep in mind that other elements putting into relief the deep formal relations between the American School and the *Mathematische Musiktheorie* (MaMuTh) have been discussed by Guerino Mazzola and Thomas Noll in a presentation that could unfortunately not find the shape of a contribution susceptible of being integrated to this book.

6. Where by "classic" we mean the presentations of the basic principles of Set Theory in handbooks such as *The Structure of Atonal Music* by Allen Forte, *Basic Atonal Theory* by John Rahn, *Composition with Pitch-Classes* by Robert Morris or *Introduction to Post-Tonal Theory* by Joseph Straus.

7. See, in particular, the book *Formes, opérations, objets*, Paris: Librairie Philosophique J. Vrin, 1994.

8. This characterizing of the dodecaphonic system was already present in his doctoral dissertation worded *The function of Set Structure in the Twelve-Tone System*. The dissertation, complemented in 1946, was only approved by Princeton University's Music Department at the beginning of the 90's, which shows of course the difficulties of the institutional dialogue between mathematics and music.

9. Has been consecrated to this issue the second issue of the Journal of Mathematics and Music entitled "The Legacy of John Clough in mathematical music theory" (guest editor: David Clampitt).

10. Let's keep in mind that the *Société Française d'Analyse Musicale* (SFAM), concerned with inciting meetings allowing broaching important theories, had invited Allen Forte as well as Célestin Deliège to participate to a debate during the First European Music Analysis Congress in Colmar, in 1989. Despite the fact that this first approach had given place to the publication of articles in the journal *Analyse Musicale*, we must admit that Forte's book, as well as *Generalized Musical Intervals and Transformations* by David Lewin, have still not been translated into French. We deeply hope being able to host their publications within this new collection.