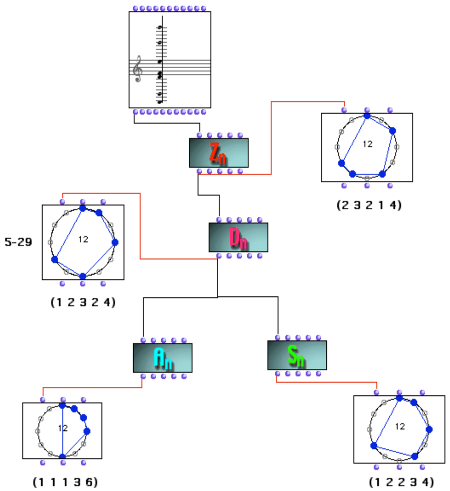
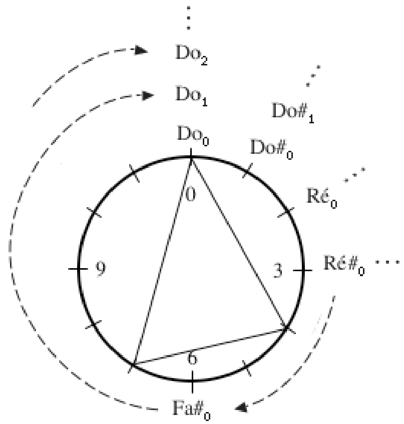


Elementi di Geometria Superiore 2



(4,3,5)

Matematica & Musica

Primo trittico:

rappresentazioni, formalizzazioni, *Set Theory*…

Moreno Andreatta

Equipe Représentations Musicales

IRCAM/CNRS

(In collaborazione con Carlos Agon e Emmanuel Amiot)

Programma del corso

1.) **Rappresentazione e formalizzazione delle strutture musicali**
2.) **Enumerazione e classificazione delle strutture musicali**
3.) **Teorie trasformazionali, diatoniche e neo-riemanniane**
4.) Tassellazioni musicali: la costruzione dei canoni ritmici a mosaico
5.) Sequenze periodiche e calcolo delle differenze finite a valori in gruppi ciclici
6.) Ramificazioni filosofiche e cognitive dell'approccio algebrico in musica

Rappresentazione, formalizzazione e enumerazione delle strutture musicali

- Rappresentazioni geometriche e formalizzazioni algebriche
  - Il *Tonnetz* di Eulero
  - Rappresentazioni circolari e toroidali
  - Teoria degli *orbifolds*
  - [Cenni dell'approccio categoriale]

- Lemma di Burnside e teoria dell'enumerazione di Polya
  - Classificazione paradigmatica degli accordi musicali (azioni del gruppo ciclico, diedrale e affine sul sistema temperato tradizionale)
  - Modi di Messiaen a trasposizione limitata
  - Serie dodecafoniche e serie omni-intervallari
  - [Asimmetria ritmica]
  - Spazi microtonali

- La *Set Theory* d'Allen Forte
  - Il vettore intervallare
  - Teorema dell'esacordo (Milton Babbitt)
  - La relazione Z [e gli insiemi omometrici]

Teorie trasformazionali, diatoniche e neo–riemanniane

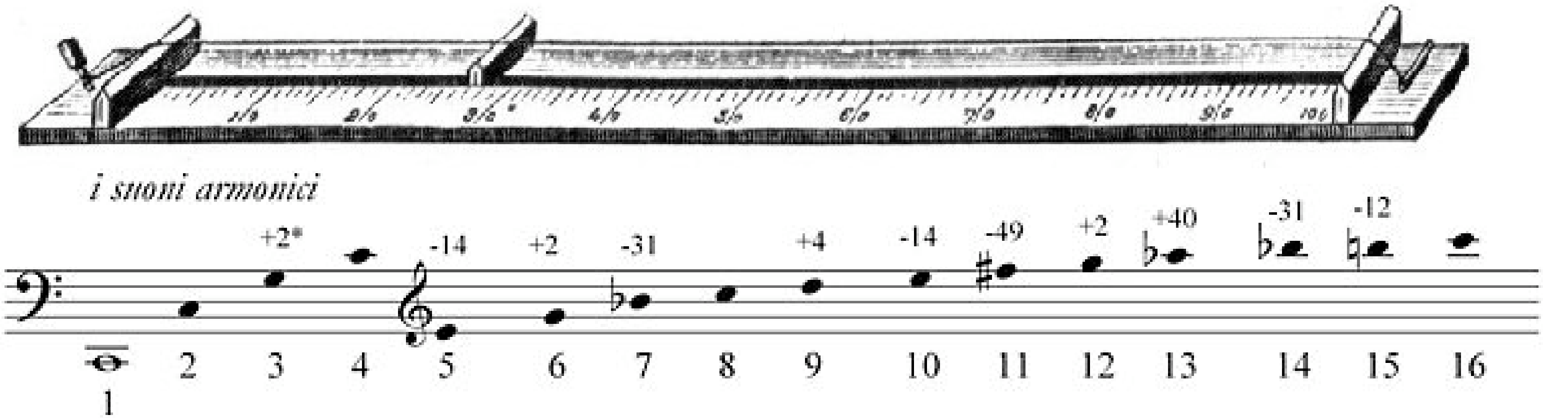
- Il sistema d'intervalli generalizzati (GIS) di David Lewin
  - La funzione intervallare e la trasformata di Fourier discreta
  - Teorema [generale] dell'esacordo

- Reticoli di Klumpenhouwer (*K-nets)*
  - Isografie forti
  - Isografie positive
  - Isografie negative

- [Teorie diatoniche]
  - Unicità della scala diatonica
  - Insiemi ripartiti in maniera massimale (*Maximally Even Sets*)
  - Scale ben formate (*Well-formed scales*)
  - Diatonismo *vs* cromatismo

- Teorie neo-riemanniane
  - Dualità trasposizione / inversione
  - [Cenni di grammatiche formali (*Christoffel words*)]

Temperamenti musicali: dal monocordo al pianoforte

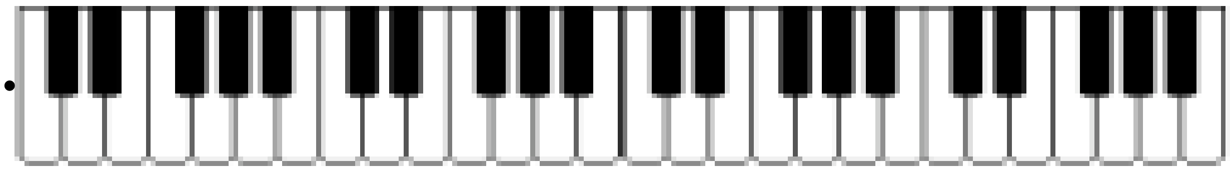


* in *cents*, confrontati con la scala temperata

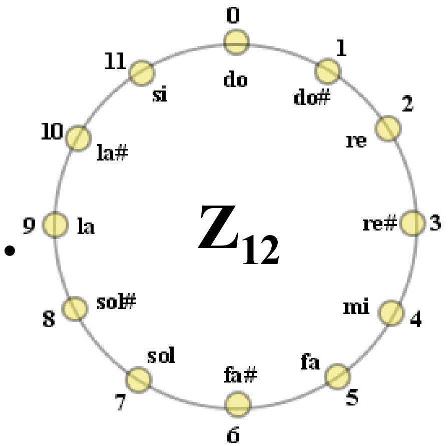
Fisica

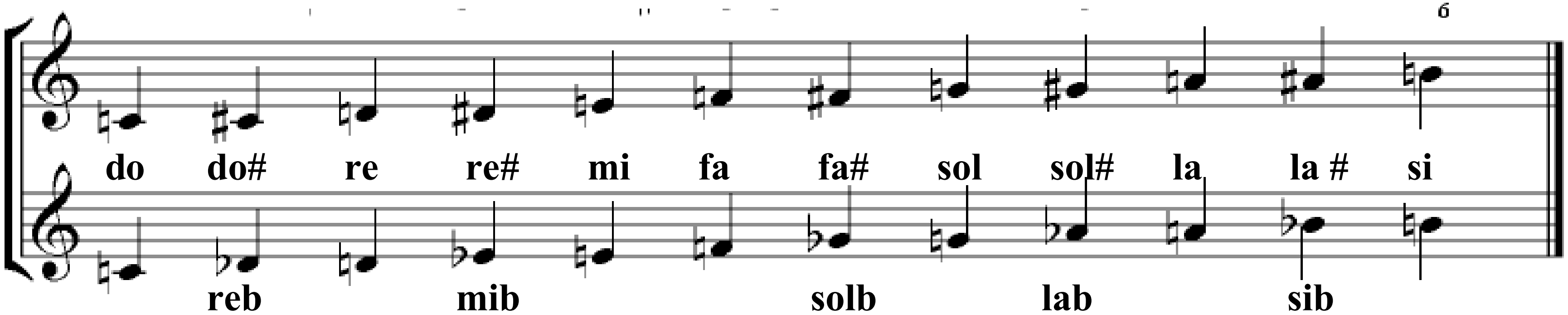
Matematica

…



…

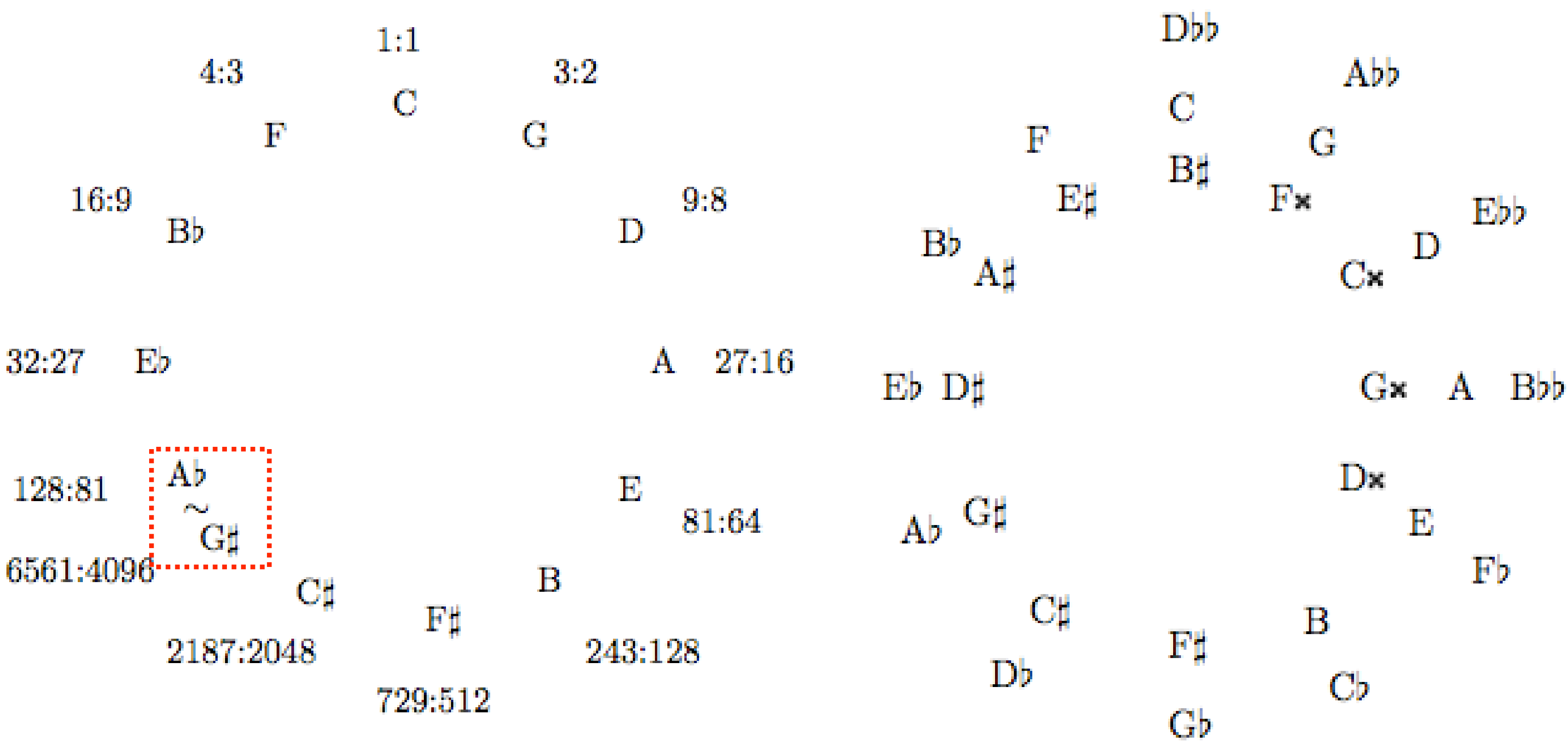




La spirale infinita del temperamento pitagorico

[Benson 2006]

note	do	re	mi	fa	so	la	ti	do
ratio	1:1	9:8	81:64	4:3	3:2	27:16	243:128	2:1



Temperamento pitagorico, intonazione giusta e temperamento equabile

TP

IG

TE

note	C	D	E	F	G	A	B	C
ratio	1:1	9:8	81:64	4:3	3:2	27:16	243:128	2:1
cents	0.000	203.910	407.820	498.045	701.955	905.865	1109.775	1200.000

TP

note	do	re	mi	fa	so	la	ti	do
ratio	1:1	9:8	5:4	4:3	3:2	5:3	15:8	2:1
cents	0.000	203.910	386.314	498.045	701.955	884.359	1088.269	1200.000

IG

[Benson 2006]

Johann Sebastian Bach e il clavicembalo ben temperato

Lehman's Bach reconstruction (2005)

$$E^{-\frac{2}{3}p} \quad B^{-\frac{2}{3}p} \quad F\sharp^{-\frac{2}{3}p} \quad C\sharp^{-\frac{2}{3}p} \quad G\sharp^{-\frac{3}{4}p}$$

$$C^{0} \quad G^{-\frac{1}{6}p} \quad D^{-\frac{1}{3}p} \quad A^{-\frac{1}{2}p} \quad E^{-\frac{2}{3}p}$$

$$E\flat^{\frac{1}{6}p} \quad B\flat^{\frac{1}{12}p} \quad F^{\frac{1}{6}p} \quad C^{0}$$

[Benson 2006]

Temperamento equabile e frazioni continue

$$\tfrac{1}{2}(1+\sqrt{5}) = 1 + \frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{1+}\cdots$$

$$\frac{\pi}{4} = \frac{1}{1+}\,\frac{1}{3+}\,\frac{4}{5+}\,\frac{9}{7+}\,\frac{16}{9+}\cdots$$

$$\begin{aligned} e &= 2.71828\,18284\,59045\,23536\,02874\,71352\,66249\,77572\,47093\ldots \\ &= 2 + \frac{1}{1+}\,\frac{1}{2+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{4+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{6+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{8+}\,\frac{1}{1+}\,\frac{1}{1+}\cdots \end{aligned}$$

$$e^{2\pi/5}\left(\sqrt{\frac{5+\sqrt{5}}{2}}-\frac{\sqrt{5}+1}{2}\right)=\frac{1}{1+}\,\frac{e^{-2\pi}}{1+}\,\frac{e^{-4\pi}}{1+}\,\frac{e^{-6\pi}}{1+}\cdots$$

[Benson 2006]

Harmonium e pianoforti generalizzati

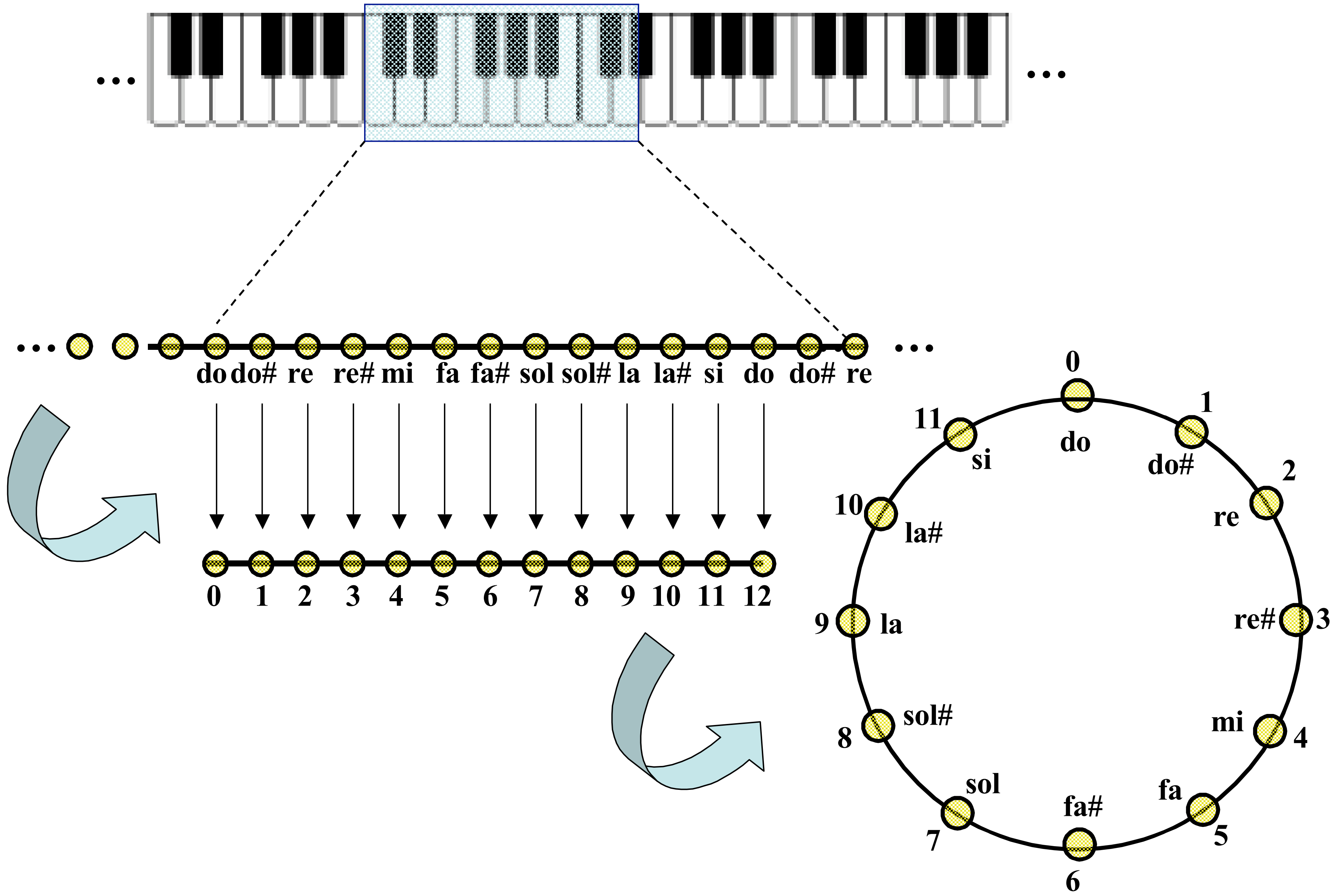
Z/31Z

Vitus Trasuntinis, 1606

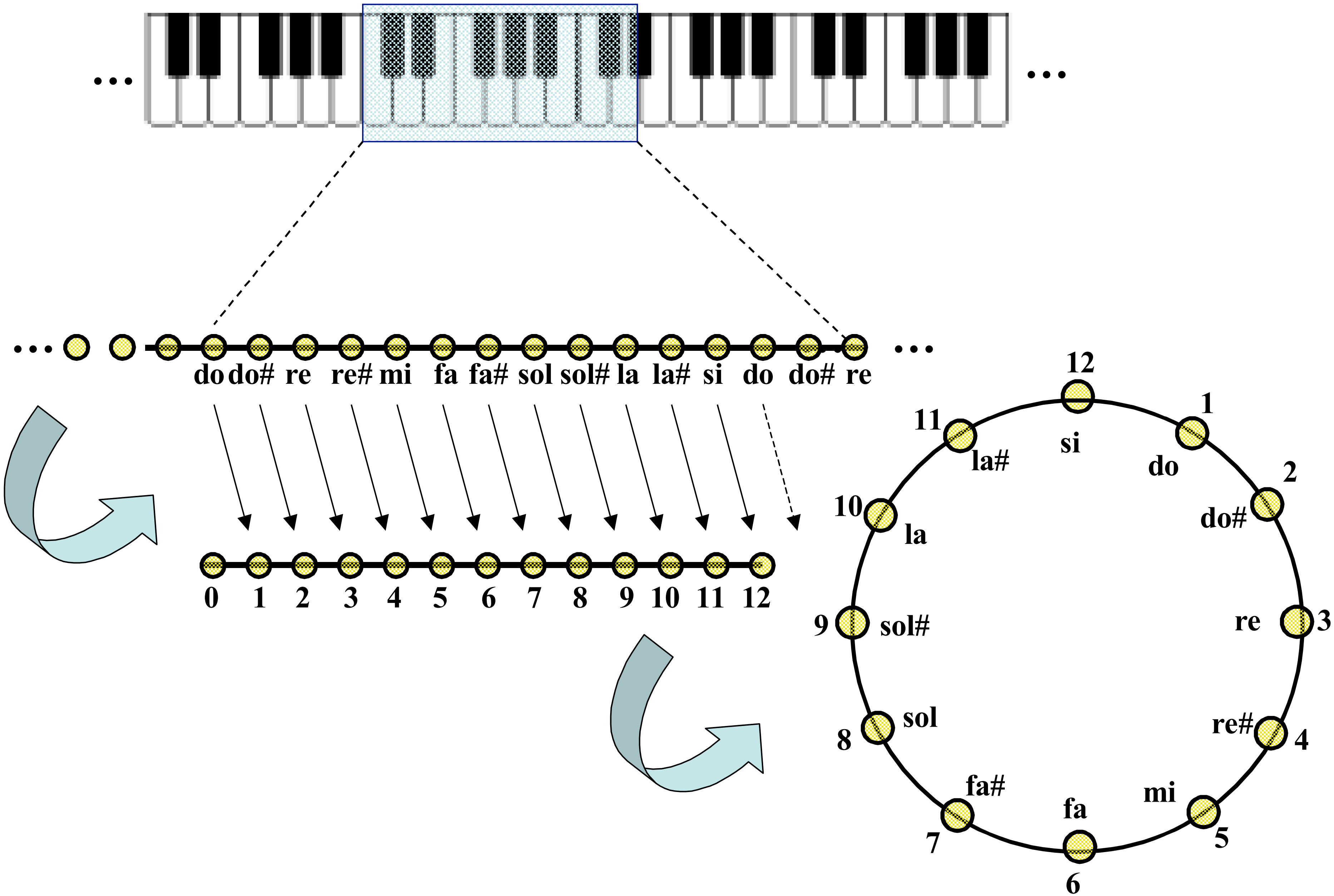
Z/53Z

Robert Bosanquet, 1876

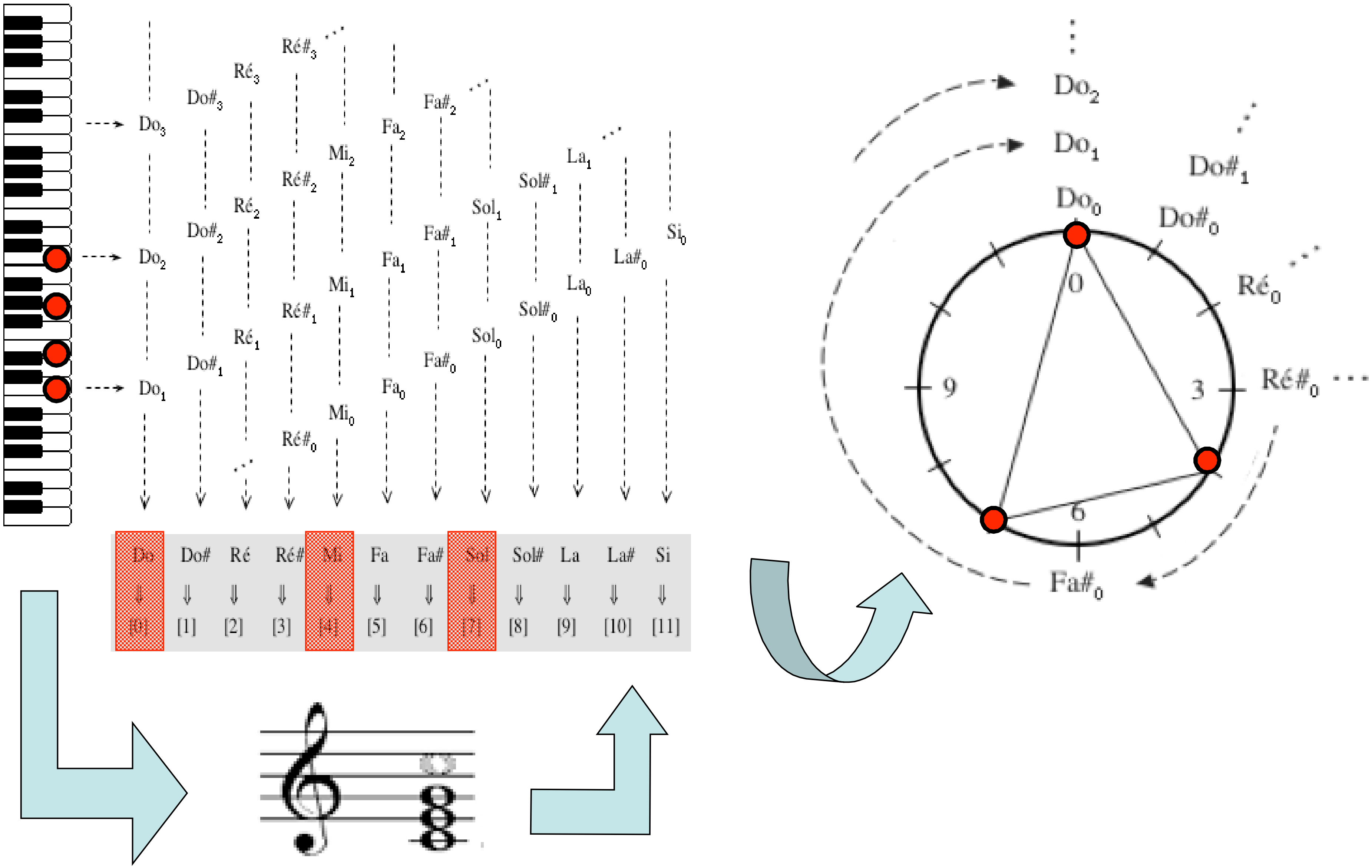
Riduzione all'ottava e congruenza modulo 12



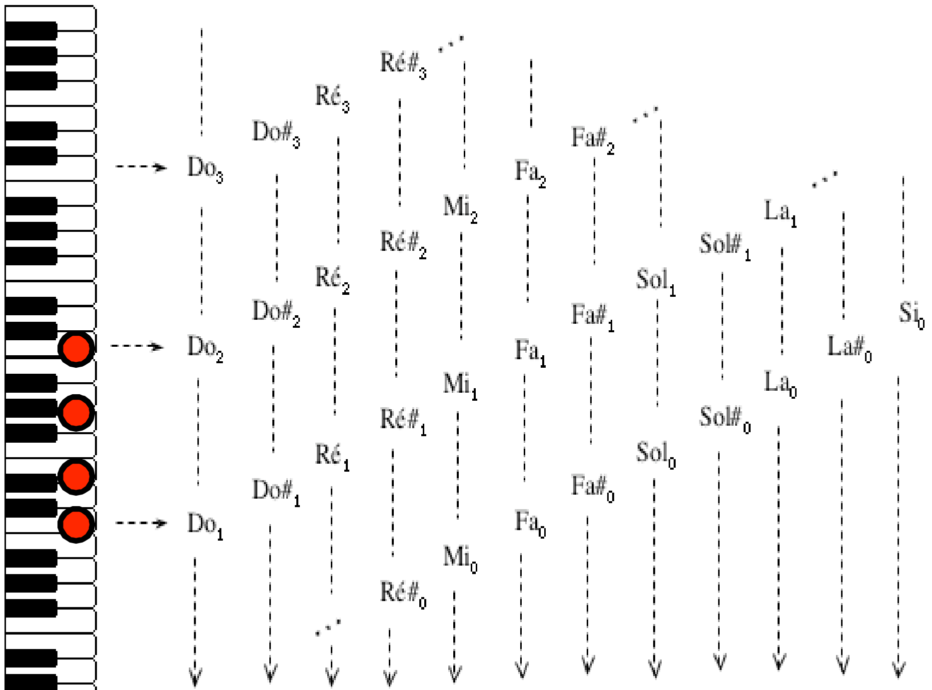
Riduzione all'ottava e congruenza modulo 12



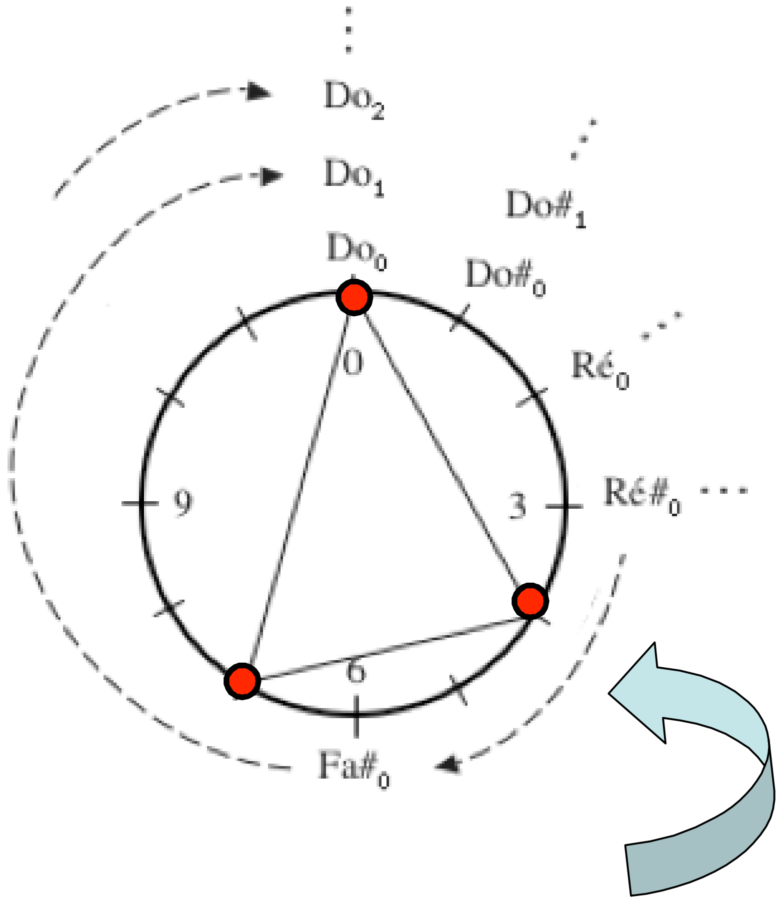
Riduzione all’ottava e congruenza modulo 12

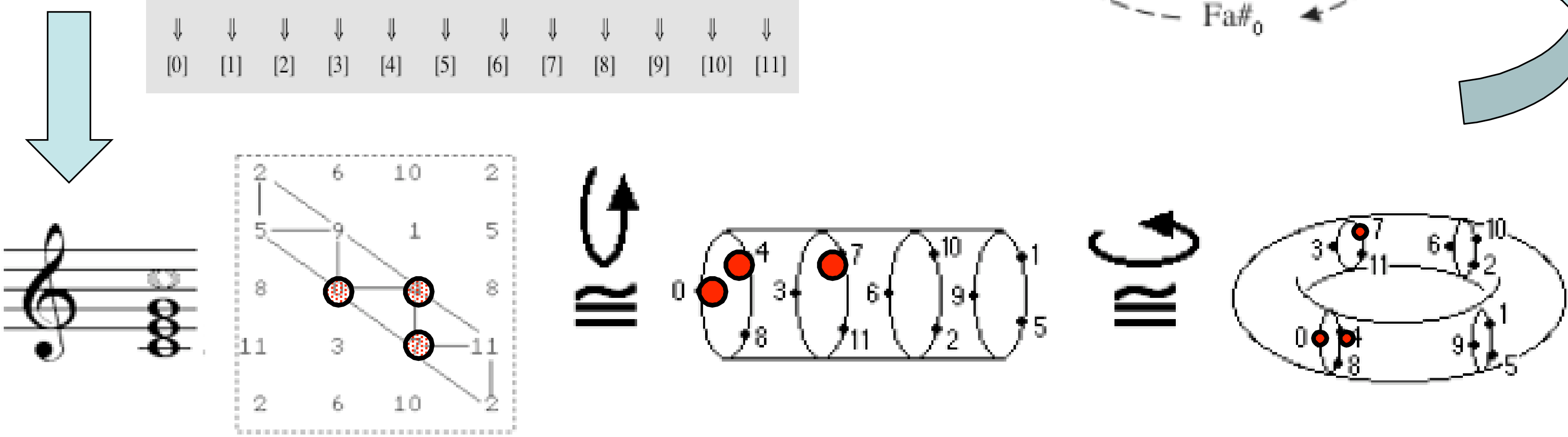


Riduzione all’ottava e congruenza modulo 12

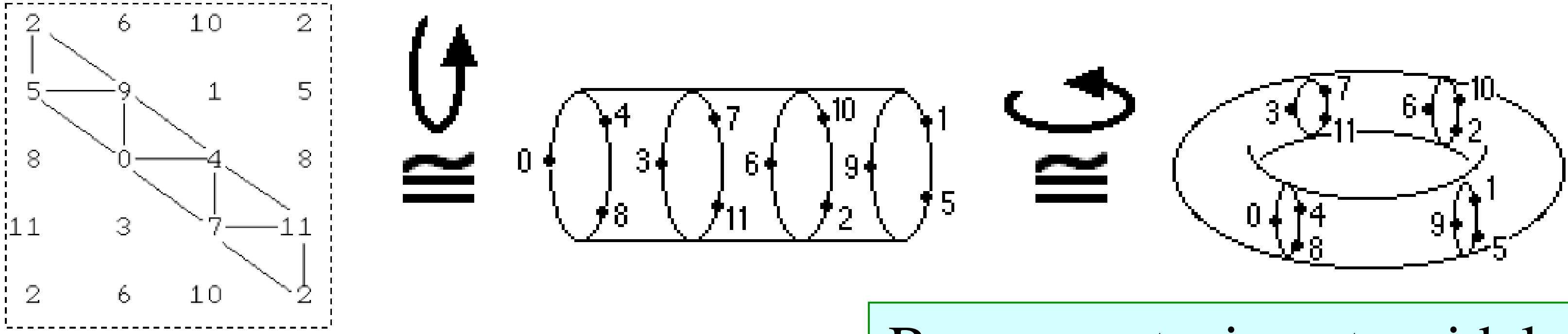


Do	Do#	Ré	Ré#	Mi	Fa	Fa#	Sol	Sol#	La	La#	Si
⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

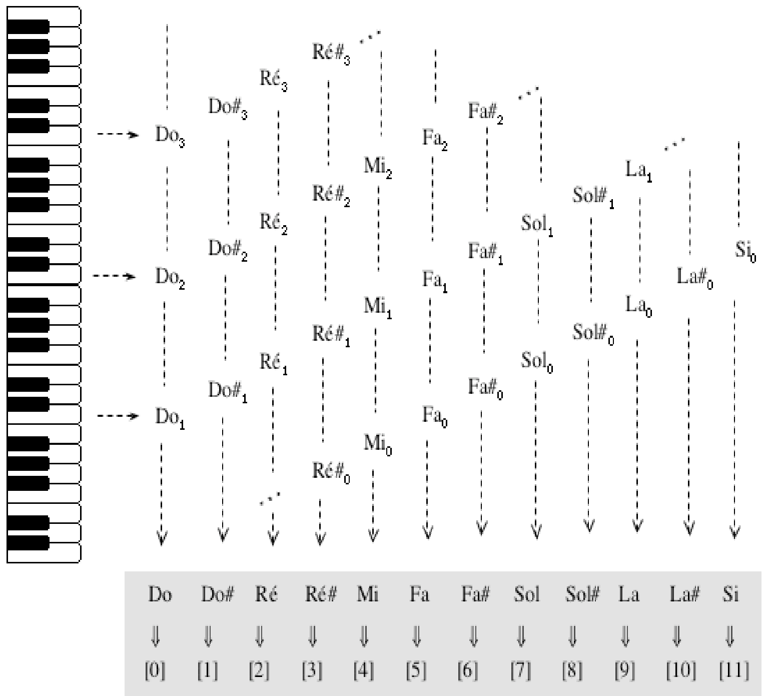




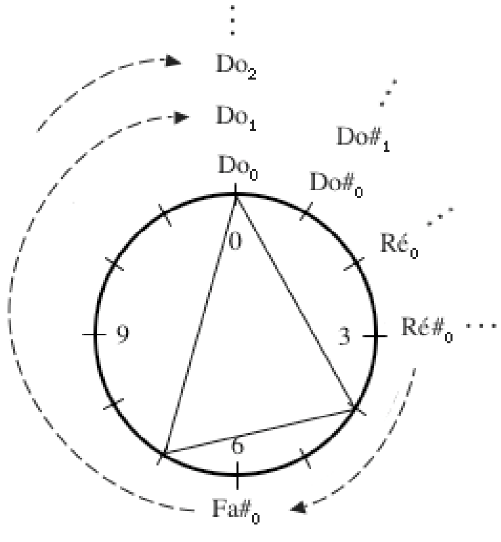
Equivalenza algebrica fra rappresentazioni geometriche



Rappresentazione toroidale



$$Z_{12} = Z_3 \times Z_4$$

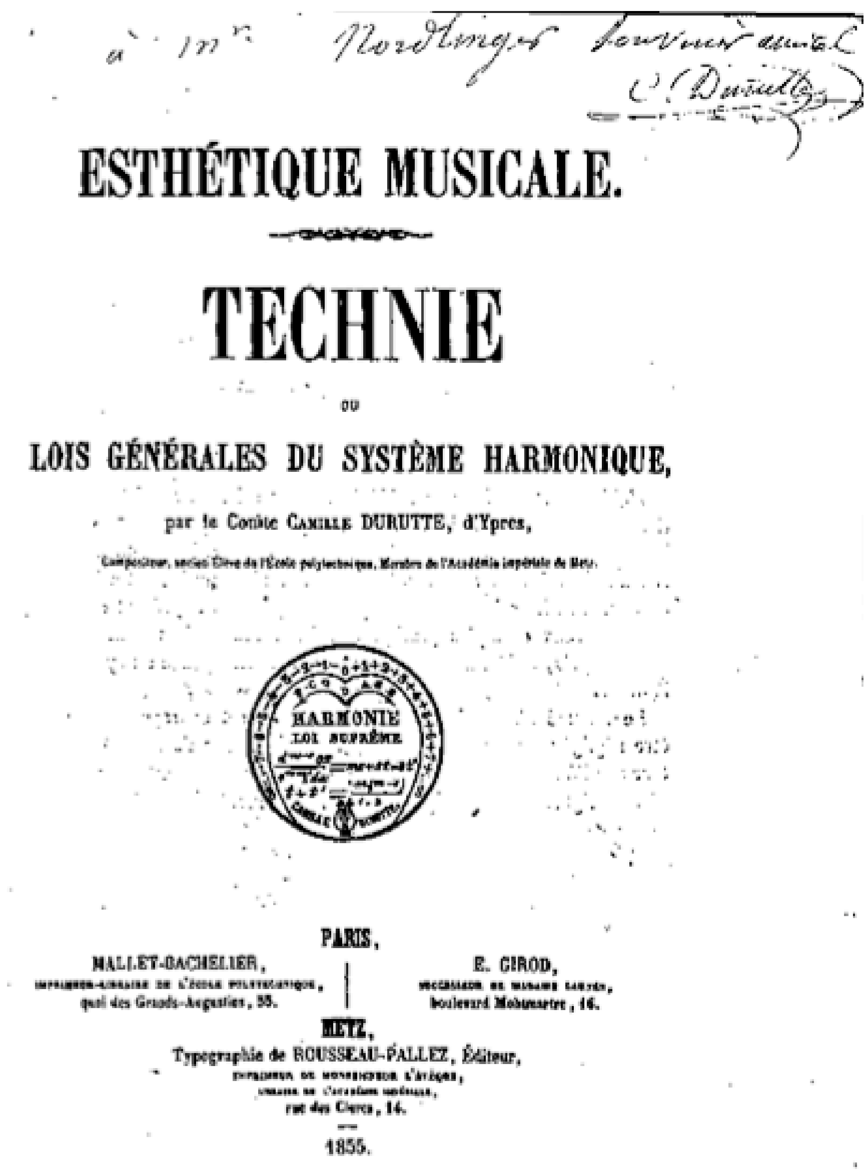


(4,3,5)

Rappresentazione circolare

=> def gruppo ciclico

La relazione di congruenza modulo 12 in musica



ESTHÉTIQUE MUSICALE.

TECHNIE

OU

LOIS GÉNÉRALES DU SYSTÈME HARMONIQUE,

par le Comte CAMILLE DURUTTE, d'Ypres,

Compositeur, ancien Élève de l'École polytechnique, Membre de l'Académie impériale de Metz.

HARMONIE LOI SUPRÊME

PARIS,

MALLET-BACHELIER, imprimeur-libraire de l'École polytechnique, quai des Grands-Augustins, 55.

E. GIROD, boulevard Montmartre, 16.

METZ,

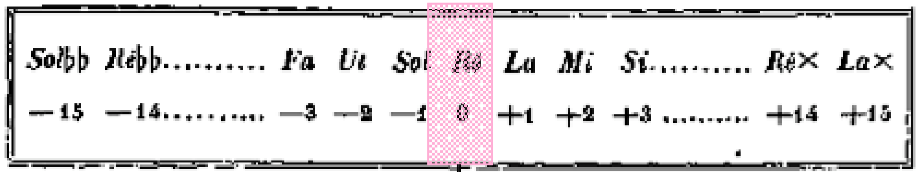
Typographie de ROUSSEAU-PALLEZ, Éditeur,

rue des Clercs, 14.

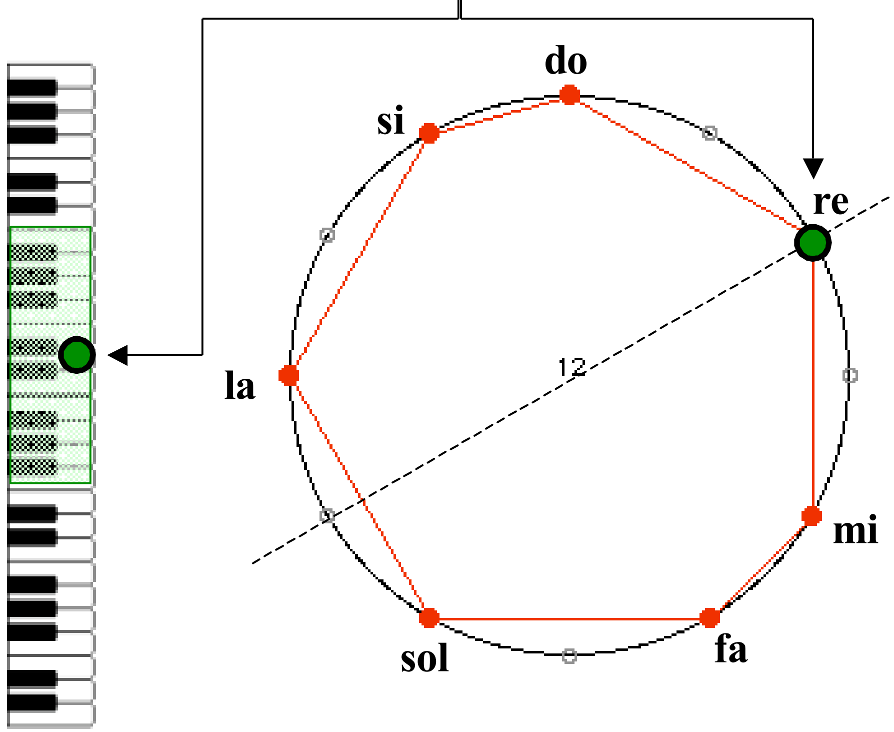
1855.

Camille Durutte:

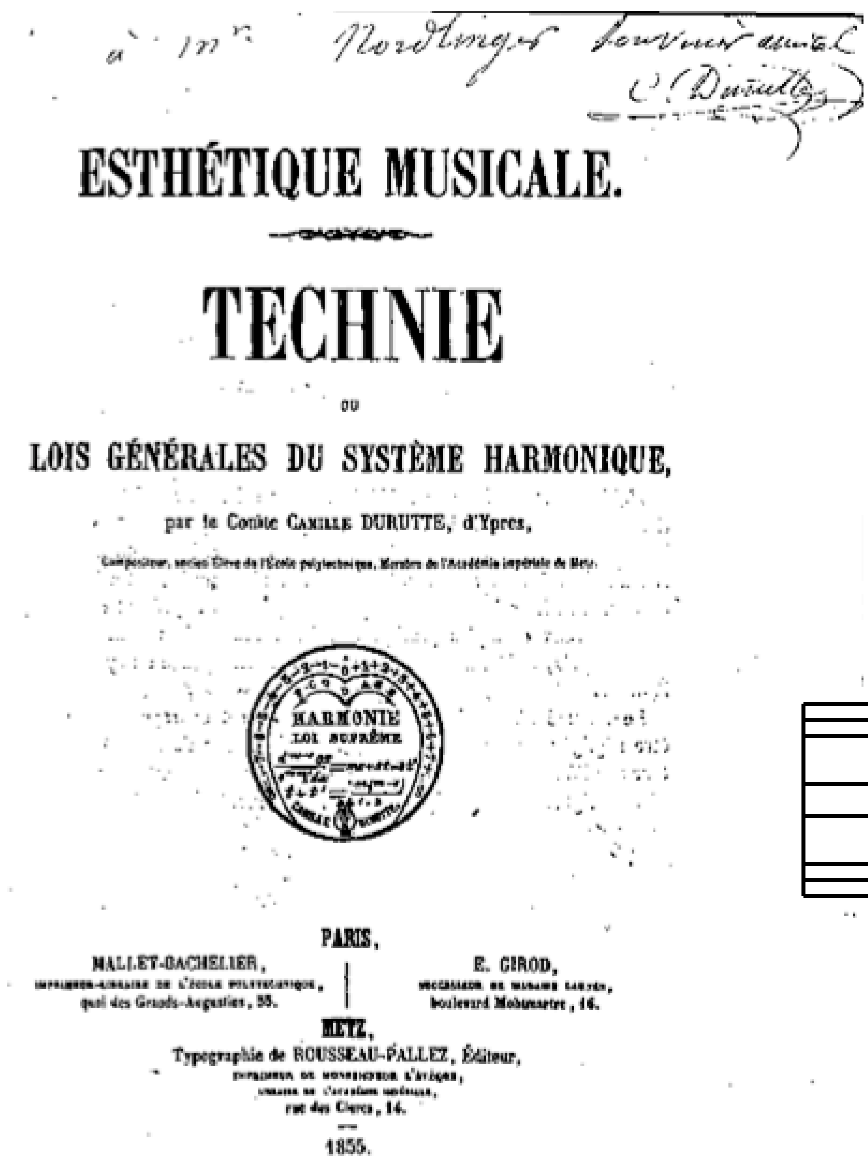
- *Technie, ou lois générales du système harmonique* (1855)
- *Résumé élémentaire de la Technie harmonique, et complément de cette Technie* (1876)



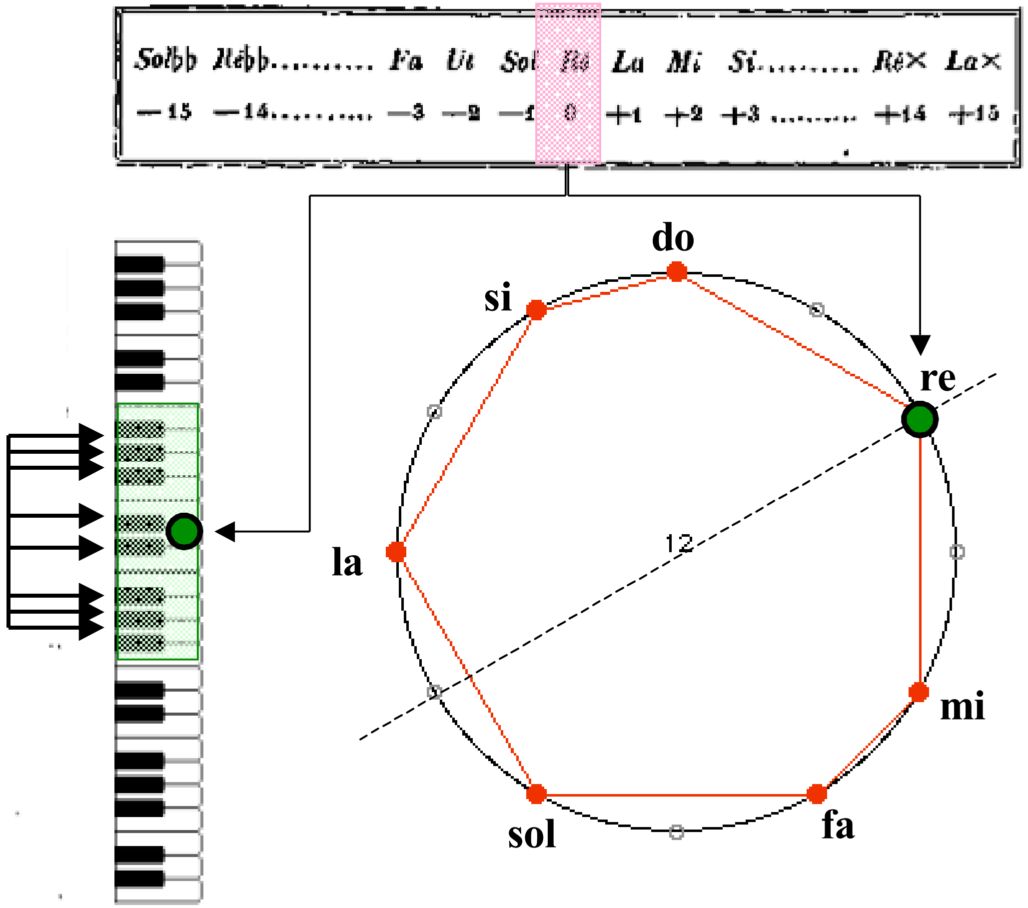
Solbb	Rébb	....	Fa	Ut	Sol	Ré	La	Mi	Si	....	Ré×	La×
—15	—14	....	—3	—2	—1	0	+1	+2	+3	....	+14	+15



La relazione di congruenza modulo 12 in musica



Camille Durutte:
- *Technie, ou lois générales du système harmonique* (1855)
- *Résumé élémentaire de la Technie harmonique, et complément de cette Technie* (1876)



La relazione di congruenza modulo 12 in musica


ESTHÉTIQUE MUSICALE.

TECHNIE

OU

LOIS GÉNÉRALES DU SYSTÈME HARMONIQUE,

par le Comte CAMILLE DURUTTE, d'Ypres,

HARMONIE
LOI SUPRÊME

PARIS,

MALLET-BACHELIER,

quai des Grands-Augustins, 55.

E. GIROD,

boulevard Montmartre, 16.

METZ,

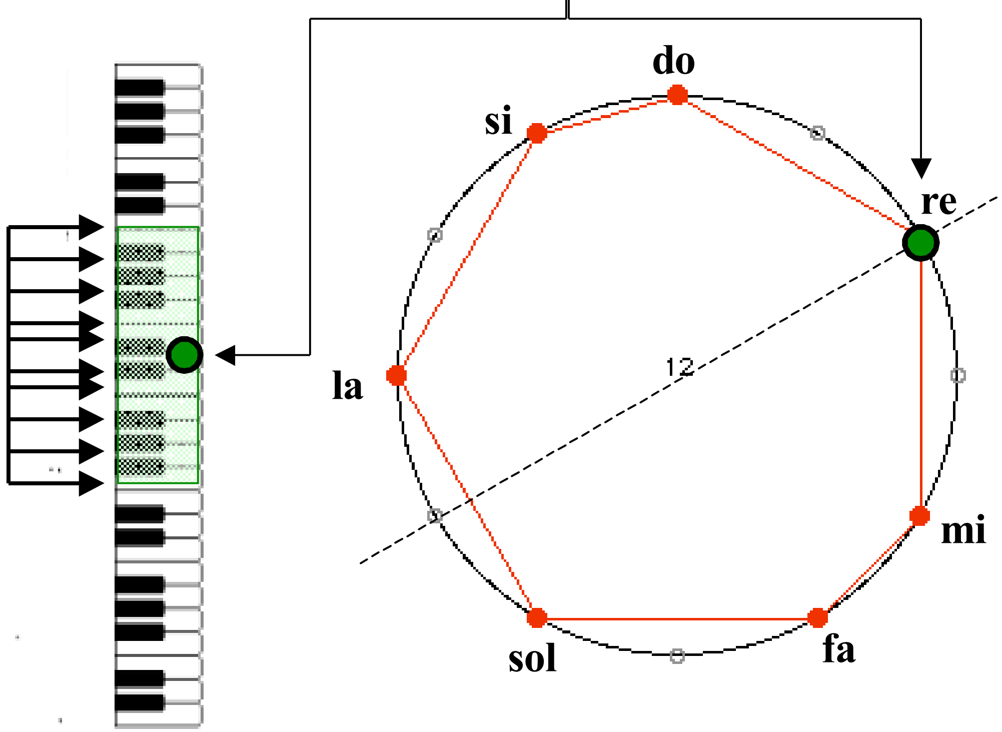
Typographie de ROUSSEAU-PALLEZ, Éditeur,

rue des Clercs, 14.

1855.

Camille Durutte:
- *Technie, ou lois générales du système harmonique* (1855)
- *Résumé élémentaire de la Technie harmonique, et complément de cette Technie* (1876)

Sol♭♭	Ré♭♭	....	Fa	Ut	Sol	Ré	La	Mi	Si	....	Ré×	La×
−15	−14	....	−3	−2	−1	0	+1	+2	+3	....	+14	+15



Simmetrie musicali e strutture matematiche

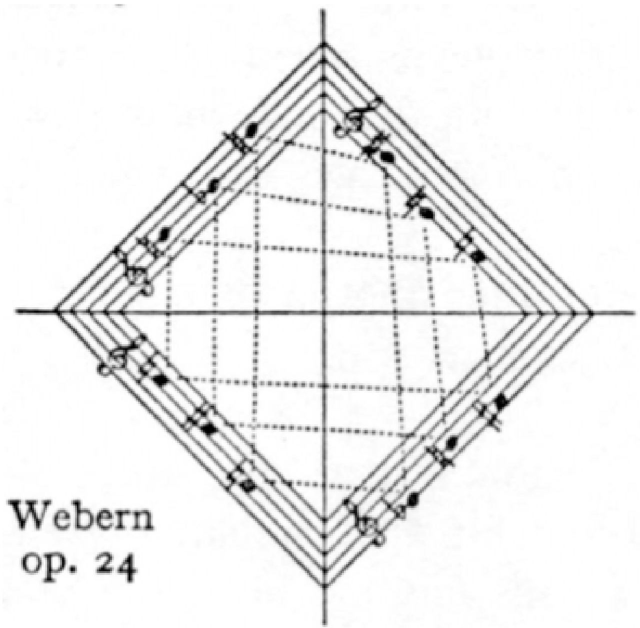
Ernst Krenek e l'approccio assiomatico in musica

- *The Relativity of Scientific Systems*
- *The Significance of Axioms*
- *Axioms in music*
- *Musical Theory and Musical Practice*

Ernst Krenek : *Über Neue Musik*, 1937 (Engl. Transl. *Music here and now,* 1939).

Physicists and mathematicians are far in advance of musicians in realizing that their respective sciences do not serve to establish a concept of the universe conforming to an objectively existent nature

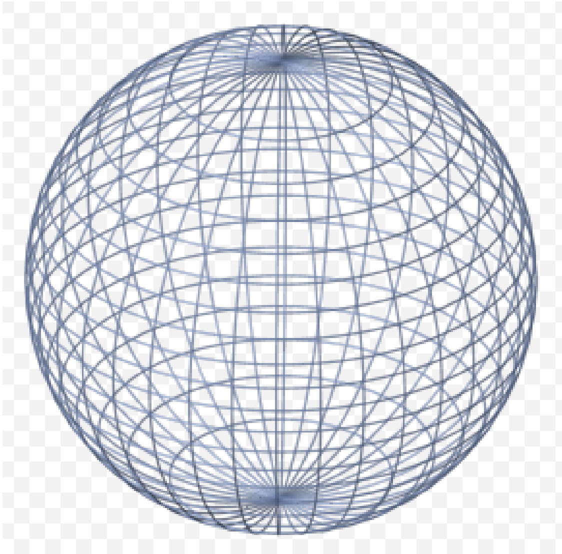
*As the study of axioms eliminates the idea that axioms are something absolute, conceiving them instead as **free propositions of the human mind**, just so would this **musical theory** free us from the concept of major/minor tonality […] as an irrevocable law of nature.*



L'approccio assiomatico in matematica

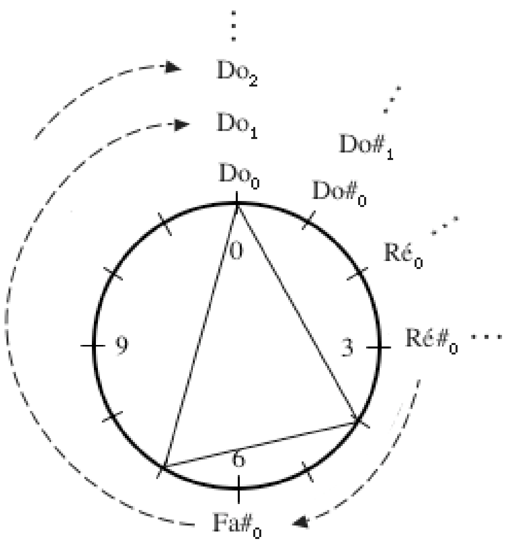
David Hilbert: i fondamenti assiomatici della geometria e il ruolo dell'intuizione

In order to be constructed in a right way, geometry [...] only needs few simple principles. These principles are called the ***axioms*** *of the geometry. [...] This study (of the axioms) goes back to the* ***logical analysis of our spatial intuition*** (Grundlage der Geometrie, 1899).

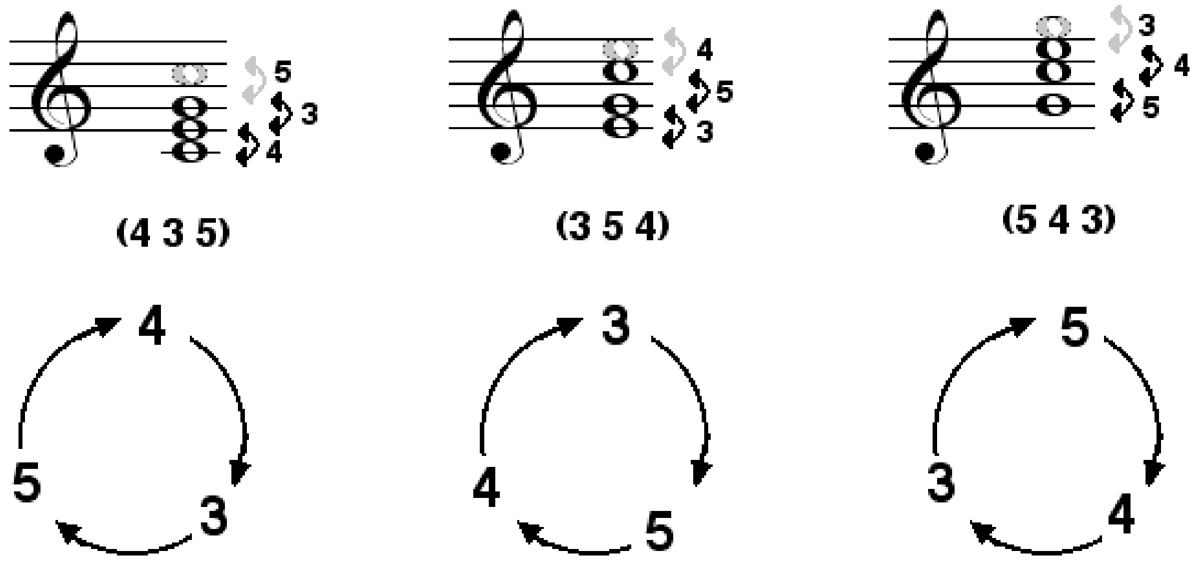


At the moment there are two tendencies in mathematics. From one side, the tendency toward abstraction aims at 'cristallizing' the logical relations inside of a study object and at organizing this material in a systematic way. But there is also a tendency towards the ***intuitive understanding*** *which aims at understanding the* ***concrete meaning of their relations*** (Anschauliche Geometrie, 1932)

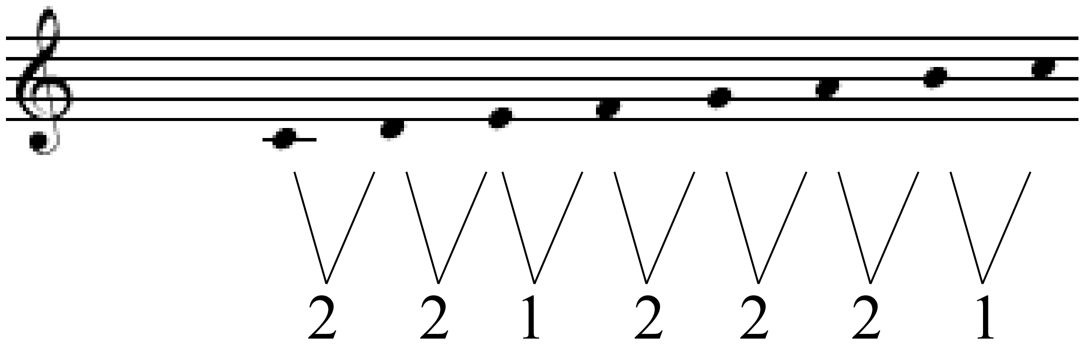
Rappresentazione circolare e struttura intervallare

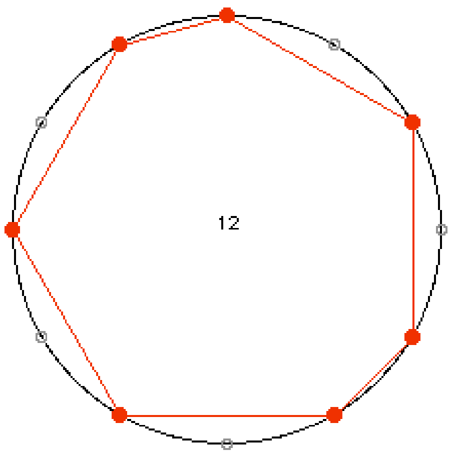


(4,3,5)

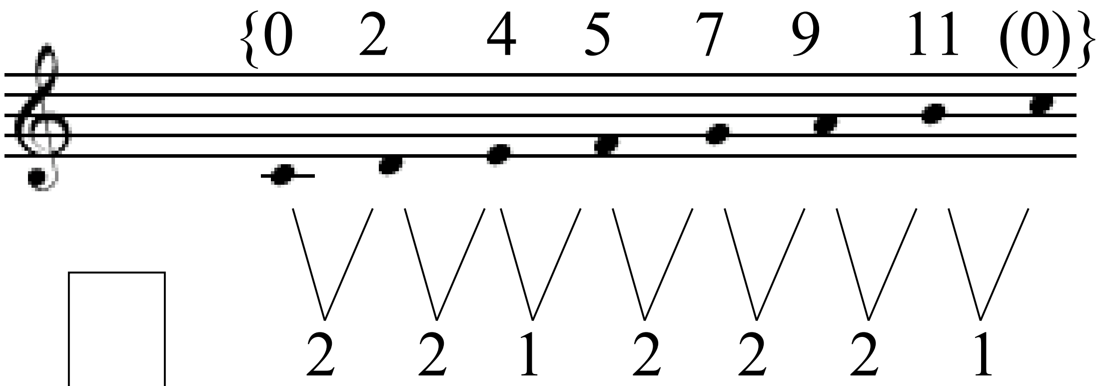


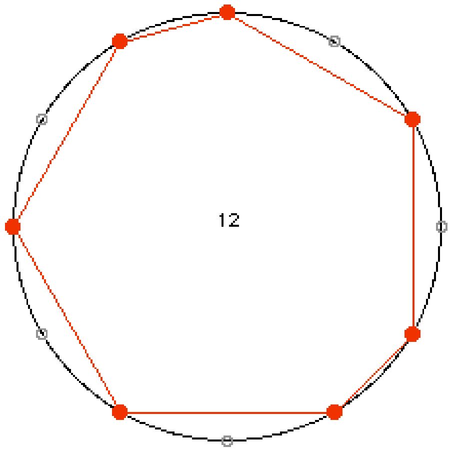
I rivolti di un accordo corrispondono alle permutazioni circolari della struttura intervallare

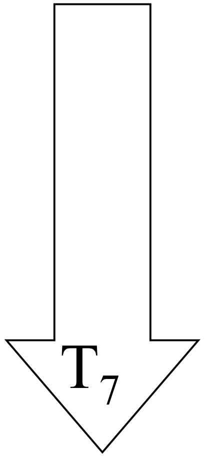




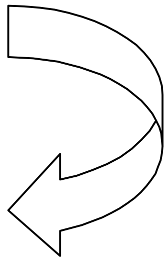
Trasformazioni geometriche: la trasposizione



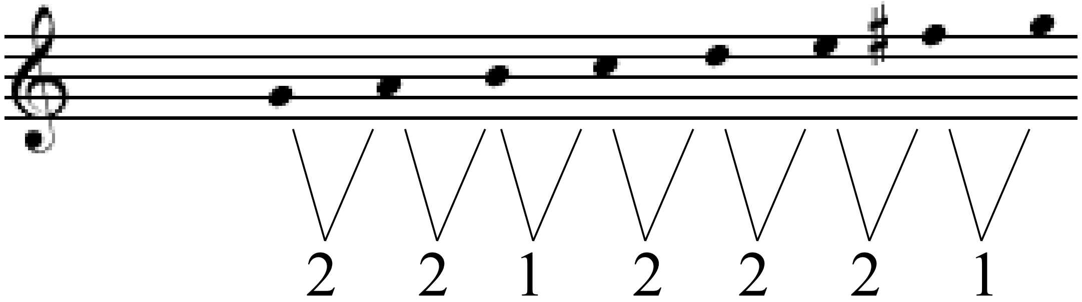


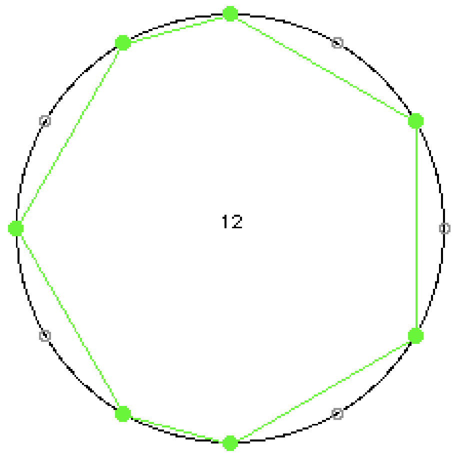


$T_7(x)=7+x$ (mod 12)



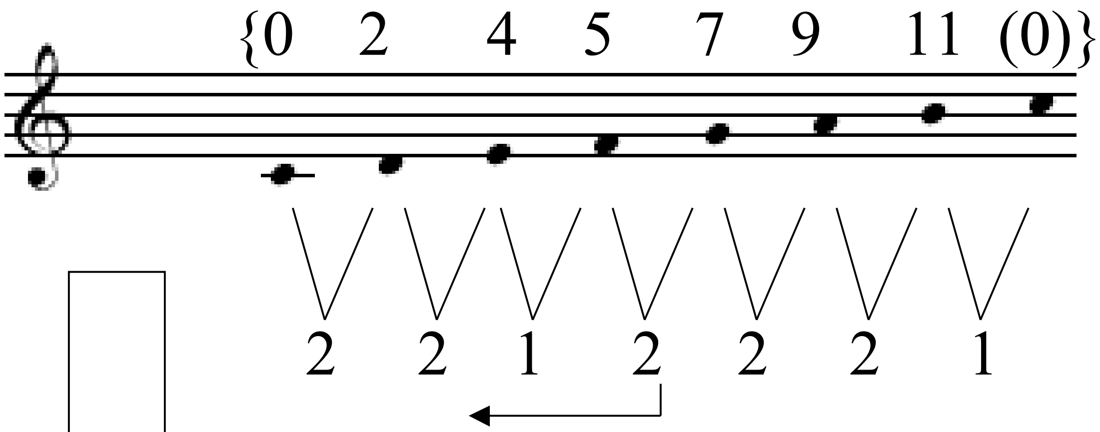
$\alpha=210°$

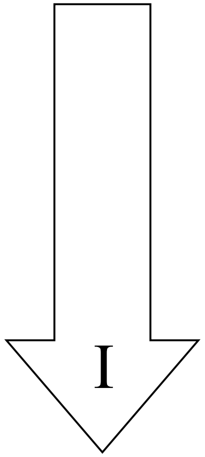




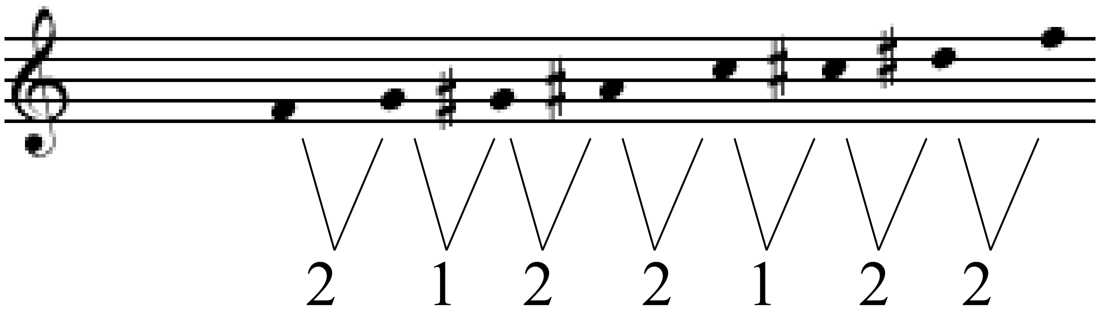
Equivalenza modulo la trasposizione

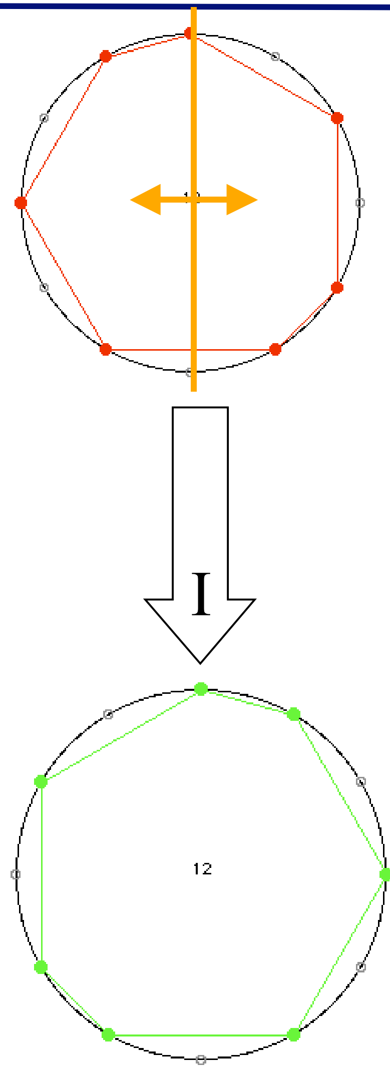
Trasformazioni geometriche: l’inversione





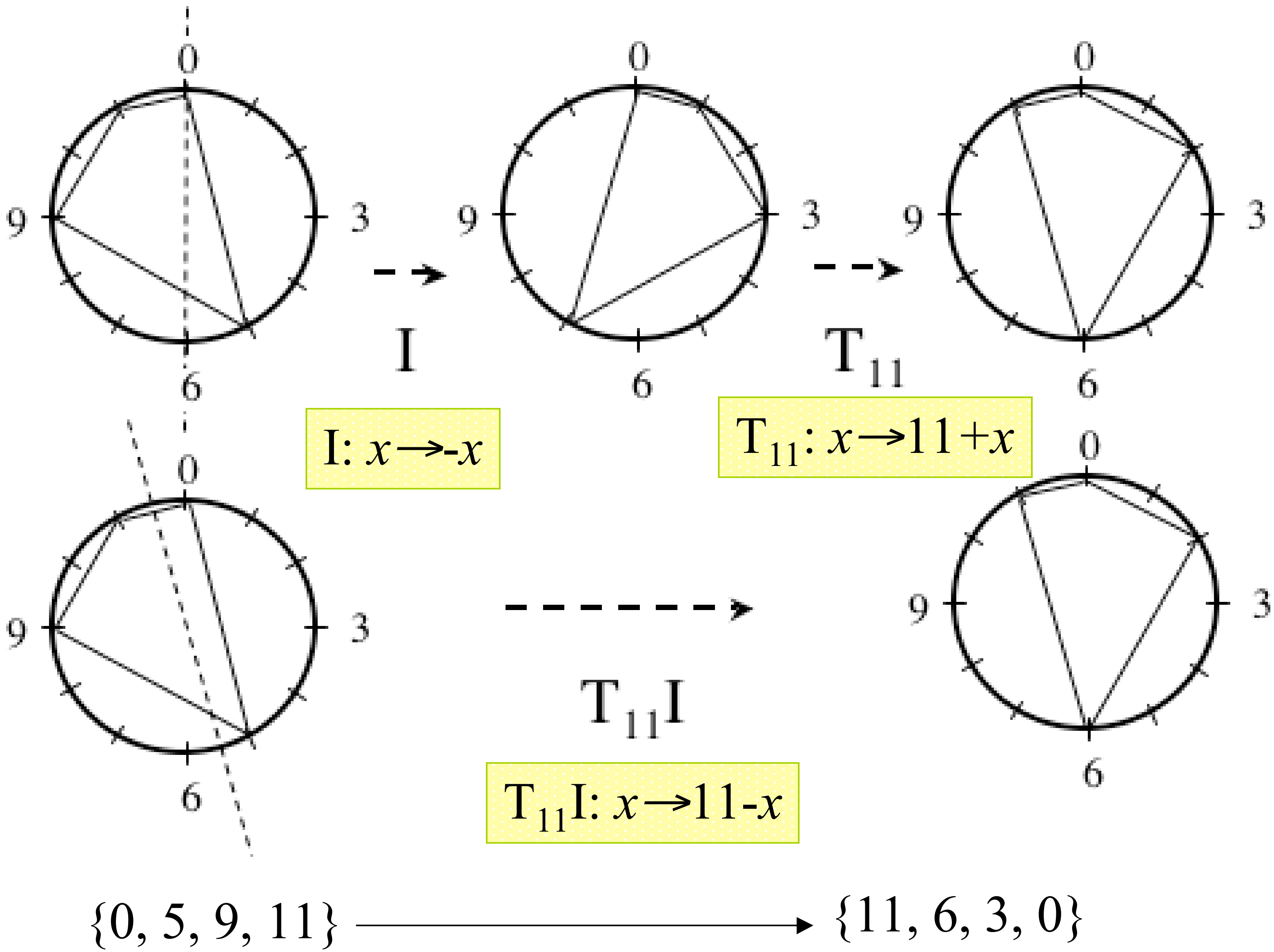
$I(x) = -x \pmod{12}$





Equivalenza modulo l’inversione

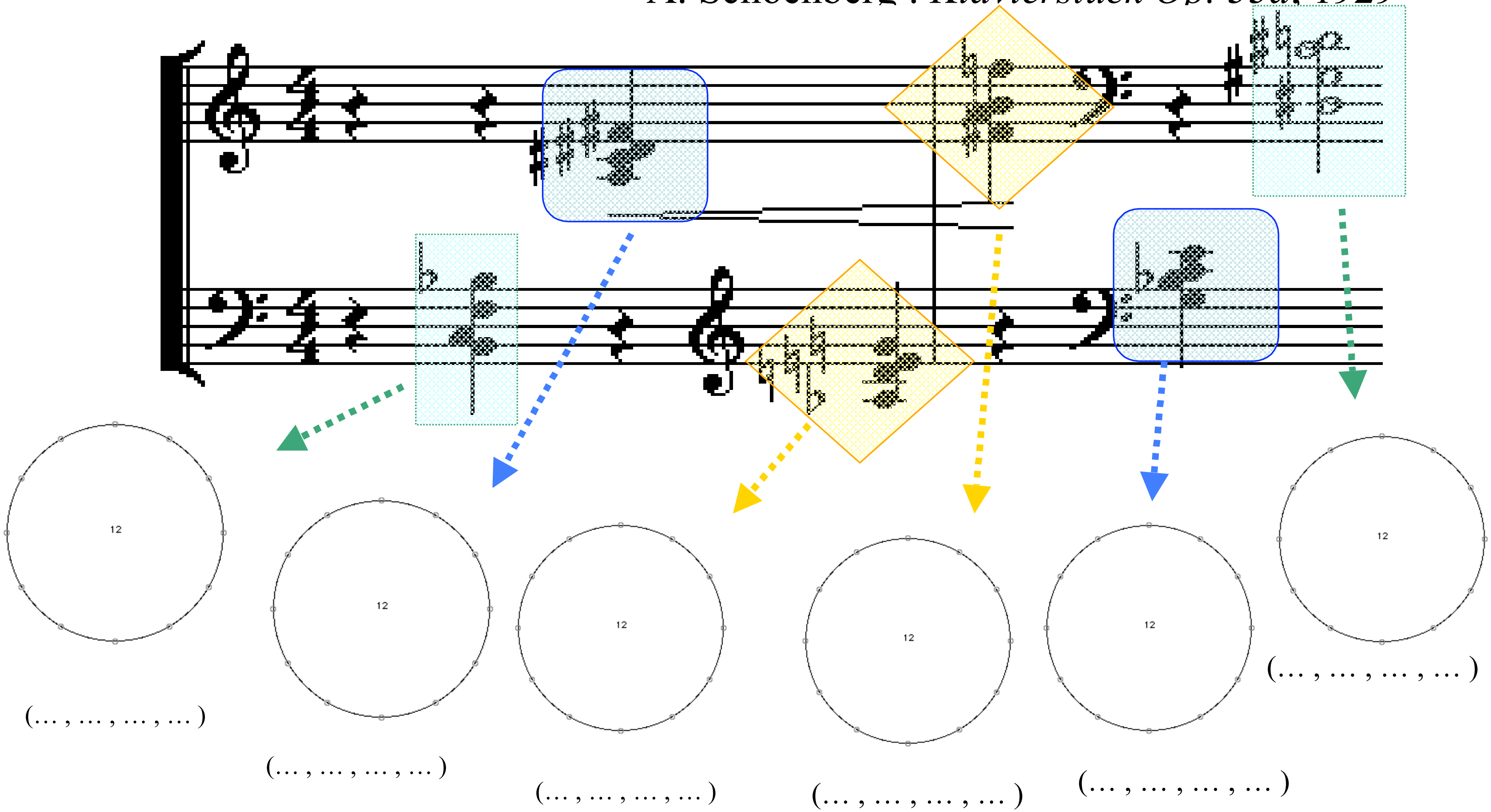
La *Set Theory*: equivalenza per trasposizione/inversione



L’analisi formalizzata o le « entités formelles » in musica

André Riotte e Marcel Mesnage

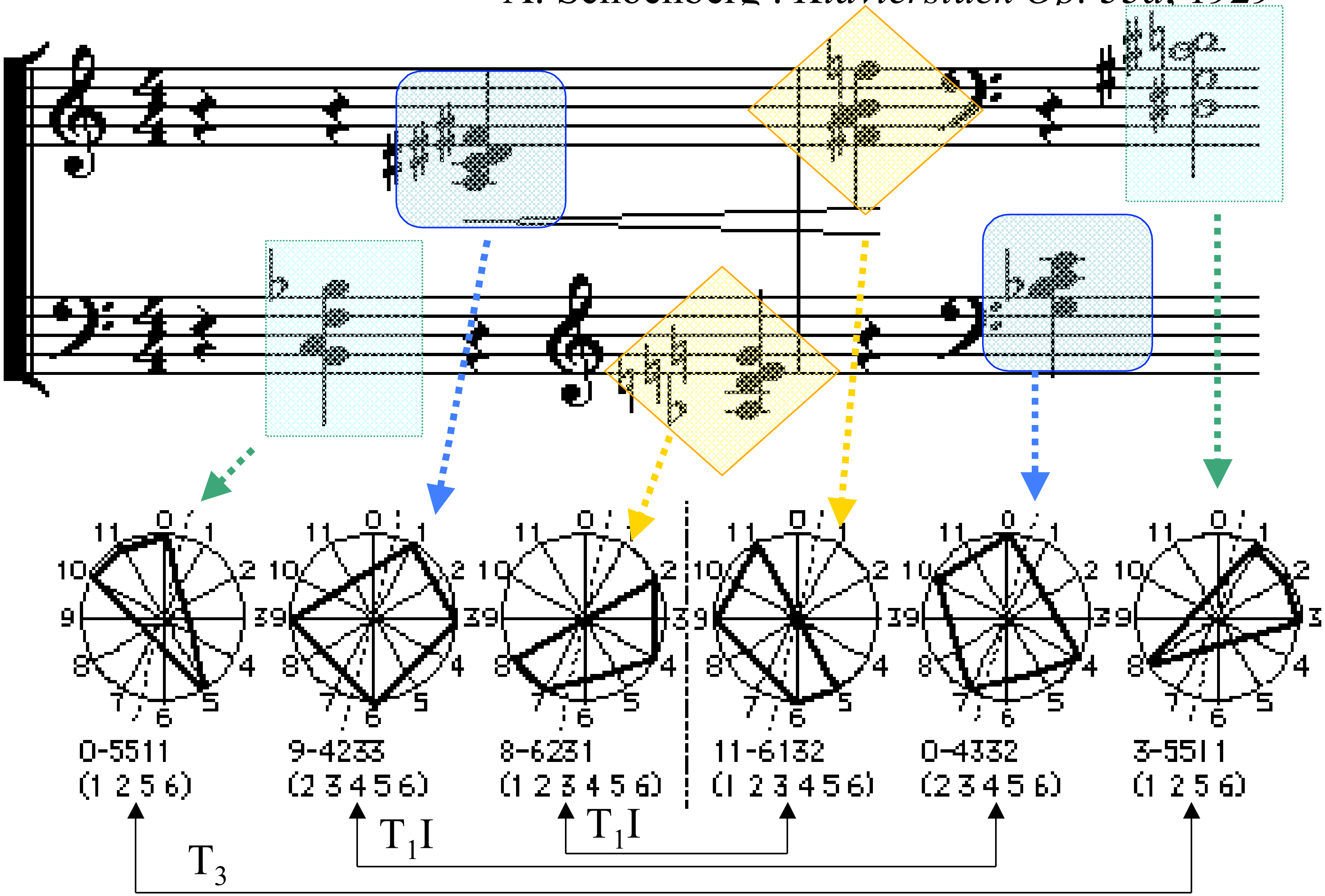
A. Schoenberg : *Klavierstück Op. 33a*, 1929



L'analisi formalizzata o le « entités formelles » in musica

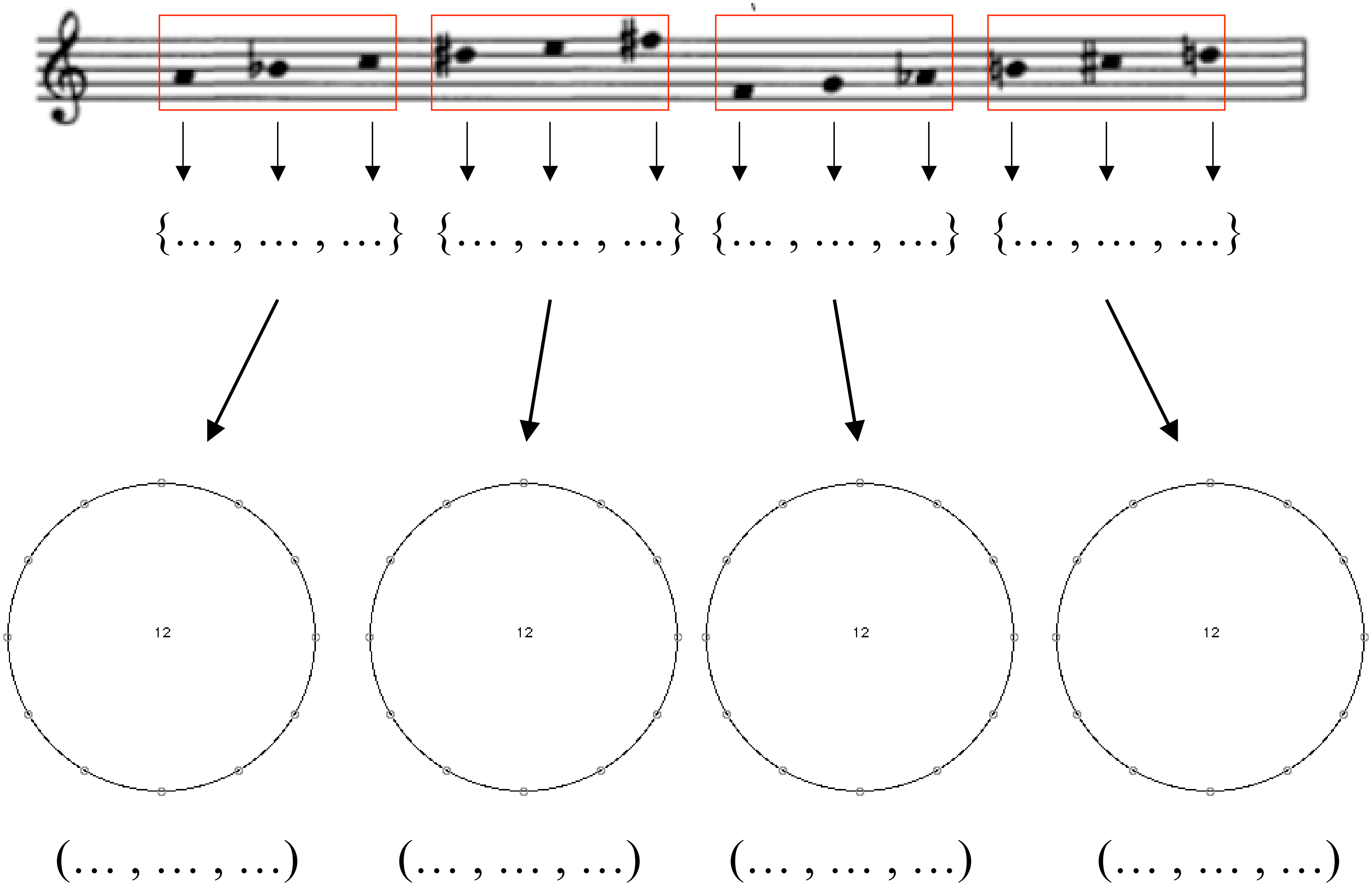
André Riotte e Marcel Mesnage

A. Schoenberg : *Klavierstück Op. 33a*, 1929



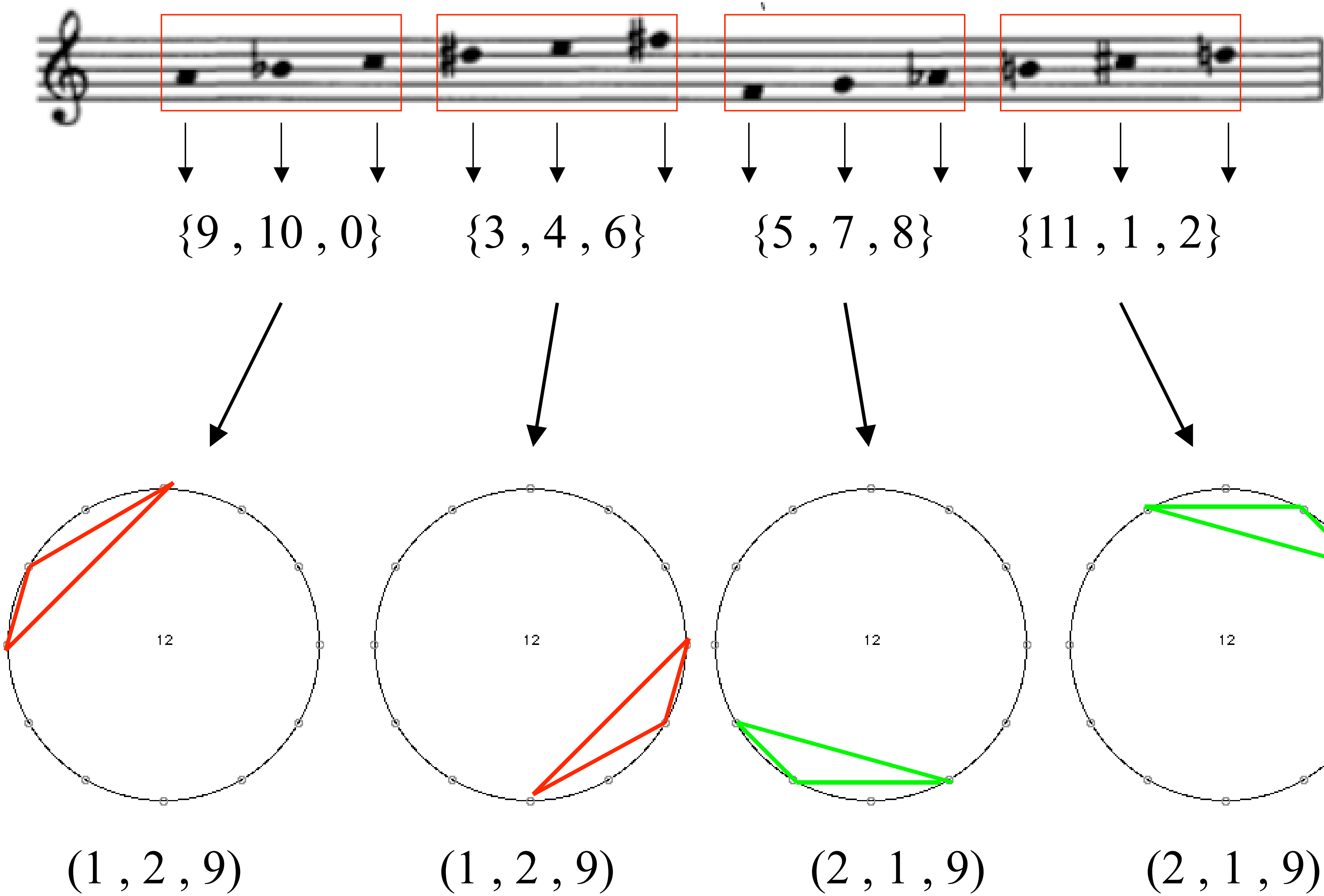
Ejercicio: encontrar las simetrias en una serie dodecafonica

Schoenberg: Serenade Op.24, Mouvement 5



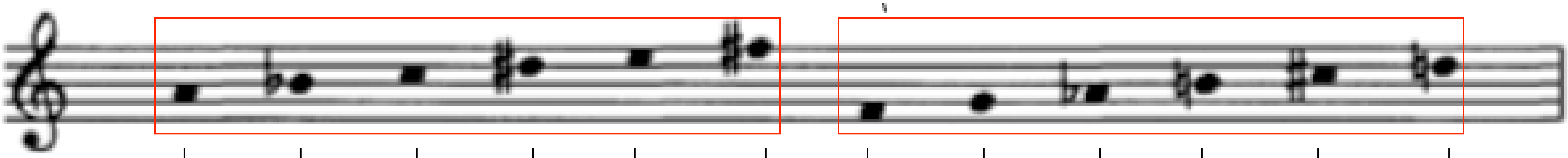
Las simetrias en una serie dodecafonica

Schoenberg: Serenade Op.24, Mouvement 5

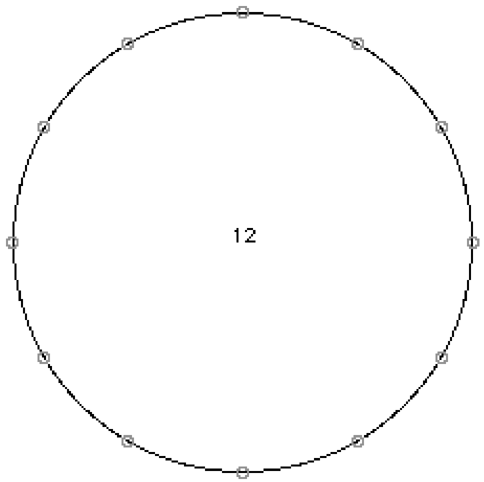


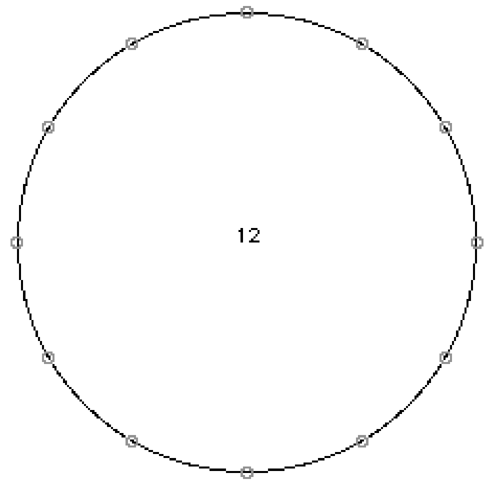
Ejercicio: encontrar las simetrias (pero con otra segmentation)

Schoenberg: Serenade Op.24, Mouvement 5



{… , … , … , … , … , … }   {… , … , … , … , … , … }

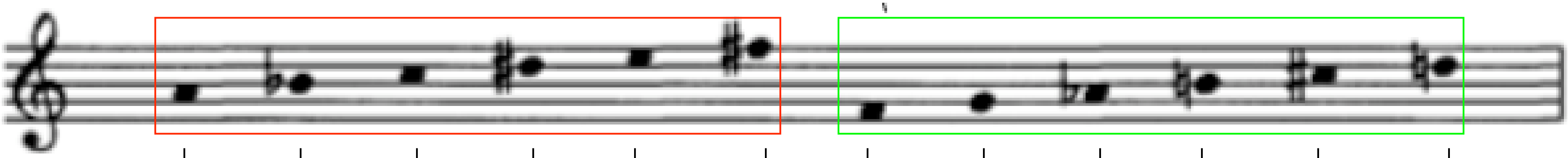




(… , … , … , … , … , …)   (… , … , … , … , … , …)

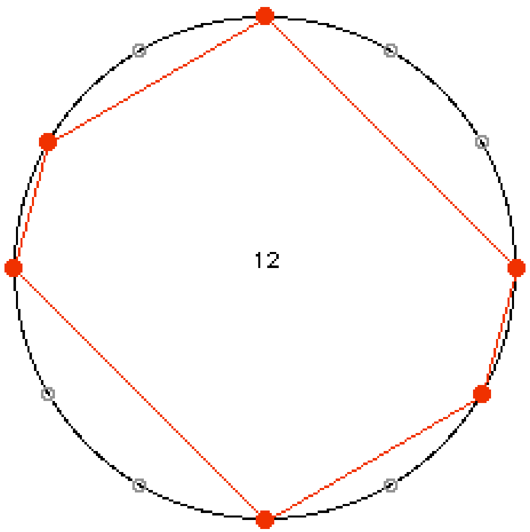
Otra simetria

Schoenberg: Serenade Op.24, Mouvement 5

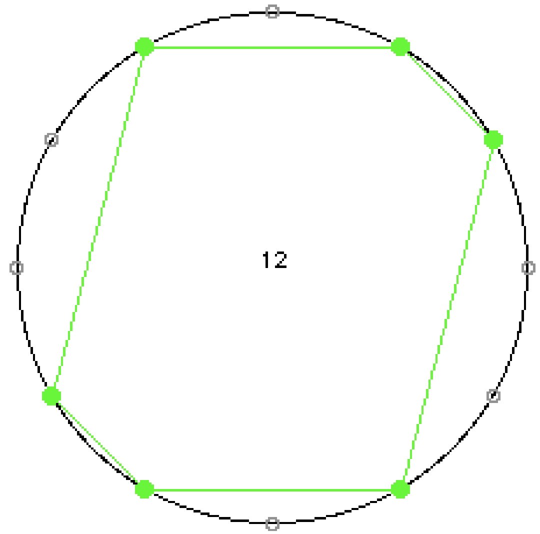


{9 , 10 , 0 , 3 , 4 , 6 }

{5 , 7 , 8 , 11 , 1 , 2 }



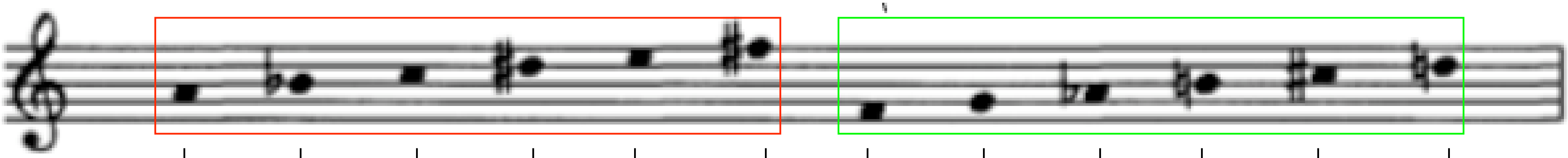
(3, 1, 2, 3, 1, 2)



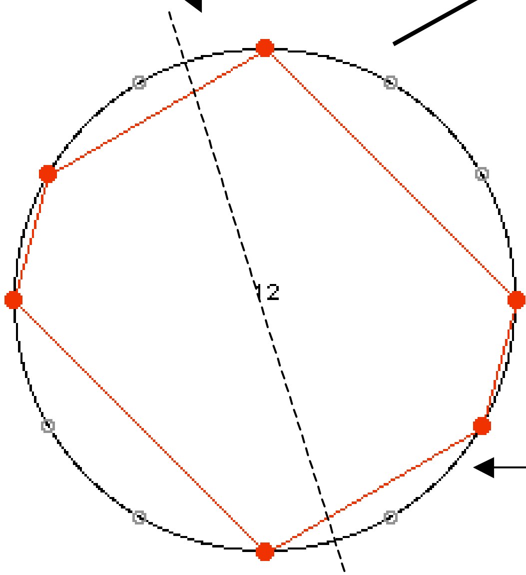
(2, 1, 3, 2, 1, 3)

Simmetria trasposizionale e “combinatorialità” esacordale

Schoenberg: Serenade Op.24, Mouvement 5



A = {9 , 10 , 0 , 3, 4 , 6} {5 , 7, 8, 11, 1, 2} =A’



(3, 1, 2, 3, 1, 2)

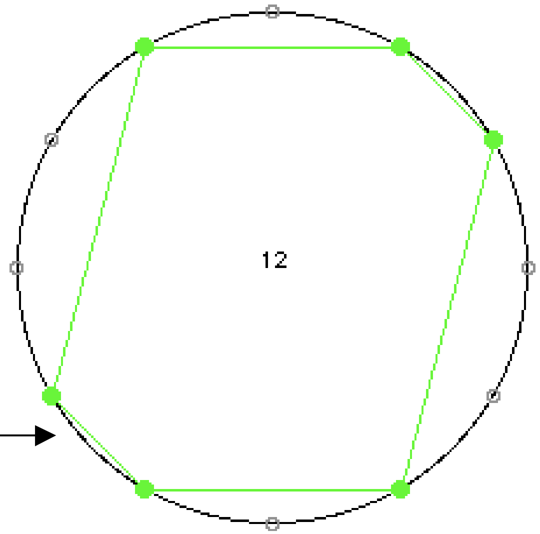
$T6\{9,10,0,3,4,6\}=$

$=\{6+9,6+10,6,6+3,6+4,6+6\}=$

$=\{3,4,6,9,10,0\}$

$T6(A)=A$

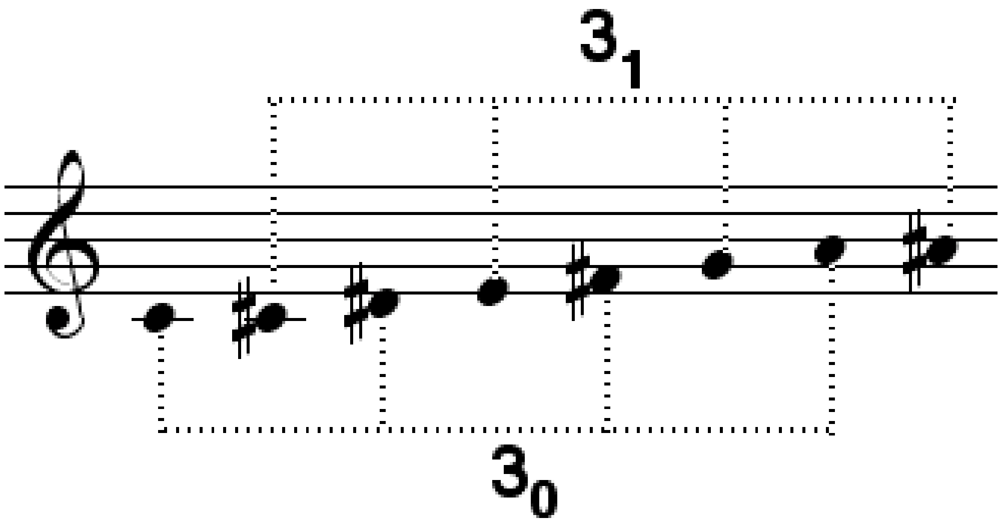
$\mathbf{I_{11} = T_{11}\, I}$

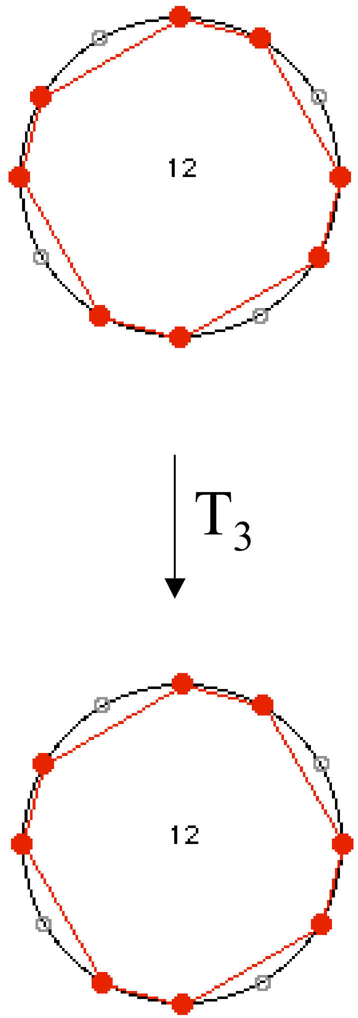


(2, 1, 3, 2, 1, 3)

Formalizzazione dei modi di Messiaen a trasposizione limitata

Quante e quali scale musicali hanno le stesse proprietà strutturali della scala ottatonica (semitono-tono)?

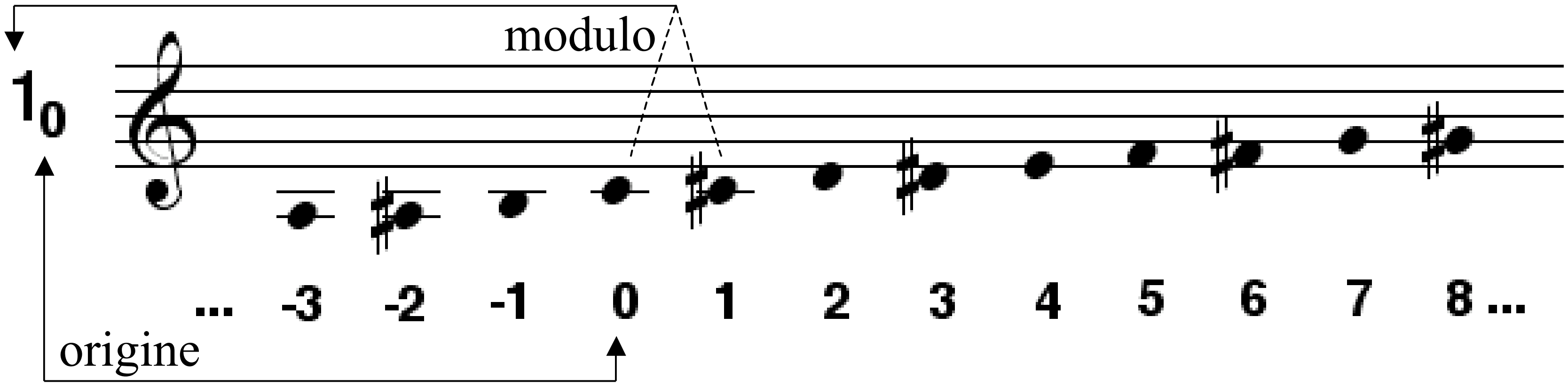


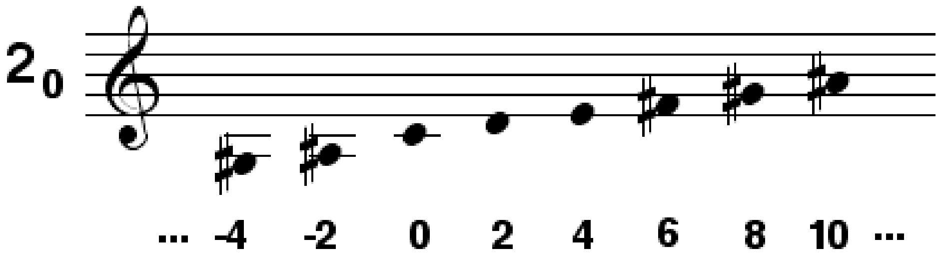


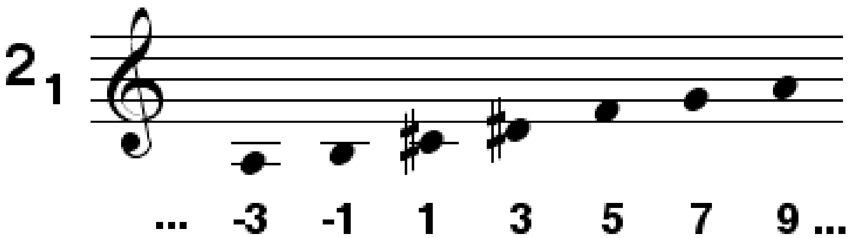
Trovare tutte le scale musicali che si ripetono esattamente ad una trasposizione T_k di k semitoni ($k \neq 0$ mod 12)

La teoria dei setacci

Formalizzazione algebrica di strutture musicali secondo Xenakis







$1_0 = 2_0 \cup 2_1$

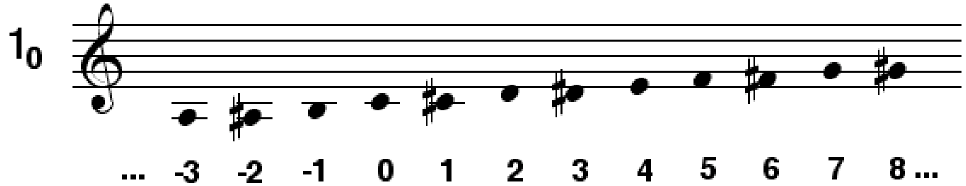
$2_0 \cap 2_1 = \varnothing$

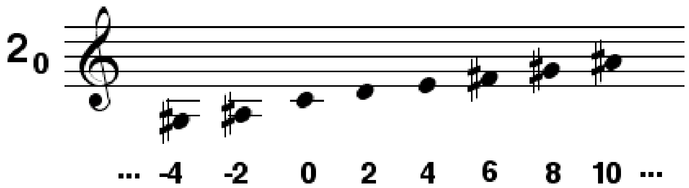
$(2_0)^c = 2_1$

$(2_1)^c = 2_0.$

La teoria dei setacci

I « Modes à transpositions limitées » d'Olivier Messiaen



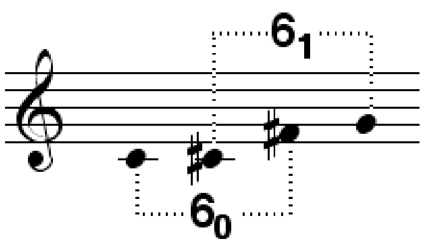


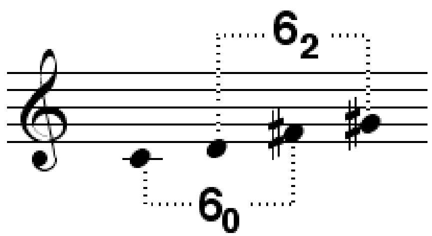
$T_m(A)=A$

$m \neq 0 \mod 12$

(3_0)	Settima diminuita
(4_0)	Triade aumentata
(6_0)	Tritono

$6_0 \cup 6_1$





$6_0 \cup 6_2$

$6_0 \cup 6_3$?

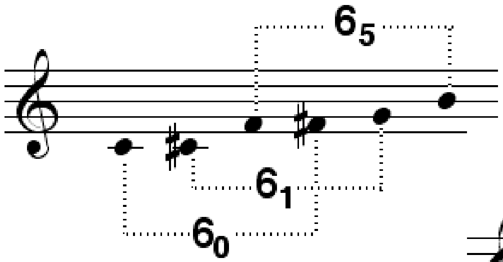
« Cribles » / Messiaen

Verso un catalogo esaustivo

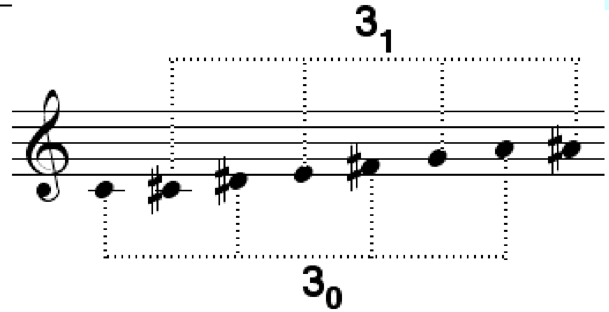
(1_0) (2_0) (3_0) (4_0) (6_0) $6_0 \cup 6_1$ $6_0 \cup 6_2$

$6_0 \cup 6_1 \cup 6_5$

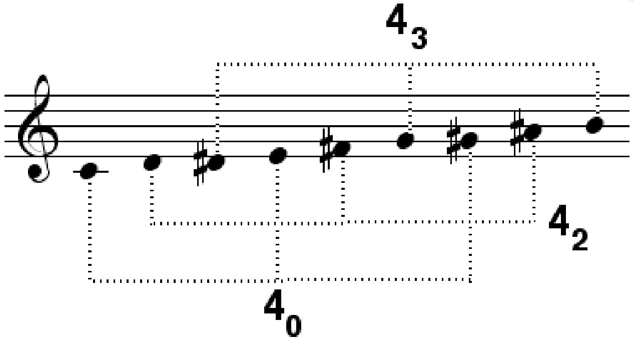
Modo n.5

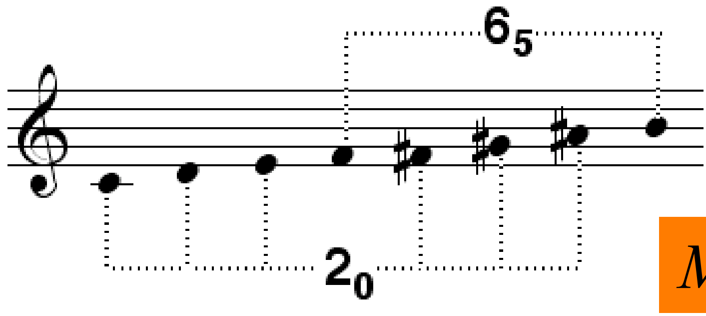


$3_0 \cup 3_1$ *Modo n.2*



$4_0 \cup 4_2 \cup 4_3$ *Modo n.3*

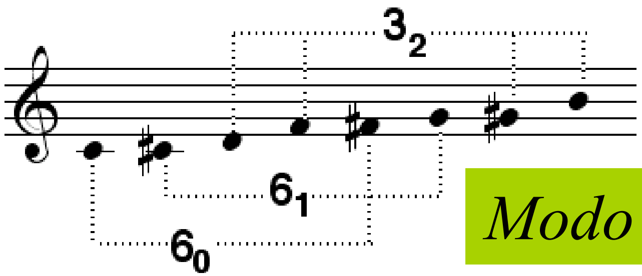




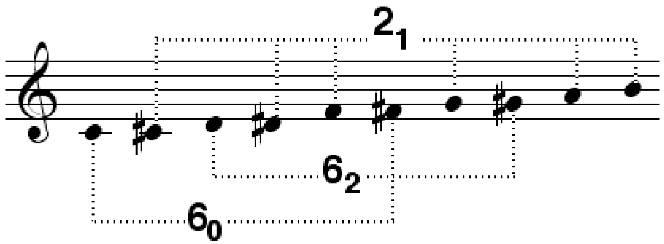
$2_0 \cup 6_5$

Modo n.6

$6_0 \cup 6_1 \cup 3_2$



Modo n.4



Modo n.7 $2_1 \cup 6_0 \cup 6_2$

(6_0) (2_0) $6_0 \cup 6_1$ $6_0 \cup 6_1 \cup 6_5$ $4_0 \cup 4_2 \cup 4_3$ $3_0 \cup 3_1$

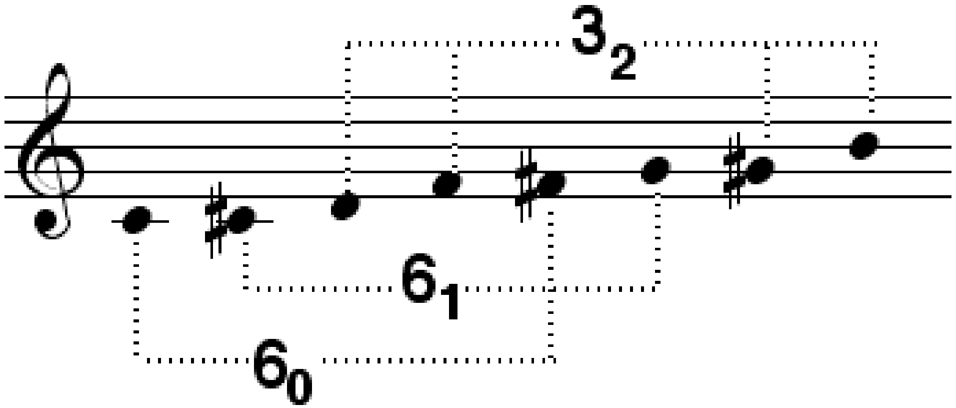
« Cribles » / Messiaen

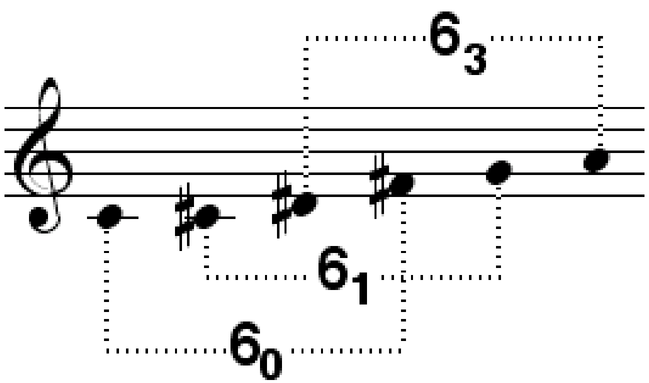
Modi dimenticati…

(1_0) (4_0)

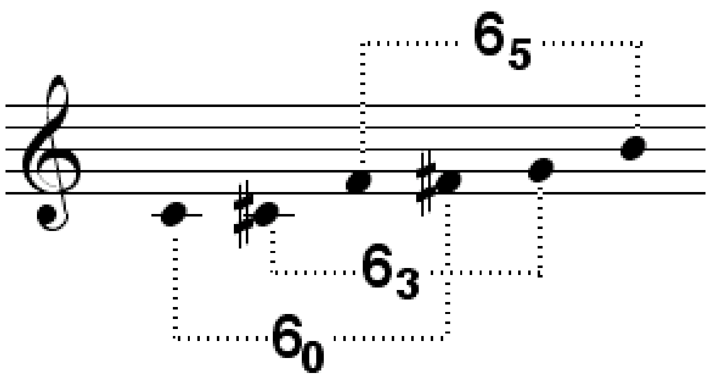
$6_0 \cup 6_2$ $6_0 \cup 6_1 \cup 3_2$ $2_0 \cup 6_5$ $2_1 \cup 6_0 \cup 6_2$ (3_0)

$6_0 \cup 6_1 \cup 3_2$





$6_0 \cup 6_1 \cup 6_3$



$6_0 \cup 6_3 \cup 6_5$

Assiomatizzazioni, teoria dei gruppi e dei setacci

« *La formalizzazione e l'assiomatizzazione costituiscono uno strumento procedurale [guide processionnel] più adatto al pensiero moderno in generale* »
(Musiques formelles, 1963)

« *...formulazione universale per ciò che riguarda la **percezione** delle altezze: lo spazio degli intervalli melodici è provvisto di una struttura di **gruppo** avente come legge di composizione interna l'addizione »*
(« La voie de la recherche et de la question », *Preuves,* n° 177, nov. 1965)

« ...teoria dei setacci [cribles], una teoria che annette le congruenze modulo n e che deriva da un'assiomatizzazione della struttura universale della musica »
(Descrittivo di *Nomos Alpha* per violoncello solo, 1966)

Portata « universale » della teoria dei setacci

« ...la teoria si può applicare ad ogni caratteristica (musicale) dotata di una struttura d'ordine totale, come le intensità, gli attacchi, le densità etc. Inoltre in un futuro immediato assisteremo ad un esplorazione della teoria e delle sue multiple applicazioni attraverso il computer, visto che la essa è completamente implementabile »

(*Arts/Sciences - Alloys*, Stuyvesant: Pendragon Press, 1985)

Isomorfismo altezze/ritmi secondo Xenakis

« [Con la teoria dei setacci] si possono costruire delle ***architetture ritmiche*** *estremamente complesse che possono arrivare persino alla distribuzione pseudo-aleatoria di punti si una retta se il periodo è sufficientemente lungo »*
(« Redécouvrir le temps », éditions de l'Université de Bruxelles, 1988)

$A = (\mathbf{13_3} \cup \mathbf{13_5} \cup \mathbf{13_7} \cup \mathbf{13_9})^c$

$B = \mathbf{11_2}$

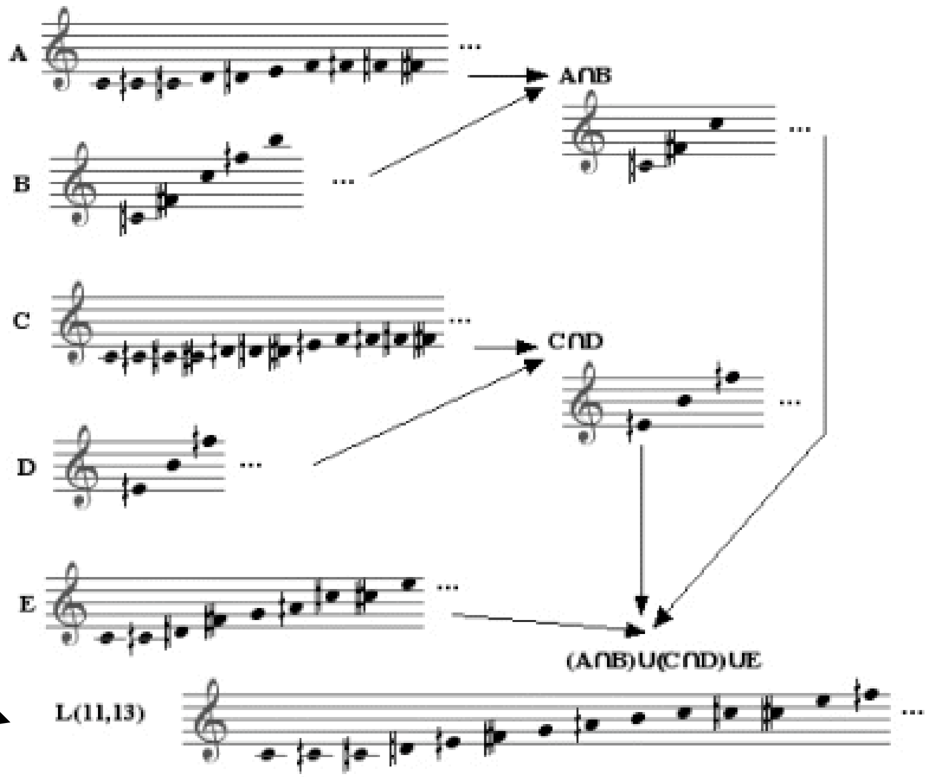
$C = (\mathbf{11_4} \cup \mathbf{11_8})^c$

$D = \mathbf{13_9}$

$E = \mathbf{13_0} \cup \mathbf{13_1} \cup \mathbf{13_6}$

(A∩B)∪(C∩D)∪E

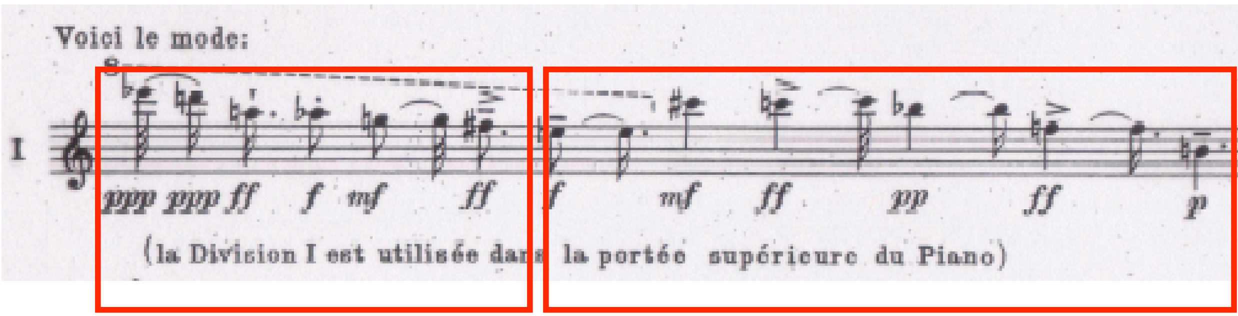
(*Nomos Alpha*, 1966)

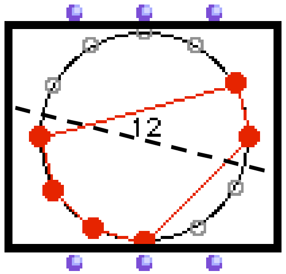


0, 2, 13, 24, 35, …

Isomorfismo altezze/ritmi in Messiaen

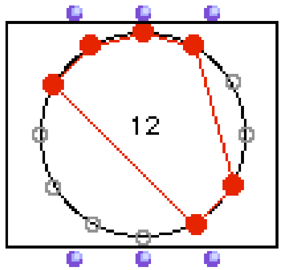
- Mode de valeurs et d'intensités (1950)





$\{3,2,9,8,7,6\} \longrightarrow \{4,5,10,11,0,1\}$

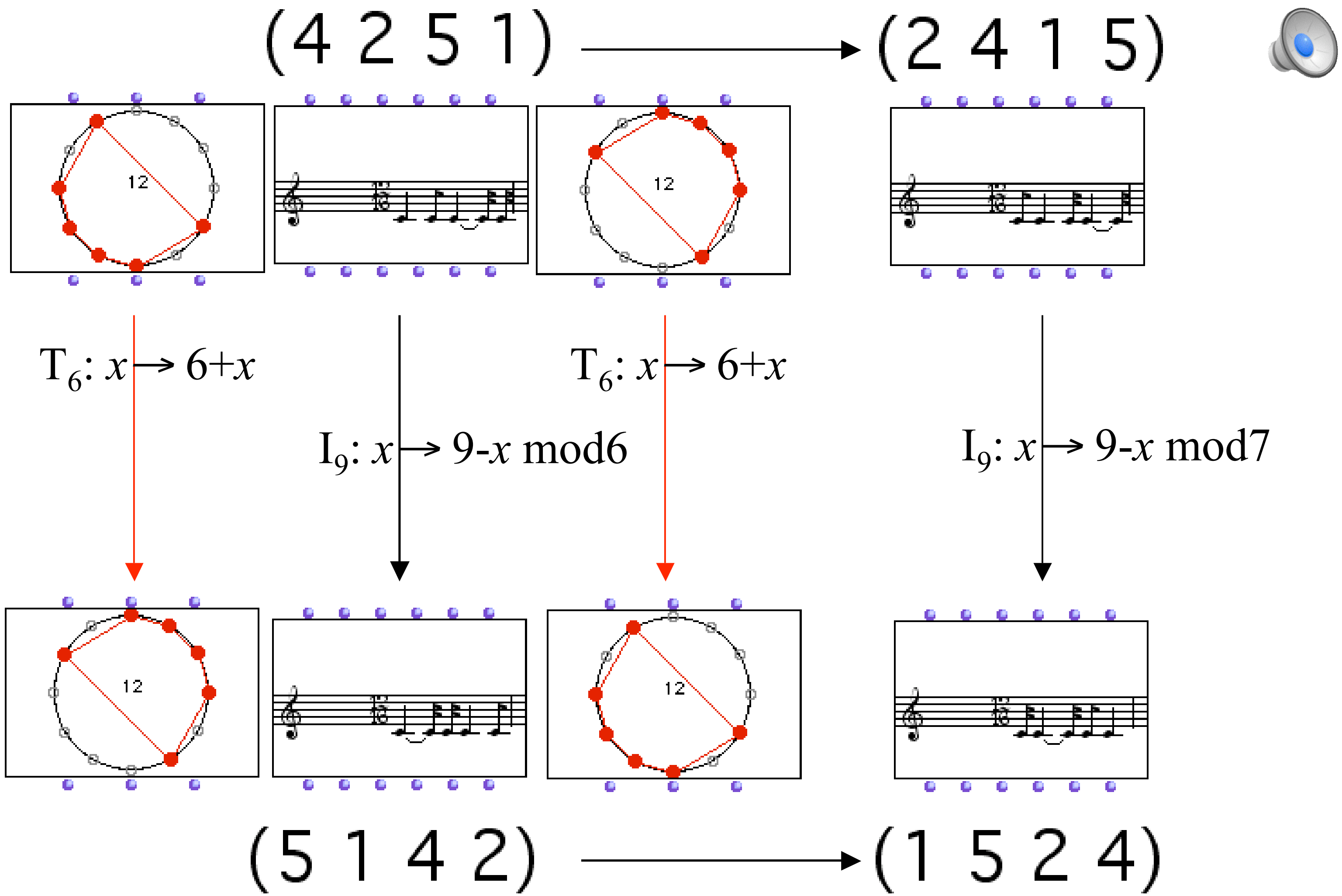
$T_7I : x \rightarrow 7\text{-}x$



Combinatorialità esacordale e interpretazioni ritmiche

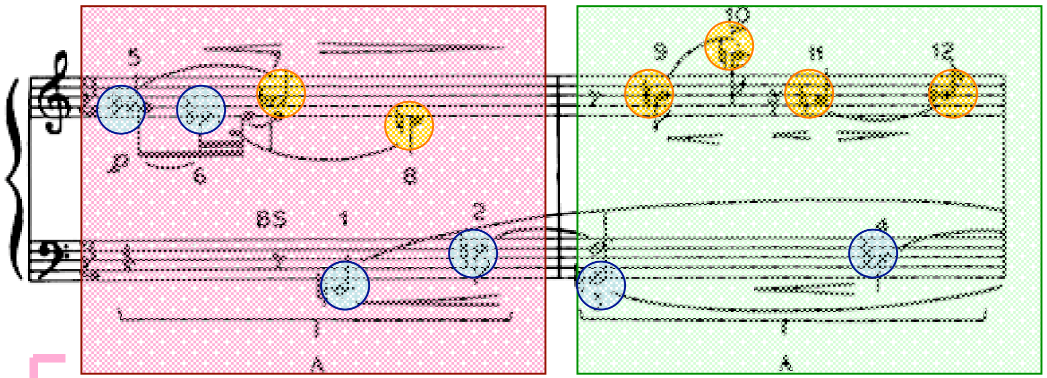
- *Three compositions for piano* (1948)

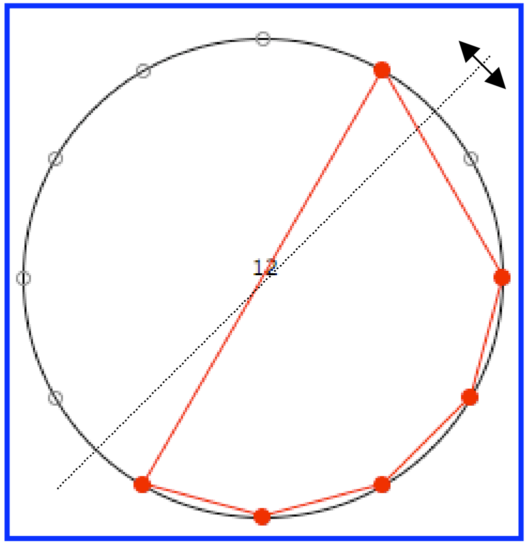
mm.14-16



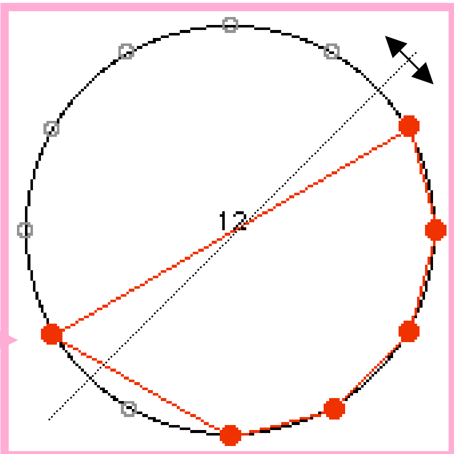
Set Theory, serialismo e equivalenza modulo trasposizione/inversione

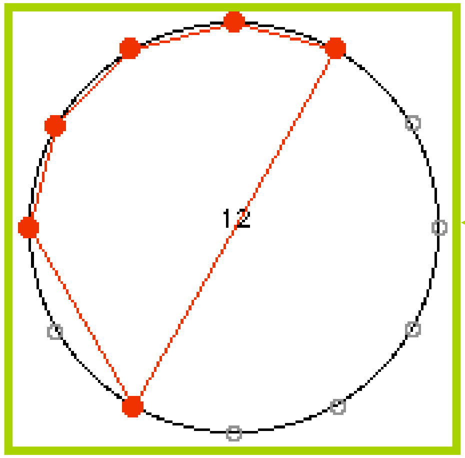
Schoenberg: Suite Op.25, Minuetto

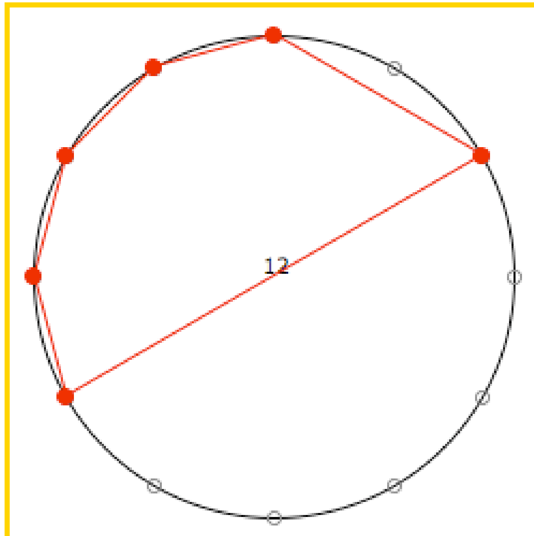




“Combinatorialità” esacordale







Operazioni dodecafoniche e strutture matematiche

Il sistema dodecafonico è "*un insieme d'elementi, **relazioni** fra gli elementi e **operazioni** sugli elementi*. […] *Un'effettiva matematizzazione avrebbe bisogno di una formulazione e di una presentazione dettata dal fatto che il sistema dodecafonico è un **gruppo di permutazioni** determinato* [shaped] *dalla struttura di questo modello matematico*"

M. Babbitt: *The function of Set Structure in the Twelve-Tone System,* PhD (1946/1992)

Operazioni dodecafoniche e strutture matematiche

Il sistema dodecafonico è "*un insieme d'elementi, **relazioni** fra gli elementi e **operazioni** sugli elementi*. […] *Un'effettiva matematizzazione avrebbe bisogno di una formulazione e di una presentazione dettata dal fatto che il sistema dodecafonico è un **gruppo di permutazioni** determinato* [shaped] *dalla struttura di questo modello matematico*"

M. Babbitt: *The function of Set Structure in the Twelve-Tone System,* PhD (1946/1992)

Operazioni dodecafoniche e strutture matematiche

Il sistema dodecafonico è "*un insieme d'elementi, **relazioni** fra gli elementi e **operazioni** sugli elementi*. […] *Un'effettiva matematizzazione avrebbe bisogno di una formulazione e di una presentazione dettata dal fatto che il sistema dodecafonico è un **gruppo di permutazioni** determinato* [shaped] *dalla struttura di questo modello matematico*"

M. Babbitt: *The function of Set Structure in the Twelve-Tone System,* PhD (1946/1992)

Operazioni dodecafoniche e strutture algebriche

+	S	I	R	RI
S	S	I	R	RI
I	I	S	RI	R
R	R	RI	S	I
RI	RI	R	I	S

Struttura di *gruppo:*

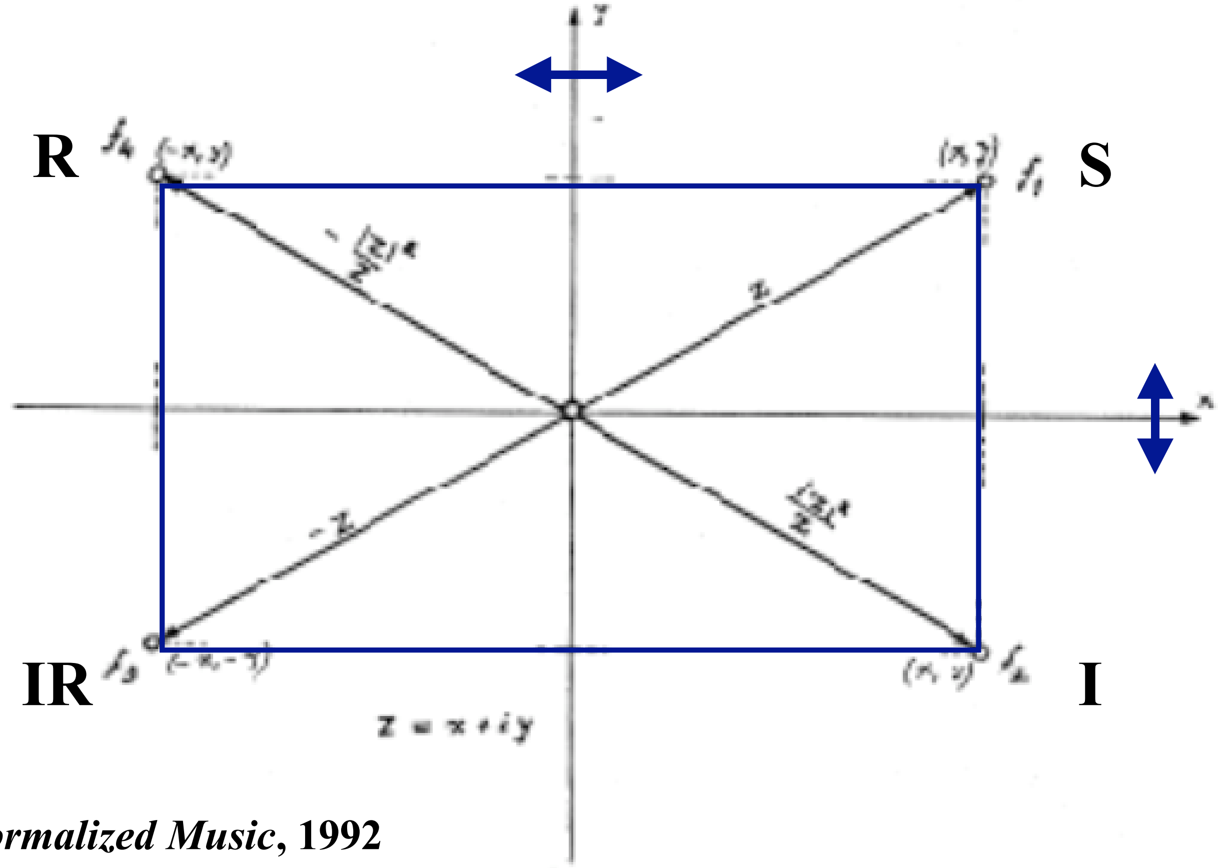
- **Chiusura**
- **Esistenza dell'elemento neutro**
- **Esistenza dell'inverso**
- **Associatività: $(a+b)+c=a+(b+c)$**

Operazioni dodecafoniche e strutture algebriche

	S	I	R	RI
S	S	I	R	RI
I	I	S	RI	R
R	R	RI	S	I
RI	RI	R	I	S

Struttura di *gruppo*

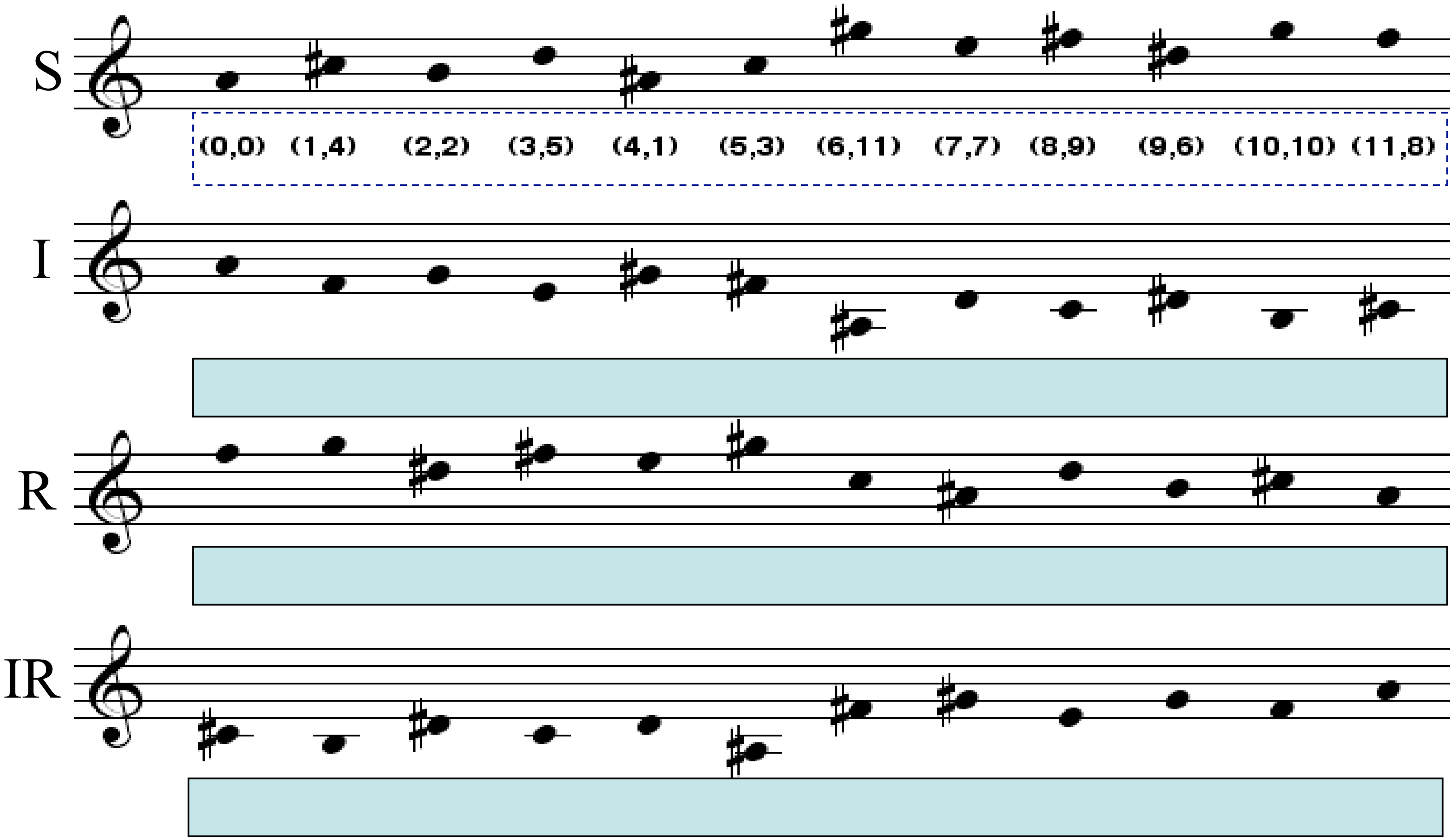
- **Chiusura**
- **Esistenza dell'elemento neutro**
- **Esistenza dell'inverso**
- **Associatività**



Iannis Xenakis, *Formalized Music*, 1992

Felix Klein

Operazioni dodecafoniche e strutture algebriche



$$\mathbf{S:}\ (a,b) \rightarrow (a,b)$$

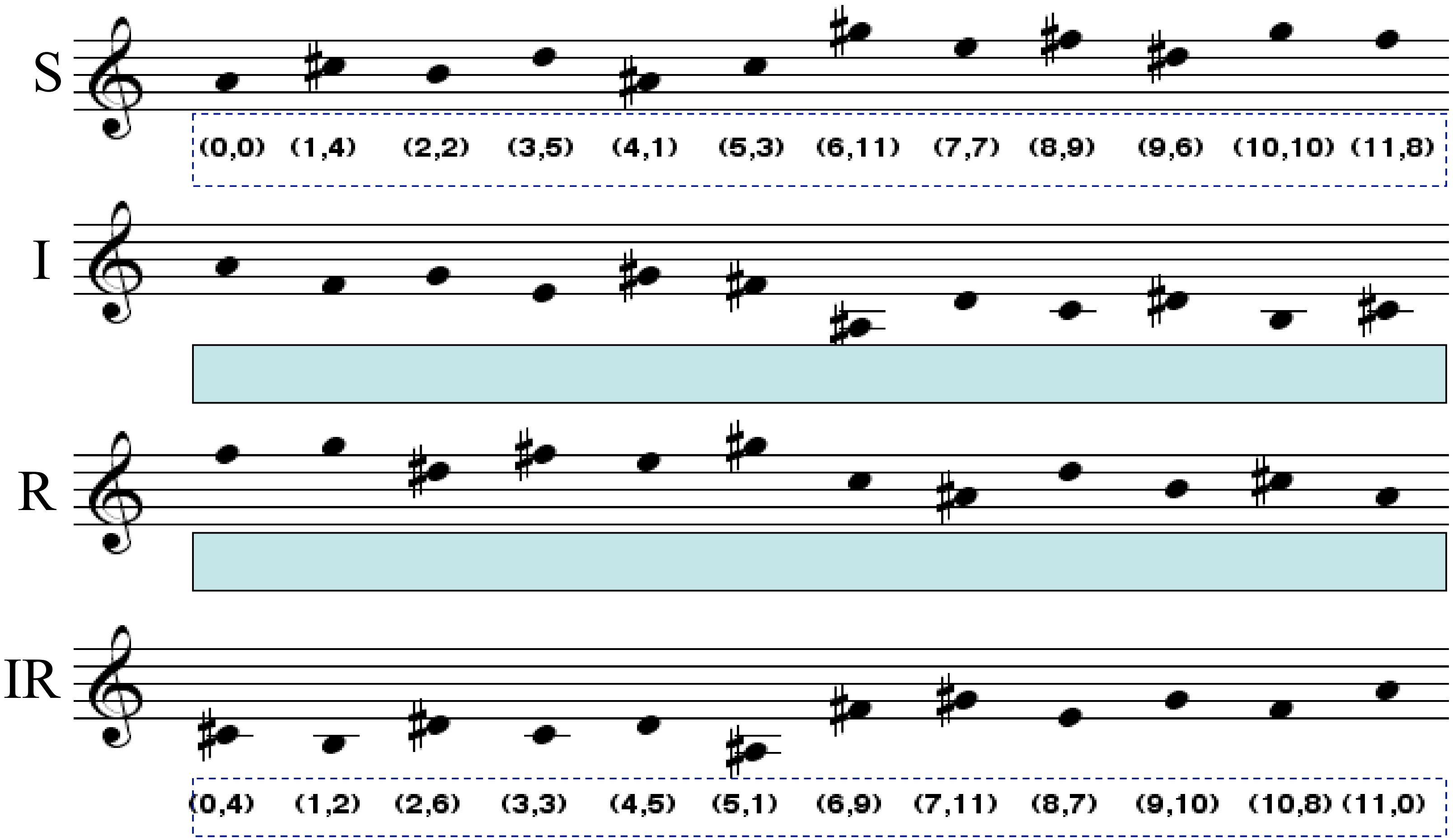
Operazioni dodecafoniche e strutture algebriche

I: $(a,b) \rightarrow (a, \mathbf{12}\text{-}\boldsymbol{b}\ \mathbf{mod.12})$

Operazioni dodecafoniche e strutture algebriche

R: $(a,b) \rightarrow (11-a,b)$.

Operazioni dodecafoniche e strutture algebriche

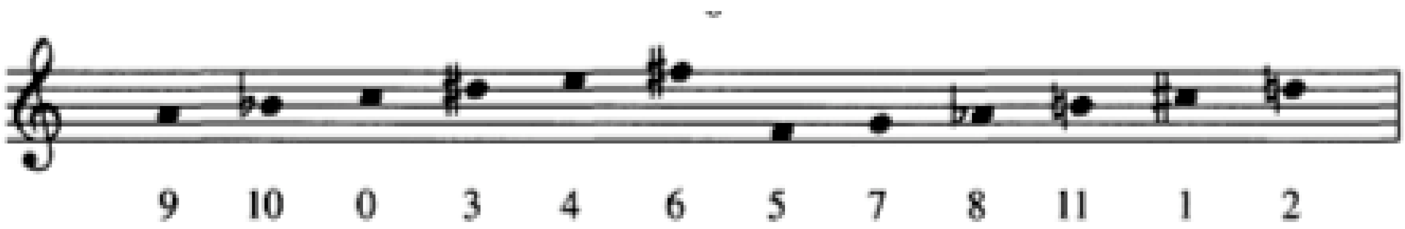


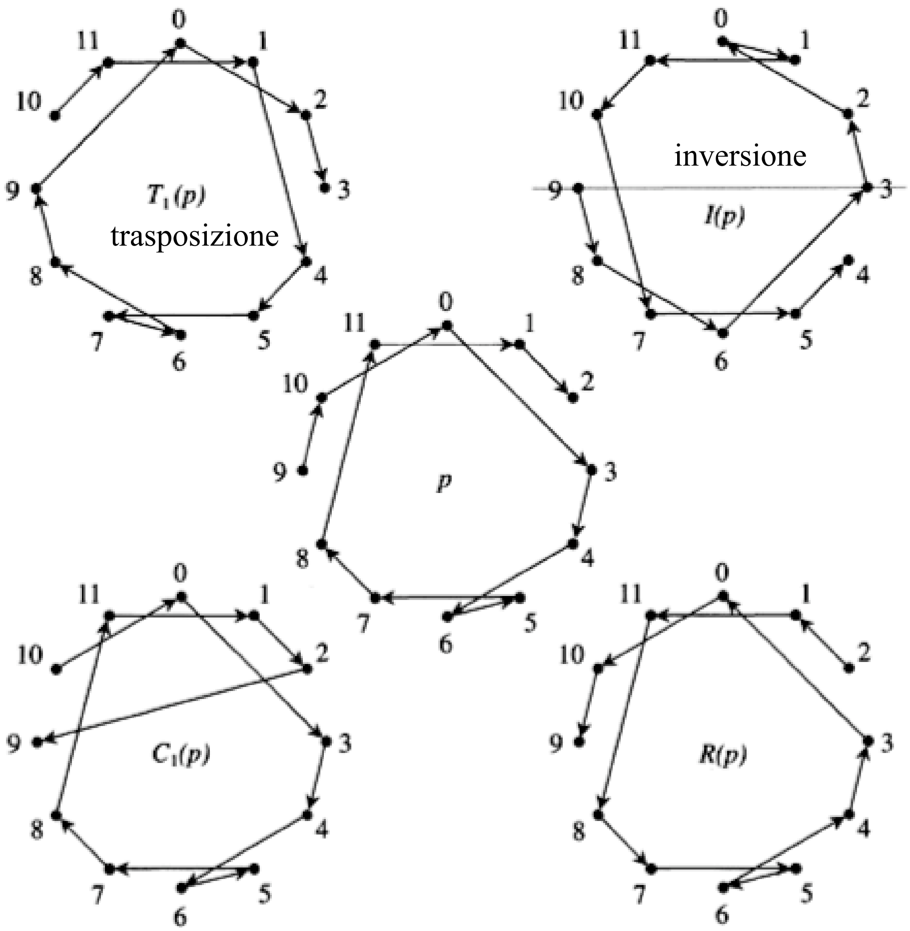
IR: $(a,b) \rightarrow (11-a,\ b \text{ mod. } 12) \downarrow (11-a,\ 12-b \text{ mod. } 12)$

=

RI: $(a,b) \rightarrow (a,\ 12-b \text{ mod. } 12) \downarrow (11-a,\ 12-b \text{ mod. } 12)$

Rappresentazioni geometriche delle trasformazioni dodecafoniche





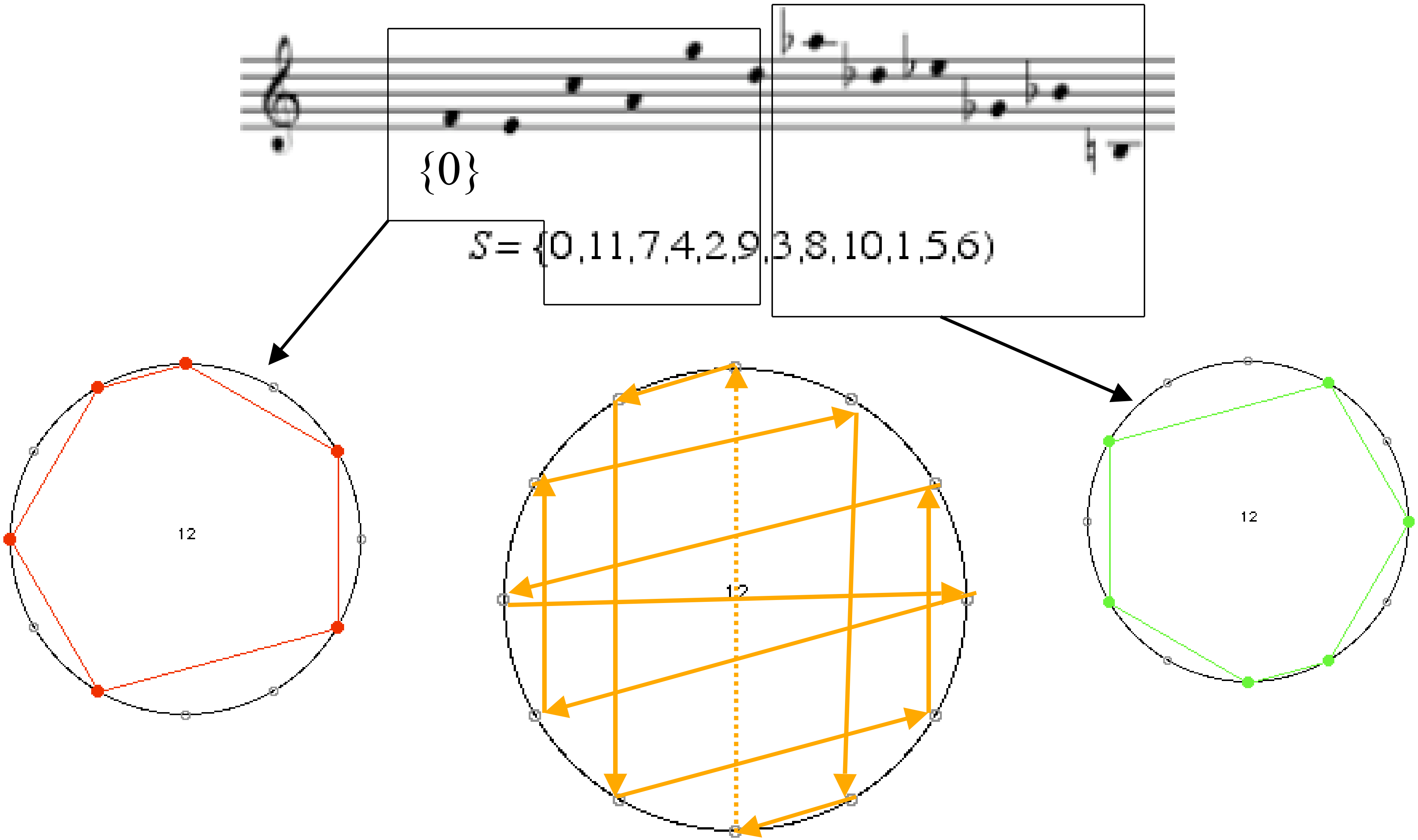
« rotazione »
(permutation circulaire)

retrogradazione

R.C. Read: « Combinatorial problems in the theory of music », *Discrete Math.*, 1997

David J. Hunter & Paul T. von Hippel : « How Rare Is Symmetry in Musical 12-Tone Rows? », *The American Mathematical Monthly*, Vol. 110, No. 2. (Feb., 2003), pp. 124-132.

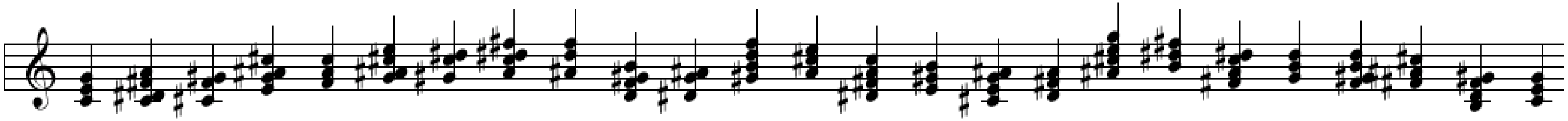
Serie dodecafoniche « omni–intervallari » (*all–interval rows*)



S* = (11, 8, 9, …)

Serie omni–intervallari e teoria della modulazione tonale

Thorvald Ötterstrom, *A Theory of Modulation*, Chicago UP, 1935

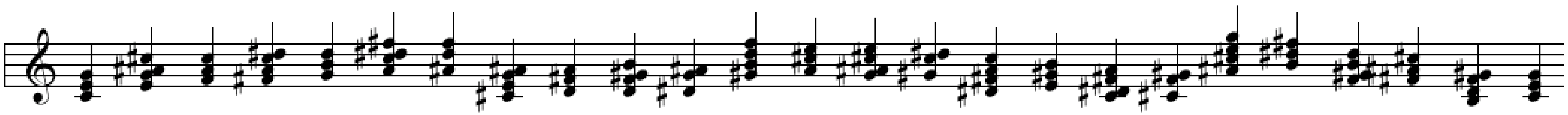


C	C#	F	G#	A#	D#	A	E	D	B	G	F#	(C)
0	1	5	8	10	3	9	4	2	11	7	6	(0)

α = [1 4 3 2 5] 6 7 10 9 8 11 (6)

←   ←

11 8 9 10 7 6 5 2 3 4 1 (6)



E(α)= [5 2 3 4 1] 6 11 8 9 10 7 (6)

=> *OpenMusic*

key-form; otherwise it is called an *acentral* key-form. By Theorem II, when b is even, there always exists a central key-form. If b is greater than 4, then by the Corollary we need to find only one-fourth of all the key-forms in order to have them all, since each one *generates* three more. To find a complete set of generating key-forms we may proceed as follows:

1. Find all the central key-forms with the element 1 in the first half of A or in its middle.
2. Find all the acentral key-forms with B longer than A and with the element 1 in the first half of A or in its middle.
3. Find all the acentral key-forms with B longer than A and with the element 1 in the first half of B or in its middle.

Complete sets of generating key-forms for $b=2, 4, 6, 8, 10$ are as follows:

$b=2$:

1

$b=4$:

123

$b=6$:

14325

$b=8$:

CENTRAL

1234567
1634527
5174632

ACENTRAL

1643752
3241576
7245136
6542137

$b=10$:

CENTRAL

$k_1=1$: 126357489
124753689
124759863
174258639
138654279
183654729
176852439
176859342

$k_2=1$: 312654897
318456297
813456792

ACENTRAL

$k_1=5$: 129574836
162548793
184597263

216534789

498512367
367512498
867512493
926513847
427513968
472518963

347521689
849571326
342571689

386524179
638592147

$k_1=5$: 125978436
135869427
175948326
185369427

765123849
985132674
485132679
265134879
935162784
435162789

475218693
865714293
365714298
425718693

975241683
925741683

$k_2=5$: 153869247
158324967
157632498

351267849
851327694
851396724
751623489
251678439
851762349
351872694
351896274

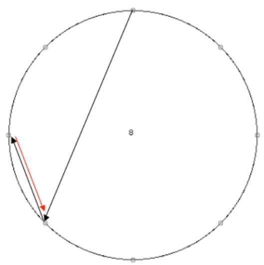
657132948
256143879
756148329
657149238
952168743
453186927

653814297
259714386
759214836
953416872
452618793

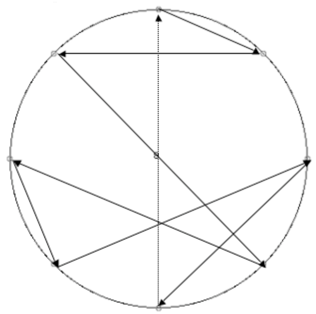
452781693
957241863

Base b	Generating Key-Forms				Total Key-Forms
	Central		Acentral	Total	
	$k_1=1$	Total			
2	1	1	0	1	1
4	1	1	0	1	2
6	1	1	0	1	4
8	2	3	4	7	28
10	8	11	55	66	264

The problem of the number of key-forms for the general base b is a problem in partitions and probably admits of no formula.



(5,1,7,4,6,3,2)



(1,6,4,3,7,5,2)

The problem of the number of key-forms for the general base *b* is a problem of partition and probably admits of no formula

Enumerazione delle serie omni-intervallari (via Burnside / Polya)

H. Fripertinger: «Enumeration in Musical Theory», *Beiträge zur Elektr. Musik*,1,1992

ACENTRAL

$k_4 = 5$:	$k_3 = 5$:	$k_2 = 5$:
129574836	125978436	153869247
162548793	135869427	158324967
184597263	175948326	157632498
	185369427	
216534789		351267849
	765123849	851327694
498512367	985132674	851396724
367512498	485132679	751623489
867512493	265134879	251678439
926513847	935162784	851762349
427513968	435162789	351872694
472518963		351896274
	475218693	
347521689	865714293	657132948
849571326	365714298	256143879
342571689	425718693	756148329
		657149238
386524179	975241683	952168743
638592147	925741683	453186927
		653814297
		259714386
		759214836
		953416872
		452618793
		452781693
		957241863

Base b	Generating Key-Forms				Total Key-Forms
	Central		Acentral	Total	
	$k_1 = 1$	Total			
2	1	1	0	1	1
4	1	1	0	1	2
6	1	1	0	1	4
8	2	3	4	7	28
10	8	11	55	66	264

Theorem 25 (Number of Patterns of All-Interval-Rows) *For $i = 1, 2, 3$ the number of patterns of all-interval-rows in regard to G_i is*

1. $\frac{1}{4}(\chi(\mathrm{id}) + \chi(\varphi_I \circ \varphi_R))$ *for* $i = 1$.
2. $\frac{1}{8}(\chi(\mathrm{id}) + \chi(\varphi_I \circ \varphi_R) + \chi(\varphi_I \circ V))$ *for* $i = 2$.
3. *For $i = 3$ we calculate*

$$\frac{1}{16}(\chi(\mathrm{id}) + \chi(\varphi_I \circ \varphi_R) + \chi(\varphi_I \circ V) + \chi(\varphi_Q \circ \varphi_R \circ V)) =$$

$$= \frac{1}{16}(3\,856 + 176 + 120 + 120) = 267.$$

G_3= <I,R,E,Q>

G_2= <I,R,E>

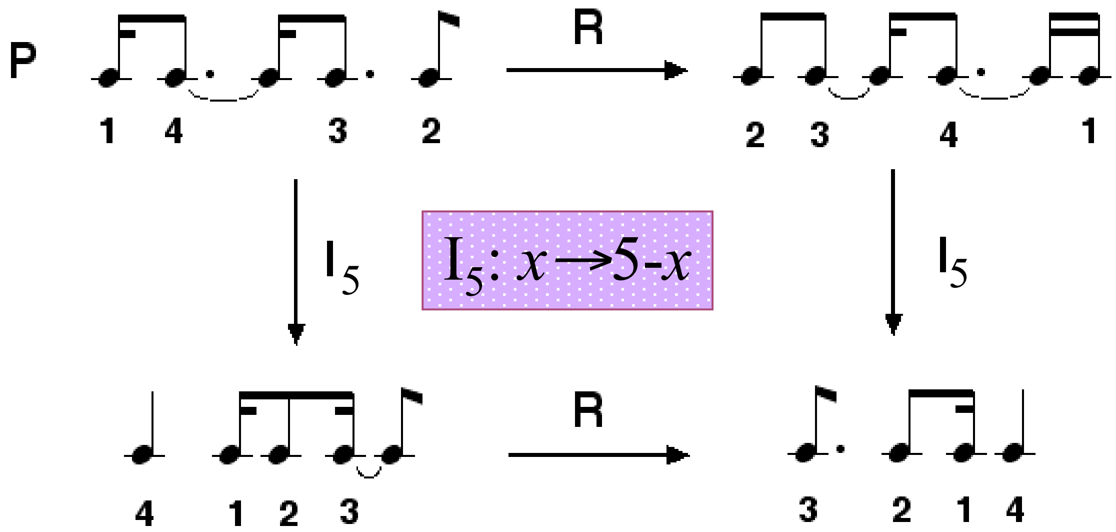
G_1= groupe de Klein

Verso una formalizzazione algebrica del serialismo integrale

- La serie delle durate temporali (*durational row*)

Composition for Four Instruments (1948)

I: $x \rightarrow 12\text{-}x$



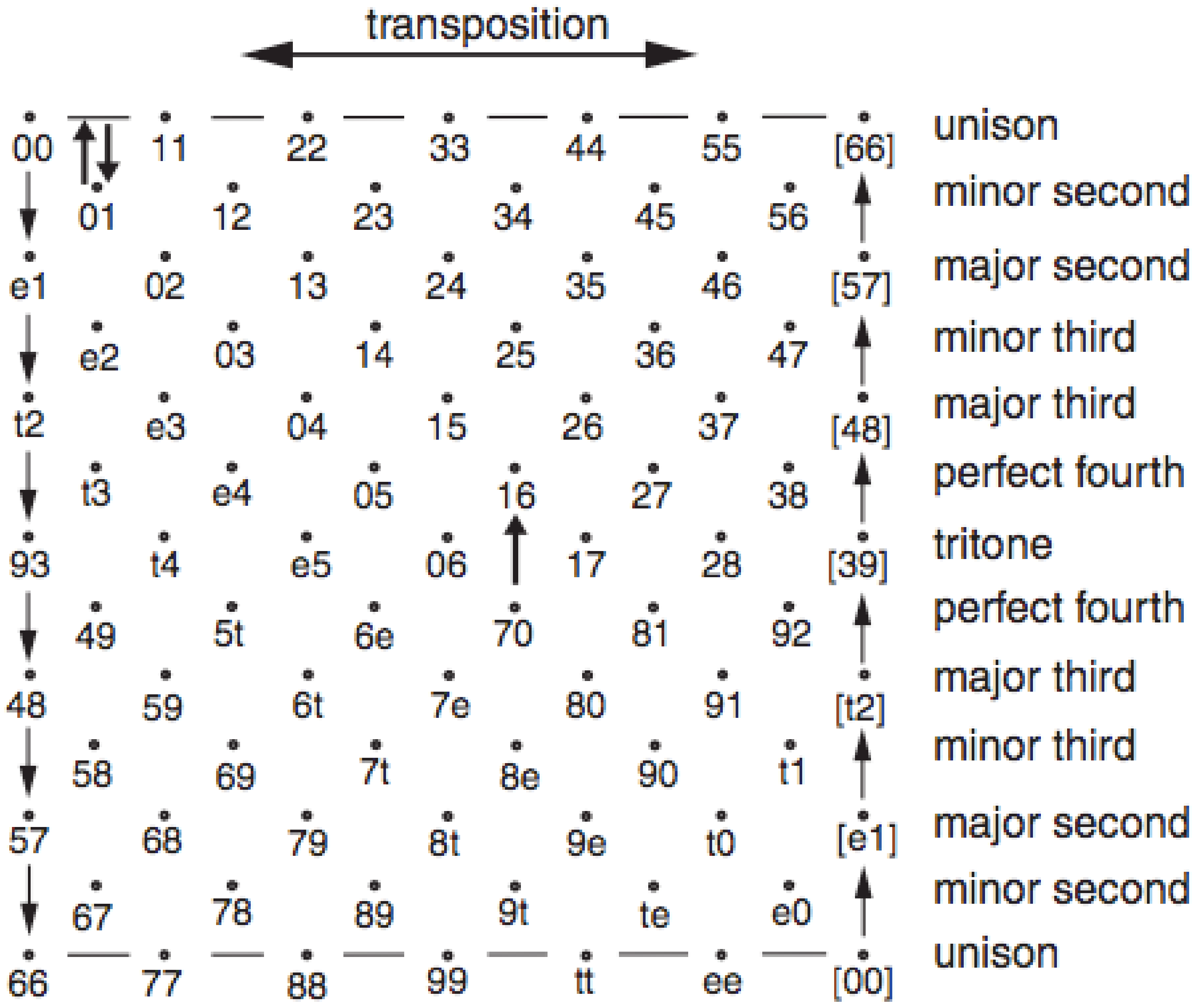
$I_5: x \rightarrow 5\text{-}x$

- Il *Time-Points System*

$I_{11}: x \rightarrow 11\text{-}x$

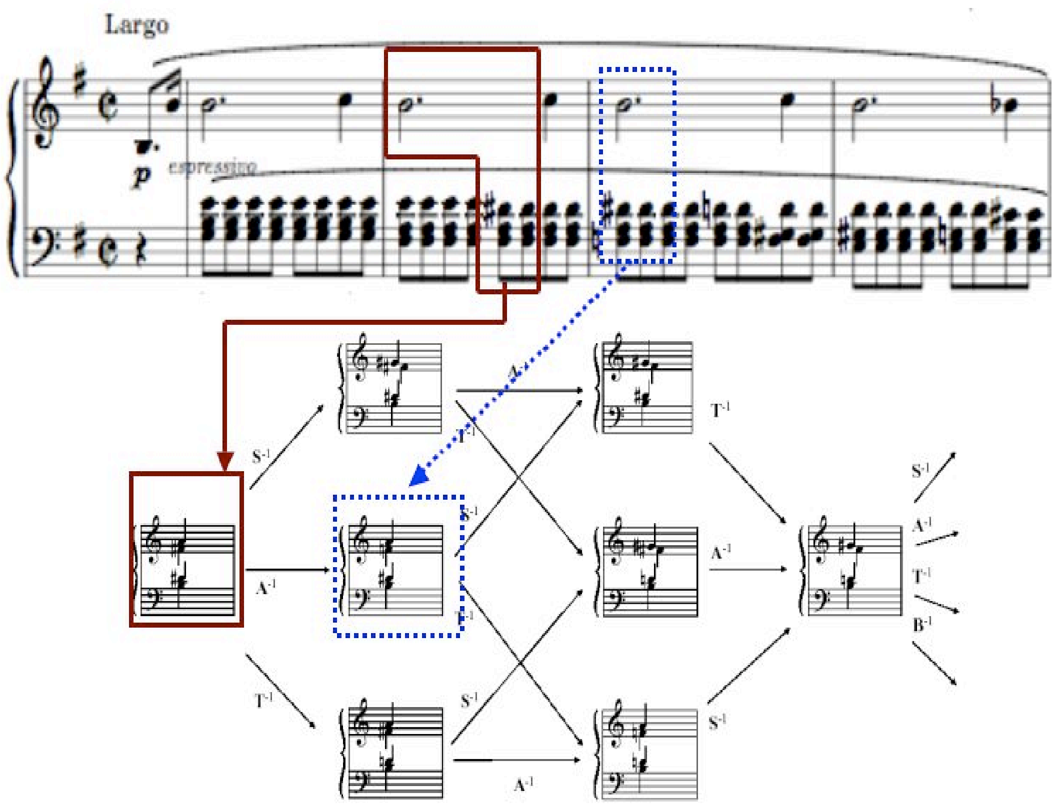
$T_{11}I: x \rightarrow 11+(12\text{-}x)$

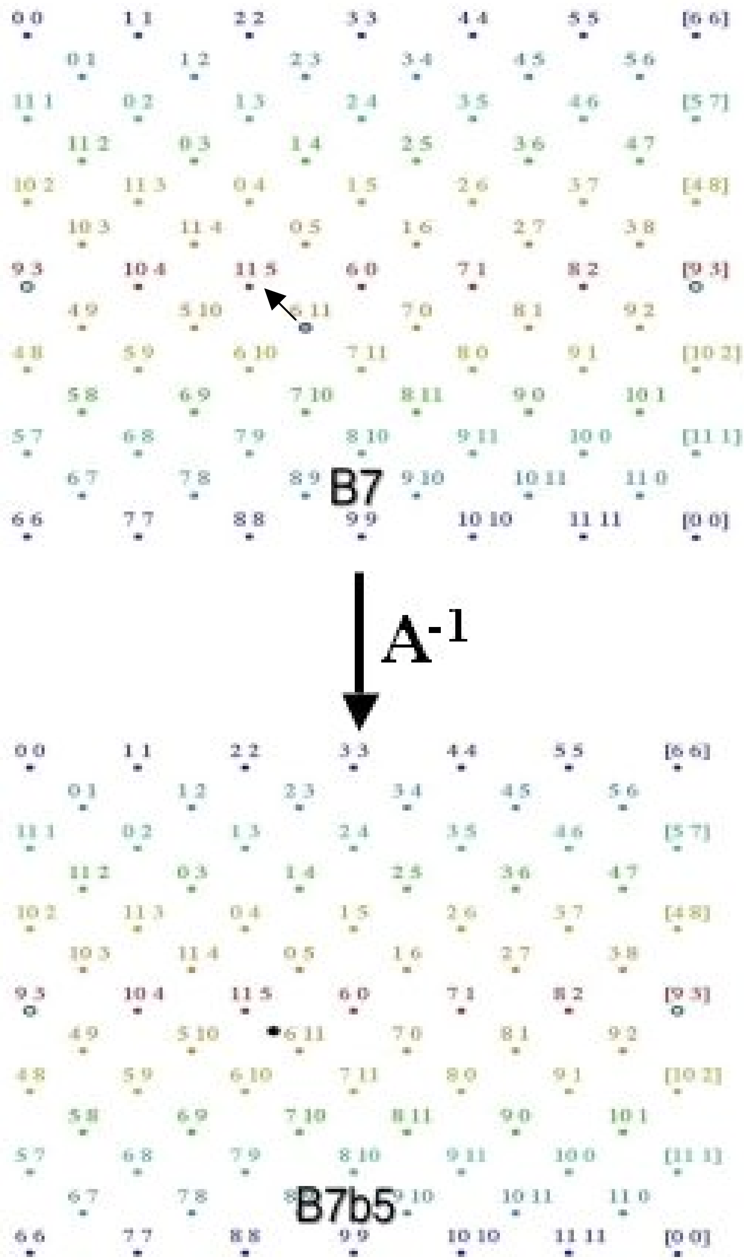
$$T^2 = (\mathbf{R}/12\mathbf{Z})^2 \longrightarrow T^2/S_2$$



Dmitri Tymoczko, « The Geometry of Musical Chords », *Science*, 313, 2006

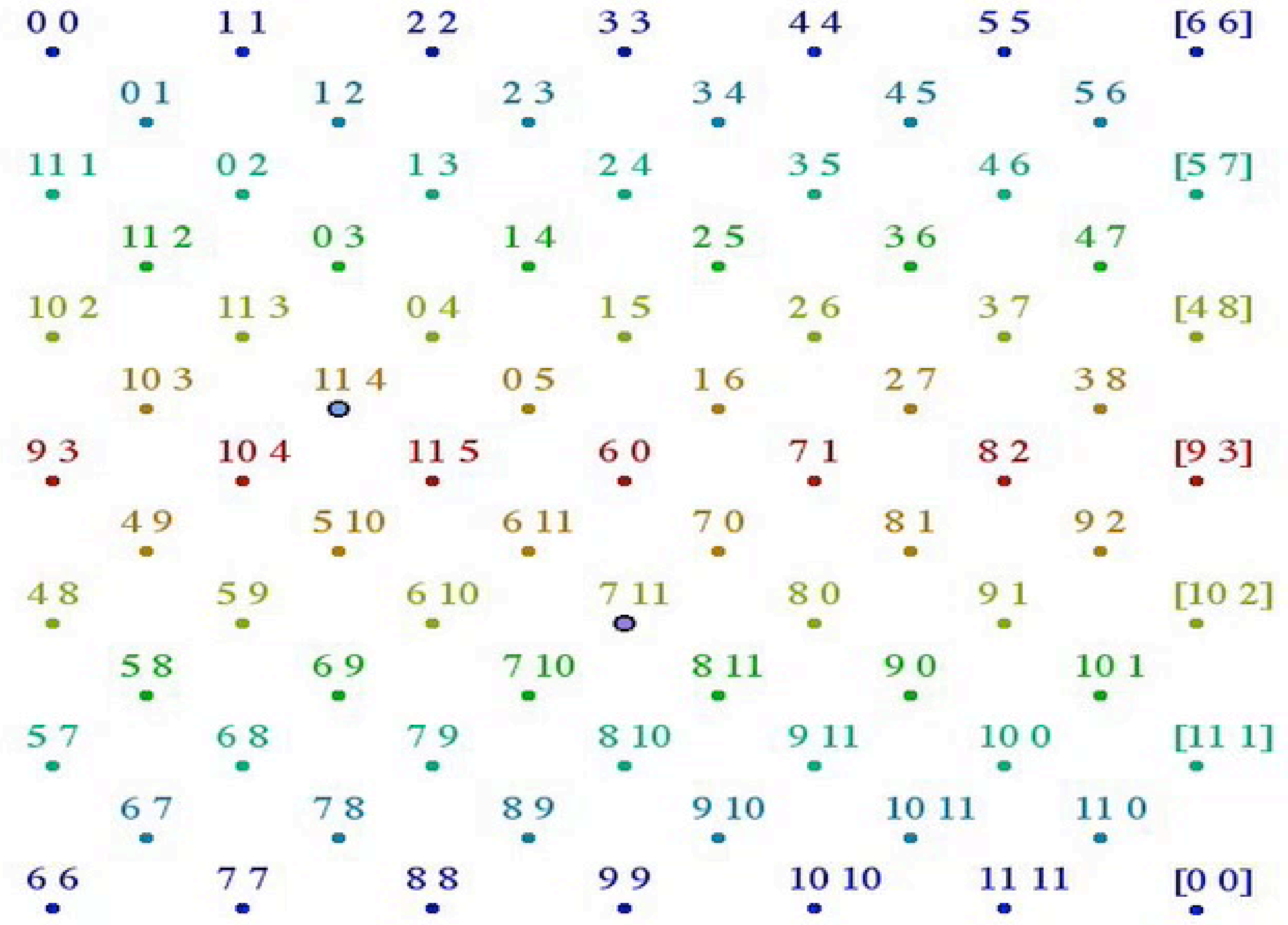
Applicazione all'analisi musicale (Chopin)



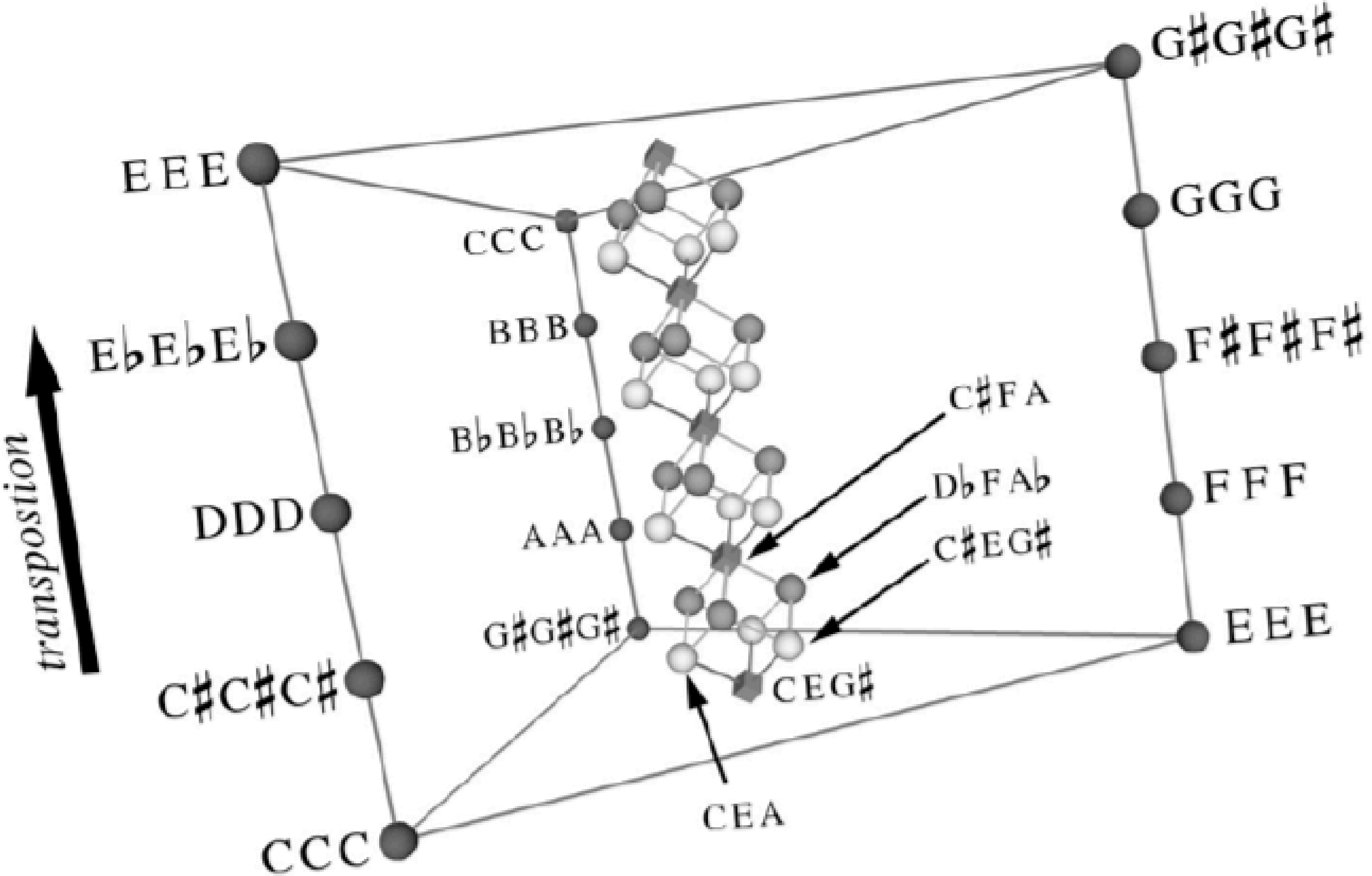


Dmitri Tymoczko :
« The Geometry of Musical Chords »,
Science, 313, 2006

$$T^2 = \mathbf{R}/12\mathbf{Z} \times \mathbf{R}/12\mathbf{Z} \longrightarrow T^2/S_2$$

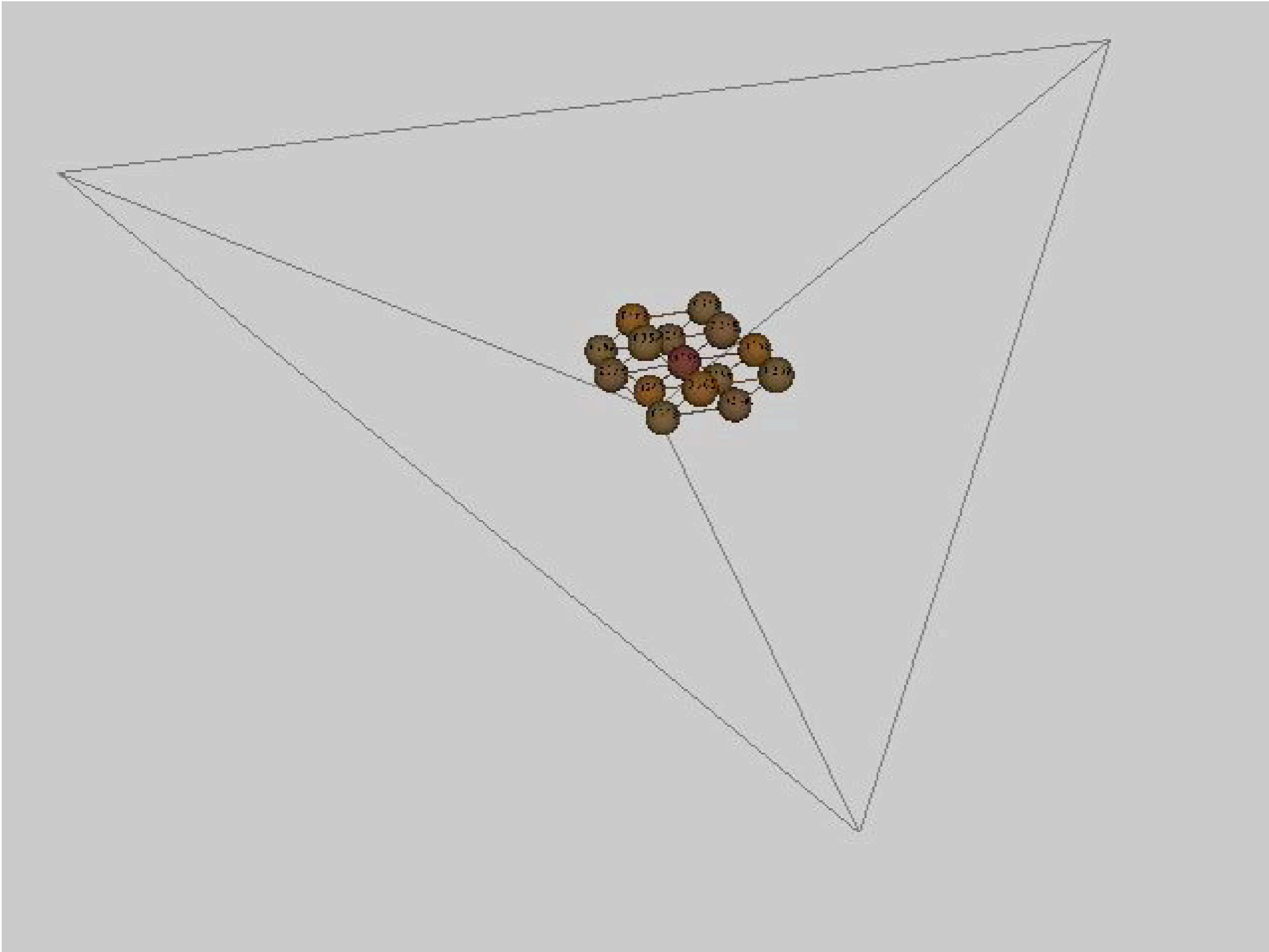


$T^3 = (\mathbf{R}/\mathbf{12Z})^\mathbf{3} \longrightarrow T^3/S_3$



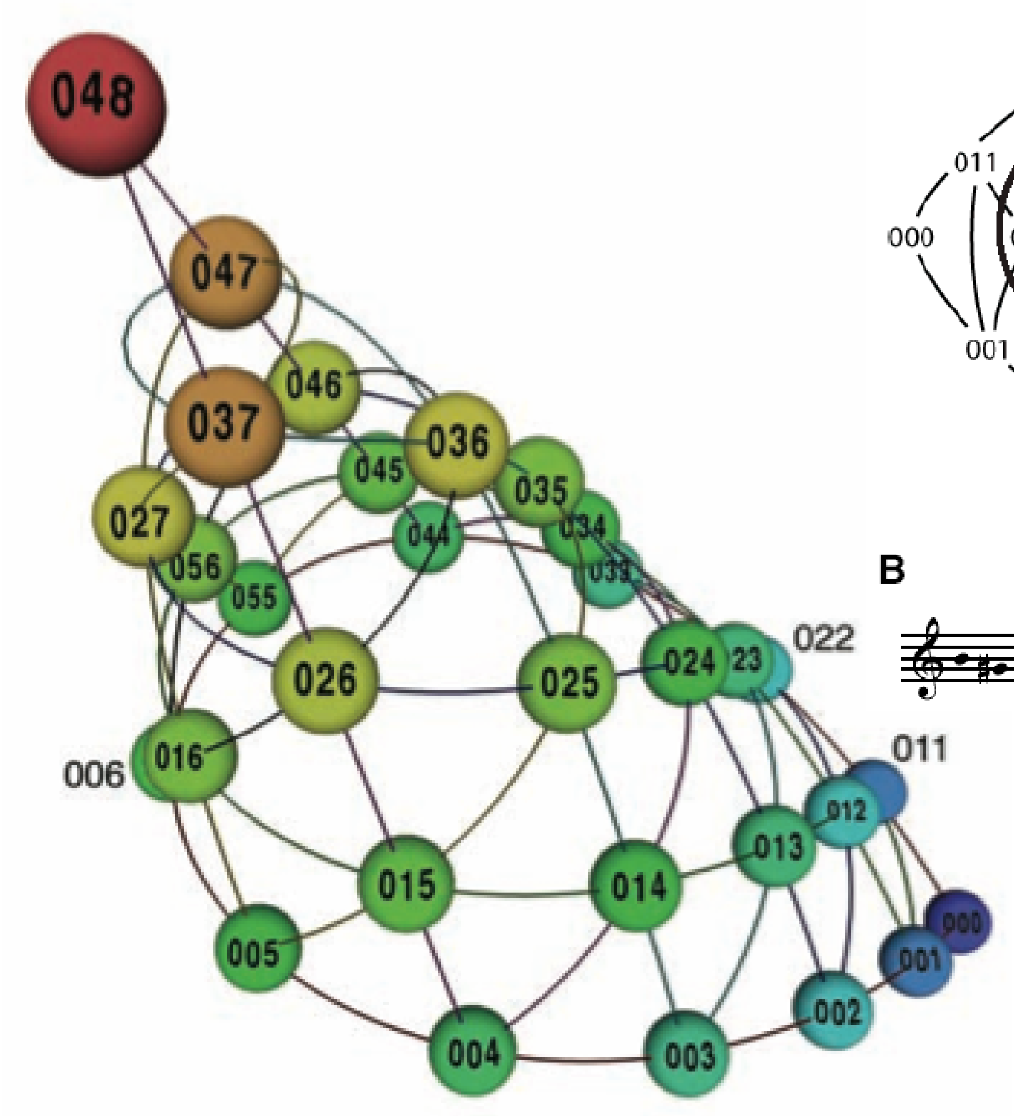
Dmitri Tymoczko, « The Geometry of Musical Chords », *Science*, 313, 2006

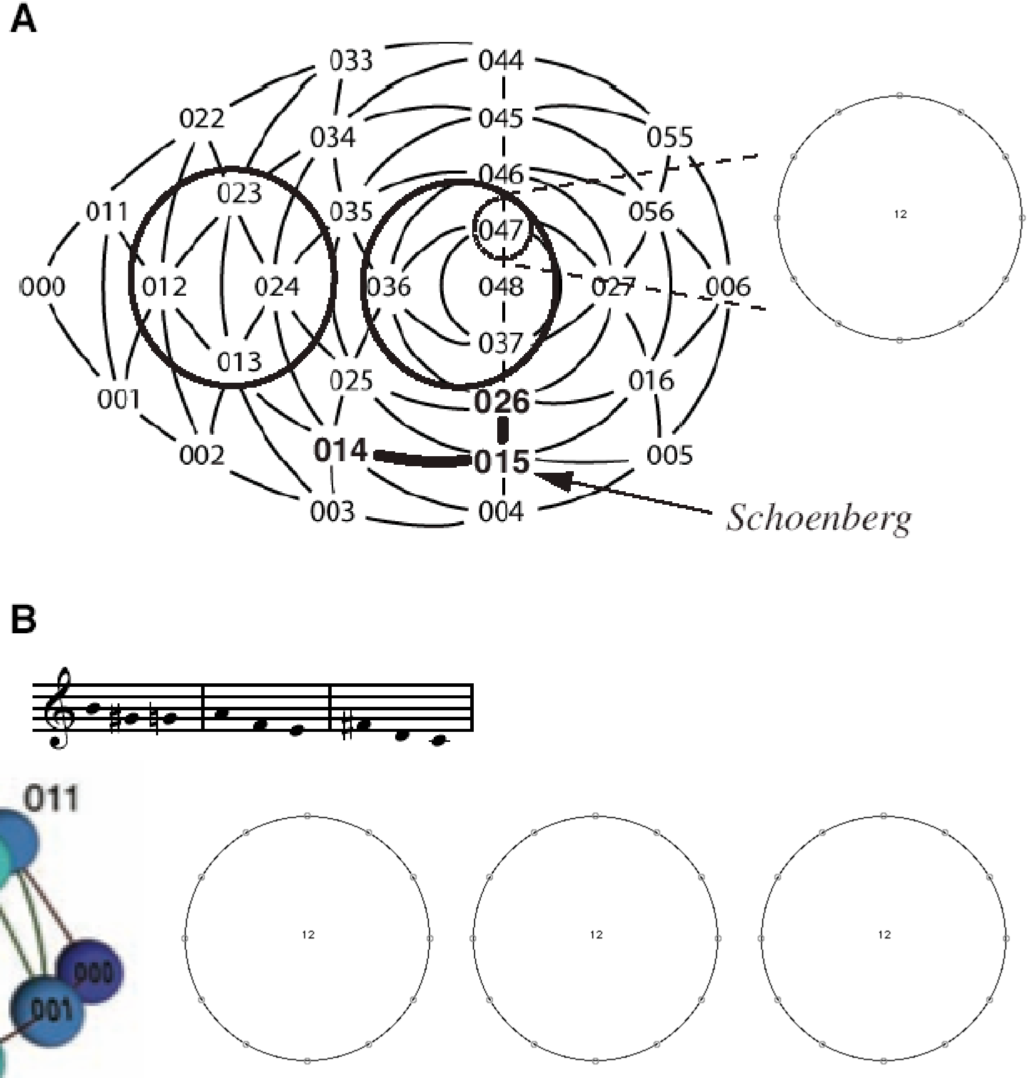
$T^4 = (\mathbf{R}/12\mathbf{Z})^4 \longrightarrow T^4/S_4$



Dmitri Tymoczko, « The Geometry of Musical Chords », *Science*, 313, 2006

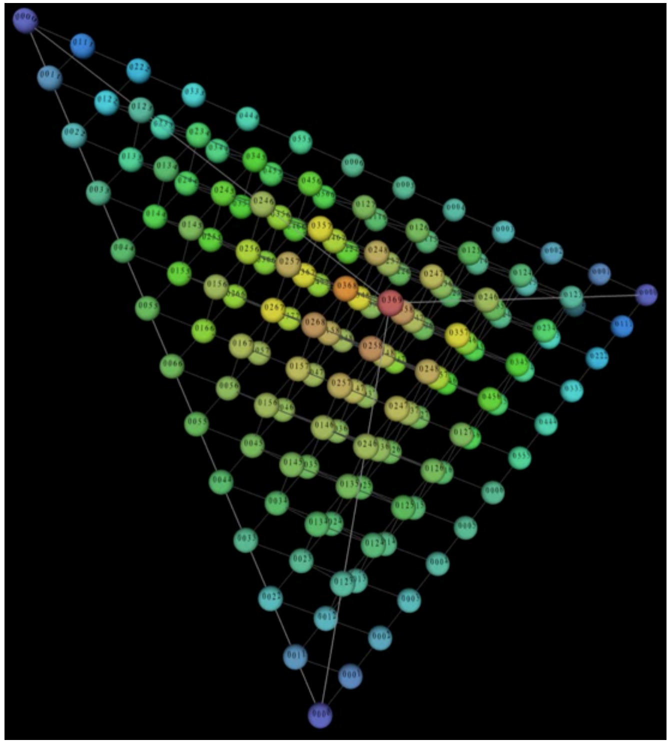
$T^2 = (\mathbf{R}/\mathbf{12Z})^2 \longrightarrow T^2/S_3$



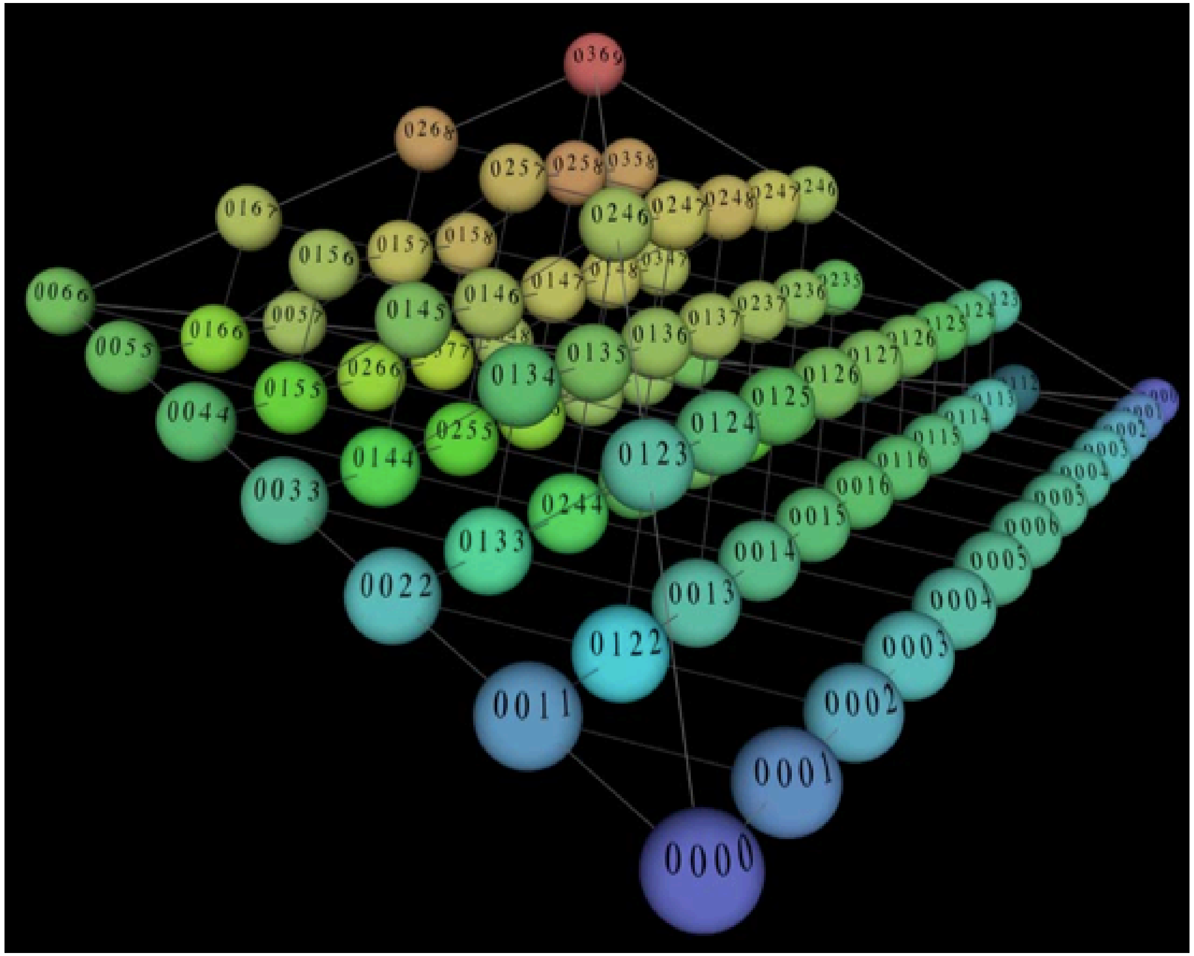


C. Callender, I. Quinn & D. Tymoczko, « Generalized Voice-Leading Spaces », *Science*, 320, 2008

T^3 / S_4

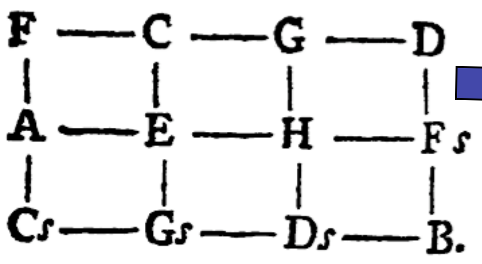


$T^3 / (S_4 \times Z_2)$

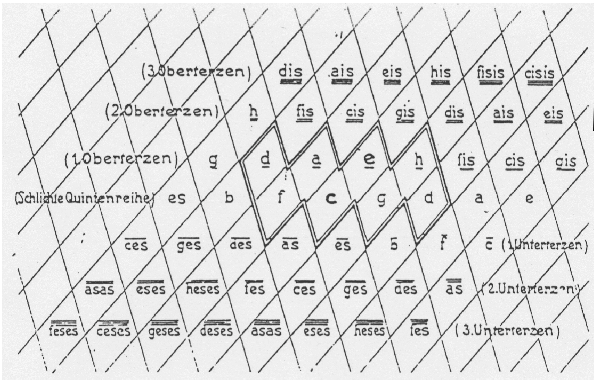


C. Callender, I. Quinn & D. Tymoczko, « Generalized Voice-Leading Spaces », *Science*, 320, 2008

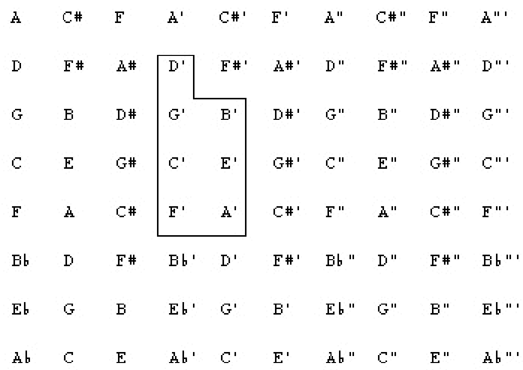
Rappresentazioni geometriche delle strutture musicali



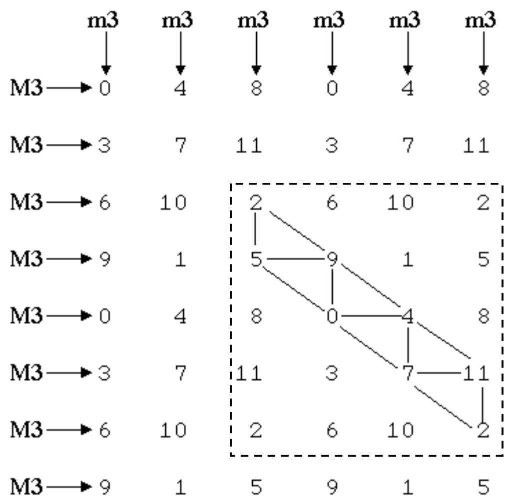
Euler : *Speculum musicum*, 1773



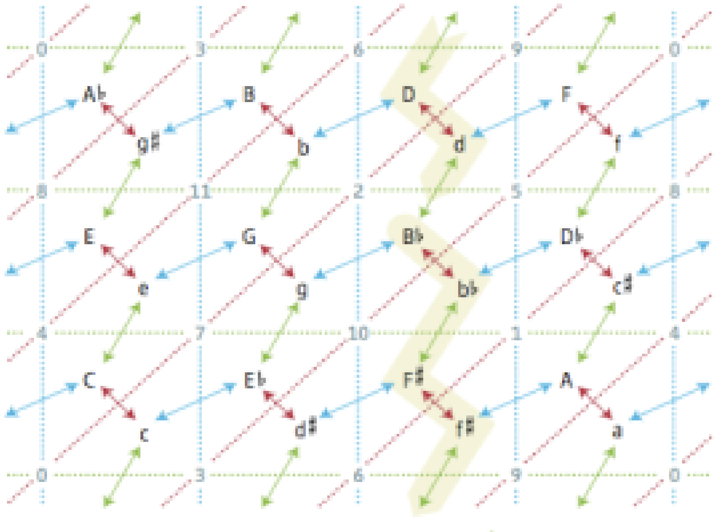
Hugo Riemann : « Ideen zu einer Lehre von den Tonvorstellung », 1914



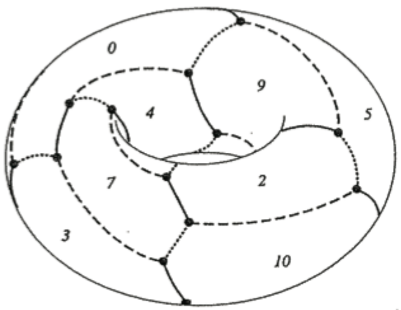
Longuet-Higgins (1962)



Balzano (1980)

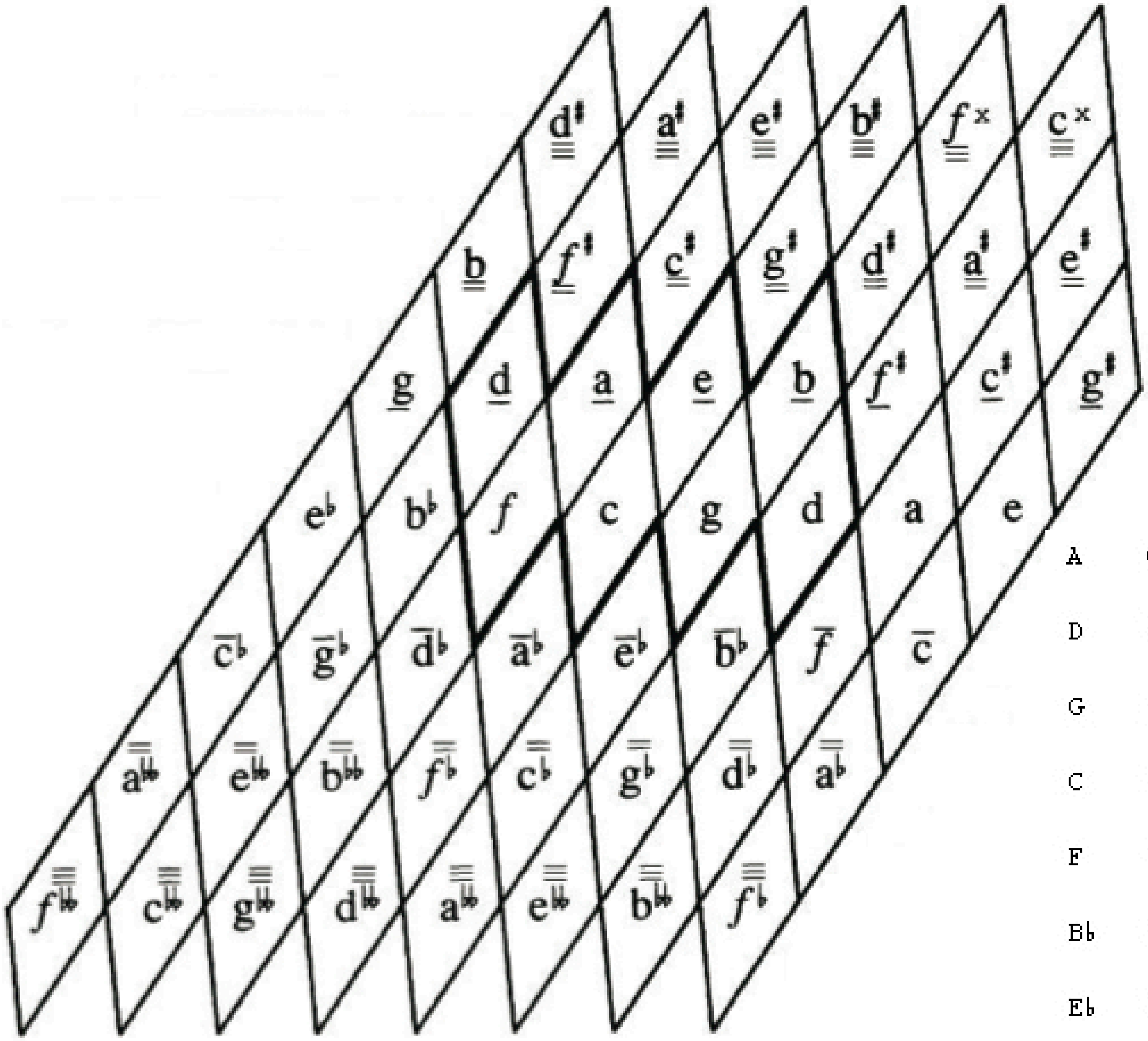


J. Hook, « Exploring Musical Space », *Science, 2006*



Douthett & Steinbach, *JMT*, 1998

Il *Tonnetz* (reticolo di note)

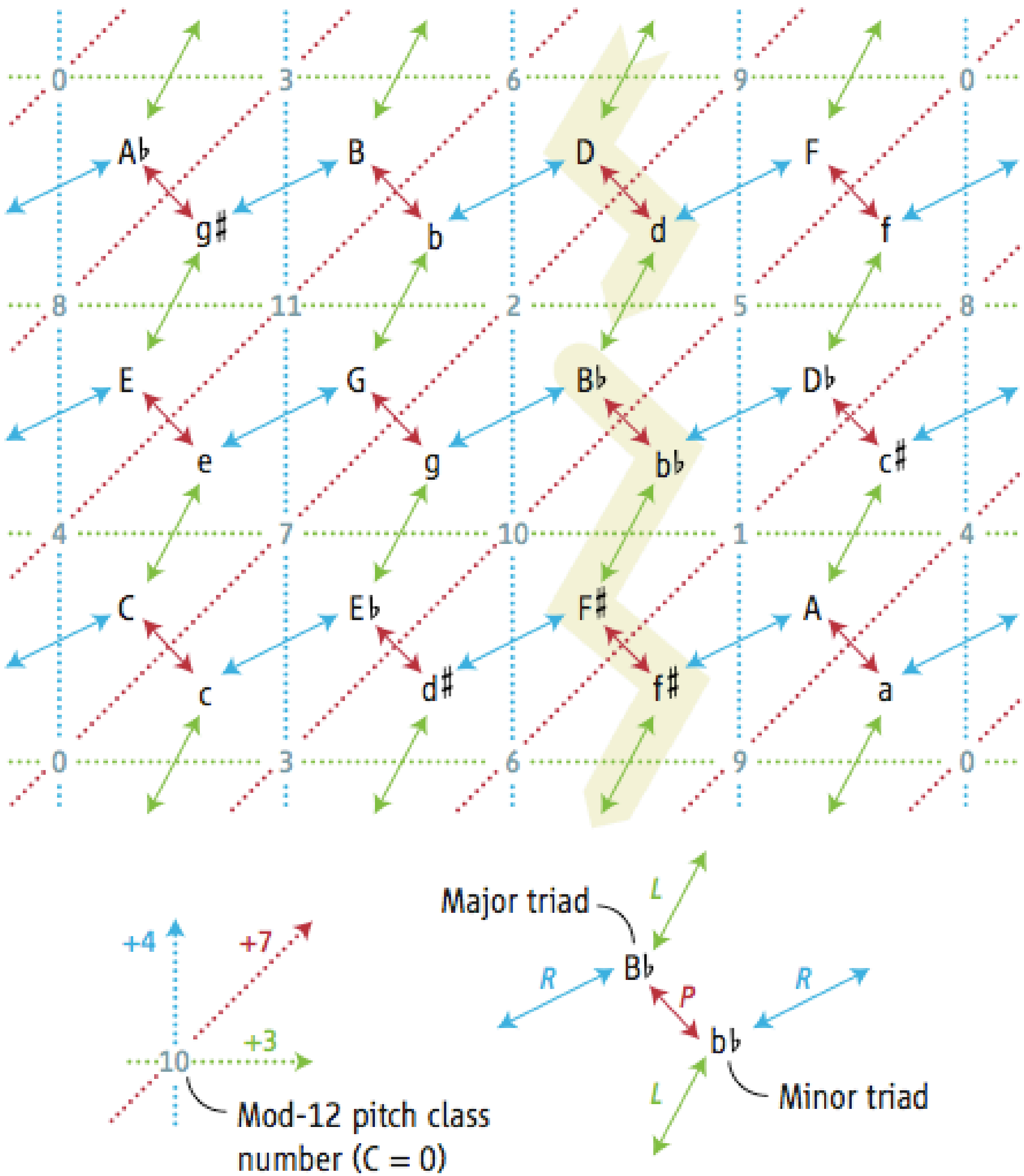


Oettingen/Riemann

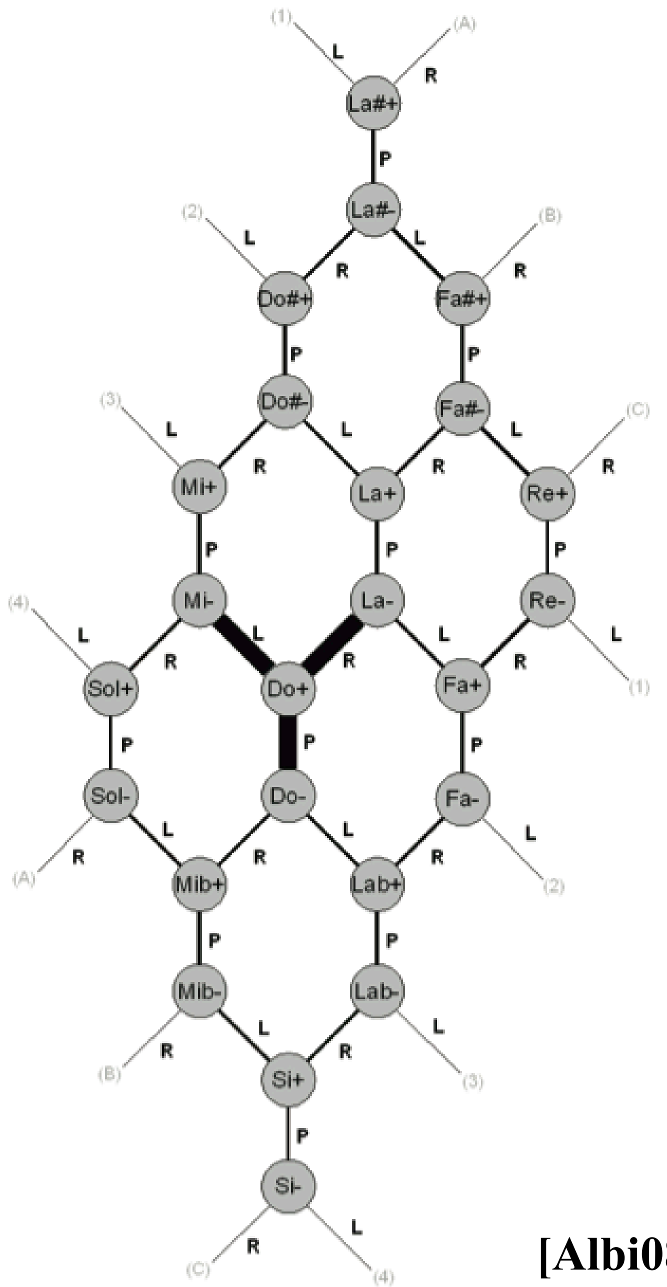
A	C#	F	A'	C#'	F'	A"	C#"	F"	A"'
D	F#	A#	D'	F#'	A#'	D"	F#"	A#"	D"'
G	B	D#	G'	B'	D#'	G"	B"	D#"	G"'
C	E	G#	C'	E'	G#'	C"	E"	G#"	C"'
F	A	C#	F'	A'	C#'	F"	A"	C#"	F"'
B♭	D	F#	B♭'	D'	F#'	B♭"	D"	F#"	B♭"'
E♭	G	B	E♭'	G'	B'	E♭"	G"	B"	E♭"'
A♭	C	E	A♭'	C'	E'	A♭"	C"	E"	A♭"'

Longuet–Higgins

Il *Tonnetz* di Oettingen/Riemann

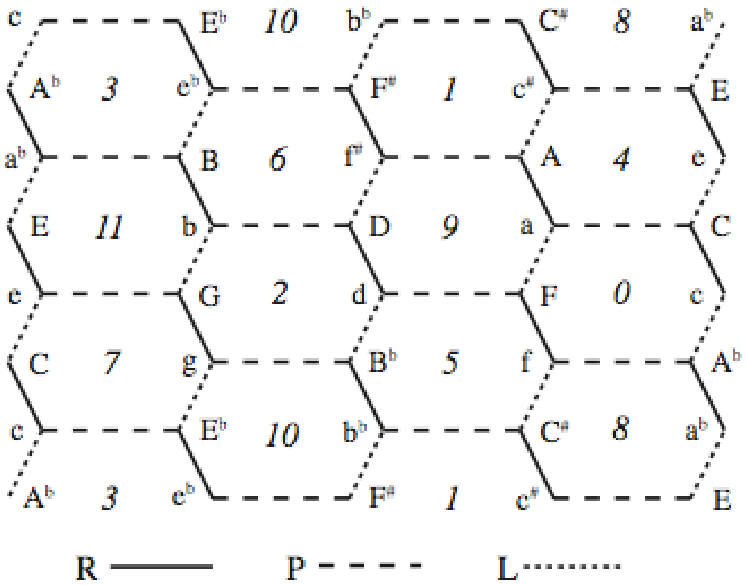


[Hook06]

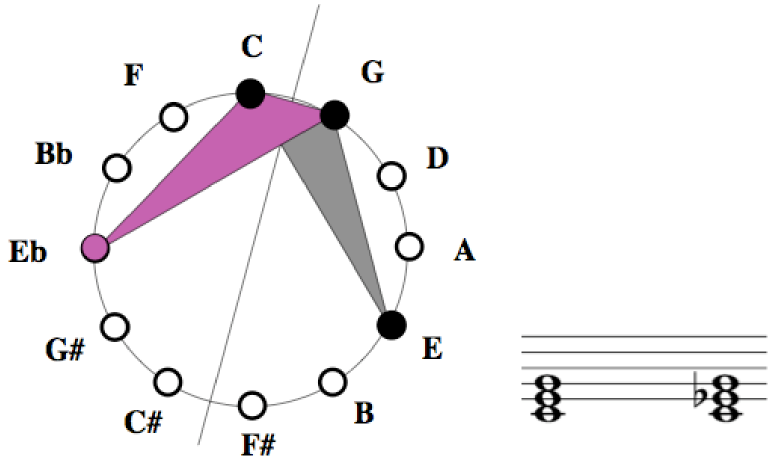


[Albi08]

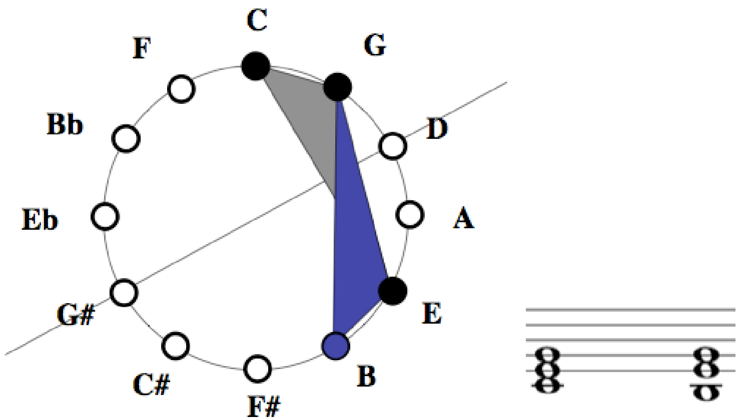
Teorie neo-Riemanniane



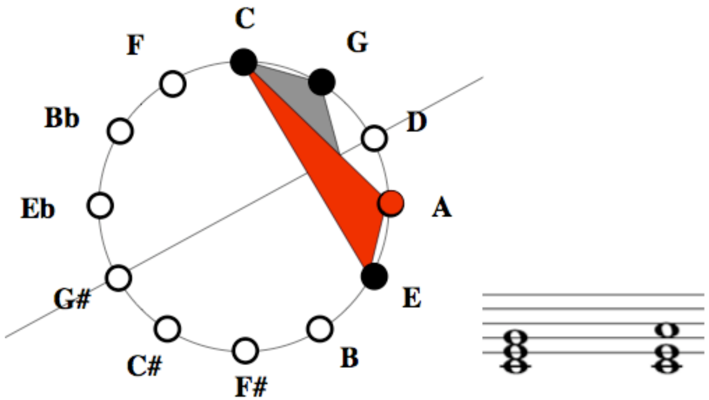
(Neo-)Riemannian Operation **P** = „**Parallel**“



(Neo-)Riemannian Operation **L** = „**Leading-Tone**“



(Neo-)Riemannian Operation **R** = „**Relative**“



Cf. Th. Noll: The Z12 Story, MaMuX, Dec. 18 2004

Una seconda definizione del gruppo diedrale D_{12}

$$\mathbf{LPR = \langle L, R, P \mid P^2 = L^2 = (LR)^{12} = 1 \rangle}$$

• LPR è il duale del gruppo T_nI generato dalle trasposizioni e inversioni (i.e. sono uno il *centralizzatore* dell'altro)
• LPR e T_nI condividono il sottogruppo abeliano massimale $\langle T_n \rangle = Z_n$
• LPR agisce in maniera semplicemente transitiva sull'insieme delle 24 triadi consonanti

[Albi08] e [Crans08]

Ricerca dei cicli hamiltoniani in **LPR = <L, R | $L^2 = (LR)^{12} = 1$ >**

C	e	G	b	D	f#	A	c#	E	g#	B	d#	F#	a#	C#	f	G#	c	D#	g	A#	d	F	a	#41
C	e	E	g#	G#	c	D#	g	G	b	B	d#	F#	a#	A#	d	D	f#	A	c#	C#	f	F	a	#62
C	c	G#	f	C#	c#	A	a	F	d	A#	a#	F#	f#	D	b	G	g	D#	d#	B	g#	E	e	#13
C	c	G#	g#	E	c#	A	a	F	f	C#	a#	F#	f#	D	d	A#	g	D#	d#	B	b	G	e	#4
C	e	E	g#	G#	c	D#	d#	B	b	G	g	A#	a#	F#	f#	D	d	F	f	C#	c#	A	a	#58
C	c	D#	g	G	b	B	d#	F#	f#	D	d	A#	a#	C#	c#	A	a	F	f	G#	g#	E	e	#19
C	c	G#	g#	B	d#	D#	g	G	b	D	d	A#	a#	F#	f#	A	a	F	f	C#	c#	E	e	#7
C	c	G#	g#	E	e	G	b	B	d#	D#	g	A#	a#	F#	f#	D	d	F	f	C#	c#	A	a	#27
C	c	D#	d#	B	b	G	g	A#	d	D	f#	F#	a#	C#	c#	A	a	F	f	G#	g#	E	e	#21
C	c	G#	g#	B	b	G	g	D#	d#	F#	a#	A#	d	D	f#	A	a	F	f	C#	c#	E	e	#8
C	c	G#	g#	E	e	G	g	D#	d#	B	b	D	f#	F#	a#	A#	d	F	f	C#	c#	A	a	#30
C	c	D#	d#	B	b	G	g	A#	a#	F#	f#	D	d	F	a	A	c#	C#	f	G#	g#	E	e	#22
C	c	G#	g#	B	b	G	g	D#	d#	F#	f#	D	d	A#	a#	C#	f	F	a	A	c#	E	e	#10
C	c	G#	g#	E	e	G	g	D#	d#	B	b	D	d	A#	a#	F#	f#	A	c#	C#	f	F	a	#31
C	c	G#	f	F	a	A	c#	C#	a#	A#	d	D	f#	F#	d#	D#	g	G	b	B	g#	E	e	#9
C	c	G#	g#	E	c#	C#	f	F	a	A	f#	F#	a#	A#	d	D	b	B	d#	D#	g	G	e	#6
C	c	D#	d#	F#	f#	A	c#	E	e	G	g	A#	a#	C#	f	G#	g#	B	b	D	d	F	a	#33
C	e	G	g	A#	a#	C#	c#	E	g#	B	b	D	d	F	f	G#	c	D#	d#	F#	f#	A	a	#44
C	c	D#	g	A#	a#	C#	c#	E	e	G	b	D	d	F	f	G#	g#	B	d#	F#	f#	A	a	#40
C	c	D#	d#	F#	a#	C#	c#	E	e	G	g	A#	d	F	f	G#	g#	B	b	D	f#	A	a	#38
C	c	D#	d#	F#	a#	C#	f	G#	g#	B	b	D	f#	A	c#	E	e	G	g	A#	d	F	a	#34
C	e	G	g	A#	a#	C#	f	G#	c	D#	d#	F#	f#	A	c#	E	g#	B	b	D	d	F	a	#42
C	e	G	b	D	d	F	f	G#	c	D#	g	A#	a#	C#	c#	E	g#	B	d#	F#	f#	A	a	#43
C	c	D#	g	A#	d	F	f	G#	g#	B	d#	F#	a#	C#	c#	E	e	G	b	D	f#	A	a	#39
C	e	G	b	B	d#	F#	a#	A#	g	D#	c	G#	g#	E	c#	C#	f	F	d	D	f#	A	a	#50
C	c	D#	g	A#	d	D	f#	A	c#	C#	a#	F#	d#	B	b	G	e	E	g#	G#	f	F	a	#37
C	c	G#	f	F	d	A#	g	D#	d#	B	g#	E	e	G	b	D	f#	F#	a#	C#	c#	A	a	#25
C	c	D#	d#	B	g#	G#	f	C#	a#	F#	f#	D	b	G	g	A#	d	F	a	A	c#	E	e	#16
C	c	G#	g#	B	b	G	e	E	c#	A	f#	D	d	A#	g	D#	d#	F#	a#	C#	f	F	a	#23
C	c	G#	f	F	a	A	f#	F#	a#	C#	c#	E	g#	B	d#	D#	g	A#	d	D	b	G	e	#1
C	e	E	g#	B	b	G	g	A#	a#	F#	d#	D#	c	G#	f	C#	c#	A	f#	D	d	F	a	#59
C	e	G	b	B	d#	F#	f#	D	d	F	f	C#	a#	A#	g	D#	c	G#	g#	E	c#	A	a	#48
C	c	D#	g	A#	d	D	f#	A	a	F	f	G#	g#	E	c#	C#	a#	F#	d#	B	b	G	e	#18
C	e	E	c#	A	f#	D	d	A#	g	G	b	B	g#	G#	c	D#	d#	F#	a#	C#	f	F	a	#60
C	c	G#	f	C#	c#	E	g#	B	d#	D#	g	A#	a#	F#	f#	A	a	F	d	D	b	G	e	#2

#41	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R
#62	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R
#13	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L
#4	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L	P	L	P	L	R	L
#58	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P	R
#19	P	R	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P	L
#7	P	L	P	R	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L
#27	P	L	P	L	P	R	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R
#21	P	R	P	L	P	L	P	R	L	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L
#8	P	L	P	R	P	L	P	L	P	R	L	P	L	P	L	R	P	L	P	L	P	R	P	L
#30	P	L	P	L	P	R	P	L	P	L	P	R	L	P	L	P	L	R	P	L	P	L	P	R
#22	P	R	P	L	P	L	P	R	P	L	P	L	P	R	L	P	L	P	L	R	P	L	P	L
#10	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P	R	L	P	L	P	L	R	P	L
#31	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P	R	L	P	L	P	L	R
#9	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P	R	L	P	L
#6	P	L	P	L	R	P	L	P	L	P	R	P	L	P	L	P	R	P	L	P	L	P	R	L
#33	P	R	P	R	P	R	L	R	P	R	P	R	P	R	L	R	P	R	P	R	P	R	L	R
#44	L	R	P	R	P	R	P	R	L	R	P	R	P	R	P	R	L	R	P	R	P	R	P	R
#40	P	R	L	R	P	R	P	R	P	R	L	R	P	R	P	R	P	R	L	R	P	R	P	R
#38	P	R	P	R	L	R	P	R	P	R	P	R	L	R	P	R	P	R	P	R	L	R	P	R
#34	P	R	P	R	L	R	L	R	P	R	P	R	L	R	L	R	P	R	P	R	L	R	L	R
#42	L	R	P	R	P	R	L	R	L	R	P	R	P	R	L	R	L	R	P	R	P	R	L	R
#43	L	R	L	R	P	R	P	R	L	R	L	R	P	R	P	R	L	R	L	R	P	R	P	R
#39	P	R	L	R	L	R	P	R	P	R	L	R	L	R	P	R	P	R	L	R	L	R	P	R
#50	L	R	L	P	L	R	L	P	R	L	R	L	P	L	R	P	L	P	R	P	L	R	P	R
#37	P	R	L	R	L	P	L	R	L	P	R	L	R	L	P	L	R	P	L	P	R	P	L	R
#25	P	L	R	P	R	L	R	L	P	L	R	L	P	R	L	R	L	P	L	R	P	L	P	R
#16	P	R	P	L	R	P	R	L	R	L	P	L	R	L	P	R	L	R	L	P	L	R	P	L
#23	P	L	P	R	P	L	R	P	R	L	R	L	P	L	R	L	P	R	L	R	L	P	L	R
#1	P	L	R	P	L	P	R	P	L	R	P	R	L	R	L	P	L	R	L	P	R	L	R	L
#59	L	P	L	R	P	L	P	R	P	L	R	P	R	L	R	L	P	L	R	L	P	R	L	R
#48	L	R	L	P	L	R	P	L	P	R	P	L	R	P	R	L	R	L	P	L	R	L	P	R
#18	P	R	L	R	L	P	L	R	P	L	P	R	P	L	R	P	R	L	R	L	P	L	R	L
#60	L	P	R	L	R	L	P	L	R	P	L	P	R	P	L	R	P	R	L	R	L	P	L	R
#2	P	L	R	L	P	R	L	R	L	P	L	R	P	L	P	R	P	L	R	P	R	L	R	L

[Albi08]

Ricerca dei cicli hamiltoniani in **LPR = <L, R | $L^2 = (LR)^{12} = 1$ >**

C	e	E	g#	B	d#	D#	c	G#	f	C#	c#	A	f#	F#	a#	A#	g	G	b	D	d	F	a	#53
C	c	D#	g	A#	a#	C#	f	G#	g#	B	d#	F#	f#	A	c#	E	e	G	b	D	d	F	a	#32
C	e	G	g	A#	d	F	f	G#	c	D#	d#	F#	a#	C#	c#	E	g#	B	b	D	f#	A	a	#45
C	c	G#	f	C#	c#	A	f#	D	b	B	g#	E	e	G	g	D#	d#	F#	a#	A#	d	F	a	#29
C	e	G	g	D#	c	G#	g#	E	c#	A	f#	F#	d#	B	b	D	d	A#	a#	C#	f	F	a	#49
C	c	G#	f	C#	a#	A#	d	F	a	A	c#	E	g#	B	b	D	f#	F#	d#	D#	g	G	e	#11
C	e	E	g#	B	d#	F#	f#	D	b	G	g	D#	c	G#	f	F	d	A#	a#	C#	c#	A	a	#54
C	c	D#	g	G	b	D	f#	A	a	F	d	A#	a#	F#	d#	B	g#	G#	f	C#	c#	E	e	#14
C	c	G#	g#	B	d#	D#	g	A#	d	F	f	C#	a#	F#	f#	D	b	G	e	E	c#	A	a	#24
C	c	D#	d#	B	b	D	f#	F#	a#	C#	f	G#	g#	E	c#	A	a	F	d	A#	g	G	e	#20
C	e	E	c#	C#	f	F	d	A#	a#	F#	d#	B	g#	G#	c	D#	g	G	b	D	f#	A	a	#55
C	c	D#	g	G	e	E	g#	G#	f	C#	c#	A	f#	D	b	B	d#	F#	a#	A#	d	F	a	#36
C	e	G	g	A#	d	D	b	B	d#	D#	c	G#	g#	E	c#	A	f#	F#	a#	C#	f	F	a	#47
C	c	G#	f	C#	a#	A#	g	D#	d#	F#	f#	D	d	F	a	A	c#	E	g#	B	b	G	e	#5
C	e	E	g#	B	d#	F#	f#	A	c#	C#	a#	A#	d	D	b	G	g	D#	c	G#	f	F	a	#57
C	c	G#	f	F	a	A	f#	D	d	A#	g	D#	d#	F#	a#	C#	c#	E	g#	B	b	G	e	#3
C	e	E	g#	B	b	G	g	A#	d	D	f#	A	c#	C#	a#	F#	d#	D#	c	G#	f	F	a	#61
C	e	E	c#	A	f#	F#	d#	B	g#	G#	c	D#	g	G	b	D	d	A#	a#	C#	f	F	a	#56
C	c	D#	g	A#	a#	F#	d#	B	b	G	e	E	g#	G#	f	C#	c#	A	f#	D	d	F	a	#35
C	c	D#	g	G	b	D	f#	F#	d#	B	g#	G#	f	C#	a#	A#	d	F	a	A	c#	E	e	#15
C	c	G#	f	C#	c#	E	g#	B	b	D	f#	A	a	F	d	A#	a#	F#	d#	D#	g	G	e	#12
C	c	G#	g#	B	d#	D#	g	A#	d	D	b	G	e	E	c#	A	f#	F#	a#	C#	f	F	a	#26
C	c	D#	g	A#	a#	C#	f	G#	g#	E	c#	A	a	F	d	D	f#	F#	d#	B	b	G	e	#17
C	e	G	g	D#	c	G#	g#	E	c#	C#	f	F	d	A#	a#	F#	d#	B	b	D	f#	A	a	#51
C	e	E	g#	B	d#	D#	c	G#	f	F	d	A#	g	G	b	D	f#	F#	a#	C#	c#	A	a	#52
C	e	G	g	A#	d	F	f	C#	a#	F#	f#	D	b	B	d#	D#	c	G#	g#	E	c#	A	a	#46
C	c	G#	f	C#	c#	A	f#	F#	a#	A#	g	D#	d#	B	g#	E	e	G	b	D	d	F	a	#28

#53	L	P	L	R	L	P	R	L	R	L	P	L	R	P	L	P	R	P	L	R	P	R	L	R
#32	P	R	L	R	P	R	L	R	P	R	L	R	P	R	L	R	P	R	L	R	P	R	L	R
#45	L	R	P	R	L	R	P	R	L	R	P	R	L	R	P	R	L	R	P	R	L	R	P	R
#29	P	L	R	L	P	L	R	L	R	P	R	L	P	R	P	L	P	R	L	P	L	R	L	R
#49	L	R	P	L	R	L	P	L	R	L	R	P	R	L	P	R	P	L	P	R	L	P	L	R
#11	P	L	R	L	R	P	L	R	L	P	L	R	L	R	P	R	L	P	R	P	L	P	R	L
#54	L	P	L	R	L	R	P	L	R	L	P	L	R	L	R	P	R	L	P	R	P	L	P	R
#14	P	R	L	P	L	R	L	R	P	L	R	L	P	L	R	L	R	P	R	L	P	R	P	L
#24	P	L	P	R	L	P	L	R	L	R	P	L	R	L	P	L	R	L	R	P	R	L	P	R
#20	P	R	P	L	P	R	L	P	L	R	L	R	P	L	R	L	P	L	R	L	R	P	R	L
#55	L	P	R	P	L	P	R	L	P	L	R	L	R	P	L	R	L	P	L	R	L	R	P	R
#36	P	R	L	P	R	P	L	P	R	L	P	L	R	L	R	P	L	R	L	P	L	R	L	R
#47	L	R	P	R	L	P	R	P	L	P	R	L	P	L	R	L	R	P	L	R	L	P	L	R
#5	P	L	R	L	R	P	R	L	P	R	P	L	P	R	L	P	L	R	L	R	P	L	R	L
#57	L	P	L	R	L	R	P	R	L	P	R	P	L	P	R	L	P	L	R	L	R	P	L	R
#3	P	L	R	P	L	P	R	L	P	L	R	L	P	R	L	R	P	R	L	R	P	L	R	L
#61	L	P	L	R	P	L	P	R	L	P	L	R	L	P	R	L	R	P	R	L	R	P	L	R
#56	L	P	R	L	R	P	R	L	R	P	L	R	L	P	L	R	P	L	P	R	L	P	L	R
#35	P	R	L	R	P	L	R	L	P	L	R	P	L	P	R	L	P	L	R	L	P	R	L	R
#15	P	R	L	P	L	R	L	P	R	L	R	P	R	L	R	P	L	R	L	P	L	R	P	L
#12	P	L	R	L	P	R	L	R	P	R	L	R	P	L	R	L	P	L	R	P	L	P	R	L
#26	P	L	P	R	L	P	L	R	L	P	R	L	R	P	R	L	R	P	L	R	L	P	L	R
#17	P	R	L	R	P	R	L	R	P	L	R	L	P	L	R	P	L	P	R	L	P	L	R	L
#51	L	R	P	L	R	L	P	L	R	P	L	P	R	L	P	L	R	L	P	R	L	R	P	R
#52	L	P	L	R	L	P	R	L	R	P	R	L	R	P	L	R	L	P	L	R	P	L	P	R
#46	L	R	P	R	L	R	P	L	R	L	P	L	R	P	L	P	R	L	P	L	R	L	P	R
#28	P	L	R	L	P	L	R	P	L	P	R	L	P	L	R	L	P	R	L	R	P	R	L	R

62 cicli hamiltoniani classificabili in 8 tipi (o modelli)

[Albi08]

Sistema d'Intervalli generalizzati/Sistema Generalizzato d'intervalli

Generalized Interval System

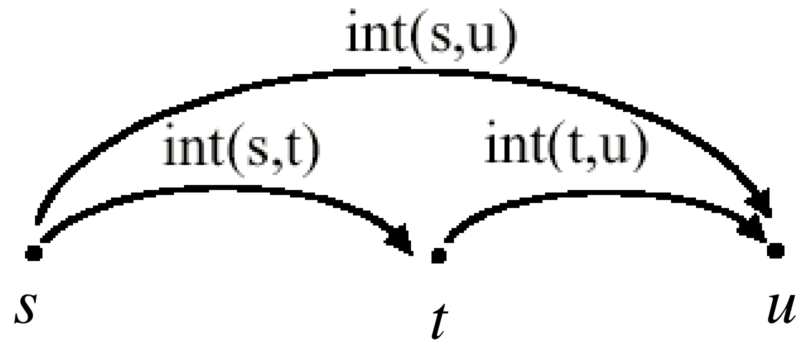
GIS = (*S*, *G*, int)

S=insieme

(*G*,•) = gruppo d'intervalli

int = funzione intervallare

$S \times S$ ---▸ G



• Dati tre elementi *s*, *t*, *u* nell'insieme *S* :

int(*s*,*t*)•int(*t*,*u*)=int(*s*,*u*)

• Per ogni elemento *s* in *S* e ogni intervallo *i* nel gruppo *G* vi è un (solo) elemento *u* dell'insieme *S* che dista un intervallo *i* dall'elemento *s*:

int(*s*,*u*)=*i*

⟺

Azione semplicemente transitiva di un gruppo su un insieme

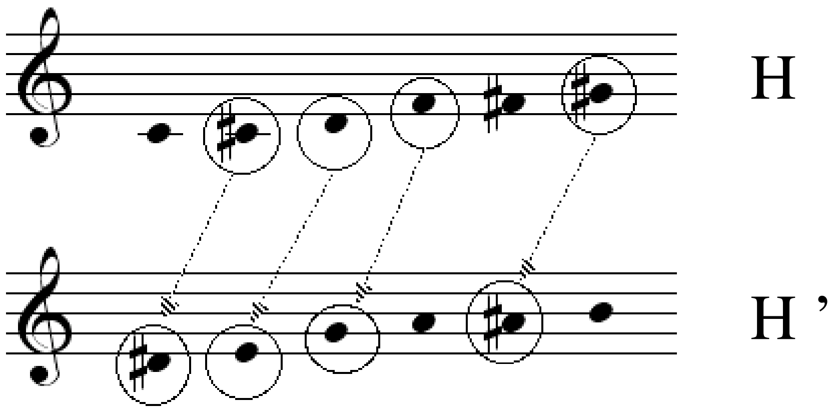
Funzione Intervallare IFUNC in un GIS

Interval Function IFUNC in a GIS

GIS = (*S*, *G*, int)

S insieme

H e H' due sottoinsiemi di *S*



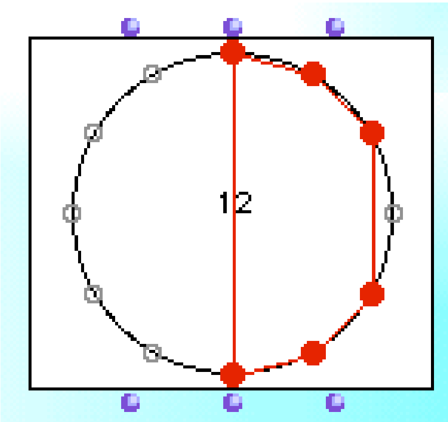
IFUNC(H, H')(i) =
= numeri di coppie (*a*,*b*) in H × H ' i cui elementi hanno distanza reciproca uguale ad *i* ovvero int(*a*,*b*) = i

IFUNC(H, H ')(2) = 4

Contenuto intervallare e trasformata di Fourier discreta

David Lewin, *Journal of Music Theory,* 1958

- Il **contenuto intervallare** (*IC*) di un accordo esprime la frequenza di apparizione di ogni intervallo (dall'unisono all'intervallo di settima maggiore)



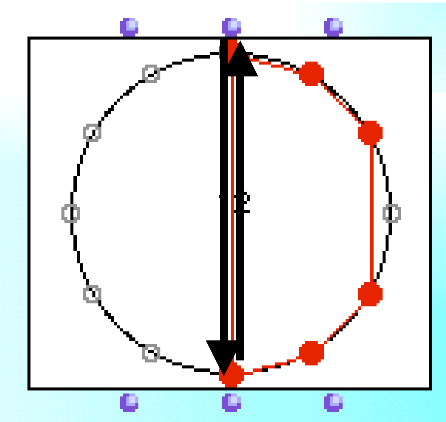
$$IV_A = \quad [4, 3, 2, 3, 2, 1]$$

$$IC_A = [6, 4, 3, 2, 3, 2, 2, 2, 3, 2, 3, 4]$$

Contenuto intervallare e trasformata di Fourier discreta

David Lewin, *Journal of Music Theory,* 1958

- Il **contenuto intervallare** (*IC*) di un accordo esprime la frequenza di apparizione di ogni intervallo (dall'unisono all'intervallo di settima maggiore)



$IV_A =$ [4, 3, 2, 3, 2, 1]

$IC_A =$ [6, 4, 3, 2, 3, 2, 2, 2, 3, 2, 3, 4]

$$IC_A(k) = \mathrm{Card}\{(x,y) \in A \times A \mid x + k = y\}$$

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

$$1_A * \tilde{1}_B(k) = \sum_i 1_A(i) \times 1_B(i-k) = \sum_{\substack{i \in A \\ i-k \in B}} 1$$

=> AMIOT

David Lewin e la trasformata di Fourier

Journal of Music Theory, 1958

- Il contenuto intervallare di due accordi *A* e B è uguale al prodotto di convoluzione delle loro funzioni caratteristiche

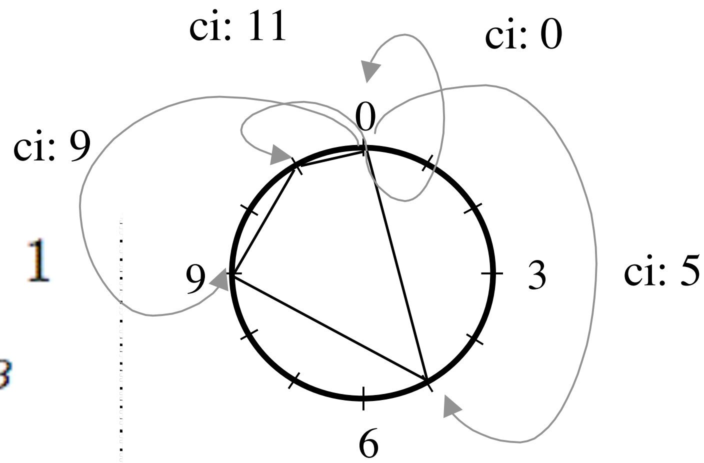
$$IC_A(k) = \text{Card}\{(x,y) \in A \times A \mid x + k = y\}$$

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

$$1_A * \tilde{1}_B(k) = \sum_i 1_A(i) \times 1_B(i-k) = \sum_{\substack{i \in A \\ i-k \in B}} 1$$

$$\mathcal{F}(1_A * \tilde{1}_B) = \mathcal{F}(1_A) \times \mathcal{F}(\tilde{1}_B)$$

$$\forall \text{k}\ \mathcal{F}(\text{IC}_{\mathbb{Z}_c \setminus \text{A}})(\text{k}) = \mathcal{F}(\text{IC}_\text{A})(\text{k})$$



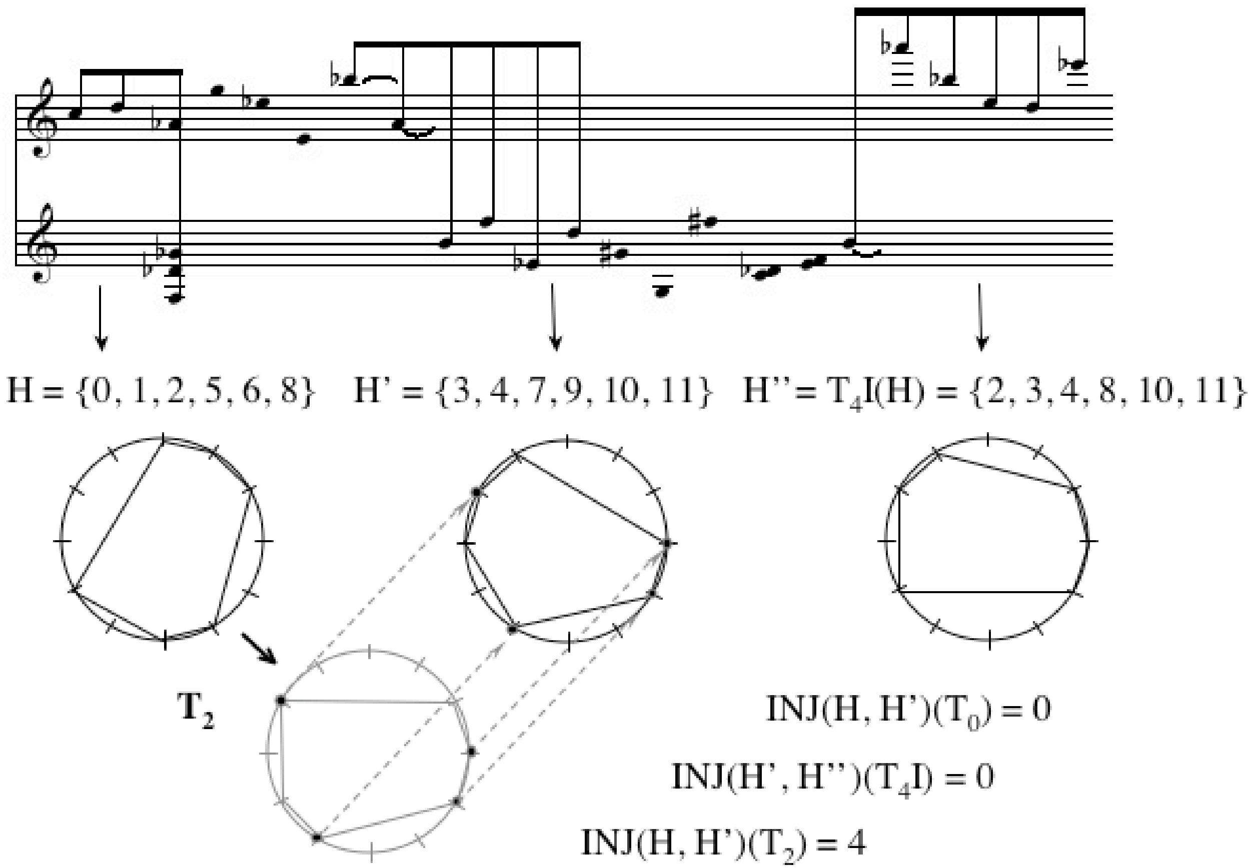
A = {0, 5, 9, 11}

$IC_A(k) = 1\ \forall\ k = 1 \ldots 11$

=> AMIOT

Funzione d'iniezione e relazione inclusione/complementare

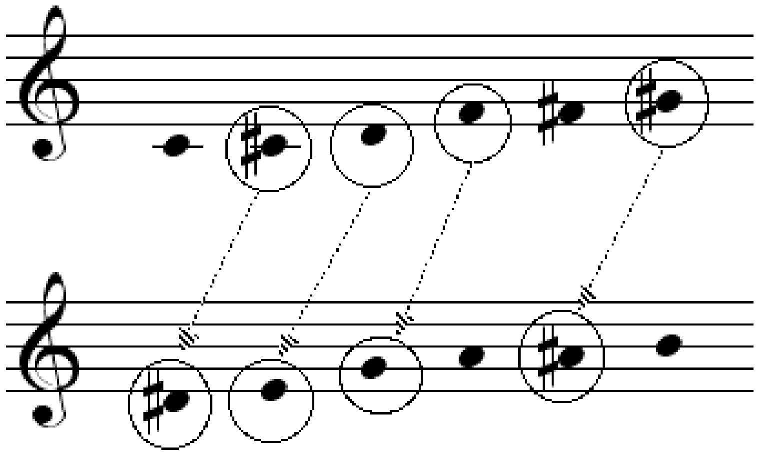
Injection function and the inclusion/complementary relation



$\mathrm{INJ}(H,H')(T_n)$ = numero di elementi a di H tali che $T_n(a) \in H'$

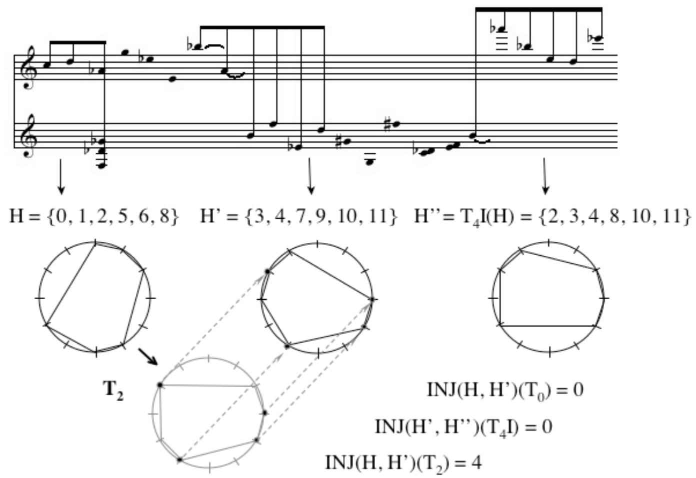
Relazione fra funzione d'iniezione e funzione intervallare

Injection Function and IFUNC



?

<-->



GIS = (S, G, int)

f trasformazione dell'insieme S

INJ(A,B)(f) = numero di elementi a di A tali che $f(a)$ appartiene a B

?

<- - - - - -

- - - - - ->

IFUNC(A,B)(i) = numero di elementi (a,b) di AxB tali che int(a,b) = i

INJ, IFUNC e il « sistema dodecafonico » di Babbitt

Injection Function, IFUNC and the twelve-tone system

« Here the basic hierarchical scope of the (twelve-tone) system is contained essentially in the simple theorem that:

Given a collection of pitches (pitch classes), the multiplicity of occurrence of any interval (…) determines the number of common pitches between the original collection and the transposition by the interval »

(Milton Babbitt, *Past and Present Concepts*, 1961)

$$\mathrm{INJ}(A,B)(\mathrm{T}_i) = \mathrm{IFUNC}(A,B)(\mathrm{i})$$

Funzione d 'Iniezione, IFUNC e teoria trasformazionale

Injection Function, IFUNC and transformational theory

« …il concetto di intervallo in un GIS può essere completamente sostituito col conetto di trasposizione in uno spazio »

« …si può quindi sostituire il concetto stesso di GIS con l'idea di uno spazio S sul quale opera un gruppo di operazioni »

(David Lewin, *Generalized Musical Intervals and Transformations,* 1987)

$$\mathrm{INJ}(A,B)(\mathrm{T_i}) \quad = \quad \mathrm{IFUNC}(A,B)(\mathrm{i})$$

Teorema generalizzato dell'esacordo
Generalized Hexachord Theorem

Un esacordo ed il suo complementare hanno lo stesso contenuto intervallare

$$\mathrm{IFUNC}(A, A)(\mathrm{i}) = \mathrm{IFUNC}(A', A')(\mathrm{i})$$

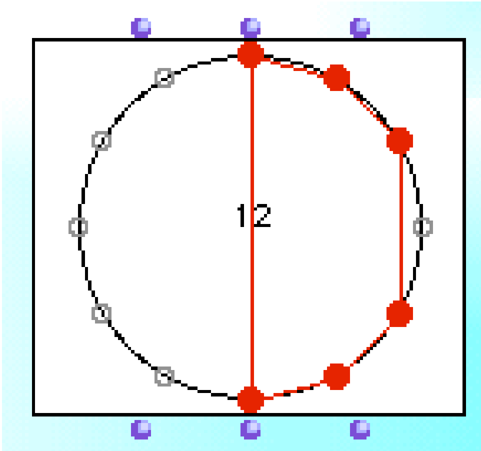
Un esacordo ed il suo complementare hanno la stessa funzione d'iniezione rispetto ad ogni applicazione biiettiva

$$\mathrm{INJ}(A, A)(f) = \mathrm{INJ}(A', A')(f)$$

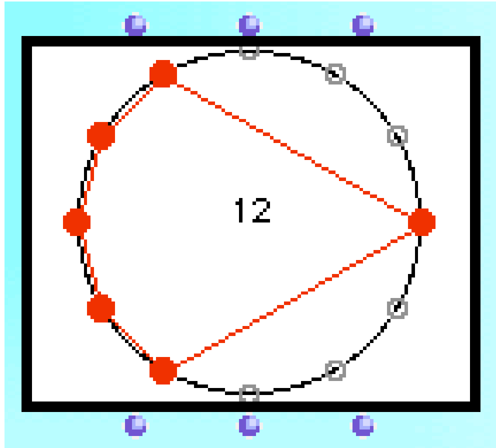
$$\mathrm{INJ}(A, A')(f) = \mathrm{INJ}(A', A)(f)$$

Teorema dell'esacordo (o teorema di Babbitt)

(Wilcox, Ralph Fox (?), Chemillier, Lewin, Mazzola, Schaub, …, Amiot [2006])



A



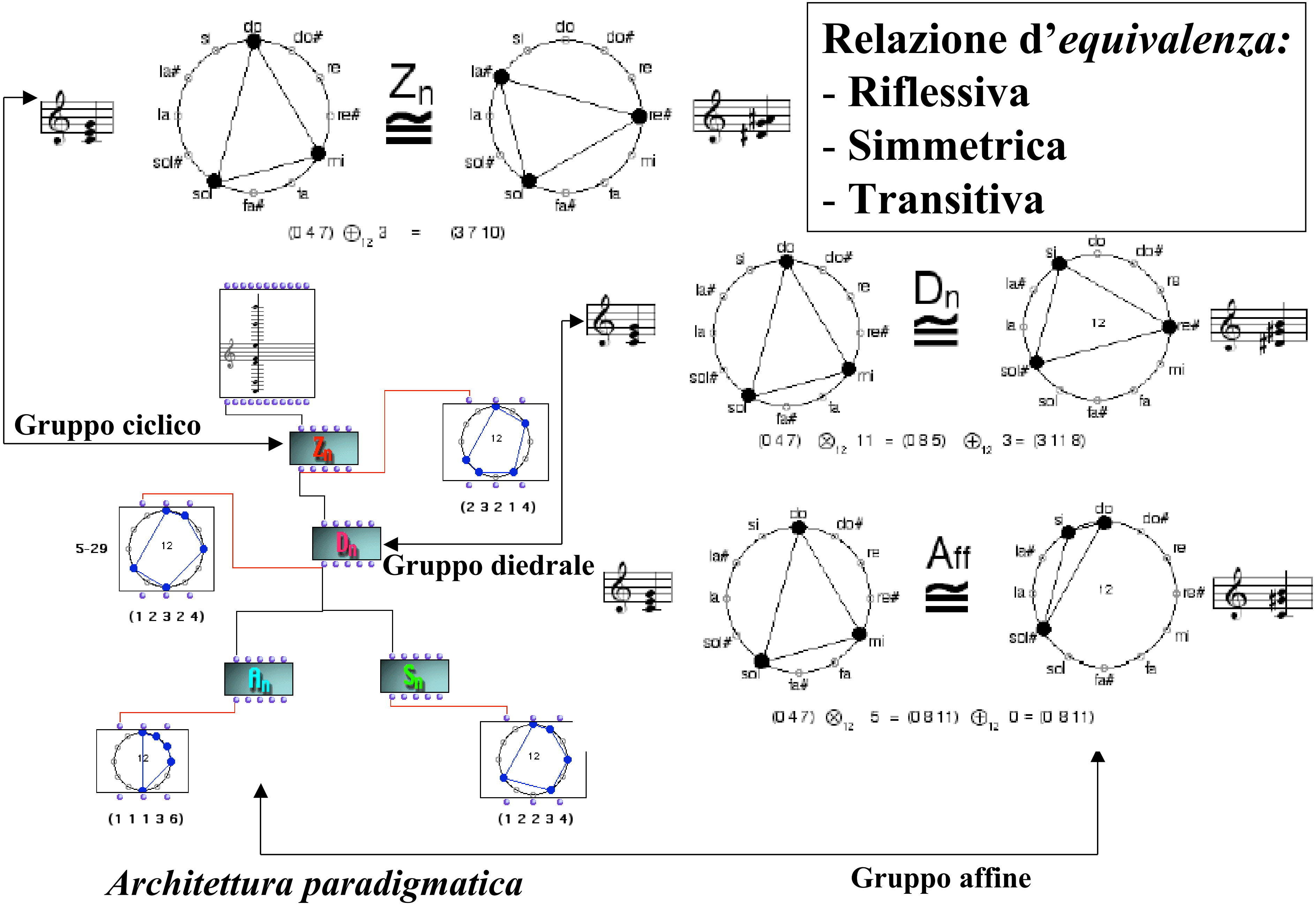
A'

$$IV(A) = [4, 3, 2, 3, 2, 1] = [4, 3, 2, 3, 2, 1] = IV(A')$$

Un esacordo e il suo complementare hanno lo stesso vettore intervallare

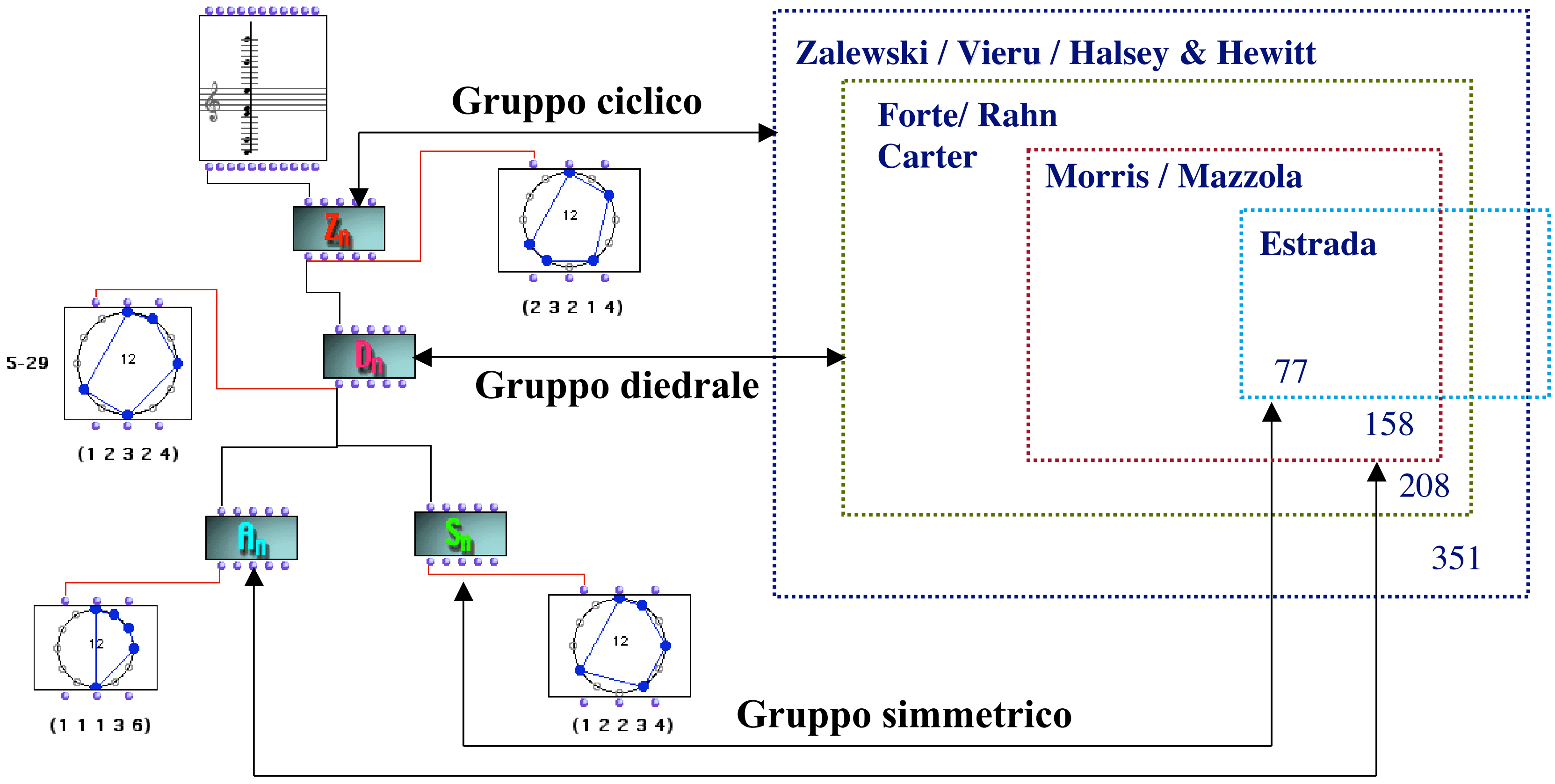
I gruppi come “paradigmi” per l’equivalenza fra accordi

Relazione d’*equivalenza:*
- **Riflessiva**
- **Simmetrica**
- **Transitiva**



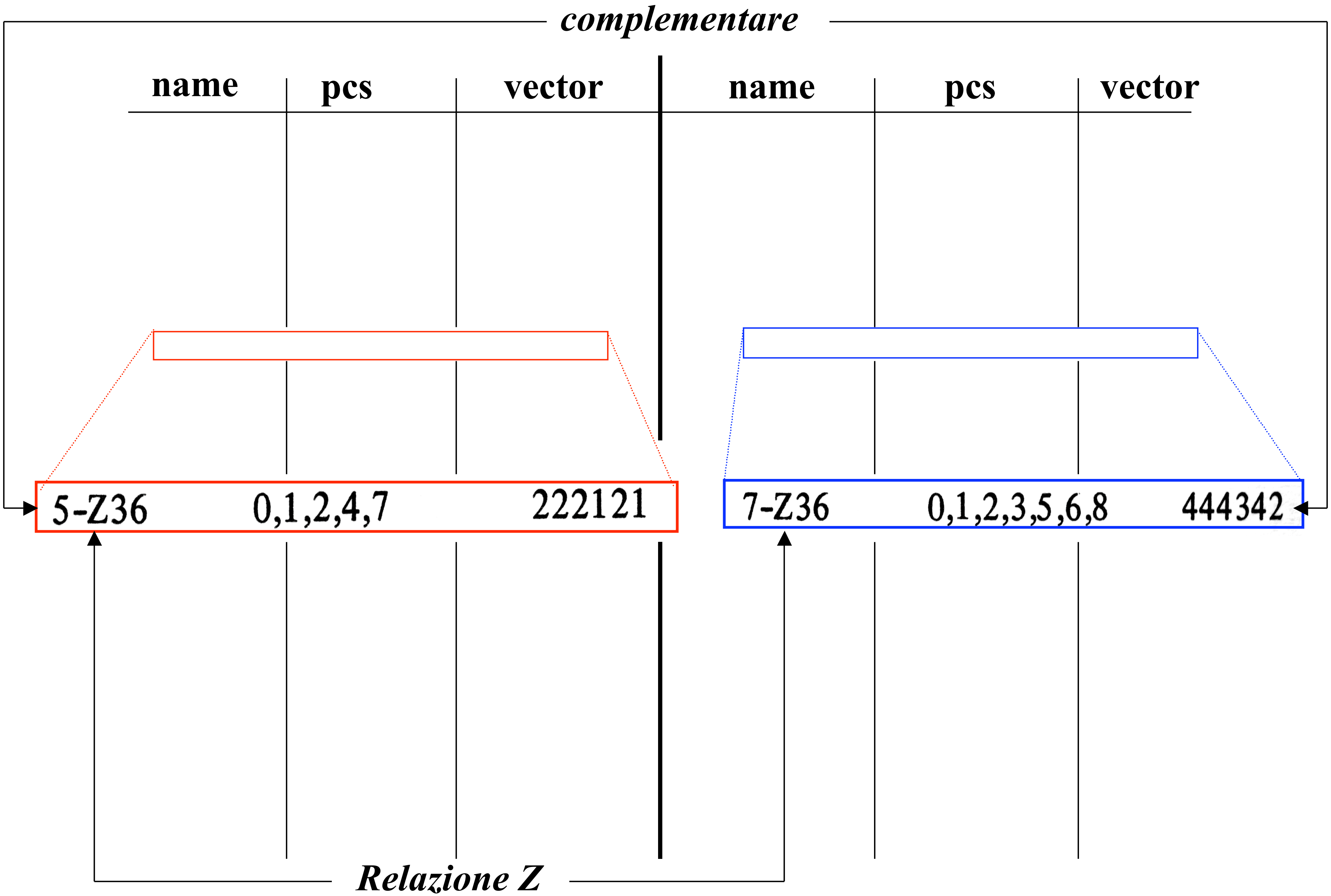
Classificazione paradigmatica delle strutture musicali

$G\backslash k$	1	2	3	4	5	6	7	8	9	10	11	12	
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1	
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1	**Set Theory**
$\mathrm{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1	

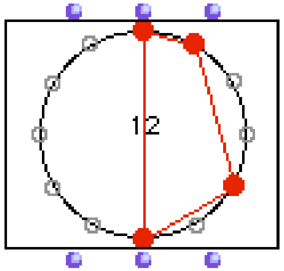
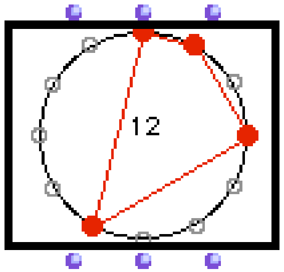


Architettura paradigmatica

La *Set Theory* d'Allen Forte: catalogo dei *pitch-class sets*

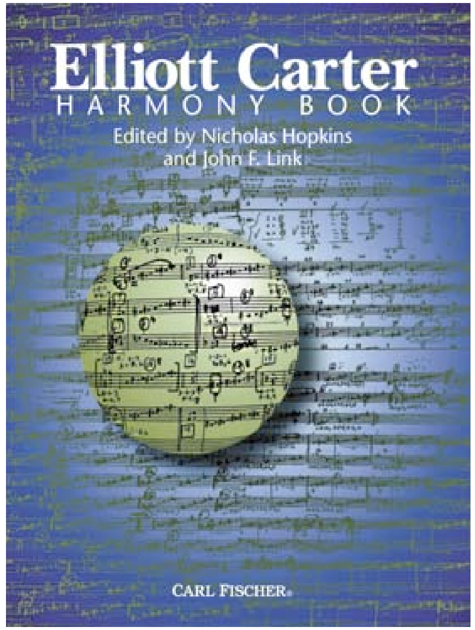


Elliott Carter's *Harmony Book* (2002)

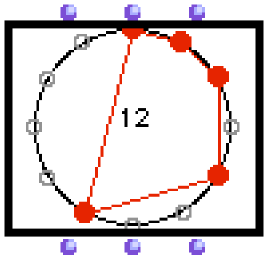
		pcset	*vecteur*	*Forte*
18		(0 1 4 6)	[111111]	4-Z15
23		(0 1 3 7)	[111111]	4-Z29

Utilizzazione (implicita) della *Z-relation*

- *Quartetto n°1* (1951)
- *Night Fantasies* (1980)
- *90+* (1994)
- …

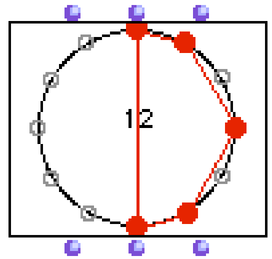


Vettore intervallare e relazione Z



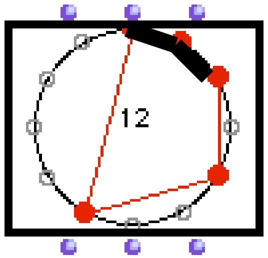
Il vettore intervallare (Forte) esprime la frequenza di apparizione di ogni intervallo (modulo il suo complementare)

5-Z36	0,1,2,4,7	222121



5-Z12

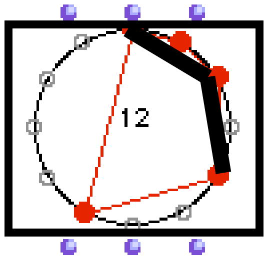
Vettore intervallare e relazione Z



Il vettore intervallare (Forte) esprime la frequenza di apparizione di ogni intervallo (modulo il suo complementare)

5-Z36	0,1,2,4,7	222121

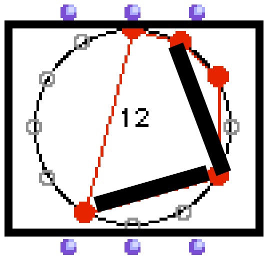
Vettore intervallare e relazione Z



Il vettore intervallare (Forte) esprime la frequenza di apparizione di ogni intervallo (modulo il suo complementare)

5-Z36	0,1,2,4,7	222121

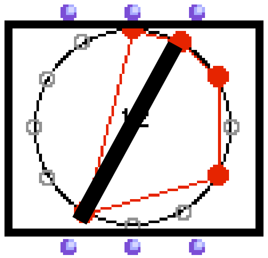
Vettore intervallare e relazione Z



Il vettore intervallare (Forte) esprime la frequenza di apparizione di ogni intervallo (modulo il suo complementare)

5-Z36	0,1,2,4,7	222121

Vettore intervallare e relazione Z



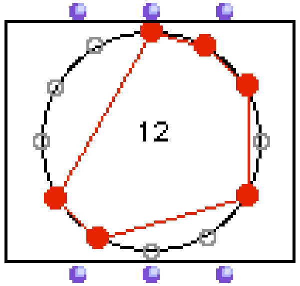
Il vettore intervallare (Forte) esprime la frequenza di apparizione di ogni intervallo (modulo il suo complementare)

5-Z36	0,1,2,4,7	222121

Elliott Carter: 90+ (1994)

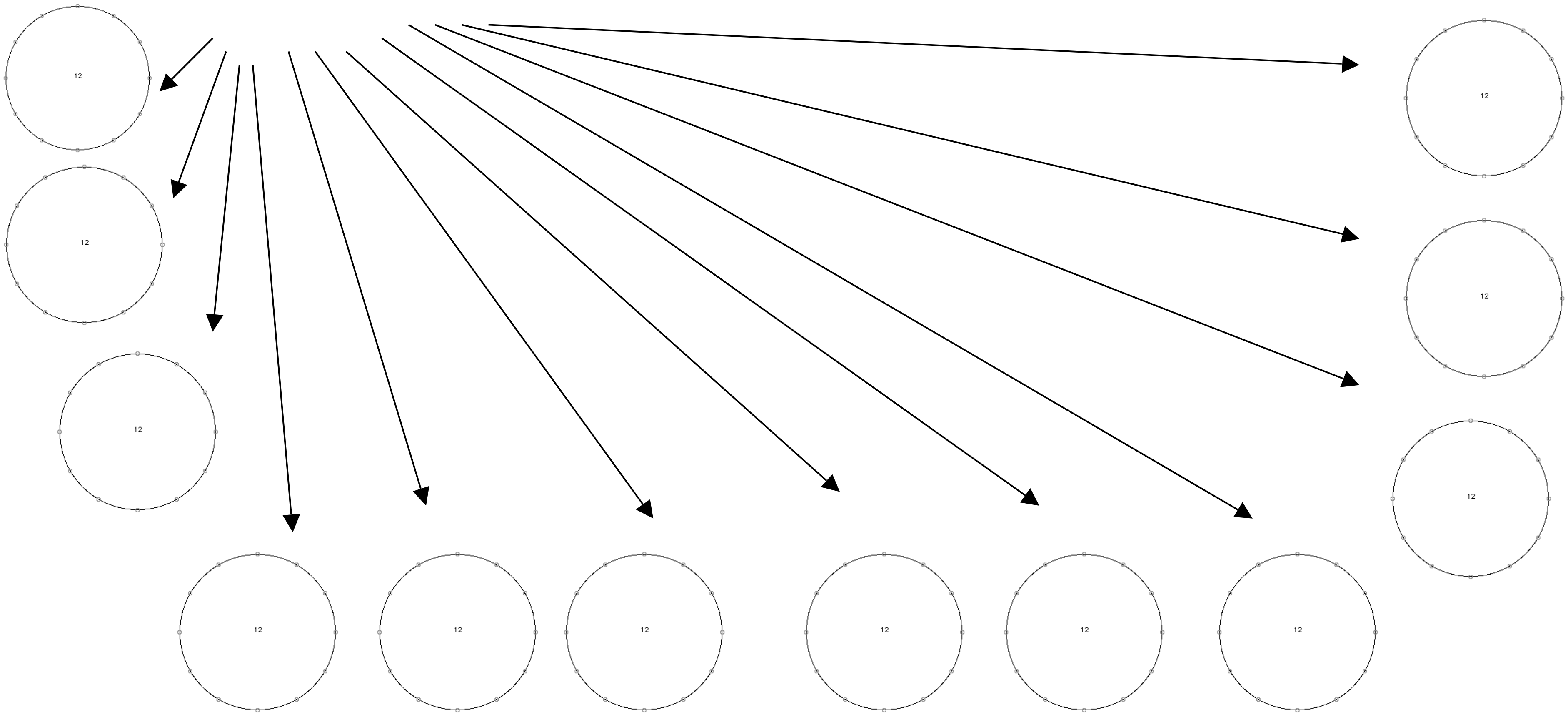
$G \backslash k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1
$\mathrm{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1

35



pcset	*vector*	*name*
(0 1 2 4 7 8)	[322332]	6-Z17

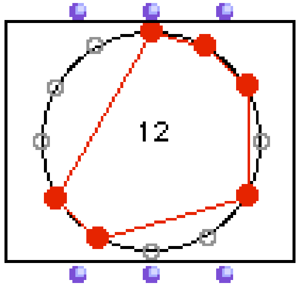
All-triad hexachord



Elliott Carter: 90+ (1994)

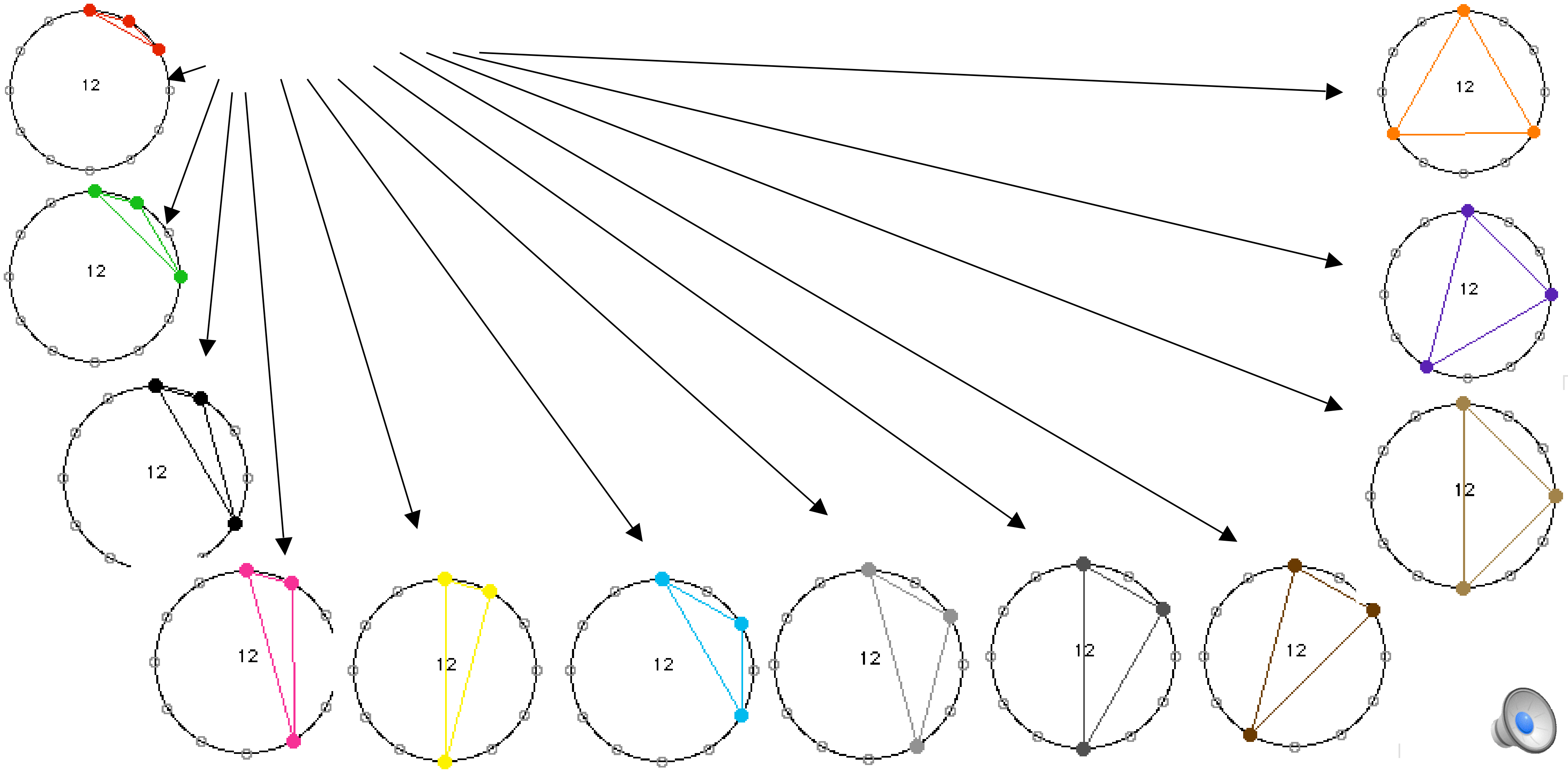
$G\backslash k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1
$\mathrm{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1

35



pcset	*vector*	*name*
(0 1 2 4 7 8)	[322332]	6-Z17

All-triad hexachord

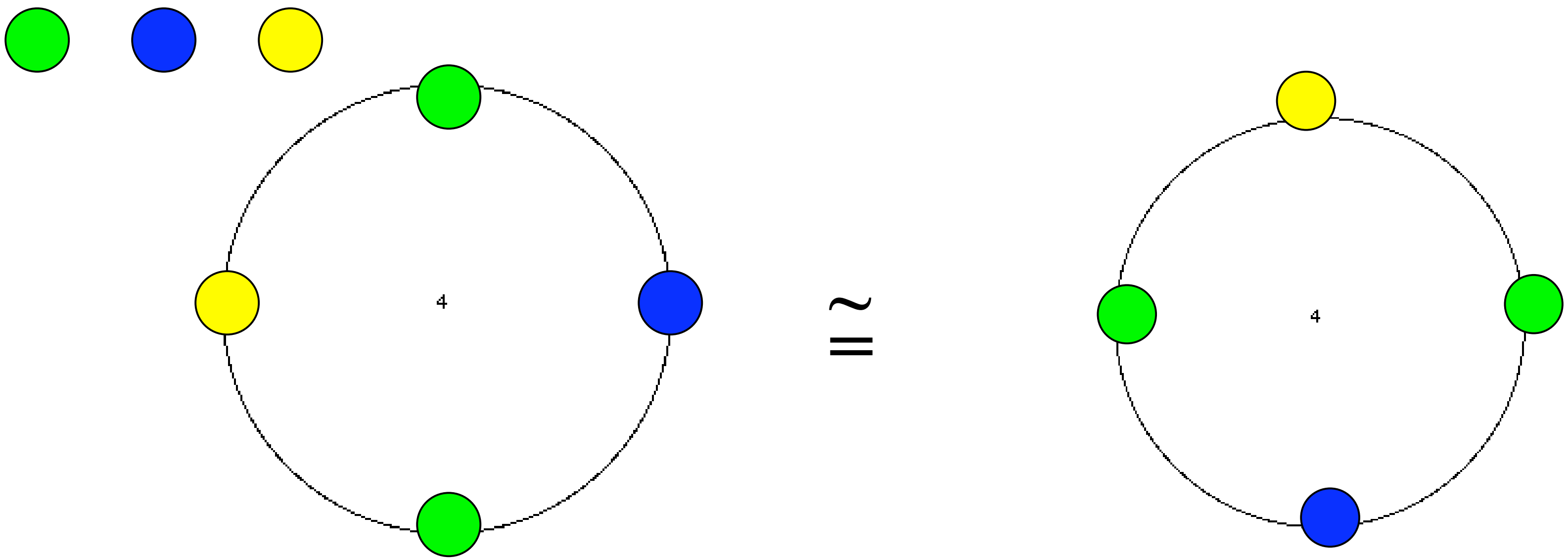


Enumerazione delle orbite rispetto all’azione di un gruppo

Lemma de Burnside

$$n = \frac{1}{|G|}\sum_{g\in G} |X^g|.$$

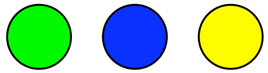
$$X^g = \{x\in X : gx=x\}$$

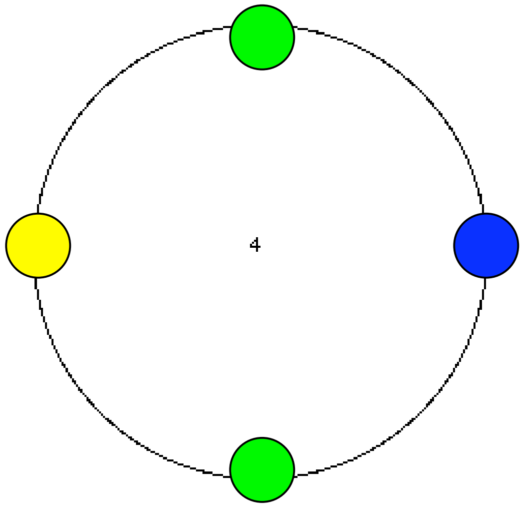


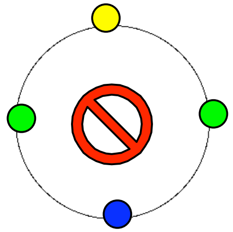
?

Trovare il numero di configurazioni possibili

Enumerazione delle orbite rispetto all'azione di **Z/nZ**







Lemma di Burnside

$$n = \frac{1}{|G|}\sum_{g\in G} |X^g|.$$

$$X^g = \{x\in X : gx=x\}$$

Azione di **Z/4Z**

T_0 = identità

T_1 = rotazione di 90°

T_2 = rotazione di 180°

T_3 = rotazione di 270°

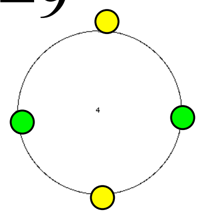
Configurazioni possibili = 3^4 = 81

T_0 fissa ogni configurazione => $|X^{T_0}|=81$

T_1 fissa ogni configurazione monocromatica => $|X^{T_1}|=3$

T_3 idem

T_2 fissa ogni configurazione «doppio-diametro » => $|X^{T_1}| = 3^2 = 9$



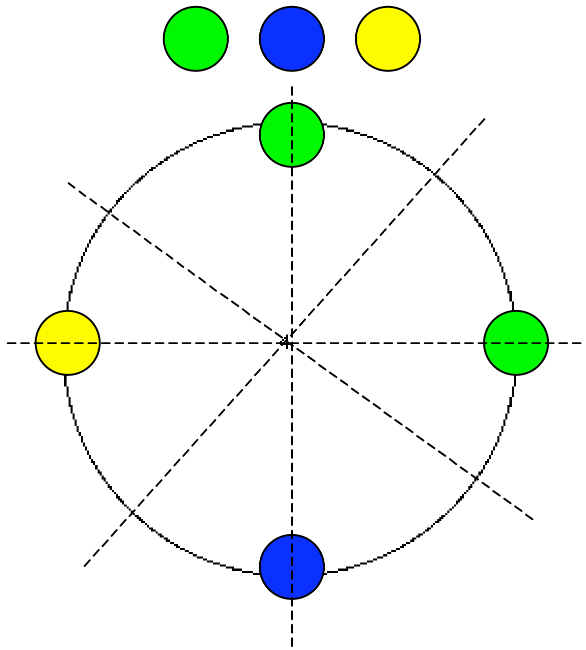
$n = 1/4\ (81+3+3+9) = 24$

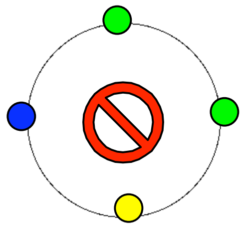
Enumerazione delle orbite rispetto all'azione di $\mathbf{D_n}$

Lemma di Burnside

$$n = \frac{1}{|G|}\sum_{g\in G} |X^g|.$$

$$X^g = \{x \in X : gx = x\}$$



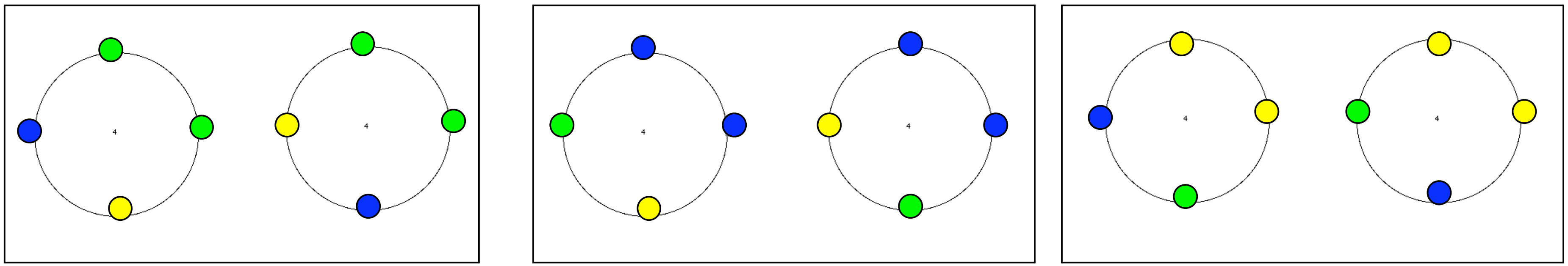


Action de $\mathbf{D_4}$

$T_0 = \text{id}$	$T_0 I = \text{inversion}$
$T_1 = \text{rot } 90°$	$T_1 I = \text{inv.}$
$T_2 = \text{rot } 180°$	$T_2 I = \text{inv.}$
$T_3 = \text{rot } 270°$	$T_3 I = \text{inv.}$

?

⇒ Verificare che il numero delle configurazioni possibili è = 21

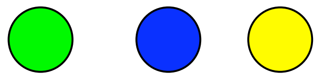


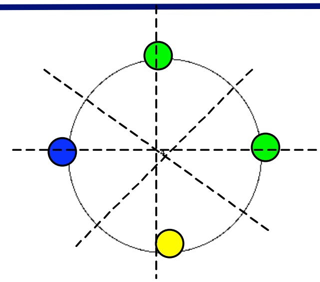
Enumerazione delle orbite rispetto all'azione di $\mathbf{D_n}$

Lemma di Burnside

$$n = \frac{1}{|G|}\sum_{g\in G}|X^g|.$$

$$X^g = \{x\in X : gx=x\}$$





Transformation	*Action*	*Cycle representation*	*No. of cycles*	*Fixed configs.*	*Cycle type*	*Cycle index*
T_0	$0\to0,\ 1\to1,\ 2\to2,\ 3\to3$	(0)(1)(2)(3)	4	$3^4 = 81$	1^4	$t_1^{\ 4}$
T_1	$0\to1\to2\to3\to0$	(0 1 2 3)	1	$3^1 = 3$	4^1	$t_4^{\ 1}$
T_2	$0\to2\to0,\ 1\to3\to1$	(0 2)(1 3)	2	$3^2 = 9$	2^2	$t_2^{\ 2}$
T_3	$0\to3\to2\to1\to0$	(0 3 2 1)	1	$3^1 = 3$	4^1	$t_4^{\ 1}$
I	$0\to0,\ 1\to3\to1,\ 2\to2$	(0)(1 3)(2)	3	$3^3 = 27$	$1^2 2^1$	$t_1^{\ 2}t_2^{\ 1}$
T_1I	$0\to1\to0,\ 2\to3\to2$	(0 1)(2 3)	2	$3^2 = 9$	2^2	$t_2^{\ 2}$
T_2I	$0\to2\to0,\ 1\to1,\ 3\to3$	(0 2)(1)(3)	3	$3^3 = 27$	$1^2 2^1$	$t_1^{\ 2}t_2^{\ 1}$
T_3I	$0\to3\to0,\ 1\to2\to1$	(0 3)(1 2)	2	$3^2 = 9$	2^2	$t_2^{\ 2}$

Julian Hook, « Why are there 29 Tetrachords? A Tutorial on Combinatorics and Enumeration in Music Theory », MTO, 13(4), 2007

Enumerazione delle orbite rispetto al gruppo ciclico $\mathbf{Z_{12}}$

Lemma di Burnside

$$n = \frac{1}{|G|}\sum_{g\in G} |X^g|.$$

$$X^g = \{x \in X : gx = x\}$$

(Benson, p. 348)

Group element	size of subset												
	0	1	2	3	4	5	6	7	8	9	10	11	12
Identity	1	12	66	220	495	792	924	792	495	220	66	12	1
$\mathbf{T}, \mathbf{T}^5, \mathbf{T}^7, \mathbf{T}^{11}$	1	0	0	0	0	0	0	0	0	0	0	0	1
$\mathbf{T}^2, \mathbf{T}^{10}$	1	0	0	0	0	0	2	0	0	0	0	0	1
$\mathbf{T}^3, \mathbf{T}^9$	1	0	0	0	3	0	0	0	3	0	0	0	1
$\mathbf{T}^4, \mathbf{T}^8$	1	0	0	4	0	0	6	0	0	4	0	0	1
$\mathbf{T}^6$	1	0	6	0	15	0	20	0	15	0	6	0	1

⇒ # accordi di 3 note = 1/12 (220 + 4·2) = 228/12 = 19

Enumerazione delle orbite (azione del gruppo diedrale $\mathbf{D_{12}}$)

Lemma di Burnside

$$n = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

$$X^g = \{x \in X : gx = x\}$$

(Benson, p. 348)

Group element	size of subset												
	0	1	2	3	4	5	6	7	8	9	10	11	12
Identity	1	12	66	220	495	792	924	792	495	220	66	12	1
$\mathbf{T}, \mathbf{T}^5, \mathbf{T}^7, \mathbf{T}^{11}$	1	0	0	0	0	0	0	0	0	0	0	0	1
$\mathbf{T}^2, \mathbf{T}^{10}$	1	0	0	0	0	0	2	0	0	0	0	0	1
$\mathbf{T}^3, \mathbf{T}^9$	1	0	0	0	3	0	0	0	3	0	0	0	1
$\mathbf{T}^4, \mathbf{T}^8$	1	0	0	4	0	0	6	0	0	4	0	0	1
$\mathbf{T}^6$	1	0	6	0	15	0	20	0	15	0	6	0	1
$\mathbf{T}^{2m}\mathbf{I}$	1	2	6	10	15	20	20	20	15	10	6	2	1
$\mathbf{T}^{2m+1}\mathbf{I}$	1	0	6	0	15	0	20	0	15	0	6	0	1

⇒ # *pitch-classes* di 3 note = 1/24 (220 + 4·2 + 10·6) = 288/24 = 12

=> *OpenMusic*

Enumerazione

Transformation	*Cycle representation*	*No. of cycles*	*Fixed configs.*	*Cycle type*	*Cycle index*
T_0	(0)(1)(2)(3)(4)(5)(6)(7)(8)(9)(A)(B)	12	$2^{12} = 4096$	1^{12}	t_1^{12}
T_1	(0 1 2 3 4 5 6 7 8 9 A B)	1	$2^1 = 2$	12^1	t_{12}^1
T_2	(0 2 4 6 8 A)(1 3 5 7 9 B)	2	$2^2 = 4$	6^2	t_6^2
T_3	(0 3 6 9)(1 4 7 A)(2 5 8 B)	3	$2^3 = 8$	4^3	t_4^3
T_4	(0 4 8)(1 5 9)(2 6 A)(3 7 B)	4	$2^4 = 16$	3^4	t_3^4
T_5	(0 5 A 3 8 1 6 B 4 9 2 7)	1	$2^1 = 2$	12^1	t_{12}^1
T_6	(0 6)(1 7)(2 8)(3 9)(4 A)(5 B)	6	$2^6 = 64$	2^6	t_2^6
T_7	(0 7 2 9 4 B 6 1 8 3 A 5)	1	$2^1 = 2$	12^1	t_{12}^1
T_8	(0 8 4)(1 9 5)(2 A 6)(3 B 7)	4	$2^4 = 16$	3^4	t_3^4
T_9	(0 9 6 3)(1 A 7 4)(2 B 8 5)	3	$2^3 = 8$	4^3	t_4^3
T_{10}	(0 A 8 6 4 2)(1 B 9 7 5 3)	2	$2^2 = 4$	6^2	t_6^2
T_{11}	(0 B A 9 8 7 6 5 4 3 2 1)	1	$2^1 = 2$	12^1	t_{12}^1
I	(0)(1 B)(2 A)(3 9)(4 8)(5 7)(6)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_1I	(0 1)(2 B)(3 A)(4 9)(5 8)(6 7)	6	$2^6 = 64$	2^6	t_2^6
T_2I	(0 2)(1)(3 B)(4 A)(5 9)(6 8)(7)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_3I	(0 3)(1 2)(4 B)(5 A)(6 9)(7 8)	6	$2^6 = 64$	2^6	t_2^6
T_4I	(0 4)(1 3)(2)(5 B)(6 A)(7 9)(8)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_5I	(0 5)(1 4)(2 3)(6 B)(7 A)(8 9)	6	$2^6 = 64$	2^6	t_2^6
T_6I	(0 6)(1 5)(2 4)(3)(7 B)(8 A)(9)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_7I	(0 7)(1 6)(2 5)(3 4)(8 B)(9 A)	6	$2^6 = 64$	2^6	t_2^6
T_8I	(0 8)(1 7)(2 6)(3 5)(4)(9 B)(A)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_9I	(0 9)(1 8)(2 7)(3 6)(4 5)(A B)	6	$2^6 = 64$	2^6	t_2^6
$T_{10}I$	(0 A)(1 9)(2 8)(3 7)(4 6)(5)(B)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
$T_{11}I$	(0 B)(1 A)(2 9)(3 8)(4 7)(5 6)	6	$2^6 = 64$	2^6	t_2^6

Lemma di Burnside

$$n = \frac{1}{|G|}\sum_{g\in G} |X^g|.$$

$X^g = \{x \in X : gx = x\}$

Azione di $\mathbf{D_{12}}$

(Hook, MTO)

d'accords = 1/12[4096+2+4+8+16+2+64+2+16+8+4+2]=352

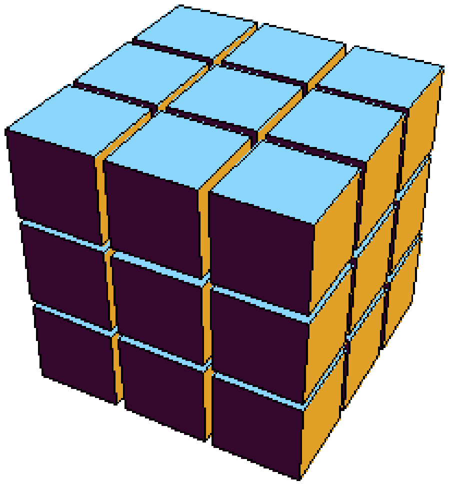
d'accords = 1/24[…] = 224

Alcune applicazioni del lemma di Burnside

1	2	3	7	8	9	4	5	6
4	5	6	1	2	3	7	8	9
7	8	9	4	5	6	1	2	3
2	3	1	8	9	7	5	6	4
5	6	4	2	3	1	8	9	7
8	9	7	5	6	4	2	3	1
3	1	2	9	7	8	6	4	5
6	4	5	3	1	2	9	7	8
9	7	8	6	4	5	3	1	2

5.472.730.538 =
5×10^9 soluzioni differenti

Bertram Felgenhauer & Fraze Jarvis, 2005
http://www.afjarvis.staff.shef.ac.uk/



901.083.404.981.813.616
$= 9 \times 10^{17}$ posizioni differenti

Turner, E. C. and Gold, K. F. "Rubik's Groups."
Amer. Math. Monthly **92**, 617-629, 1985

Formule d’enumerazione d’accordi in un sistema temperato

(Reiner, 1985)

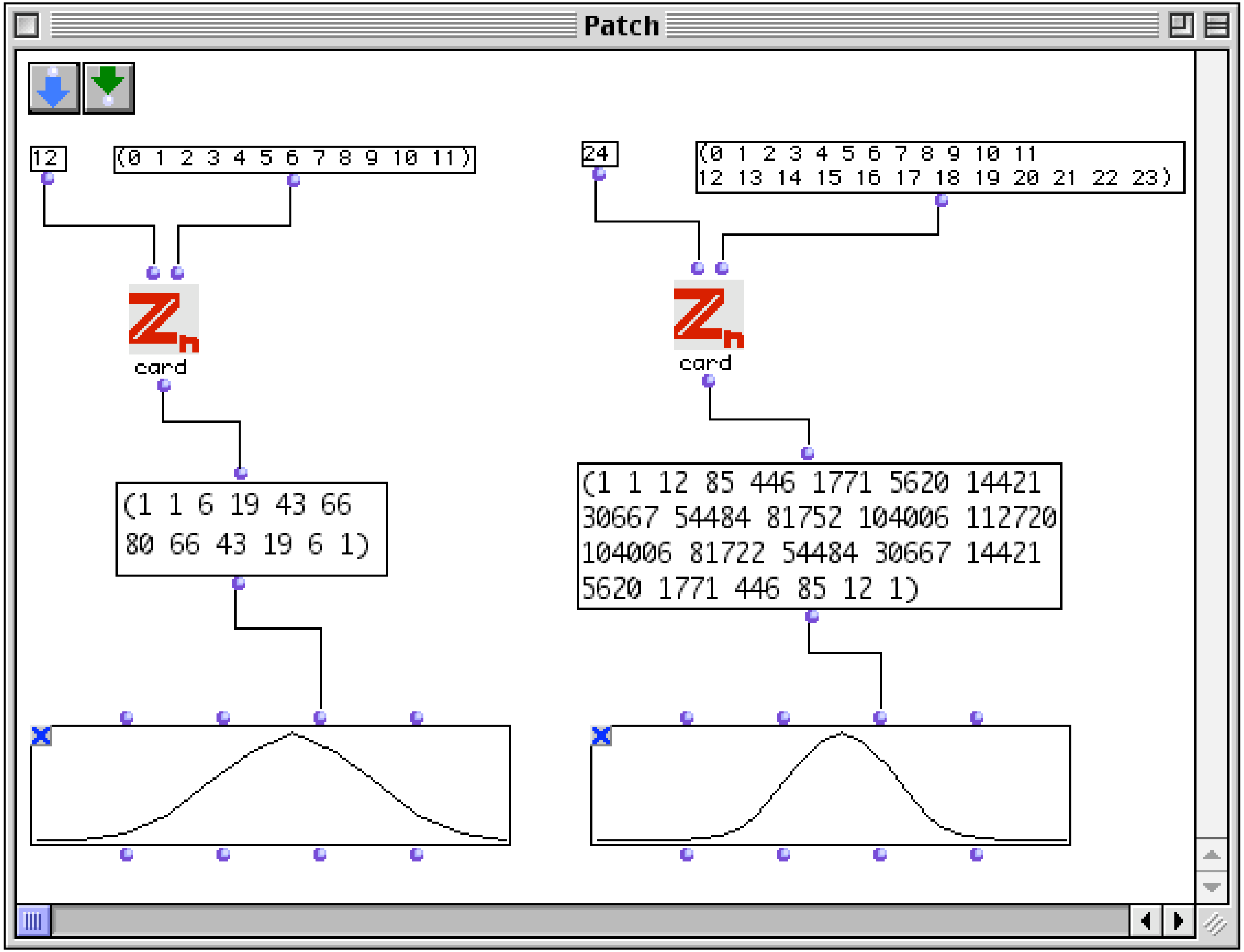
Z_n $$\#\text{ of }k\text{-chords} = \frac{1}{n}\sum_{j|(n,k)} \phi(j)\binom{n/j}{k/j} = \frac{1}{n}\Phi_n(k),$$

D_n $$\#\text{ of }k\text{-chords} = \begin{cases} \frac{1}{2n}\left[\Phi_n(k) + n\binom{(n-1)/2}{[k/2]}\right], & \text{if } n \text{ is odd,} \\ \frac{1}{2n}\left[\Phi_n(k) + n\binom{n/2}{k/2}\right], & \text{if } n \text{ is even and } k \text{ is even,} \\ \frac{1}{2n}\left[\Phi_n(k) + n\binom{(n/2)-1}{[k/2]}\right], & \text{if } n \text{ is even and } k \text{ is odd.} \end{cases}$$

- D. Halsey & E. Hewitt: « Eine gruppentheoretische Methode in der Musik-theorie », *Jahresber. Der Dt. Math.-Vereinigung*, 80, 1978.
- **D. Reiner: «Enumeration in Music Theory», *Amer. Math. Month.* 92:51-54, 1985**
- H. Fripertinger: «Enumeration in Musical Theory», *Beiträge zur Elektr. Musik*,1,1992
- R.C. Read: « Combinatorial problems in the theory of music », *Discrete Math.*, 1997
- H. Fripertinger: « Enumeration of mosaics », *Discrete Math.*, 1999
- H. Fripertinger: « Enumeration of non-isomorphic canons », *Tatra Mt. Math. Publ.*, 2001
- M. Broué : « Les tonalités musicales vues par un mathématicien », *Le temps des savoirs, Revue de l'Institut Universitaire de France,* 2002
- David J. Hunter & Paul T. von Hippel : « How Rare Is Symmetry in Musical 12-Tone Rows? », *The American Mathematical Monthly*, Vol. 110, No. 2., Feb., 2003
- H. Fripertinger: « Tiling problems in music theory », in *Perspectives in Mathematical and Computational Music Theory* (Mazzola, Noll, Puebla ed., Epos, 2004)
- Rachel W. Hall & P. Klingsberg: « Asymmetric Rhythms, Tiling Canons, and Burnside’s Lemma », *Bridge Proceedings*, 2004
- …

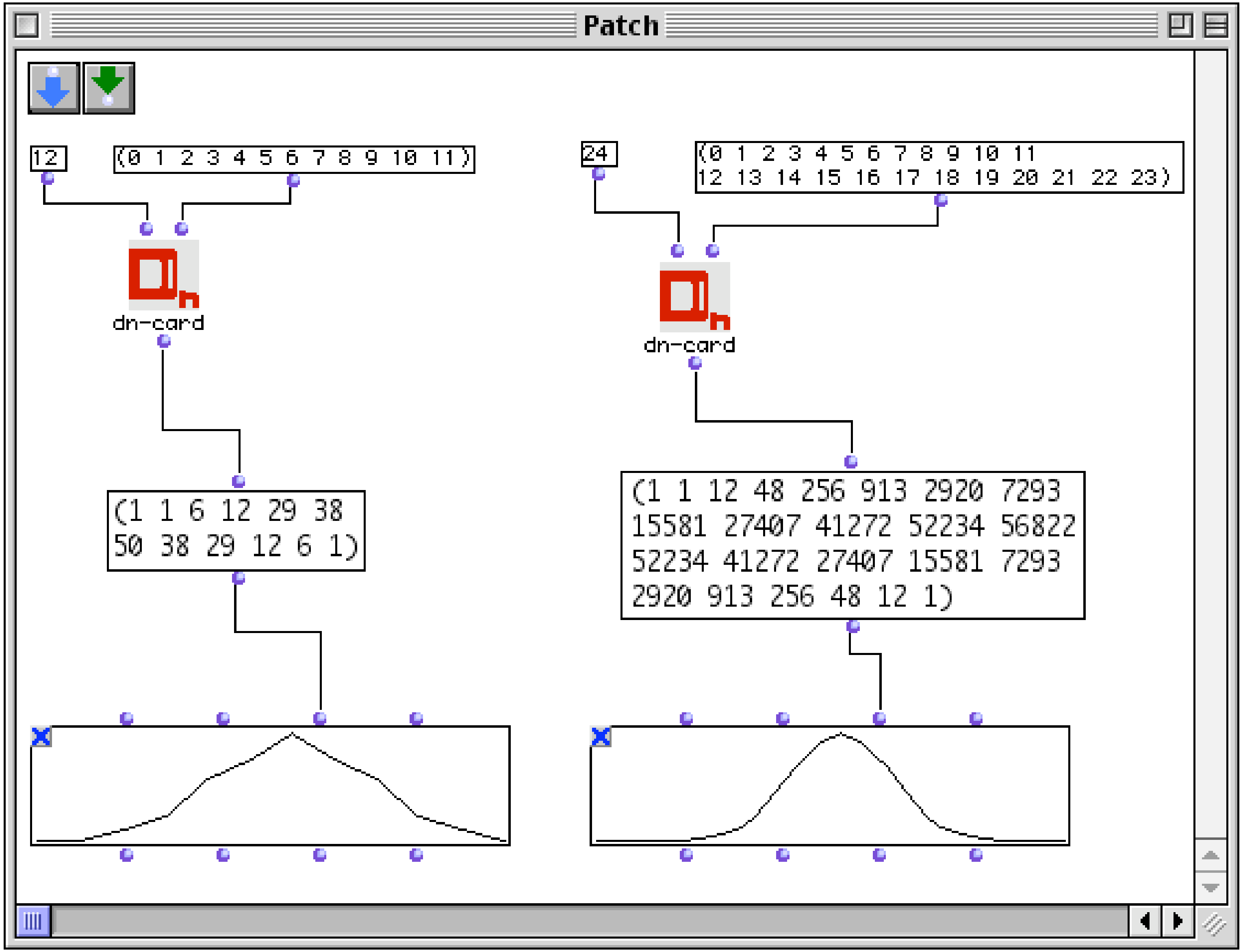
Aspetti computazionali della *Set Theory*

Enumerazione delle strutture musicali nello spazio temperato: ***Z/nZ***



Aspetti computazionali della *Set Theory*

Enumerazione delle strutture musicali nello spazio temperato: ***Dn***



Enumerazione degli accordi in un sistema temperato

(Fripertinger, 1992 / 1999) (Mazzola, *Topos of Music*, 2003)

$G\backslash k$	1	2	3	4	5	6	7	8	9	10	11	12
C_{12}	1	6	19	43	66	80	66	43	19	6	1	1
D_{12}	1	6	12	29	38	50	38	29	12	6	1	1
$\mathrm{Aff}_1(Z_{12})$	1	5	9	21	25	34	25	21	9	5	1	1

$F_1(x)=1x$
$F_2(x)=7x$
$F_3(x)=-2-x$
$F_4(x)=-2+5x$

	F_1	F_2	F_3	F_4
F_1	F_1	F_2	F_3	F_4
F_2	F_2	F_1	F_4	F_3
F_3	F_3	F_4	F_1	F_2
F_4	F_4	F_3	F_2	F_1

	S	I	R	RI
S	S	I	R	RI
I	I	S	RI	R
R	R	RI	S	I
RI	RI	R	I	S

Chord Classes

Class Nr.	*Representative* $Nr. = \bullet$, $\widehat{Nr.} = \circ$	*Group of Symmetries*	*Conj. Class*	♯ *End.* $Nr.\vert\widehat{Nr.}$
1	●●●●●●●●●●●●	$\overrightarrow{GL}(\mathbb{Z}_{12})$	19	28|28
		One/Eleven Element		
2	●○○○○○○○○○○○	$\mathbb{Z}_{12}^{\times}$	8	1|31
		Two/Ten Elements		
3	●●○○○○○○○○○○	$\langle -1e^{-1} \rangle$	3	3|23
3.1	●○○○○●○○○○○○			
4	●○●○○○○○○○○○	$\{1, 7, -1e^{-2}, 5e^{-2}\}$	8	3|25
5	●○○●○○○○○○○○	$\{1, 5, 7e^{-3}, -1e^{-3}\}$	8	3|19
6	●○○○●○○○○○○○	$\{1, 7, 5e^{8}, -1e^{8}\}$	8	3|31
7	●○○○○○●○○○○○	$\mathbb{Z}_{12}^{\times} \ltimes e^{6\mathbb{Z}_{12}}$	13	3|28
		Three/Nine Elements		
8	●●●○○○○○○○○○	$\langle -1e^{-2} \rangle$	2	4|14
8.1	●○●○○○○●○○○○			
9	●●○●○○○○○○○○	$\{1\}$	1	4|30
9.1	●○○○○○○○○○○○			
10	●●○○●○○○○○○○	$\{1\}$	1	8|36
10.1	●○○●○○○●○○○○			
11	●●○○○●○○○○○○	$\langle 5 \rangle$	4	4|20
12	●●○○○○●○○○○○	$\langle 7e^{6} \rangle$	6	5|29
13	●○●○●○○○○○○○	$\{1, 7, -1e^{8}, 5e^{8}\}$	8	4|18
14	●○●○○○●○○○○○	$\langle 7 \rangle$	6	8|31
15	●○○●○○●○○○○○	$\{1, 5, -1e^{6}, 7e^{6}\}$	8	5|32
16	●○○○●○○○●○○○	$\mathbb{Z}_{12}^{\times} \ltimes e^{4\mathbb{Z}_{12}}$	15	4|20

Enumerazione dei modi di Messiaen

R.C. Read: « Combinatorial problems in the theory of music », *Discrete Math.*, 1997

M. Broué : « Les tonalités musicales vues par un mathématicien », 2002

$$A_n = \sum_{k|n} \mu\left(\frac{n}{k}\right) 2^k$$

$$s_d(n) = \sum_{\{e\,;\,(e|(n/d))\}} \mu\left(\frac{n/d}{e}\right) 2^e$$

$\mu(k)=0$ se k è divisibile per un quadrato

$\mu(k)=(-1)^m$ se k è il prodotto di m numeri primi distinti

$A_6=\mu(6)2+\mu(3)2^2+\mu(2)2^3+\mu(1)2^6=$

$= (-1)^2 2 + (-1)2^2 + (-1)\ 2^3 + 2^2 =$

$= 2 - 4 - 8 + 64 =$

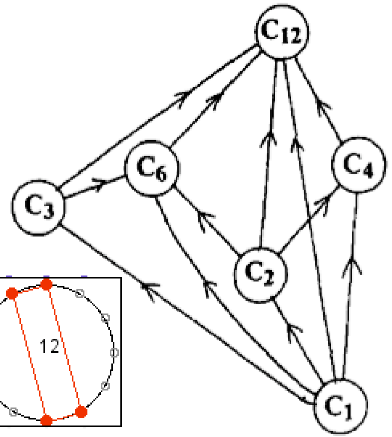
$= 54$

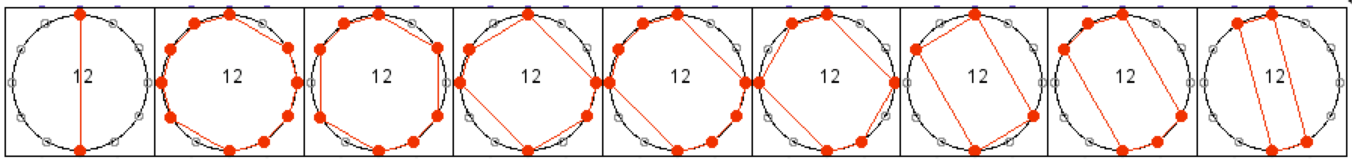
54/6 = 9

12/6 = 2

Table 1

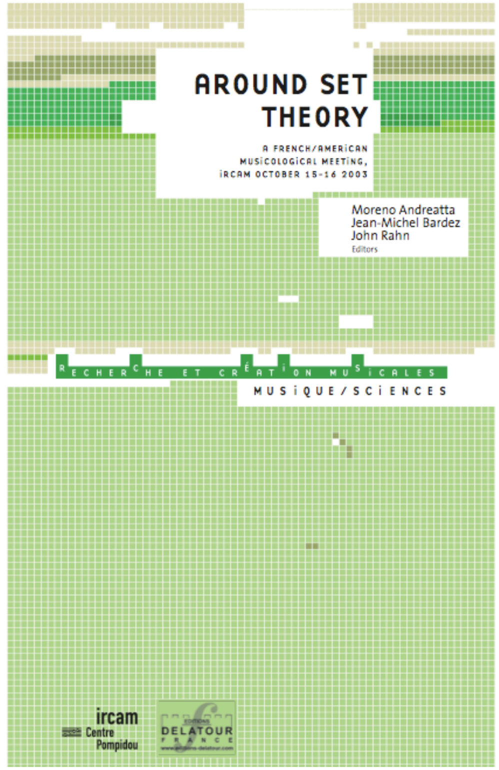
Number of notes / Symmetry	0	1	2	3	4	5	6	7	8	9	10	11	12
1		1	5	18	40	66	75	66	40	18	5	1	
2			1		2		3		2		1		
3				1			1			1			
4					1				1				
6							1						
12	1												1
All scales	1	1	6	19	43	66	80	66	43	19	6	1	1





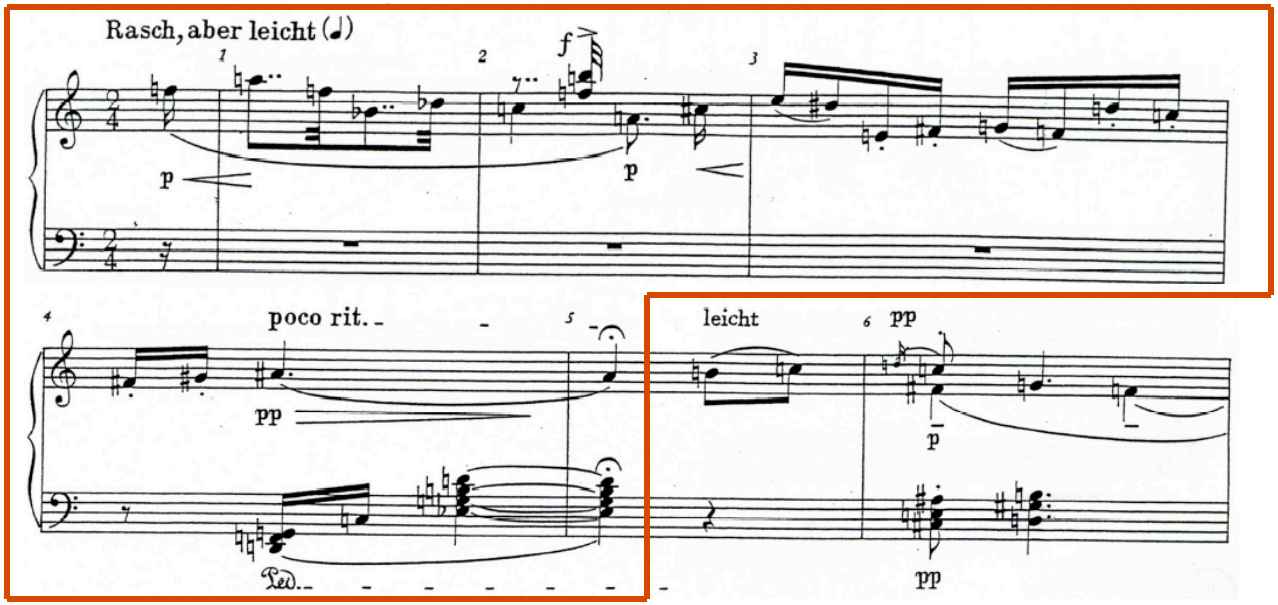
=> *OpenMusic*

Arnold Schoenberg: *Klavierstück* op. 19 n° 4

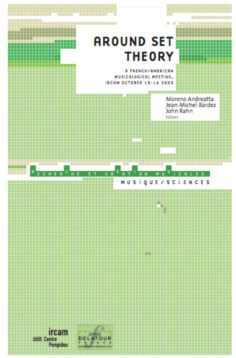


Allen Forte, "Schoenberg's Op.19 n°4: A Set-theoretical perspective"

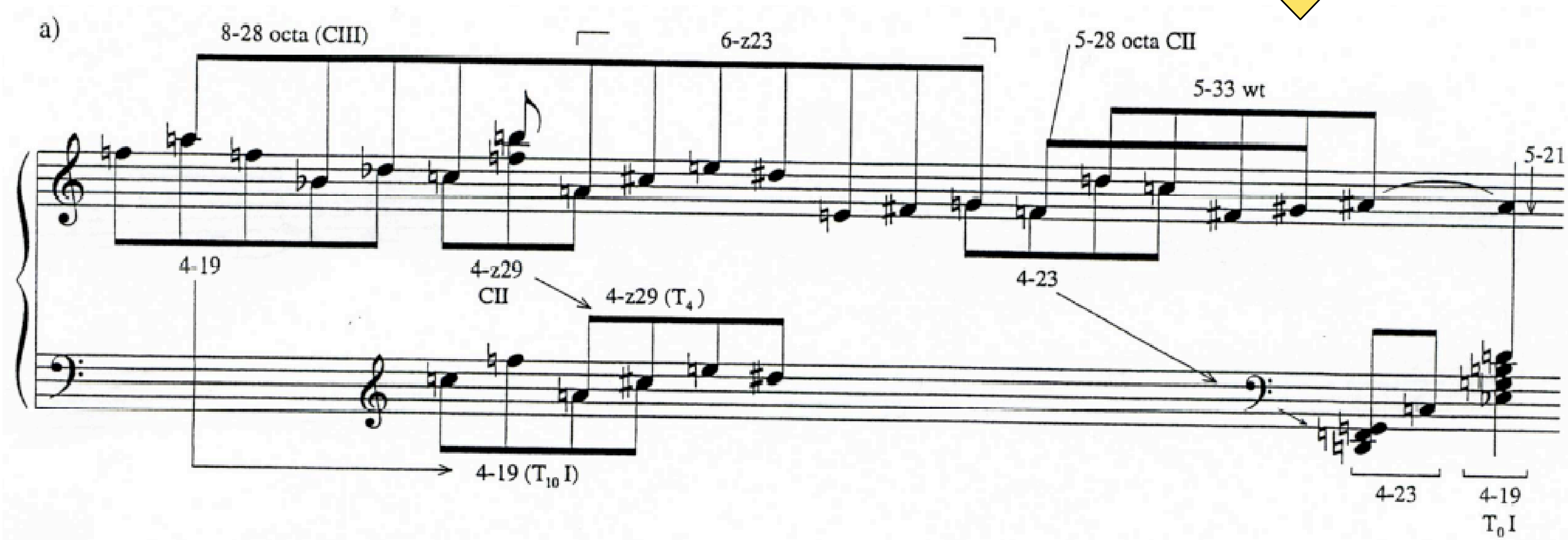
Analisi d'Allen Forte



Arnold Schoenberg: *Klavierstück* op. 19 n° 4



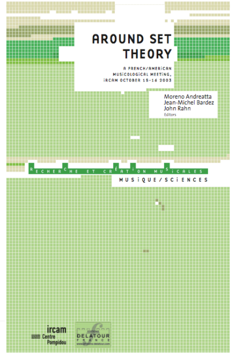
riduzione

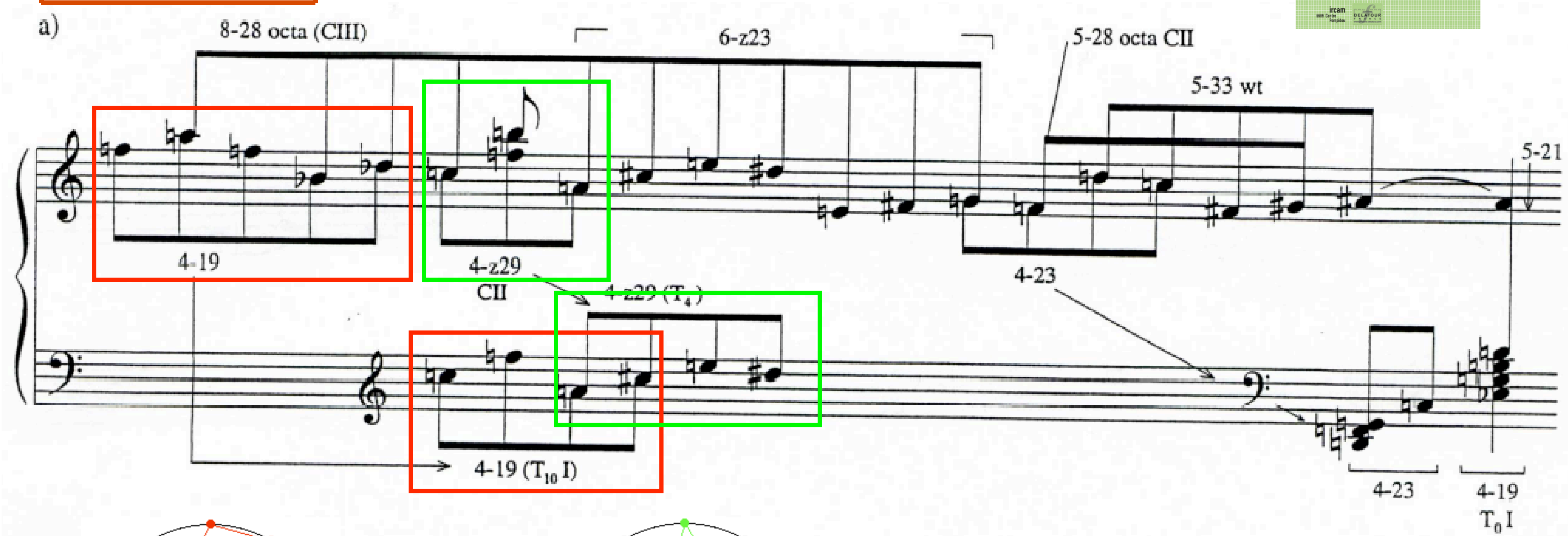


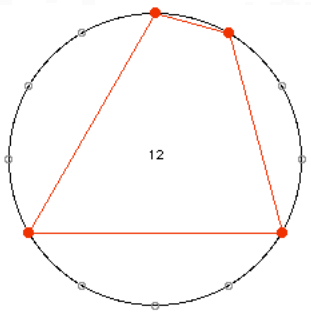
Analisi d'Allen Forte: visualizzazione

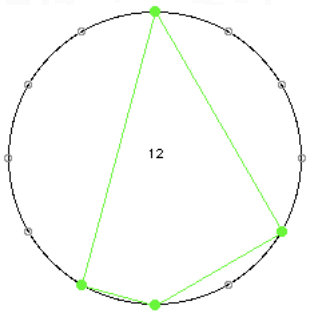
riduzione

Arnold Schoenberg: *Klavierstück* op. 19 n° 4

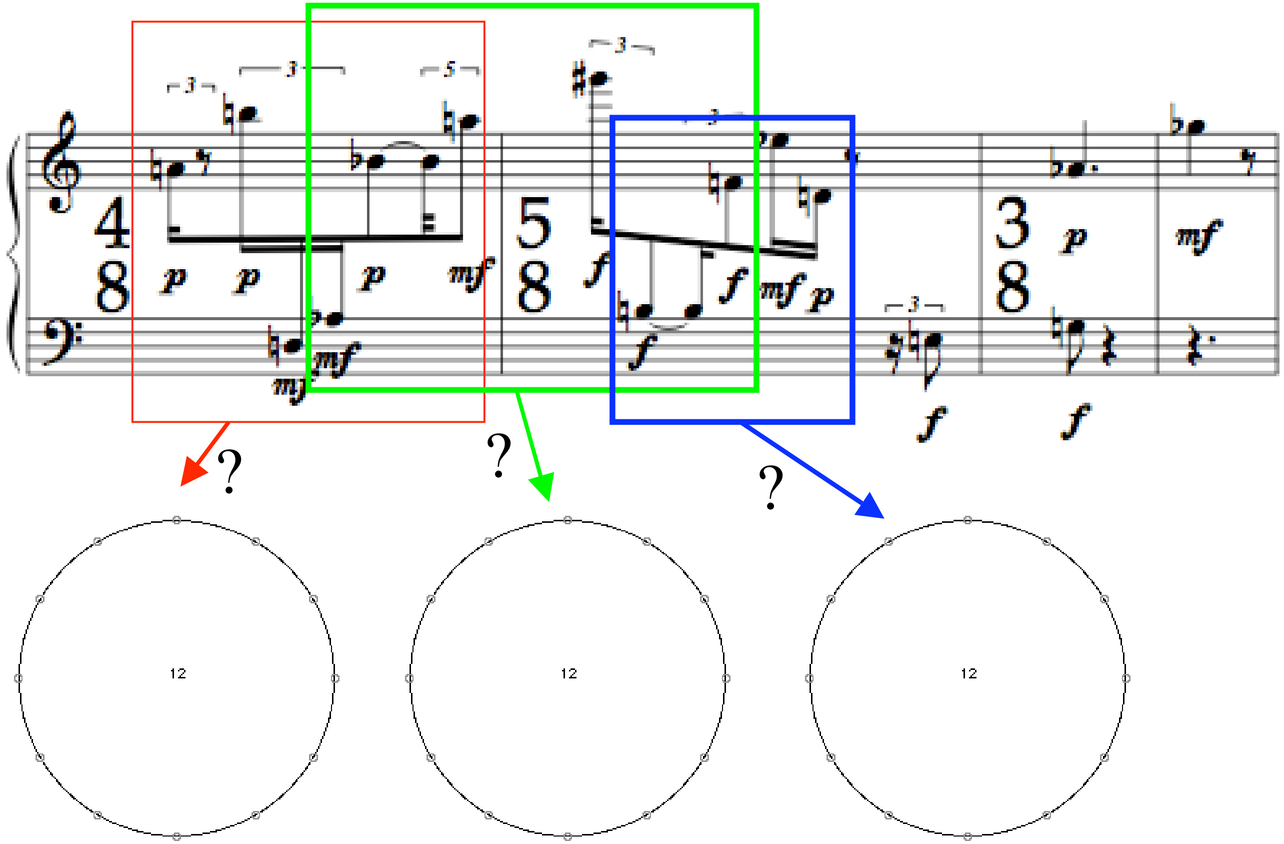








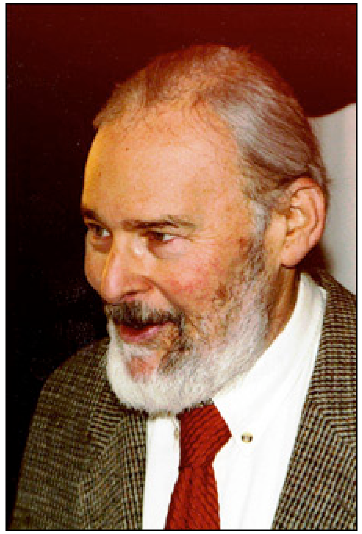
« *Making* and *Using* a Pcset Network for Stockhausen's Klavierstück III »



Henck

Kontarsky

Tudor

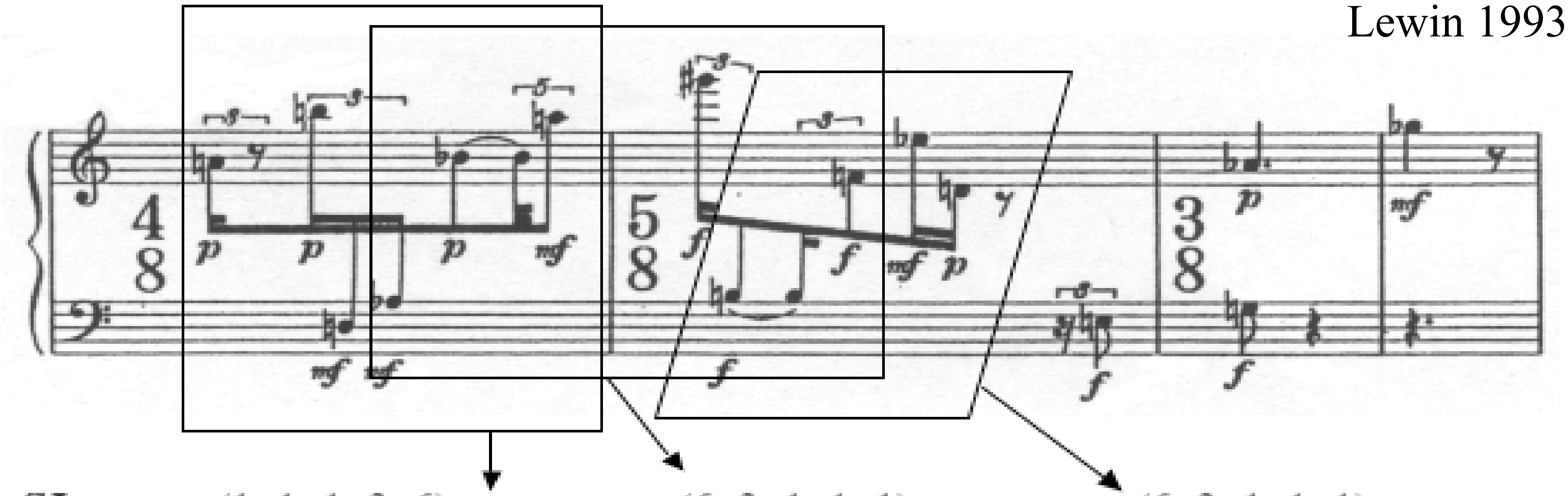


« The most ‘theoretical’ of the four essays, it focuses on the forms of one pentachord reasonably ubiquitous in the piece. A special **group of transformations** is developed, one suggested by the musical interrelations of the pentachord forms. Using that group, the essay arranges **all pentachord forms** of the music into a **spatial configuration** that illustrates network structure, for this particular phenomenon, over the entire piece. »

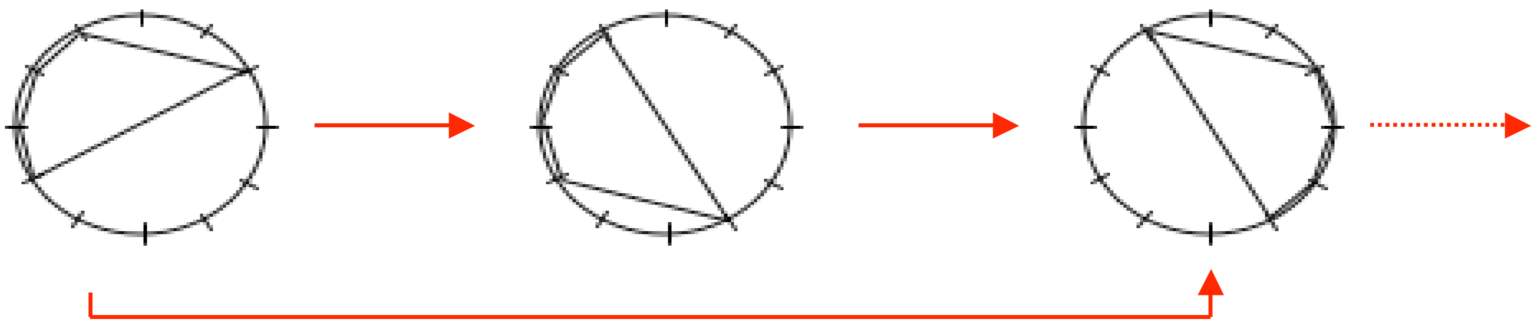
David Lewin, *Musical Form and Transformation,* YUP 1993

« *Making* and *Using* a Pcset Network for Stockhausen's Klavierstück III »

Lewin 1993

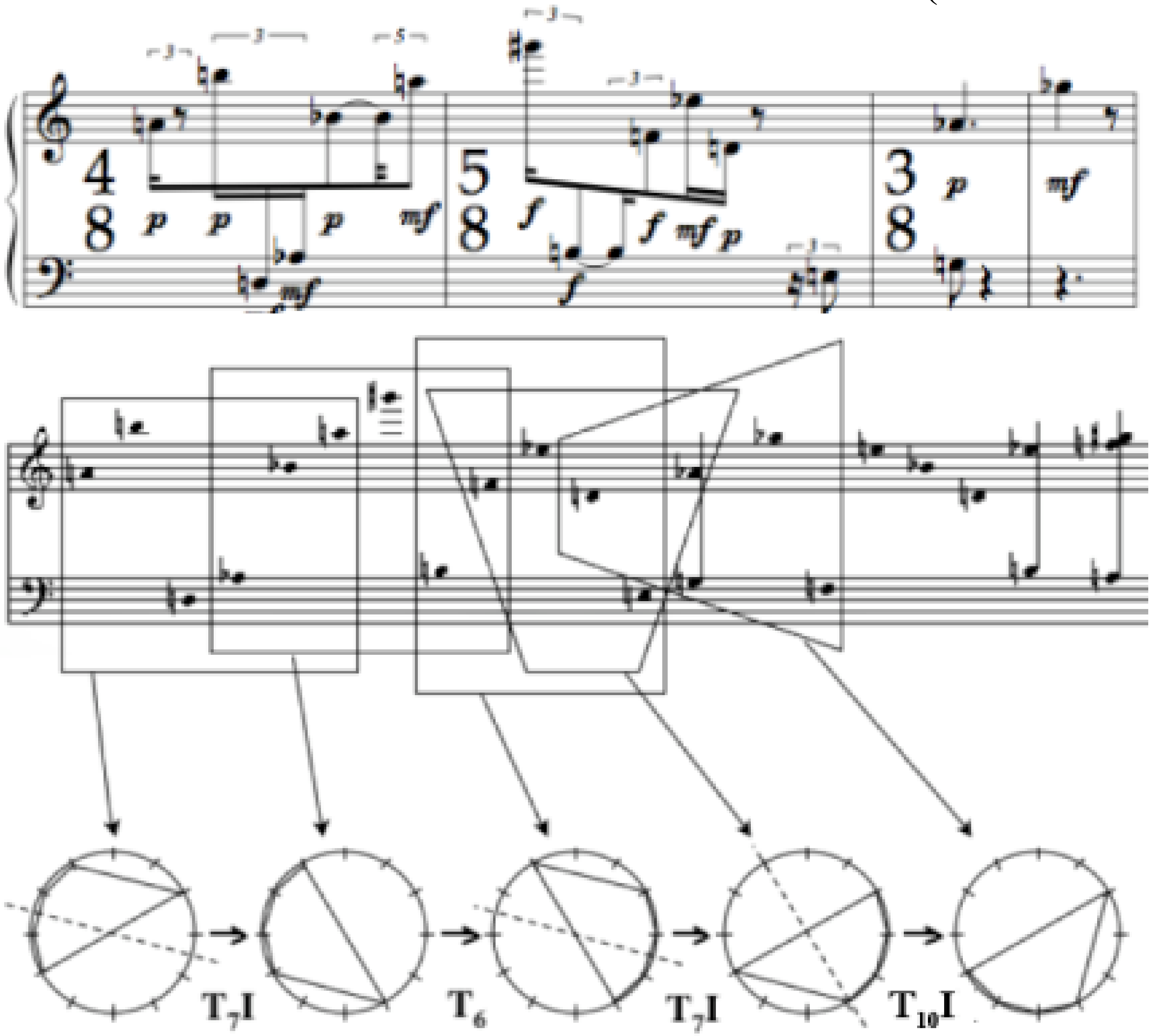


SI:	(1, 1, 1, 3, 6)	(6, 3, 1, 1, 1)	(6, 3, 1, 1, 1)
IFUNC:	[5 3 2 2 1 1 1 1 1 2 2 3]	[5 3 2 2 1 1 1 1 1 2 2 3]	[5 3 2 2 1 1 1 1 1 2 2 3]
VI:	[3 2 2 1 1 1]	[3 2 2 1 1 1]	[3 2 2 1 1 1]



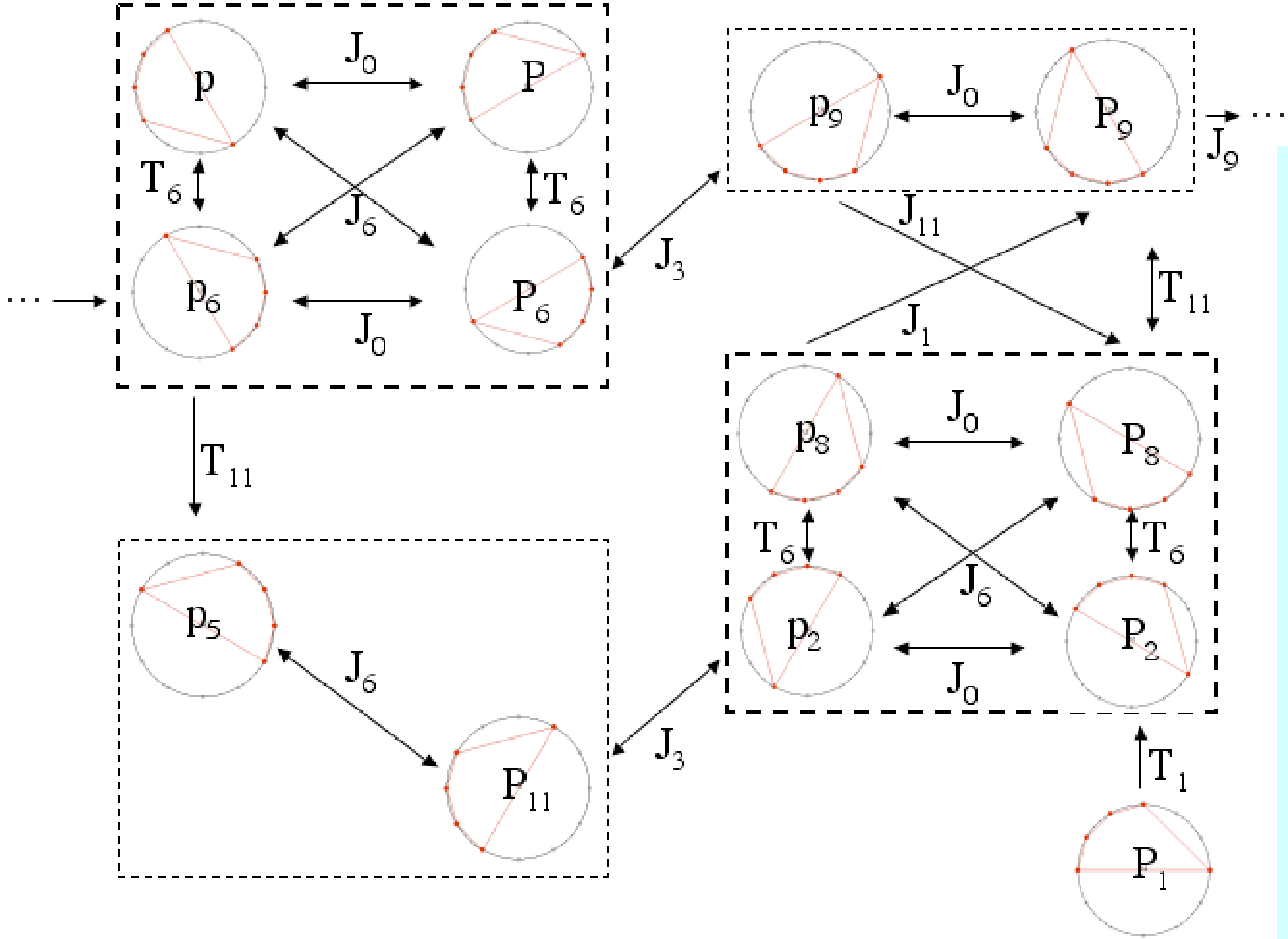
Segmentazione per « imbricazione »: progressione trasformazionale

Stockhausen: *Klavierstück III* (Analisi di D. Lewin)



Analisi trasformazionale: reticolo

Stockhausen: *Klavierstück III* (Analisi di D. Lewin)



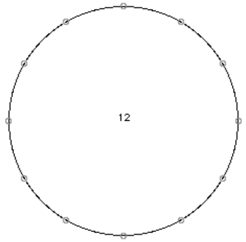
« […] the sequence of events moves within a clearly defined world of possible relationships, and because - in so moving - it makes the abstract space of such a world accessible to our sensibilities. That is to say that the story projects what one would traditionally call form. »

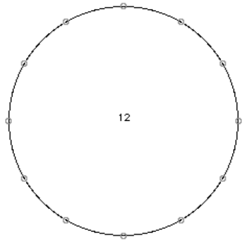
Klumpenhouwer Networks (K-nets)

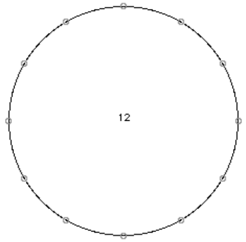
Xavier Hascher: « Liszt et les sources de la notion d'agrégat ». Analyse Musicale, 43, 2002

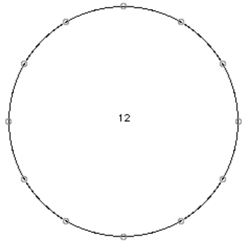
Ex. 1 - « Ladislaus Teleki » (*Historische ungarische Bildnisse* n° 4), mes. 1-7
Les agrégats dans la classification de Forte

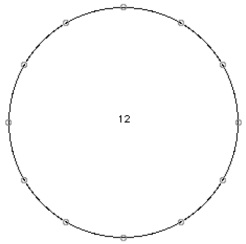
Ex. 2 - Formes premières des agrégats utilisés dans l'ex. 1







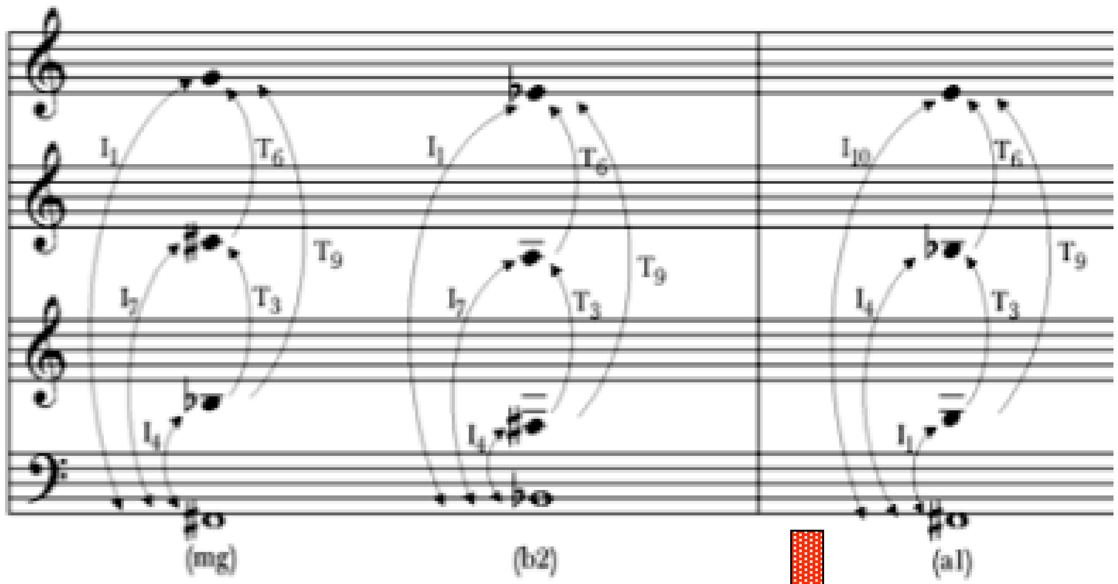


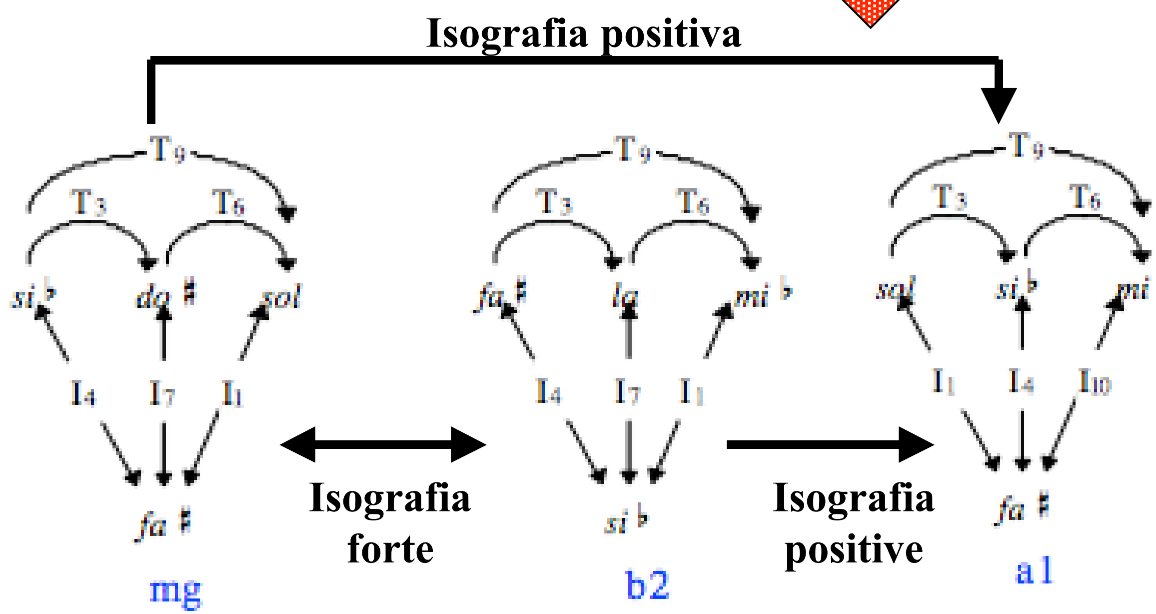


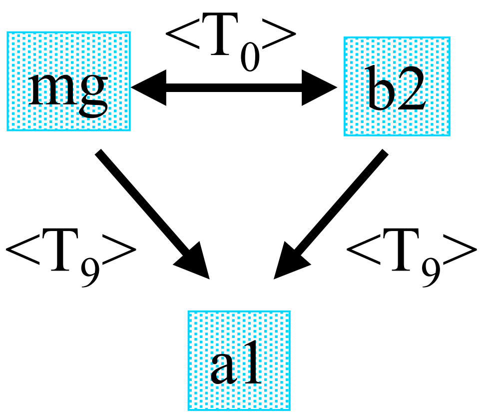
Klumpenhouwer Networks (K-nets)

Xavier Hascher: « Liszt et les sources de la notion d'agrégat », Analyse Musicale, 43, 2002

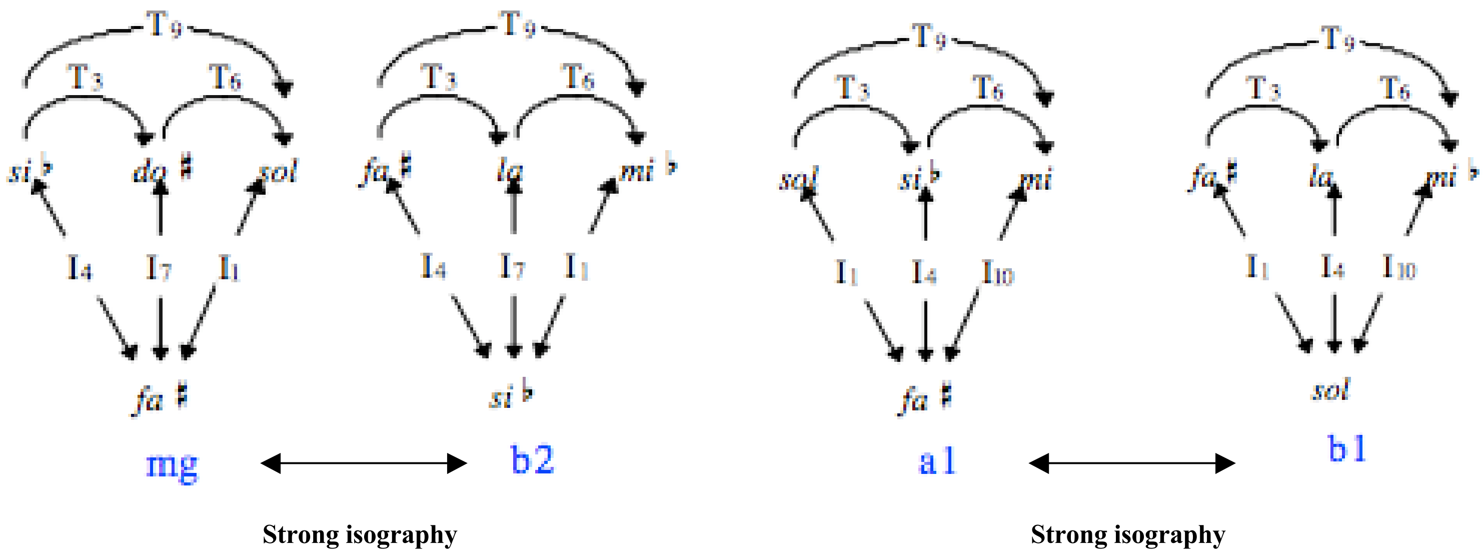
Ex. 1 - « Ladislaus Teleki » (*Historische ungarische Bildnisse* n° 4), mes. 1-7
Les agrégats dans la classification de Forte

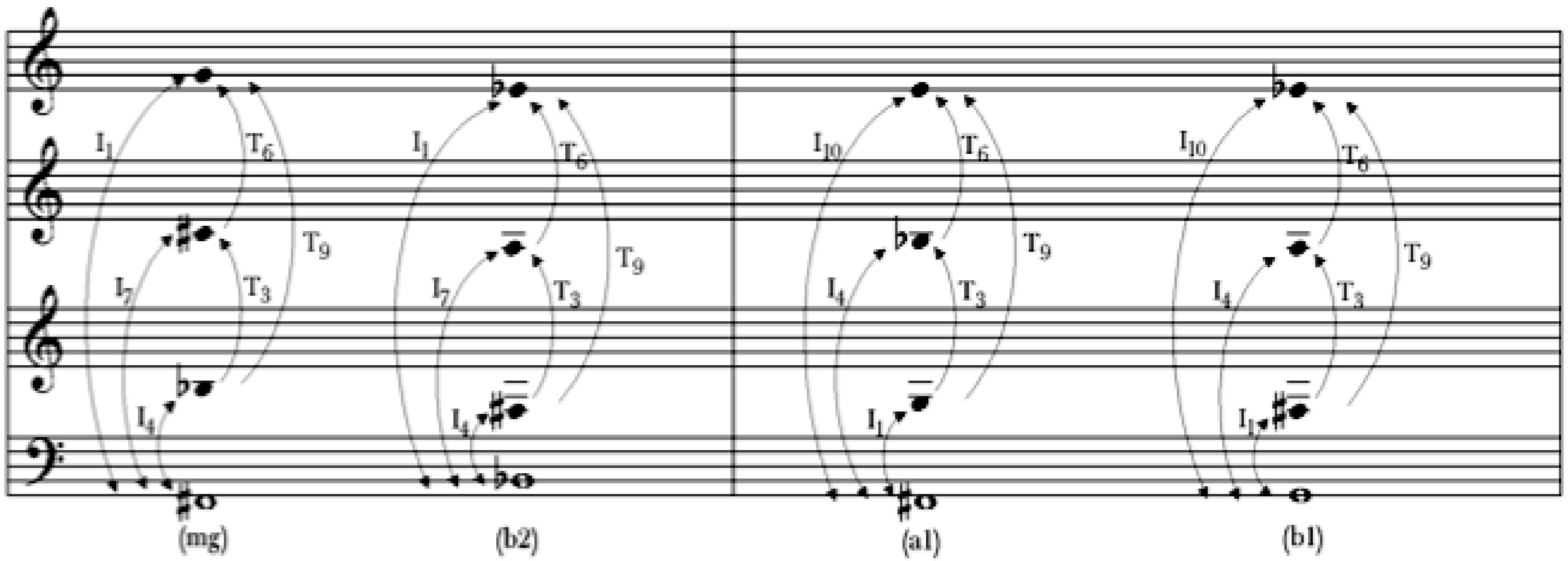






Klumpenhouwer Networks (K-nets)

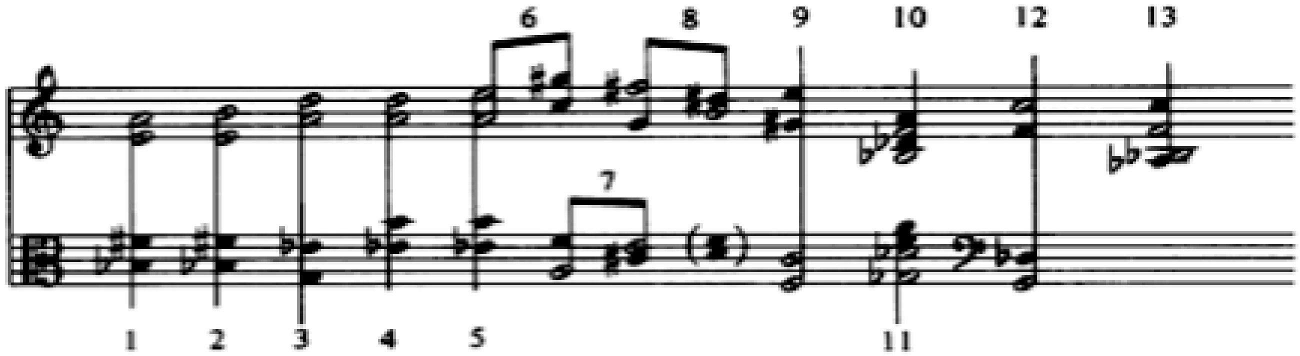


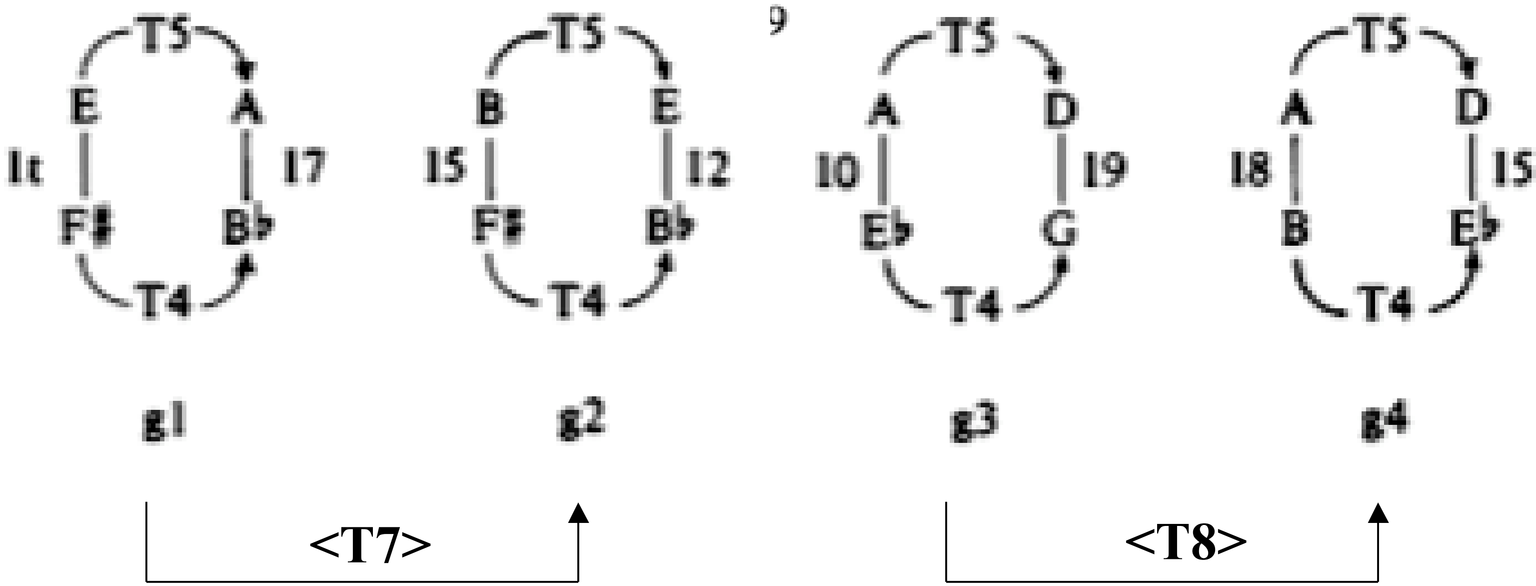


Ex. 3 - Les agrégats engendrant des réseaux en isographie forte

Klumpenhouwer Networks (K-nets)

David Lewin: «A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », JMT, 1994



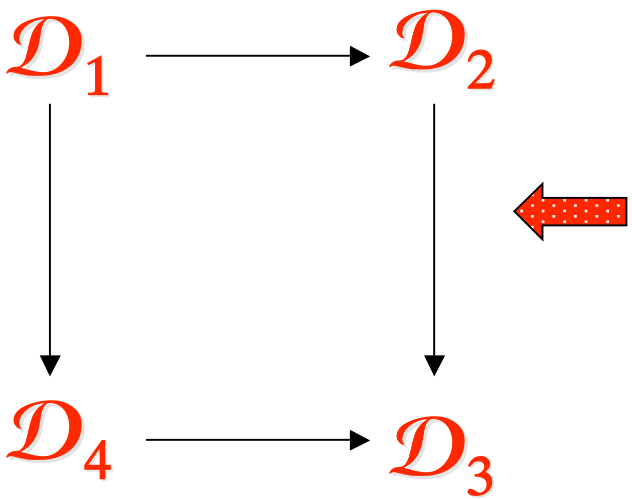


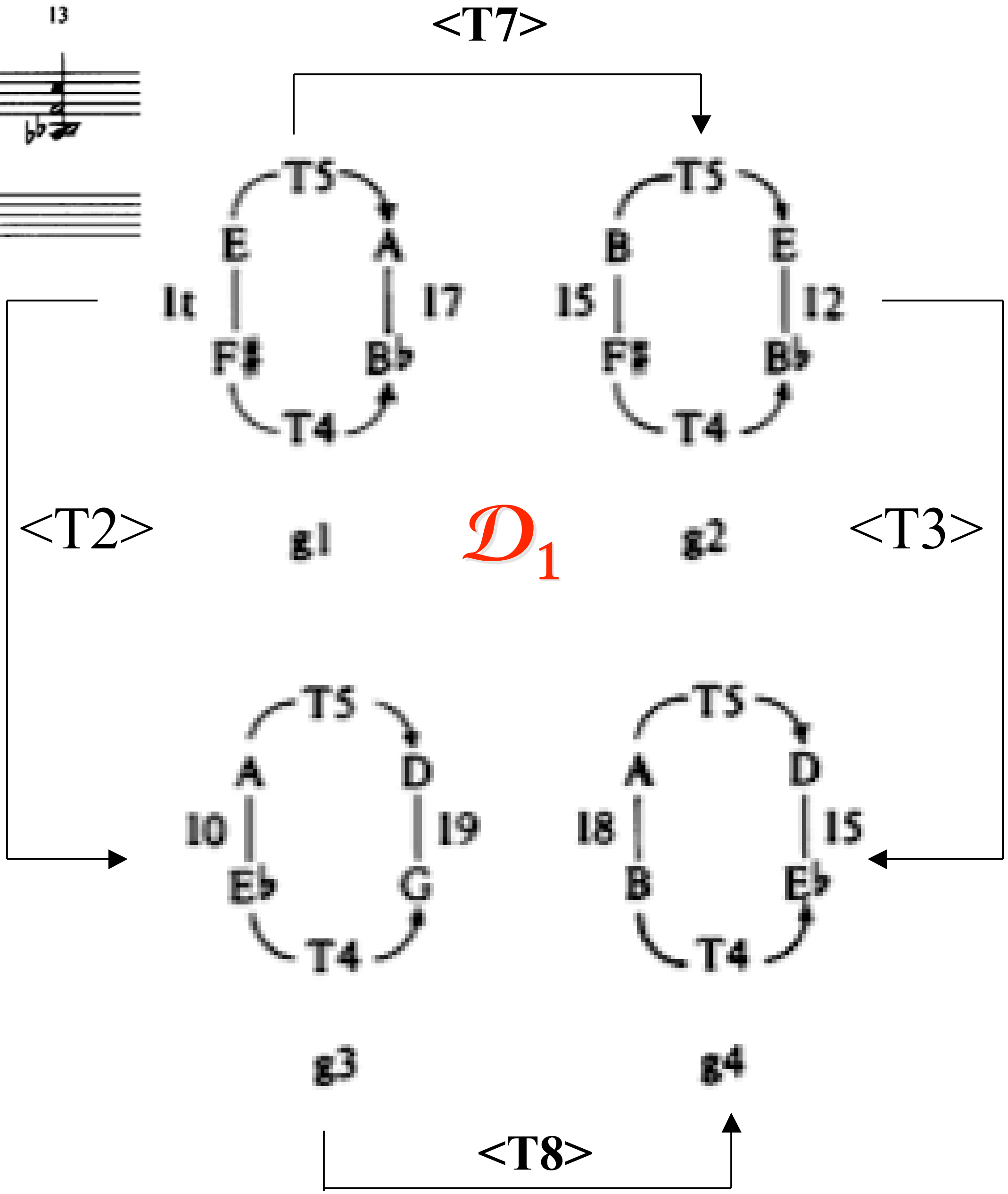
Isographie positive **Isographie positive**

Klumpenhouwer Networks (K-nets)

David Lewin: «A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », JMT, 1994

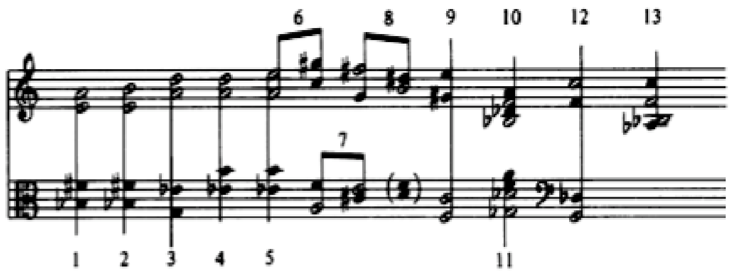
Example 9



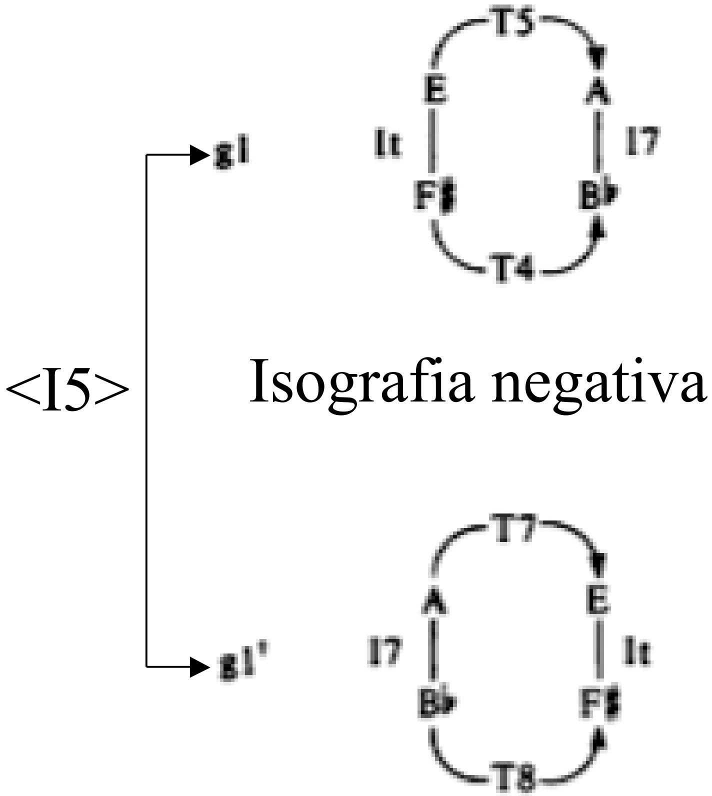


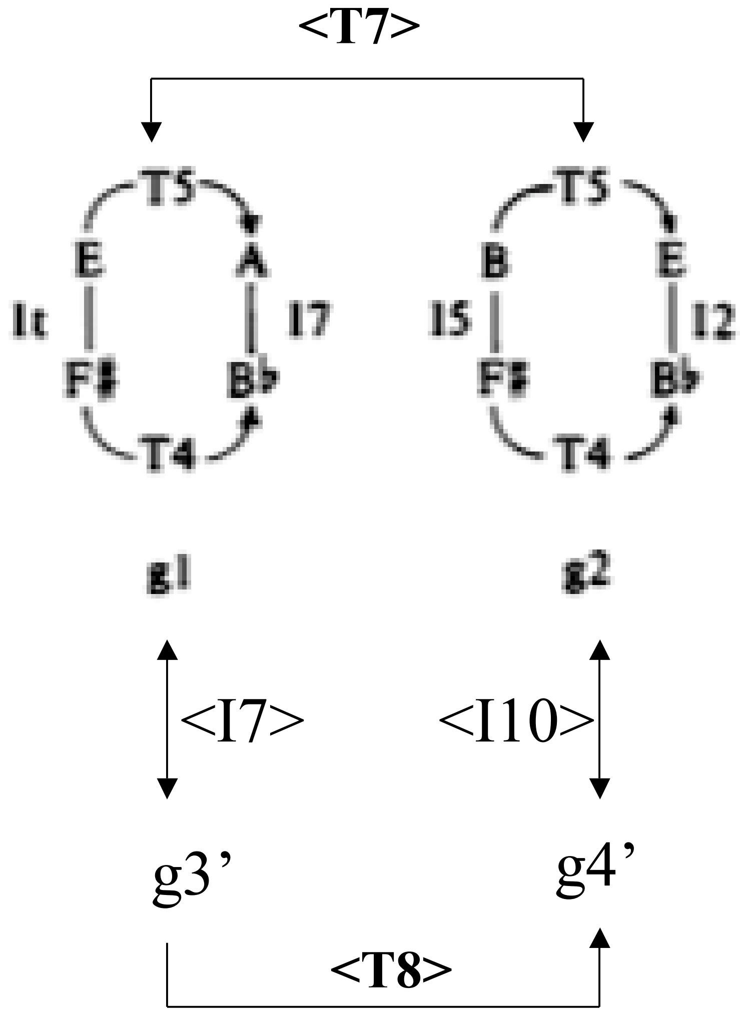
Klumpenhouwer Networks (K-nets)

David Lewin: «A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », JMT, 1994

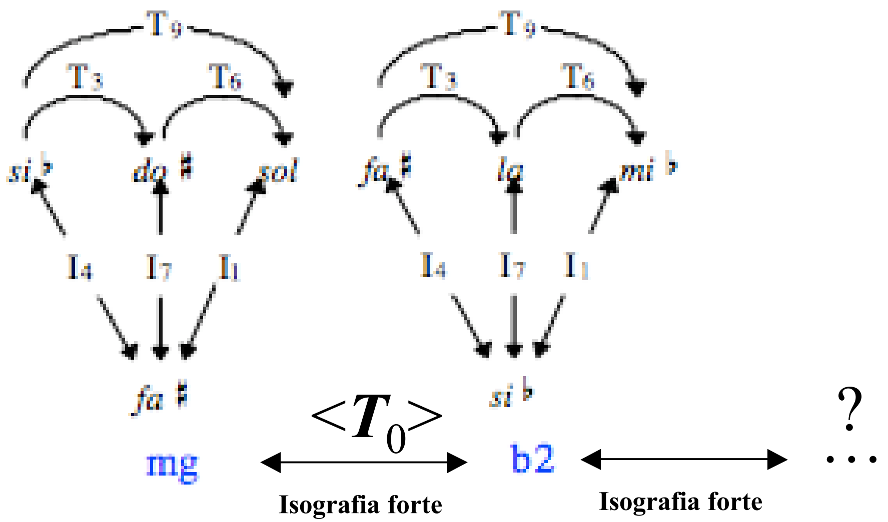


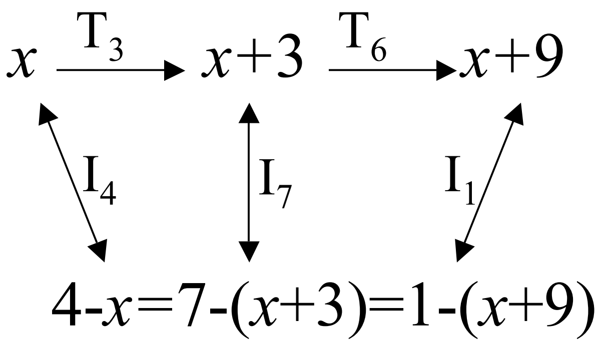
Example 9



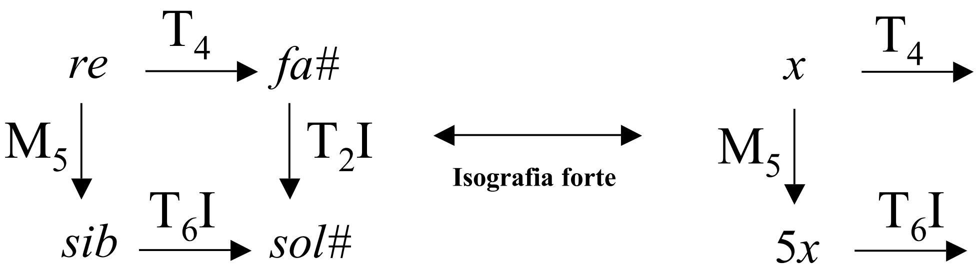


Enumerazione dei K-nets in relazione d'isografia forte



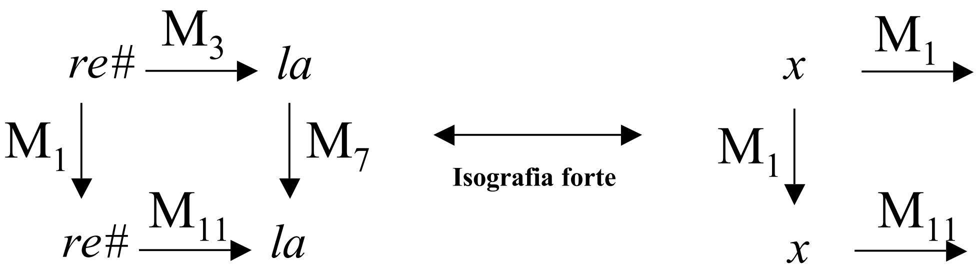


⇨ 12 soluzioni



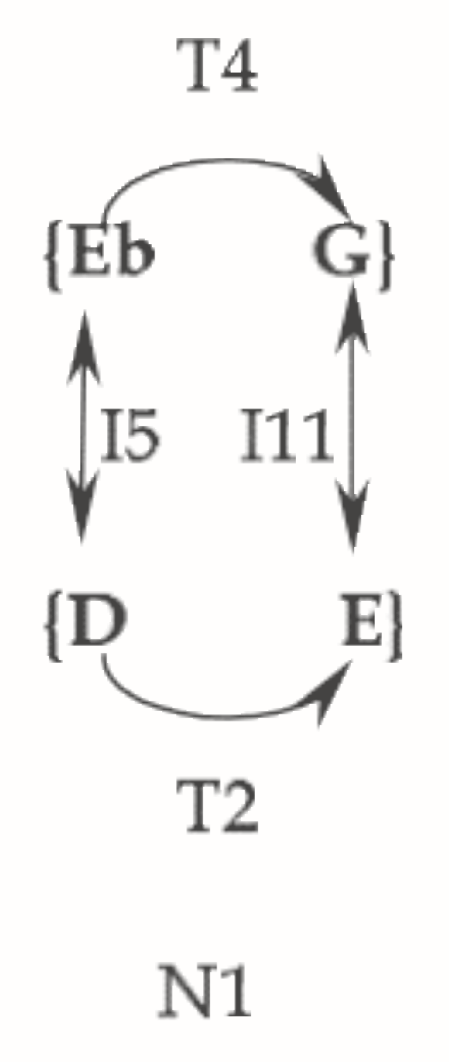
$6-5x=2-(x+4) \Longrightarrow 8=4x \Longrightarrow x=2, 5, 8, 11$

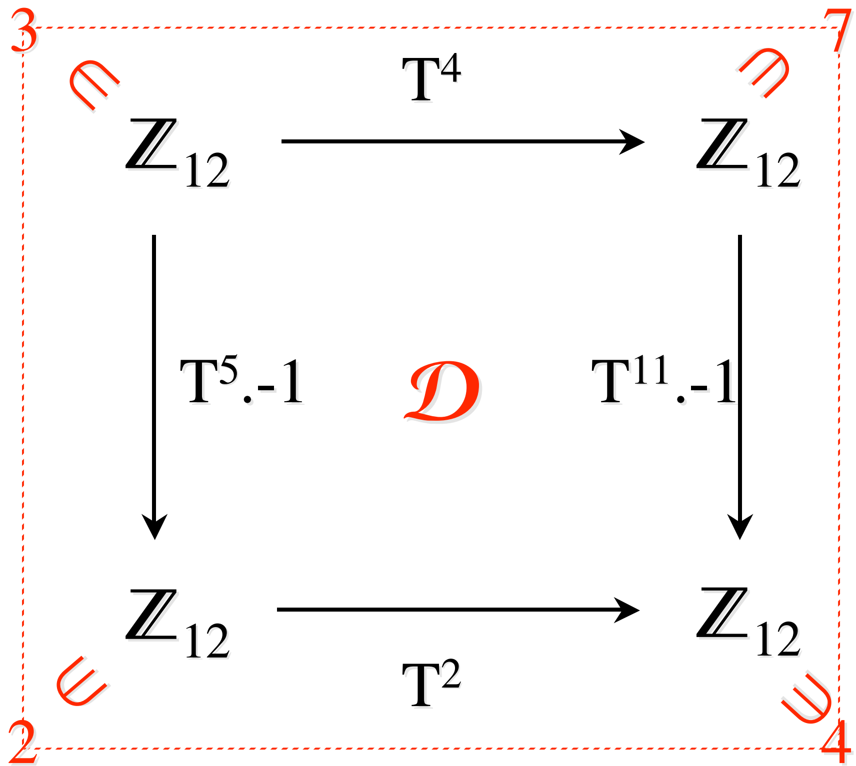
⇨ 4 soluzioni



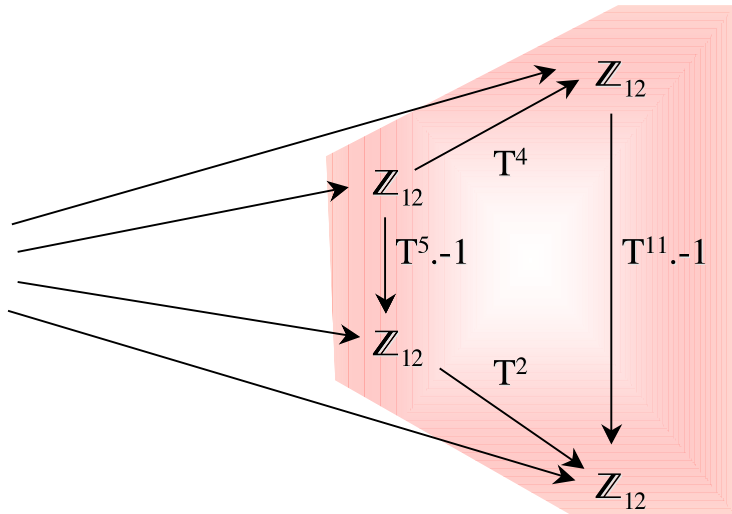
$11x=7x \Longrightarrow 4x=0 \Longrightarrow x=0, 3, 6, 9$

⇨ 4 soluzioni





$$(3, 7, 2, 4) \in \lim(\mathcal{D})$$



$\mathcal{Z}_i = \mathbb{Z}_{12}$

$f_{ij}^{t} \in \mathcal{Z}_i @ \mathcal{Z}_j$

$\lim(\mathcal{D})$ = family of strongly-isographic networks

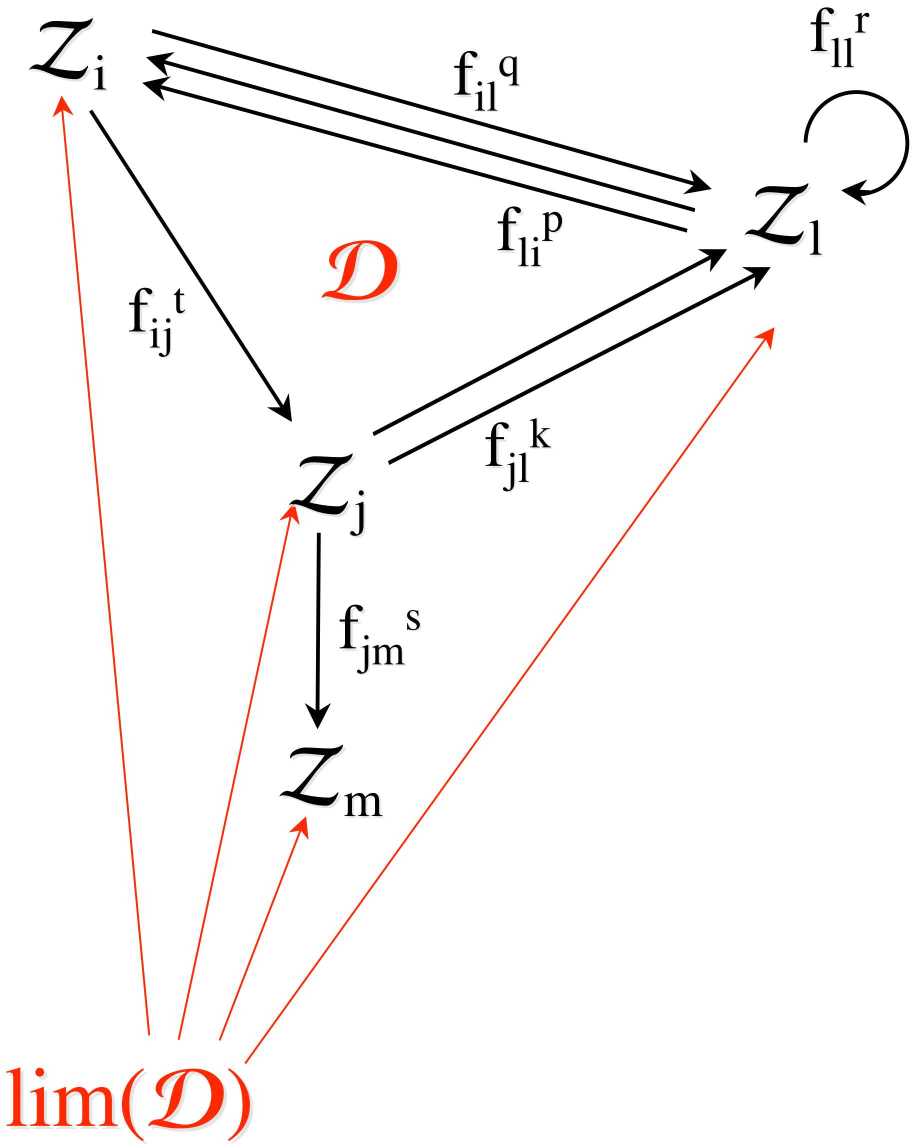
G. Mazzola & M. Andreatta: **From a Categorical Point of View: K-nets as Limit Denotators**, *PNM*, 2006

Fact:

$\lim(\mathcal{D}) \approx U$

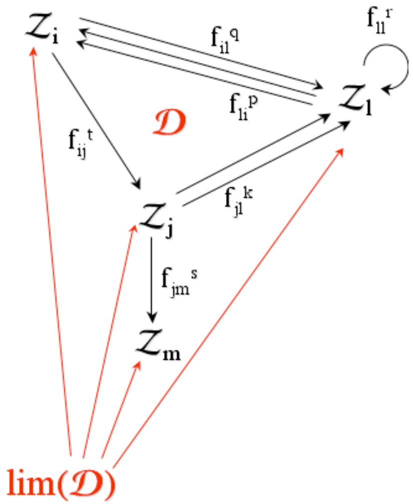
$U =$ (empty or) subgroup of $(\mathbb{Z}_{12})^n$

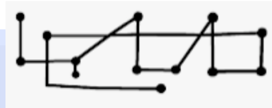
If f_{**}^{*} = isomorphisms card (U) (= 0 or) divides 12



$$\mathcal{Z}_i = \mathbb{Z}_{12}$$

$$f_{ij}^t \in \mathcal{Z}_i @ \mathcal{Z}_j$$

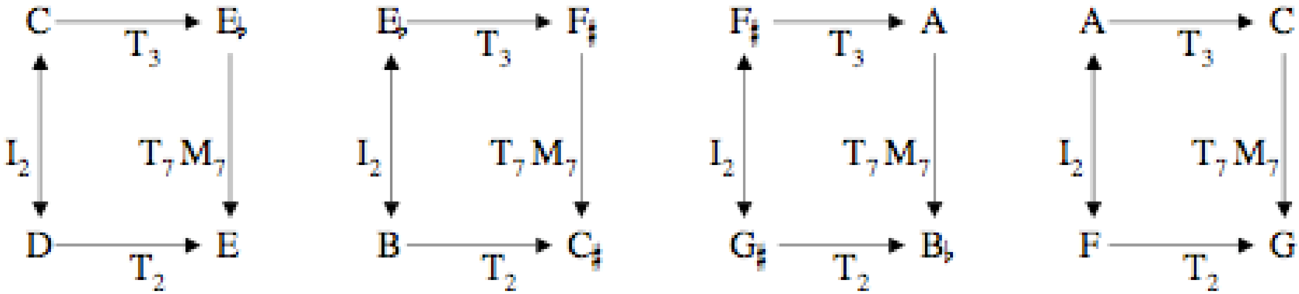




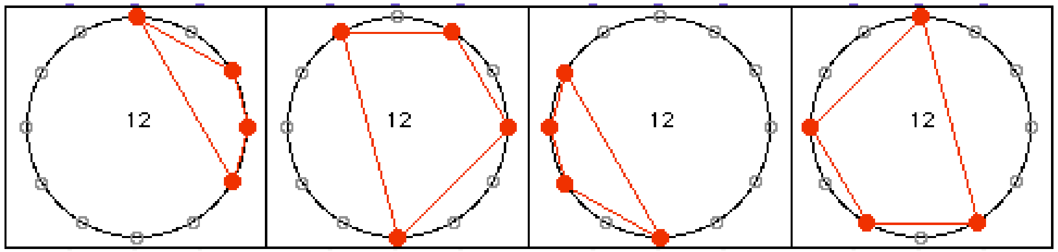
Fact:
lim($\mathcal{D}$) ≈ U

U = (empty or) subgroup of $(\mathbb{Z}_{12})^n$

If $f_{**}^{\ *}$ = isomorphisms
card (U) (= 0 or) divides 12

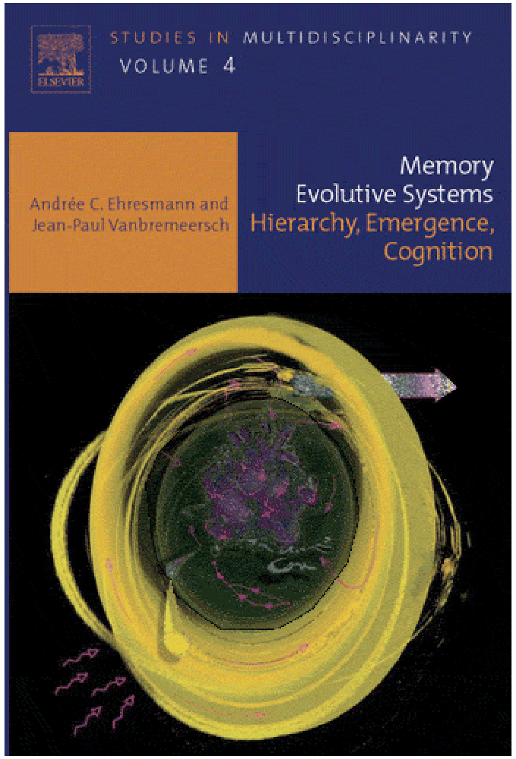


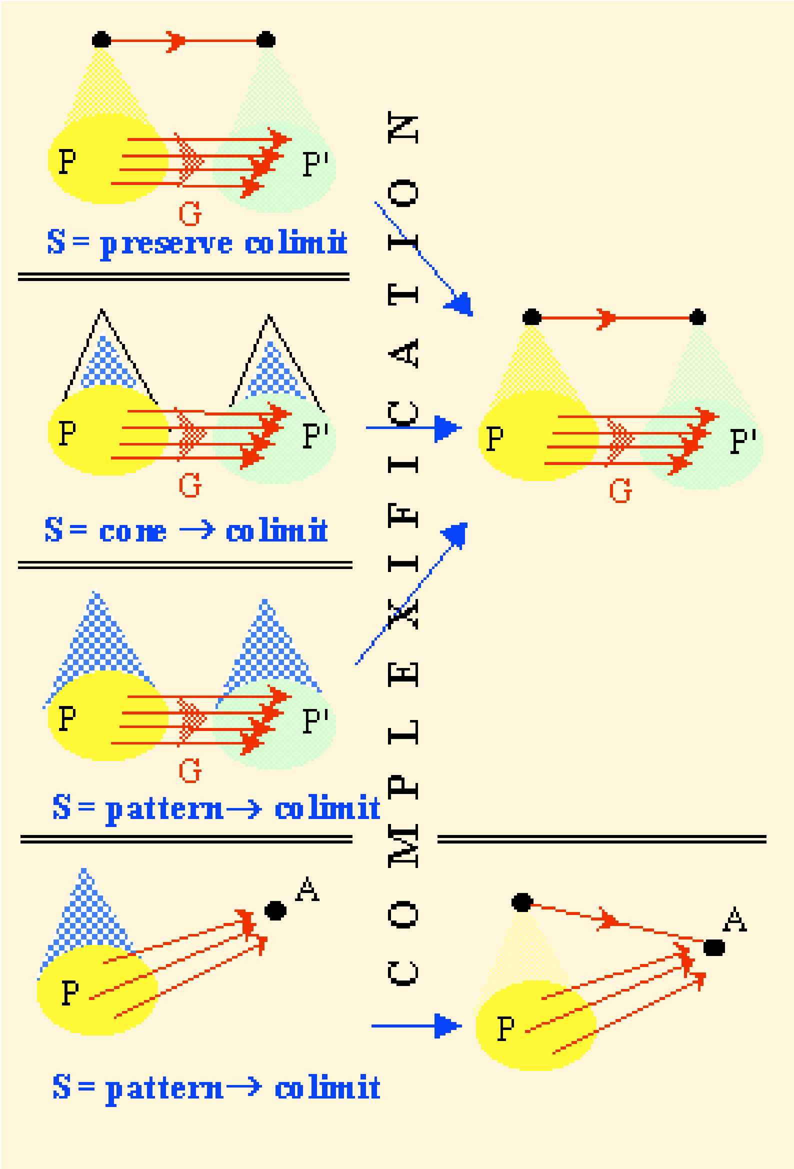
EXAMPLE 6: THE FOUR SOLUTIONS (STRONGLY ISOGRAPHIC K-NETS) OF THIS DIAGRAM ILLUSTRATE THAT THE CARDINALITY OF THE SOLUTION SET IS A DIVISOR OF 12. HERE, THE OPERATOR M_7 DENOTES THE MULTIPLICATION BY 7



« Memory Evolutive Systems » e neuroni categoriali

A 'simple' **cat-neuron** emerges as the colimit in a complexification of Neur of a pattern of neurons which has no colimit neuron in Neur, but acts as a synchronous coherent assembly of neurons in the sense of Hebb. An iteration of the process leads to cat-neurons of order 2 which correspond to a super-assembly (or 'assembly of assemblies') of neurons, which cannot be reduced to a (large) synchronous assembly of simple neurons. Higher order cat-neurons in successive complexifications represent super-super-assemblies, and so on.





Verso un' « algebra degli oggetti mentali » (Changeux) in musica

Problème corps/esprit

La **représentation** d'un **état mental**, tel un processus cognitif complexe, par un **cat-neurone** d'ordre supérieur conduit à une nouvelle approche du problème philosophique de l'identité entre **états mentaux et états physiques du cerveau**. En effet, un état physique, tel qu'il est vu par imagerie médicale, correspond à l'activation d'une simple assemblée de neurones (modélisée par un cat-neurone simple). Mais un cat-neurone d'ordre supérieur n'est pas (directement) réductible à une telle assemblée, bien qu'il soit construit par des complexifications successives à partir du niveau des neurones, et qu'il ait des ramifications jusqu'à ce niveau. Ainsi son activation exige plusieurs étapes, passant par les niveaux intermédiaires d'une de ses ramifications, jusqu'au niveau des états physiques; et, à chaque étape, elle peut se propager par l'une ou l'autre des décompositions non-équivalentes d'objets multifaces, avec balancement entre elles qui peut être d'origine aléatoire (bruit) ou contrôlé. Bien que ce processus représente un "évènement" physique bien déterminé, il ne s'identifie pas à un "état" physique: **on peut dire que les états mentaux émergent de manière dynamique (au travers du déploiement graduel d'une ramification) des états physiques, sans leur être identiques.** Ceci définirait un monisme émergentiste au sens de Bunge.

Système Evolutif à Mémoire