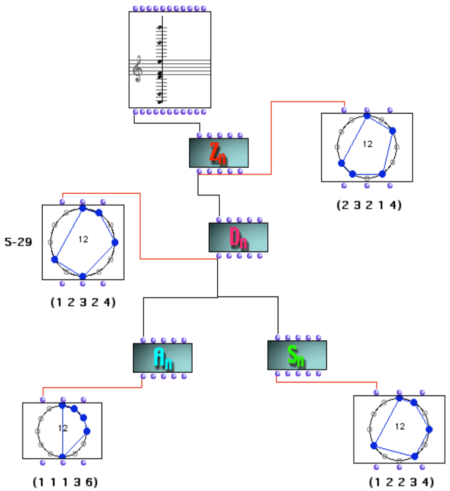
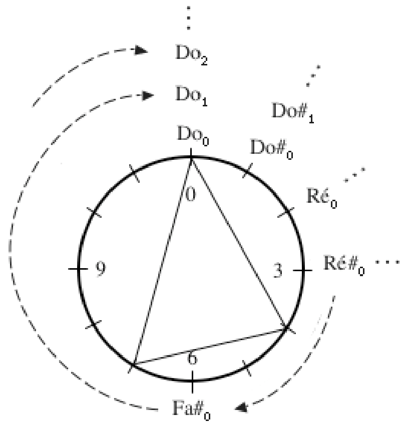


Elementi di Geometria Superiore 2



(4,3,5)

Matematica & Musica

(Un terzo del) Terzo trittico:
sequenze periodiche e aspetti filosofici del rapporto matematica/musica

Moreno Andreatta
Equipe Représentations Musicales
IRCAM/CNRS
(In collaborazione con Carlos Agon e Emmanuel Amiot)

Programma del corso

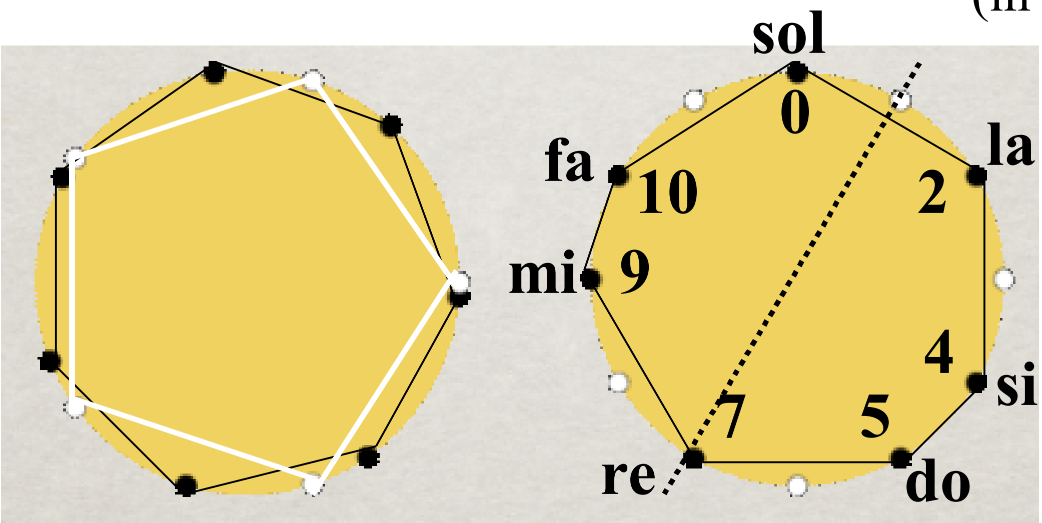
1.) Rappresentazione e formalizzazione delle strutture musicali
2.) Enumerazione e classificazione delle strutture musicali
3.) Teorie trasformazionali, diatoniche e neo-riemanniane
4.) Tassellazioni musicali: la costruzione dei canoni ritmici a mosaico
5.) Sequenze periodiche e calcolo delle differenze finite a valori in gruppi ciclici
6.) Ramificazioni filosofiche e cognitive dell'approccio algebrico in musica

Alcune precisioni e complementi

1.) Sulla trasformata di Fourier discreta: applicazione allo studio delle strutture diatoniche (Emmanuel Amiot) e alla classificazione non paradigmatica delle strutture musicali (modello computazionale in *OpenMusic*)

Transformata di Fourier e *Maximally-Even Sets*

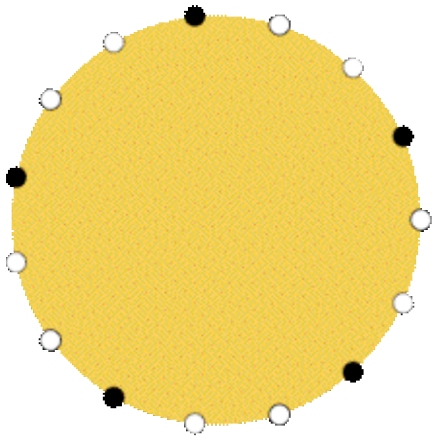
(in collaborazione con Emmanuel Amiot)



Scala diatonica:
{0, 2, 4, 5, 7, 9, 10}

Scala pentatonica:
{1, 3, 6, 8, 11}

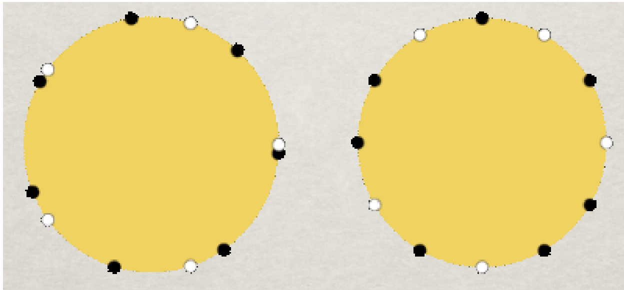
$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$



$| F_A(5) | = 1+1+1+1+1 = 5$

In generale, $| F_A(t) | \leq \#A$

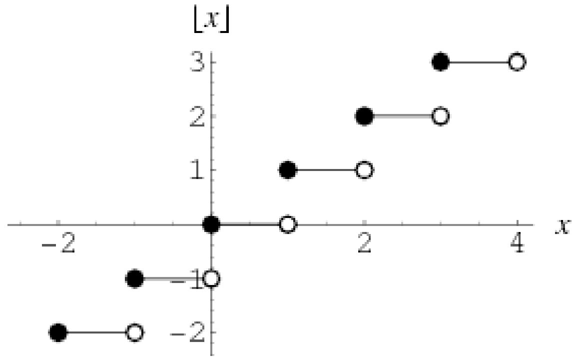
La scala diatonica come insieme di ripartizione massimale
(Maximally Even Sets)



Definition (Clough-Myerson-Douthett) A set A with cardinality d in a given equal tempered space $\mathbf{Z}_{c.}$ is maximally even if $A = \{a_k\}$

$$a_k = J^{\alpha}_{c,d}(k) = \left\lfloor \frac{kc+\alpha}{d} \right\rfloor$$

where $\alpha \in \mathbf{R}$
$\lfloor x \rfloor$ is the integer part of x



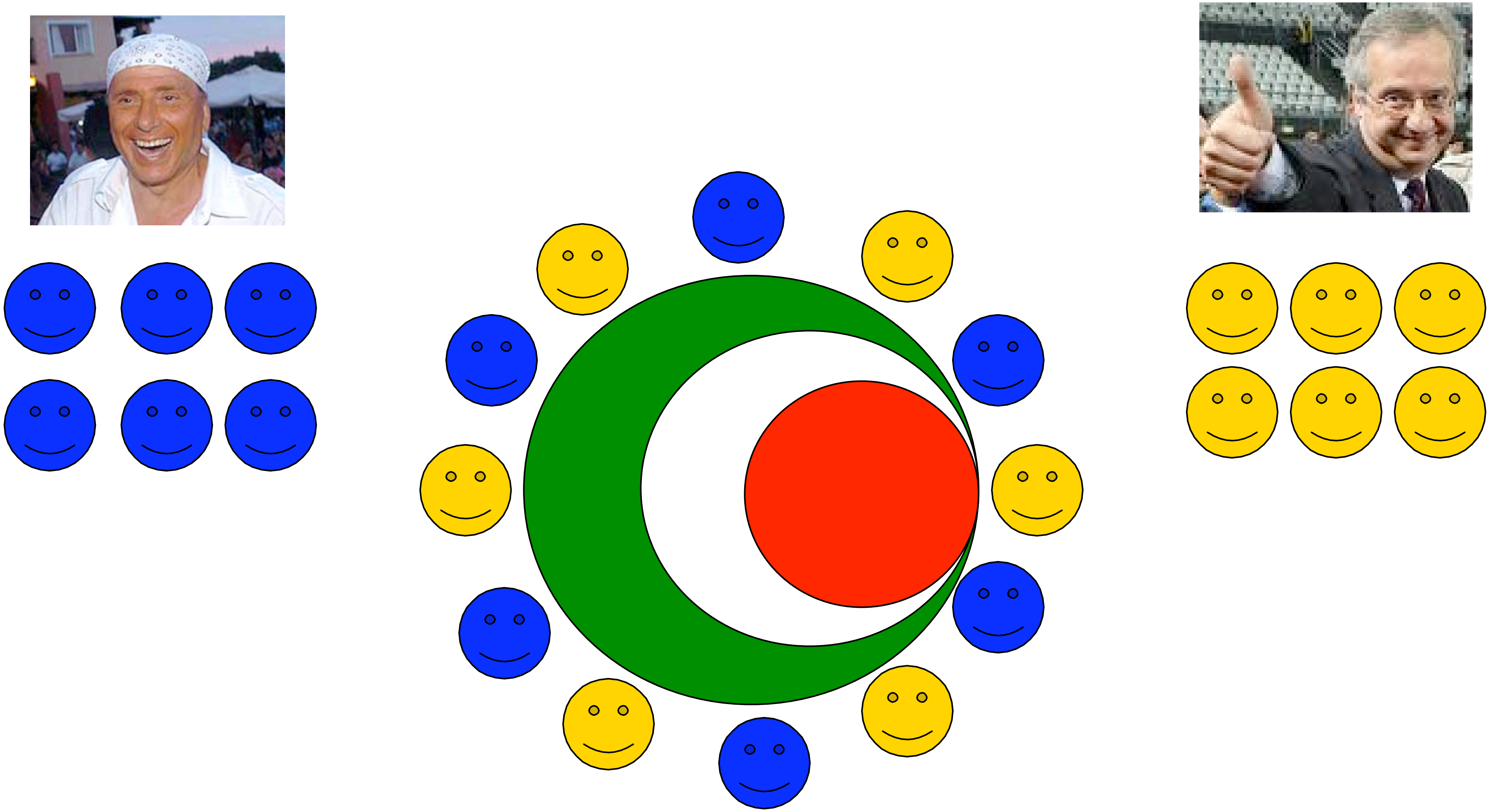
$$J^{5}_{12,7} = \left\{ \left\lfloor \frac{12k+5}{7} \right\rfloor \right\}_{k=0}^{6} = \{0,2,4,5,7,9,11\}$$

Definition (Amiot, 2005) A set A with cardinality d given equal tempered space $\mathbf{Z}_{c.}$ is maximally even if $|F_A(d)| \geq |F_B(d)|$ for all subsets B of cardinality d in $\mathbf{Z}_{c.}$

where $F_{set}(t) := \sum_{k \in set} e^{2i\pi kt/12}$

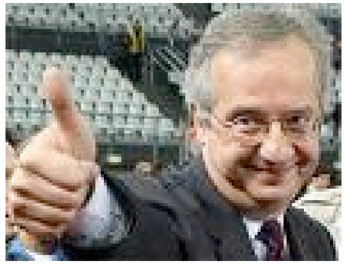
La scala diatonica come *ME-set*

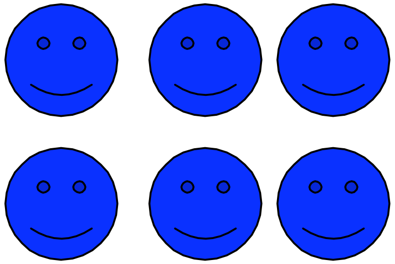
The dinner table problem (Italian version)

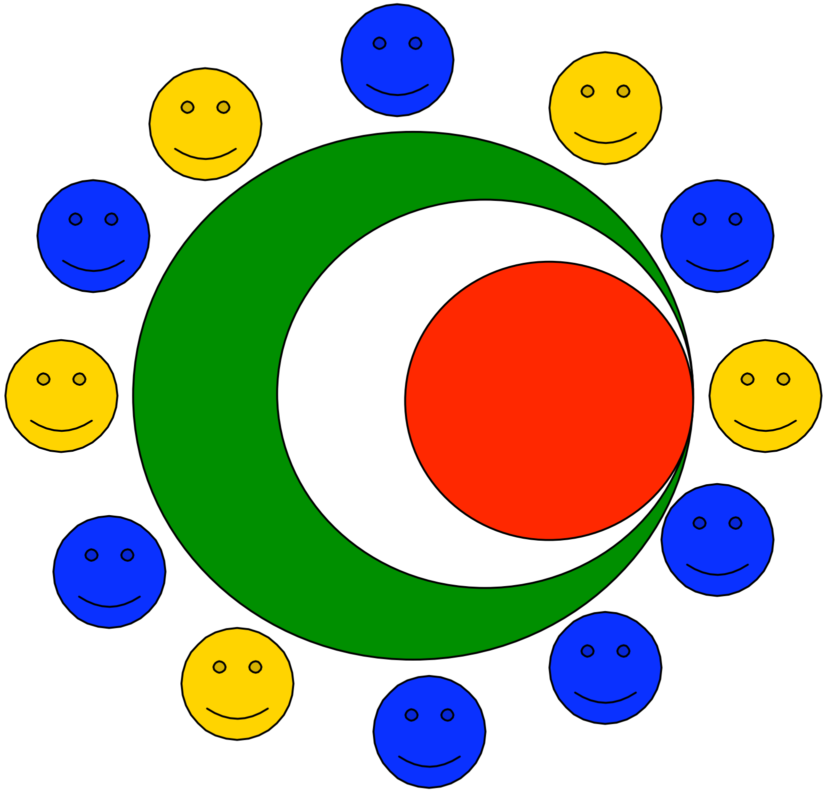


La scala diatonica come *ME-set*

The dinner table problem (Italian version)

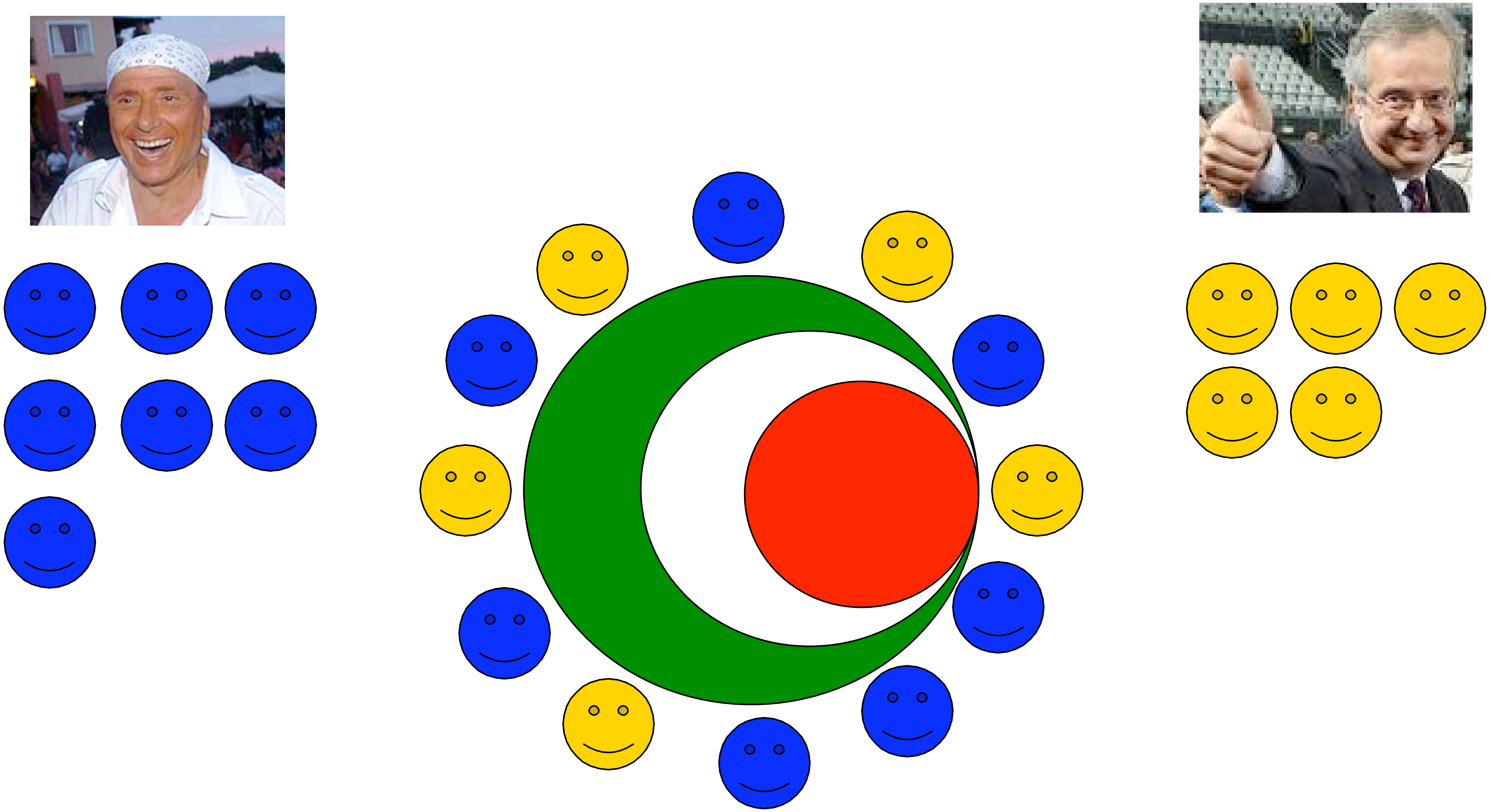






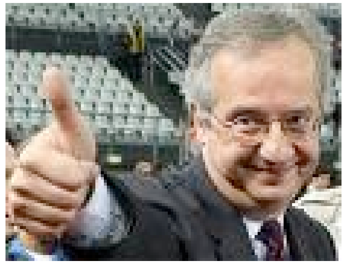
La scala diatonica come *ME-set*

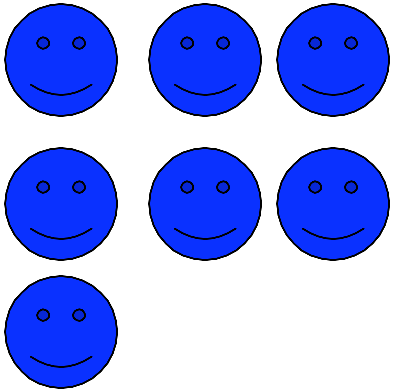
<u>The dinner table problem</u> (Italian version)

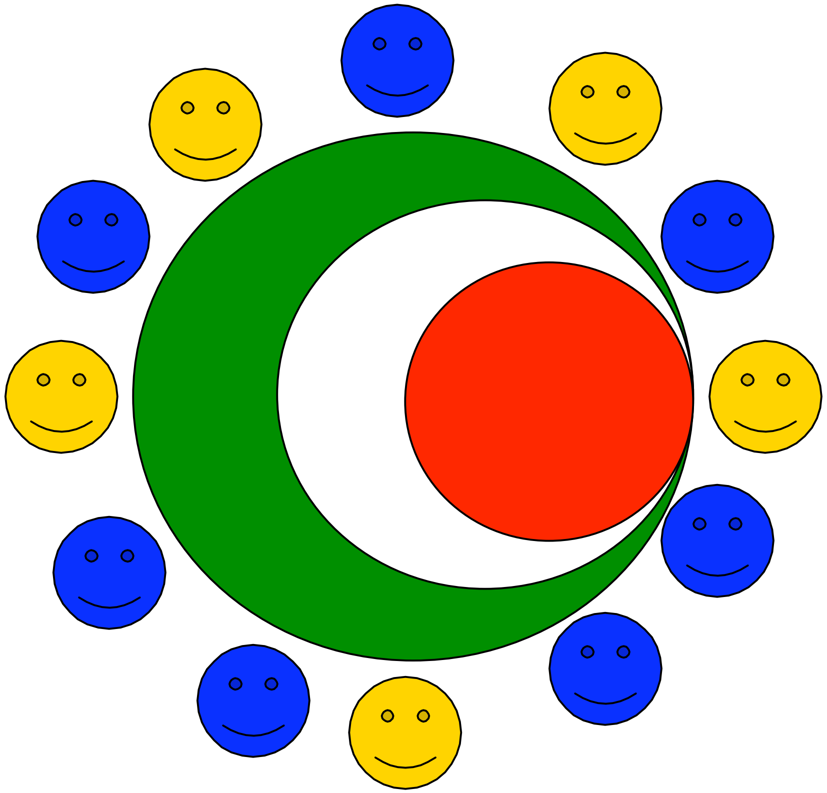


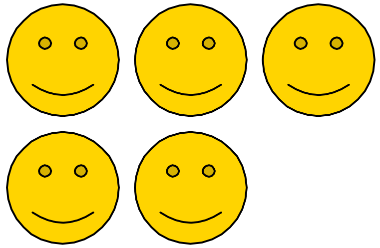
La scala diatonica come *ME-set*

<u>The dinner table problem</u> (Italian version)



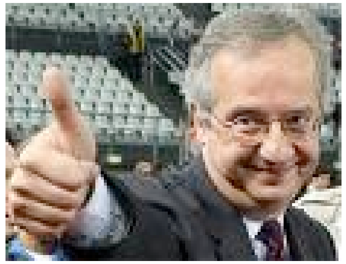


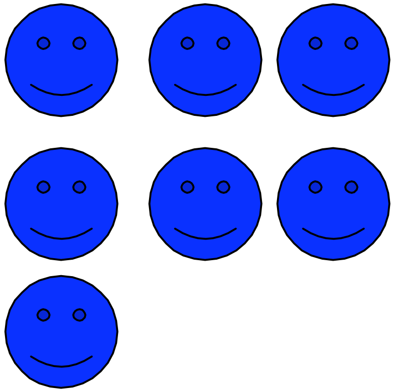


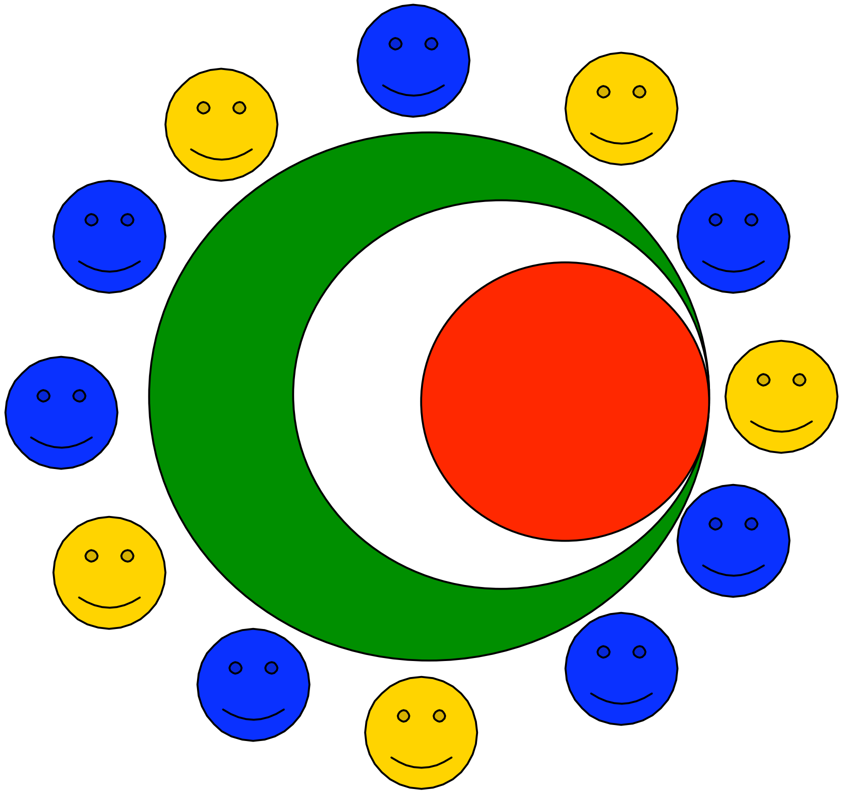


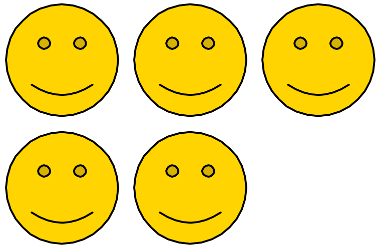
La scala diatonica come *ME-set*

<u>The dinner table problem</u> (Italian version)



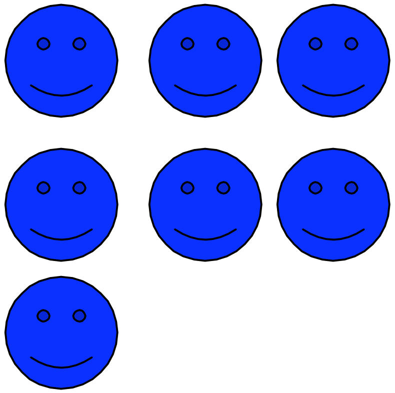


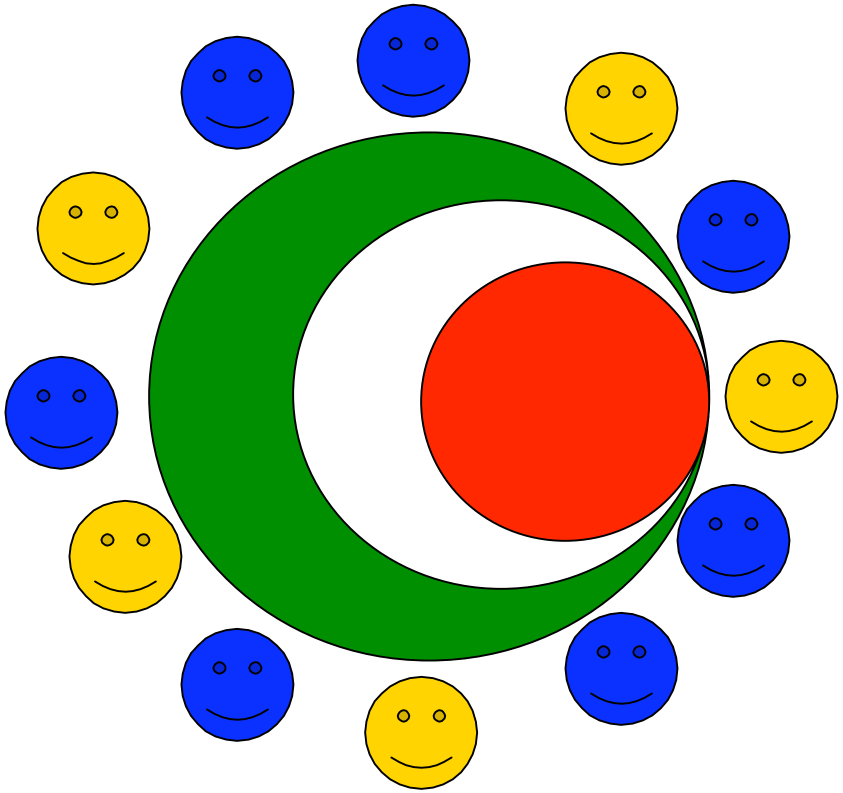


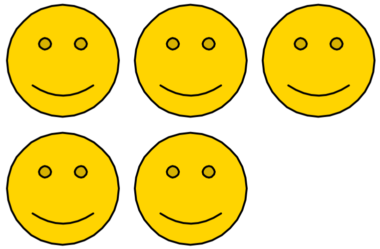


La scala diatonica come *ME-set*

<u>The dinner table problem</u> (Italian version)

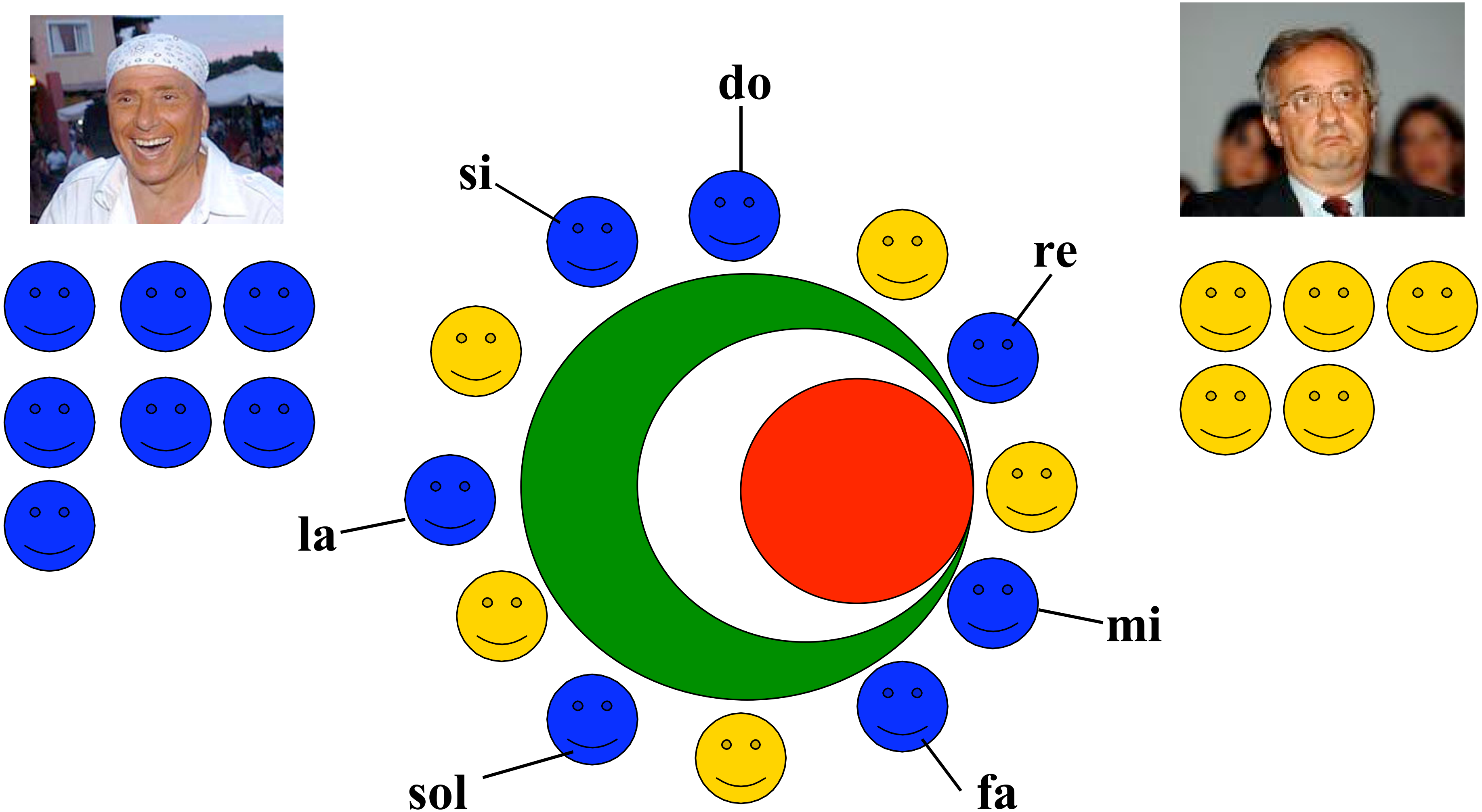






La scala diatonica come *ME-set*

The dinner table problem (Italian version)

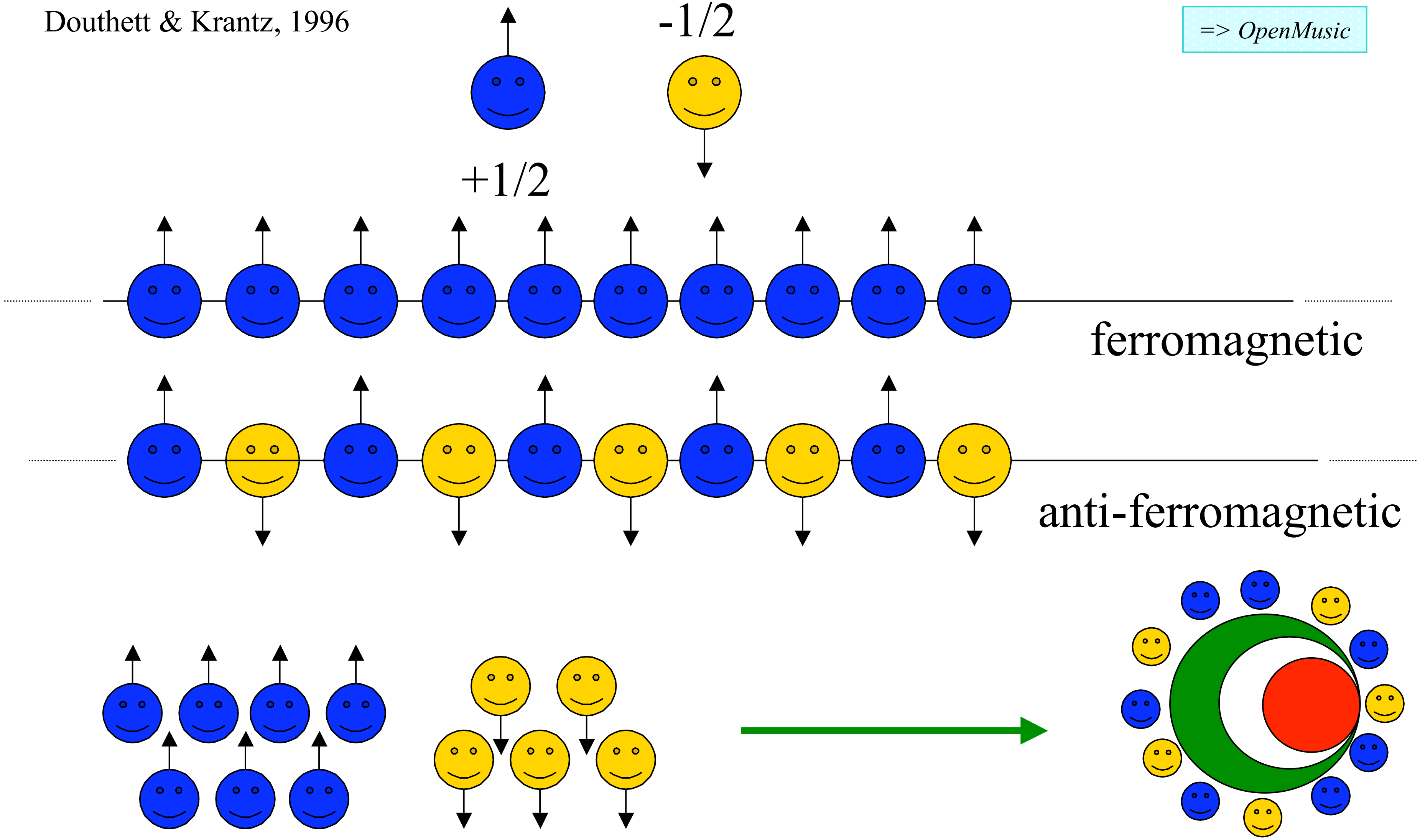


Jack Douthett &Richard Krantz, “Energy extremes and spin configurations for the one-dimensional antiferromagnetic Ising model with arbitrary-range interaction”, *J. Math. Phys.* 37 (7), July 1996

The one-dimensional antiferromagnetic spin-1/2 Ising Model

Douthett & Krantz, 1996

=> *OpenMusic*



TILING

Let $Z_A = \{ t \in \mathbb{Z}_c, F_A(t)=0 \}$

A tiles $\mathbb{Z}_c$ when equivalently:

- There exists B, $A \oplus B = \mathbb{Z}_c$
- $1_A \star 1_B = 1$
- $F_A \times F_B(t) = 1 + e^{-2i\pi t/c} + \ldots e^{-2i\pi t(c-1)/c}$ (0 unless t=0)
- $Z_A \cup Z_B = \{1,2 \ldots c-1\}$ AND Card A $\times$ Card B = c
- $IC_A \star IC_B = IC(\mathbb{Z}_c)=c$ and Card A $\times$ Card B = c

La relazione Z e la tassellazione

A musical offering:

- ***Theorem***:
  If A tiles with B and A' has the same IC, then A' tiles with B, too.

Le congetture di Minkowski/Fuglede e i canoni ritmici a mosaico

- Minkowski's conjecture (1896/1907)
- Hajos algebraic solution (1942)
- Hajos quasi-periodic conjecture
- The classification of Hajos groups (Hajos, de Bruijn, Sands, …)
- Classification of factorizations for non-Hajos groups (Vuza, Andreatta, Agon, Amiot, Fripertinger, …)
- …

- The Tiling of the line problem and Fuglede's Conjecture (**Tijdeman**, **Coven-Meyerowitz**, Lagarias, **Laba**, Kolountzakis…)
- Given a finite set that tiles **Z**, what will be the period (Kolountzakis, Steinberger, …)
- Fuglede's Conjecture and Vuza's Canons (Amiot, 2004)
- …

• R. Tijdeman: "Decomposition of the Integers as a direct sum of two subsets", *Number Theory*, Cambridge University Press, 1995. The fundamental Lemma:
A tiles $\mathbf{Z}_n$ => pA tiles $\mathbf{Z}_n$ when $<p,n>=1$

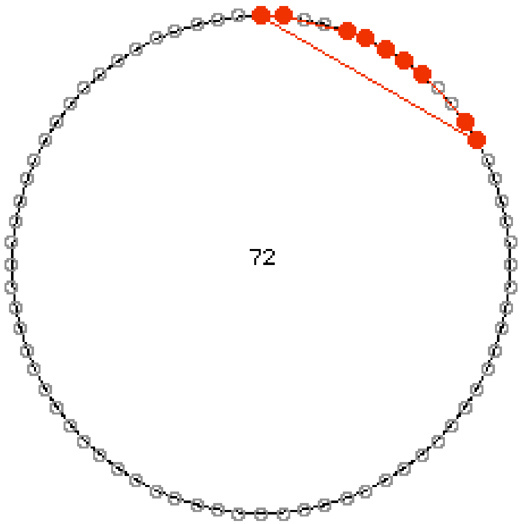
• I. Laba : "The spectral set conjecture and multiplicative properties of roots of polynomials", *J. Lond Math Soc*, 2002
T1 + T2 => spectral
T2 => spectral
spectral => T1

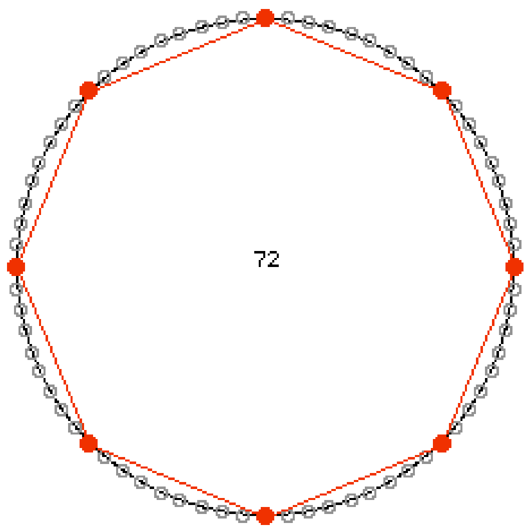
• E. Coven & A. Meyerowitz: "Tiling the integers with translates of one finite set", *J. Algebra*, 212, pp.161-174, 1999
T1 + T2 => tile
Tile => T1

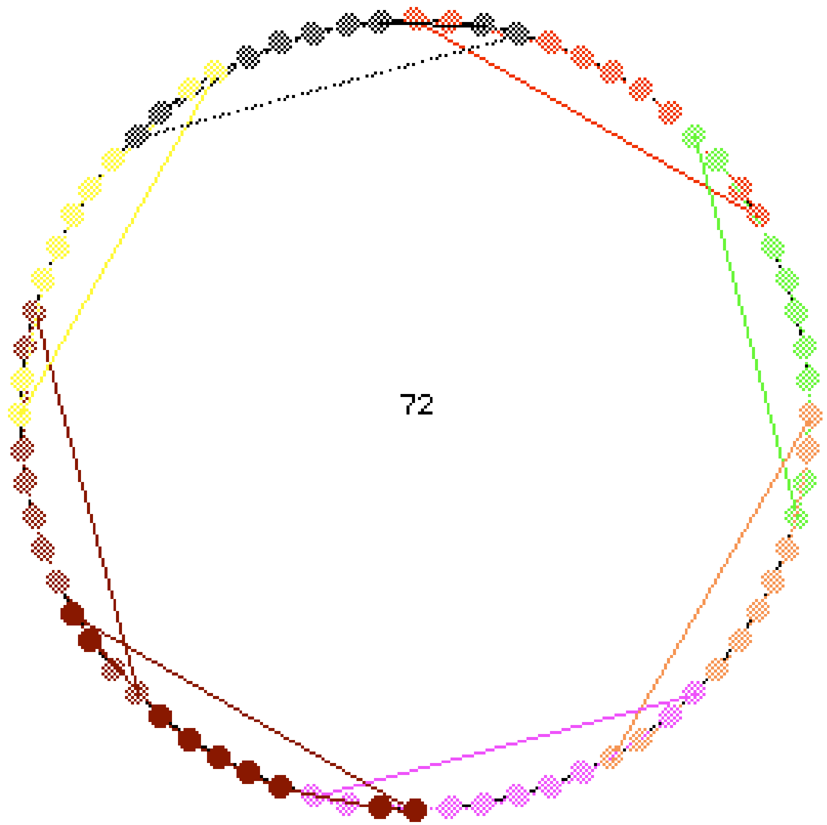
• E. Amiot : "A propos des canons rythmiques", *Gazette des Mathématiciens,* n°106, Octobre 2005.
if A tiles with period n and $\mathbf{Z}_n$ is Hajos
=> A has T2 (=> A is spectral)

Se A tassella ma non è spettrale => A è il ritmo di un canone di Vuza

La classe dei canoni ciclotomici





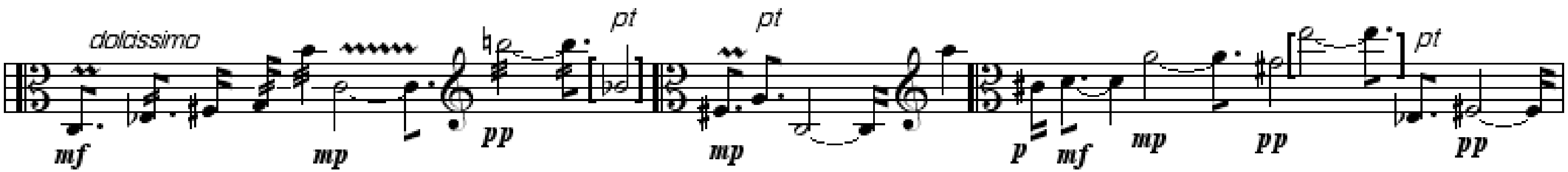


- E. Amiot, M. Andreatta, C. Agon: « Tiling the (musical) line with polynomial: some theoretical and implementational aspects », *ICMC*, Barcelona, 2005, pp.227-230.

Sequenze periodiche e calcolo delle differenze finite

$Df(x)=f(x)-f(x-1)$

$$
\begin{array}{rcl}
f & = & 7\ 11\ 10\ 11\ 7\ 2\ 7\ 11\ 10\ 11\ 7\ 2\ 7\ 11\ldots \\
Df & = & 4\ 11\ 1\ 8\ 7\ 5\ 4\ 11\ 1\ 8\ 7\ 5\ 4\ 11\ldots \\
D^2f & = & 11\ 7\ 2\ 7\ 11\ 0\ 11\ 7\ 2\ 7\ 11\ 0\ldots \\
D^3f & = & 1\ 8\ 7\ 5\ 4\ 11\ 1\ 8\ 7\ 5\ 4\ 11\ldots \\
D^kf & = & \ldots
\end{array}
$$



V	0	3	8	7	11	0	11	10	6	9	0	9	1	2	9	8	4	3	6
VIII	0	0	0	0	3	3	7	2	0	0	0	6	3	3	3	4	8	0	0
IV	3	3	4	4	1	11	11	8	3	3	9	4	1	7	11	8	11	3	9
IX	0	0	0	0	0	3	6	[1]	3	3	3	3	9	0	3	6	[10]	6	6
IV	0	10	3	9	10	0	9	7	0	6	7	9	6	4	9	3	4	6	3

Anatol Vieru: *Zone d'oubli* pour alto (1973)

Sequenze riducibili e riproduttibili

=> *OpenMusic*

f = 11 6 7 2 3 10 11 6 …

\ / \ /

Df = 7 1 7 1 7 1 7 1…

\ /

$D^2 f$ = 6 6 6 6 6…

\ /

$D^4 f$ = 0 0 0

Sequenze riducibili

$\exists\, k \geq 1$ tel que $D^k f = 0$

f = 7 11 10 11 7 2 7 11 …

\ / \ /

Df = 4 11 1 8 7 5 4 11 …

\ /

$D^2 f$ = 11 7 2 7 11 0 11 7 …

\ /

$D^4 f$ = 1 8 7 5 4 11 1 8 …

\ /

$D^5 f$ = 7 11 10 11 7 2 7 11 …

Sequenze riproducibili

$\exists\, k \geq 1$ tel que $D^k f = f$

Teorema di decomposizione: Ogni sequenza periodica (a valori in un gruppo ciclico **Z/nZ**) può essere decomposta in maniera unica in una somma di una sequenza riducibile e di una sequenza riproducibile (2001)

Quelques traits caractéristiques du positivisme logique

James A. Davis : *Positivistic Philosophy and the Foundation of Atonal Music Theory*, 1993

• Empiricisme rigide qui mène à une émulation de la science dans sa méthodologie et sa terminologie

• Utilisation de l'analyse linguistique et logique (en particulier le recours à la logique formelle).

• Principe de vérification et reconstruction rationnelle

• Refus de l'interprétation subjective et rejet/élimination de la métaphysique

« There is no field of experience which cannot, in principle, be brought under some form of scientific law, and no type of speculative knowledge about the world which is, in principle, beyond the power of science to give […] **The propositions of philosophy are not factual, but linguistic in character** – that is, they do not describe the behavior of physical, or even mental, objects ; they express definitions, or the formal consequences of definitions. Accordingly, we may say that **philosophy is a department of logic**. » [AYER, 1952]

« Ce qui caractérise le néopositivisme logistique […] est la réduction de la philosophie à l'étude syntaxique des énoncés scientifiques »

Albert Lautman : *Les mathématiques, les idées et le réel physique*, Vrin 2006 (en particulier le compte-rendu du Congrès International de philosophie des sciences, 1935)

Le transfert des idées du positivisme logique en musique

• Empiricisme rigide qui mène à une émulation de la science dans sa méthodologie et sa terminologie

• Rejet ou élimination de la métaphysique

• Utilisation de l'analyse linguistique et logique (en particulier le recours à la logique formelle). Principe de vérification

• Refus de l'interprétation subjective

« For the essential elements of the above characterizations, involving the correlations of the syntactic and semantic domains, the notion of analysis, and – perhaps most significantly – the requirements of linguistic formulation and the differentiation among predicate types, beyond strongly suggesting that the proper object of our assigned investigation may be – in the light of these criteria – a vacuous class, and strongly reminding us of the systematic obligations attending our own **necessarily verbal presentation** and discussion of the presumed subject, provide the important reminder that **there is but one kind of language, one kind of method for the verbal formulation of 'concepts' and the verbal analysis of such formulations : 'scientific' language and 'scientific' method** »

M. Babbitt : « Past and Present Concepts », 1961

Le transfert des idées du positivisme logique en musique

• Empiricisme rigide qui mène à une émulation de la science dans sa méthodologie et sa terminologie

• Rejet ou élimination de la métaphysique

• Utilisation de l'analyse linguistique et logique (en particulier le recours à la logique formelle). Principe de vérification

• Refus de l'interprétation subjective

« Progressively from the concept to the law (synthetic generality) we arrive at the deductively interrelated system of laws that is a *theory*, statable as a **connected set of axioms, definitions, and theorems, the proof of which are derived by means of an appropriate logic**. A *musical theory* reduces, or should reduce, to such a **formal theory** when uninterpreted predicates and operations are substituted for the terms and operations designating musical observables »

M. Babbitt : « Past and Present Concepts », 1961

Limites d'une conception logiciste de la tradition américaine

Analyse plus fine des relations entre le positivisme logique et la tradition américaine, e.g. principe de « reconstruction rationnelle »

« What epistemology intends is to construct thinking processes in a way in which they ought to occur if they are to be ranged in a consistent system; or to construct justifiable sets of operations which can be intercalated between the starting-point and the issue of though-processes, replacing the real intermediate links. Epistemology thus considers a **logical substitute rather than real processes**.
For this logical substitute the term ***rational reconstruction*** has been introduced. »

Hans Reichenbach : *Experience and Prediction*, 1938

« *Pour développer le concept de **structure** qui est au fondement de la **théorie de la constitution**, nous partons de la différence entre deux types de description des objets d'un domaine quelconque. Nous appelons ces deux types de description, description de **propriété** et description de **relation**. [...] La description de relation se trouve au commencement de tout le système de constitution et forme ainsi la base de la science dans son ensemble. En outre, le but de toute **théorie scientifique** est de devenir une **pure description de relation** quant à son contenu.* »

R. Carnap : *La construction logique du monde*, 1928 (tr. Fr. 2002)

Architettura paradigmatica e strutture algebriche in musica

« [C'est la notion de groupe qui] donne un sens précis à l'idée de structure d'un ensemble [et] permet de déterminer les éléments efficaces des transformations en réduisant en quelque sorte à son schéma opératoire le domaine envisagé. […]L'objet véritable de la science est le ***système des relations*** *et non pas les termes supposés qu'il relie. […] Intégrer les résultats - symbolisés - d'une* ***expérience*** *nouvelle revient […] à créer un canevas nouveau, un* ***groupe de transformations*** *plus complexe et plus compréhensif »*

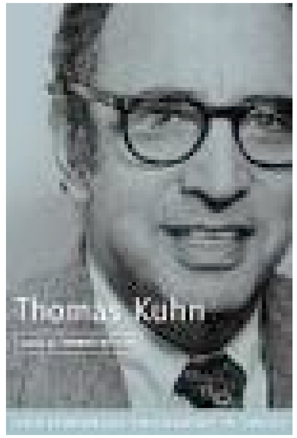
G.-G. Granger : « Pygmalion. Réflexions sur la pensée formelle », 1947

Felix Klein

Ernst Cassirer

Gilles-Gaston Granger

Jean Piaget

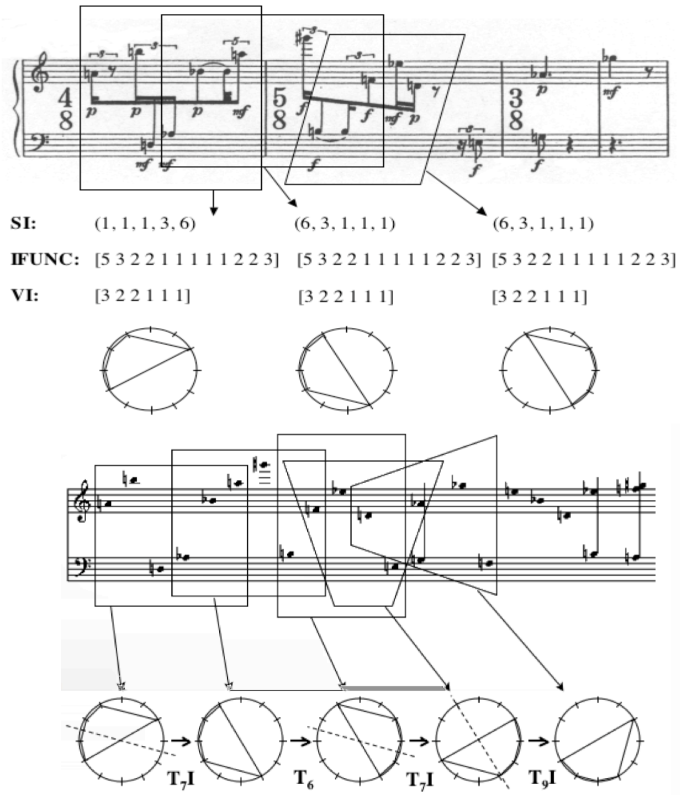


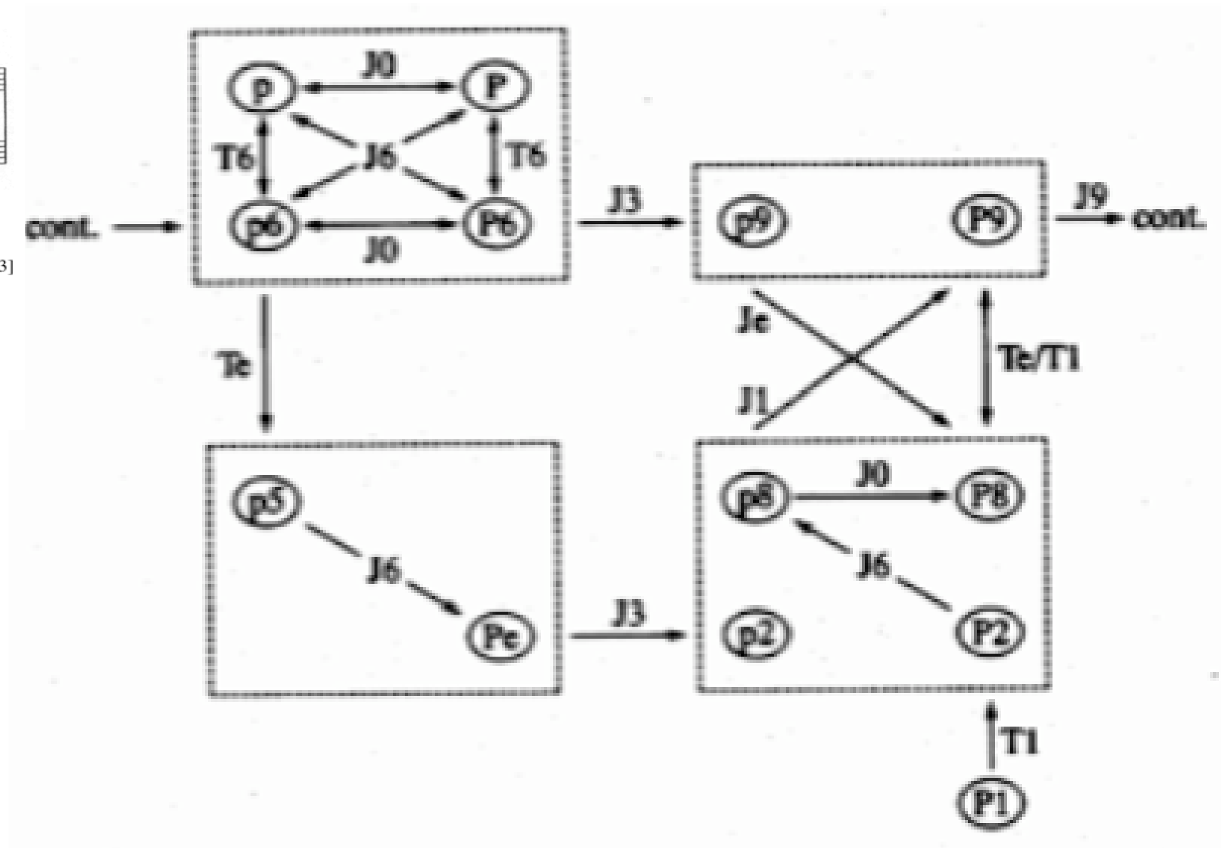
Thomas Kuhn

« [Le système dodécaphonique] peut être caractérisée complètement en explicitant les éléments, les ***relations*** *[…] entre ces éléments et les* ***opération****s sur les éléments ainsi reliés. […] Toute considération sur les opérations du système doit procéder de la conscience de leur nature permutationnelle »*

M. Babbitt : « Twelve-Tone Invariants as Compositional Determinants », 1960

Progression transformationnelle *vs* réseau transformationnel





> « *A **rational reconstruction** of a work or works, which is a **theory** of the work or works, is an **explanation** not, assuredly, of the 'actual' process of construction, but of how the work or works may be construed by a hearer, how the 'given' may be 'taken'* »

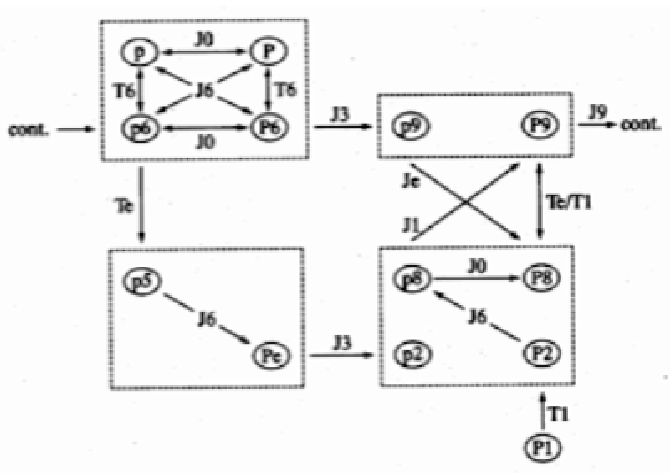
M. Babbitt : « Contemporary Music Composition and Music Theory as Contemporary Intellectual History », 1972

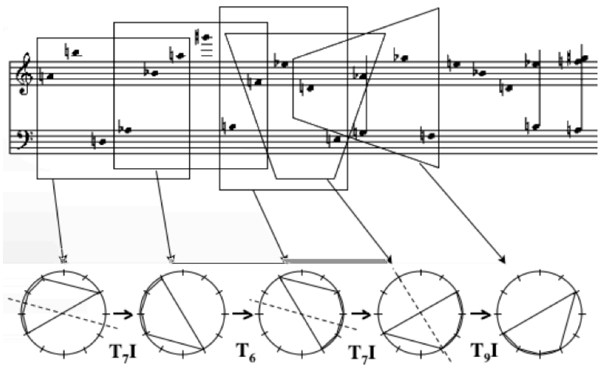
Perspectives…

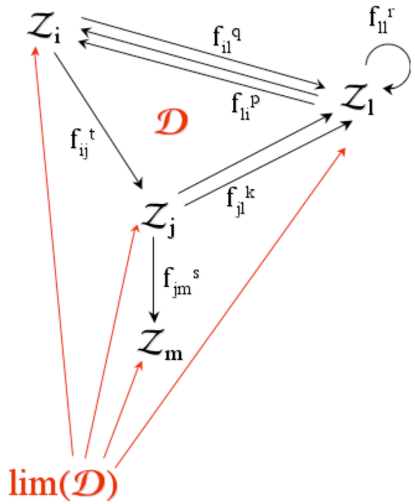
- Analyse plus fine des relations entre le positivisme logique et la tradition américaine
  - Relecture du principe de « reconstruction rationnelle » et, plus en général, des thèses de *La construction logique du monde* (Dualité de l'objectal et de l'opératoire chez Gaston-Granger)
  - Articulation entre théorie et composition
  - Articulation entre théorie et analyse musicale

« Tous les musiciens s'accordent à penser qu'à la base de l'élément émotionnel de la musique se trouve un **puissant élément formel**. Il se peut qu'il soit susceptible de cette même étude [théorie des groupes] qui s'est révélée si féconde pour l'art décoratif. Mais dans ce cas, nous n'avons sans doute pas encore découvert les outils mathématiques appropriés »

Hermann Weyl, *Symmetry*, Princeton University Press, 1952

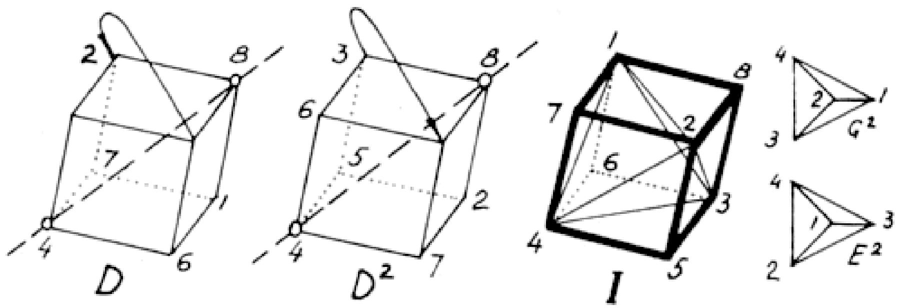






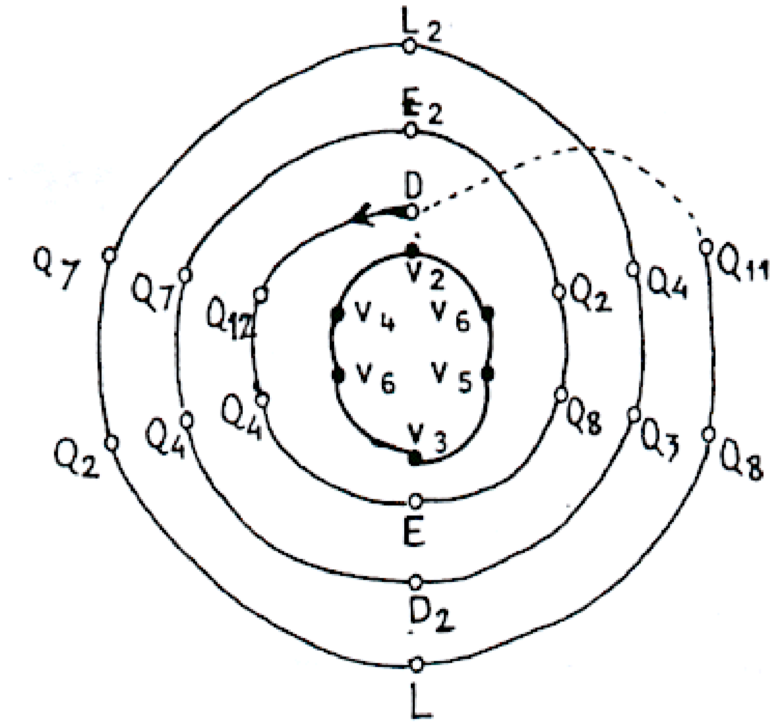
Analisi musicale computazionale: *Nomos Alpha* di I. Xenakis

La questione delle simmetrie (identità spaziali) e delle periodicità (identità nel tempo) ha un ruolo fondamentale nella musica, a tutti i livelli, da quello dei campioni sonori della sintesi del suono mediante computer, fino all'architettura di un intero brano musicale



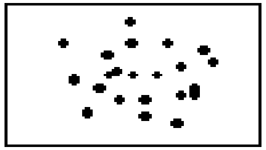
***Nomos Alpha* (1966)**

Musique symbolique pour violoncelle seul, possède une architecture "hors-temps" fondée sur la théorie des groupes de transformations.



Analisi musicale computazionale: *Nomos Alpha* di I. Xenakis

S1



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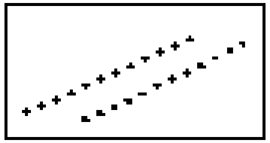

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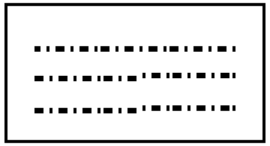

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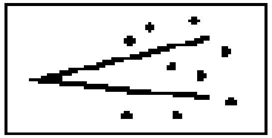

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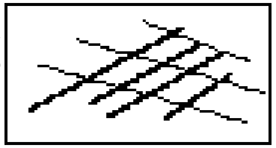

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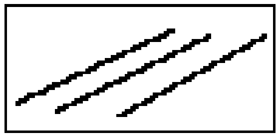

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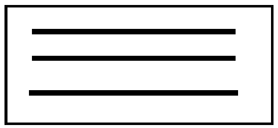

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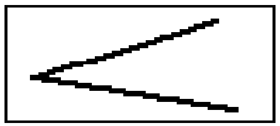

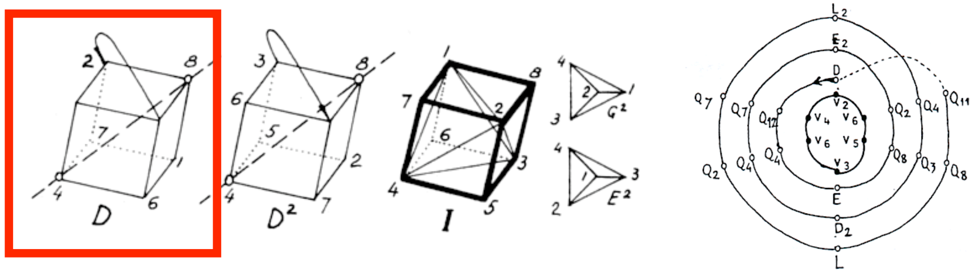

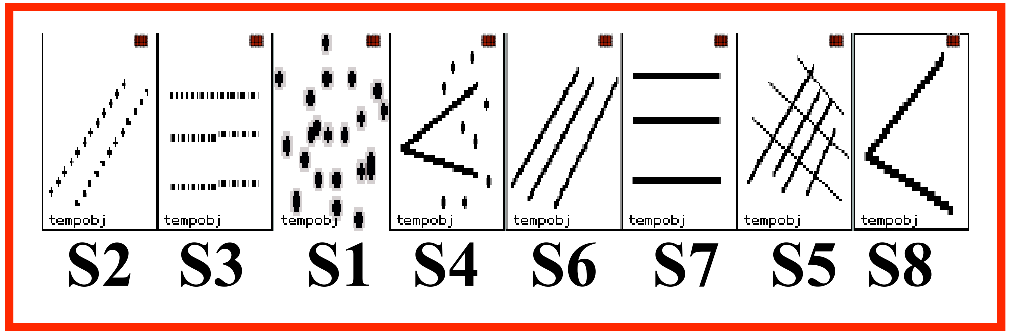

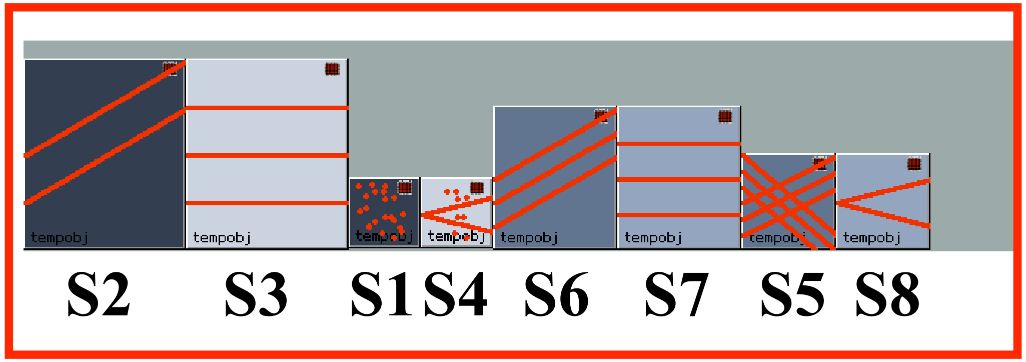

Musique symbolique pour violoncelle seul, possède une architecture "hors-temps" fondée sur la théorie des groupes de transformations.

I	12345678
A	21436587
B	34127856
C	43218765
D	23146758
D^2	31247568
E	24316875
E^2	41328576
G	32417685
G^2	42138657
L	13425786
L^2	14235867
Q_1	78653421
Q_2	76583214
Q_3	86754231
Q_4	67852341
Q_5	68572413
Q_6	65782134
Q_7	87564312
Q_8	75863142
Q_9	58761432
Q_{10}	57681324
Q_{11}	85674123
Q_{12}	56871243

TABLE (MOSAIC) OF COHERENCES

Philosophy (in the etymological sense)

Thrust towards truth, revelation. Quest in everything, interrogation, harsh criticism, active knowledge through creativity.

Chapters (in the sense of the methods followed)

Partially inferential and experimental	Entirely inferential and experimental	Other methods to come
ARTS (VISUAL, SONIC, MIXED . . .)	SCIENCES (OF MAN, NATURAL) PHYSICS, MATHEMATICS, LOGIC	?

This is why the arts are freer, and can therefore guide the sciences, which are entirely inferential and experimental.

Categories of Questions (fragmentation of the directions leading to creative knowledge, to philosophy)

REALITY (EXISTENTIALITY); CAUSALITY; INFERENCE: CONNEXITY; COMPACTNESS; TEMPORAL AND SPATIAL UBIQUITY AS A CONSEQUENCE OF NEW MENTAL STRUCTURES; INDETERMINISM . . . ← bi-pole → . . . DETERMINISM; . .

↓

Families of Solutions or Procedures (of the above categories)

?

INDETERMINISM → FREE STOCHASTIC materialized by a computer program; MARKOVIAN

bi-pole → SETS; GAMES

DETERMINISM → GROUPS

↓

Pieces (examples of particular realization)

?

FREE STOCHASTIC: ACHORRIPSIS, ST/10–1, 080262, ST/48–1, 240162, ATRÉES, MORSIMA-AMORSIMA

MARKOVIAN: ANALOGIQUE A, ANALOGIQUE B, SYRMOS

SETS: HERMA

GAMES: DUEL, STRATÉGIE

GROUPS: AKRATA, NOMOS ALPHA, NOMOS GAMMA

Classes of Sonic Elements (sounds that are heard and recognized as a whole, and classified with respect to their sources)

ORCHESTRAL, ELECTRONIC (produced by analogue devices), CONCRETE (microphone collected), DIGITAL (realized with computers and digital-to-analogue converters), . . .

Microsounds

Forms and structures in the pressure-time space, recognition of the classes to which microsounds belong or which microstructures produce.

Microsound types result from questions and solutions that were adopted at the CATEGORIES, FAMILIES, and PIECES levels.

Table (mosaic) of Coherences

Philosophy (in the etymological sense)

Thrust towards truth, revelation. Quest in everything, interrogation, harsh criticism, active knowledge through creativity.

Chapters (in the sense of the methods followed)

Partially inferential and experimental	↔	Entirely inferential and experimental	Other methods to come
ARTS (VISUAL, SONIC, MIXED . . .)		SCIENCES (OF MAN, NATURAL) PHYSICS, MATHEMATICS, LOGIC	?

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↓

Families of Solutions or Procedures (of the above categories)

?

↓

Pieces (examples of particular realization)

?

- INDETERMINISM → FREE STOCHASTIC materialized by a computer program ↓ ACHORRIPSIS, ST/10–1, 080262, ST/48–1, 240162, ATRÉES, MORSIMA-AMORSIMA
- INDETERMINISM → MARKOVIAN ↓ ANALOGIQUE A, ANALOGIQUE B, SYRMOS
- bi-pole → SETS ↓ HERMA
- bi-pole → GAMES ↓ DUEL, STRATÉGIE
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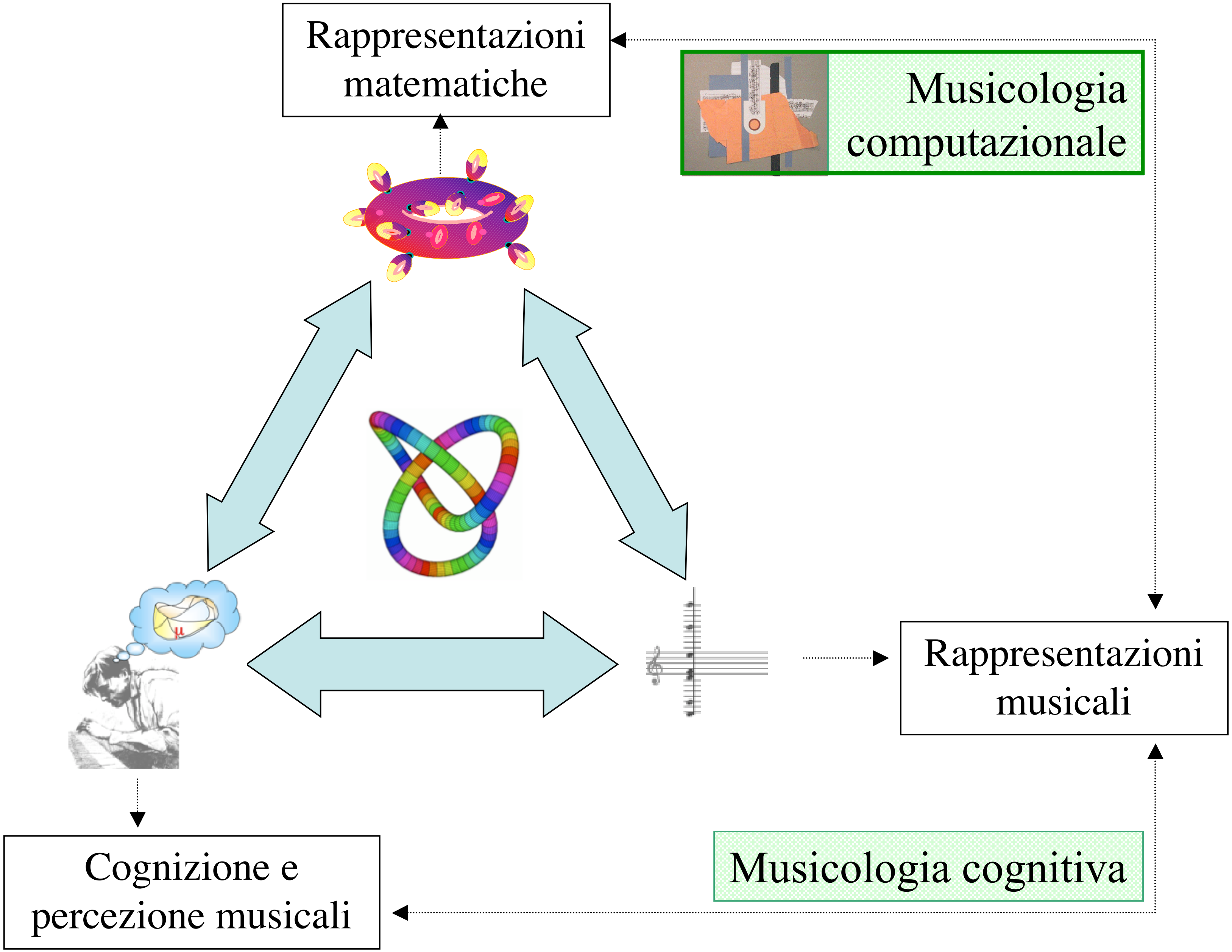
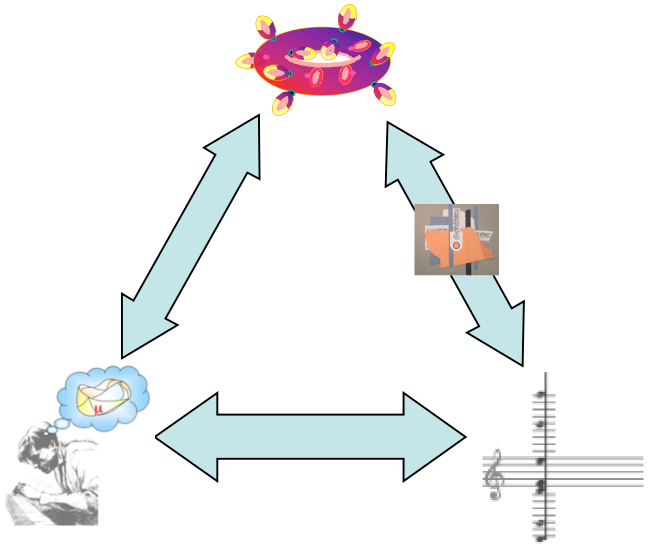
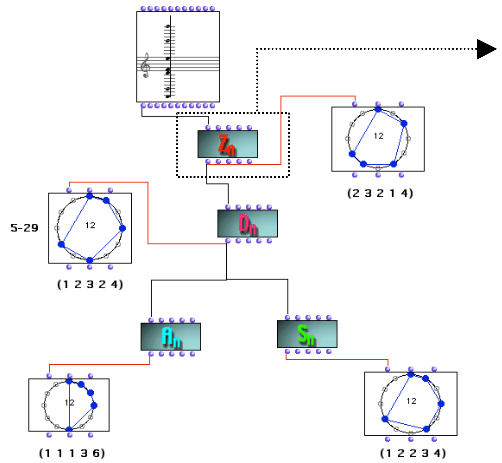
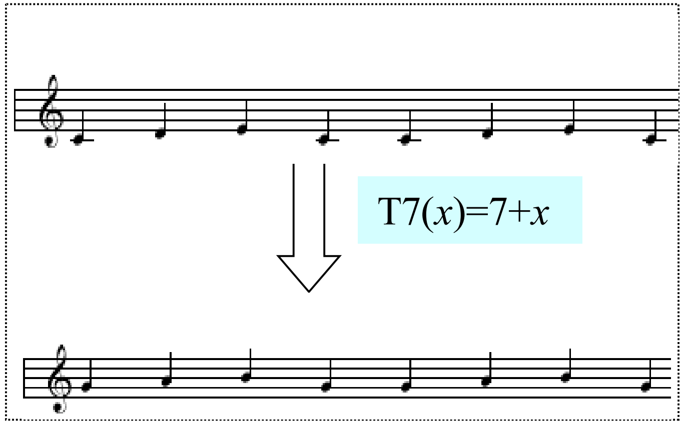
Matematica/Musica & Cognizione/Percezione



Ramificazioni percettive e cognitive dei metodi algebrici in musica



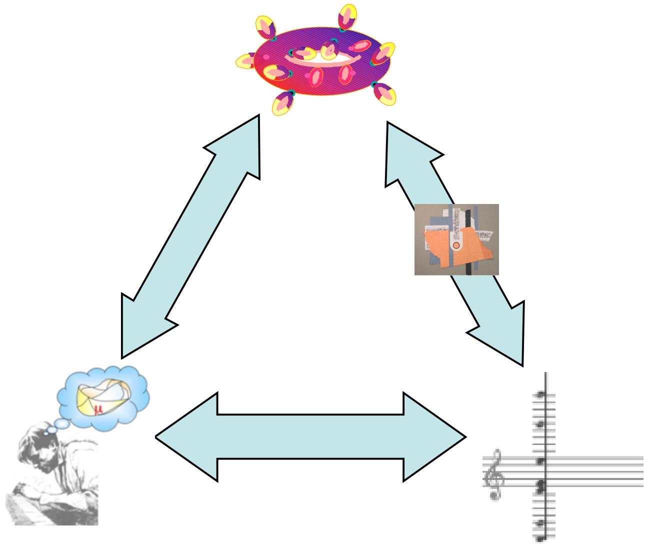


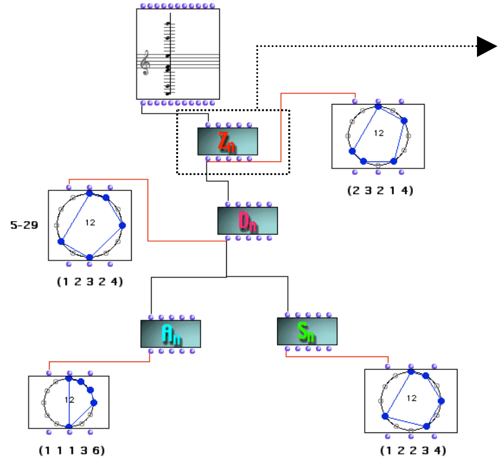


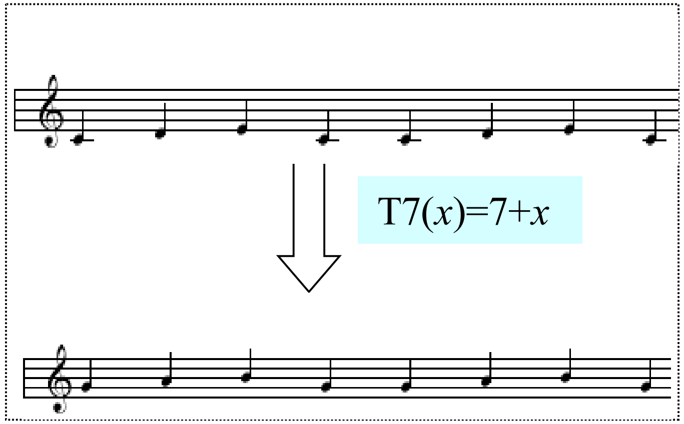
The nature of a given geometry is […] defined by the *reference* to a determinate **group** and the way in which spatial forms are related within that type of geometry. [*Cf. Felix Klein Erlangen Program - 1872*][…] We may raise the question whether there are any concepts and principles that are, although in different ways and different degrees of distinctness, necessary conditions for both the *constitution* of the **perceptual world** and the construction of the universe of geometrical thought. It seems to me that the concept of **group** and the concept of **invariance** are such principles.

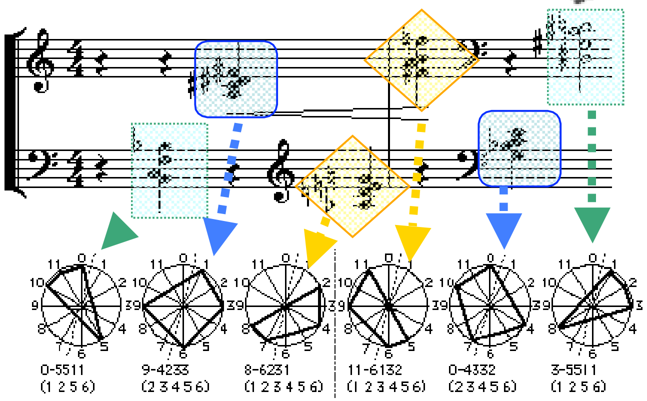
E. Cassirer, "The concept of group and the theory of perception", 1944

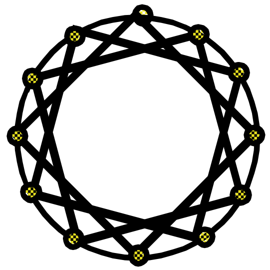
Ramificazioni percettive e cognitive dei metodi algebrici in musica

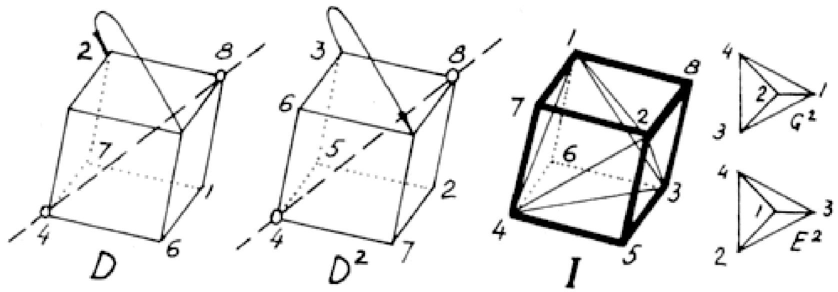












Il carattere singolare dell'esperienza musicale è dovuto in parte alle strutture particolari di ***gruppo*** *che la musica rende accessibile* [consciamente o inconsciamente] *all'ascoltatore.*

G. Balzano : « The group-theoretic description of 12-fold and microtonal pitch systems », 1980

Programma del corso

1.) Rappresentazione e formalizzazione delle strutture musicali
2.) Enumerazione e classificazione delle strutture musicali
3.) Teorie trasformazionali, diatoniche e neo-riemanniane
4.) Tessellazioni musicali: la costruzione dei canoni ritmici a mosaico
5.) Sequenze periodiche e calcolo delle differenze finite a valori in gruppi ciclici
[6.) Ramificazioni filosofiche e cognitive dell'approccio algebrico in musica]

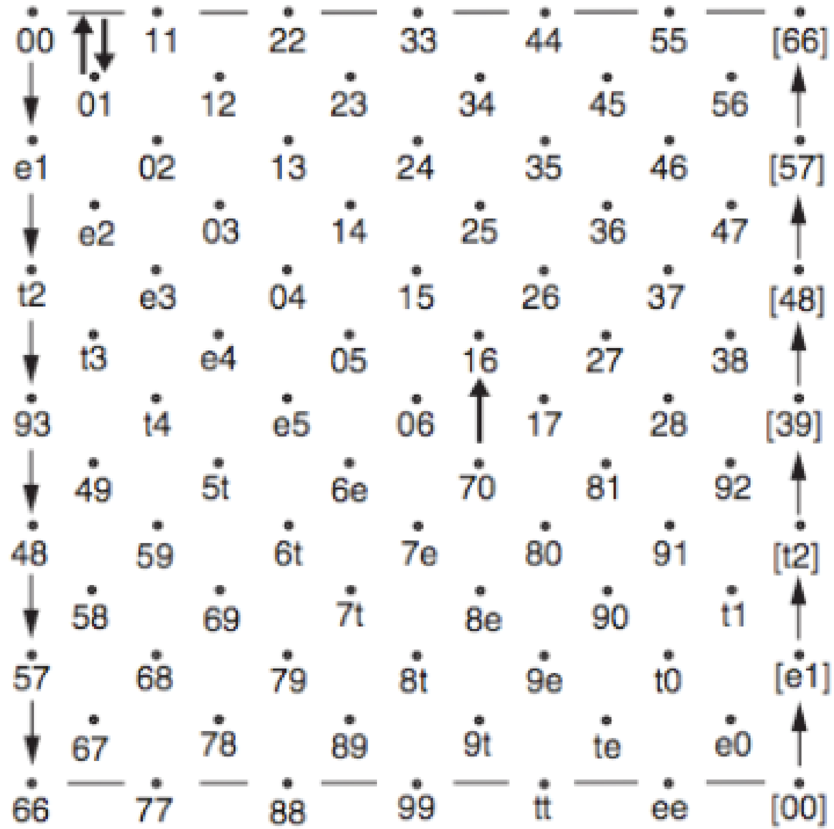
Rappresentazione e formalizzazione delle strutture musicali

○ Rappresentazioni geometriche e formalizzazioni algebriche

- Il *Tonnetz* di Eulero
- Rappresentazioni circolari e toroidali
- **Teoria degli *orbifolds***
- **Teoria dei nodi**

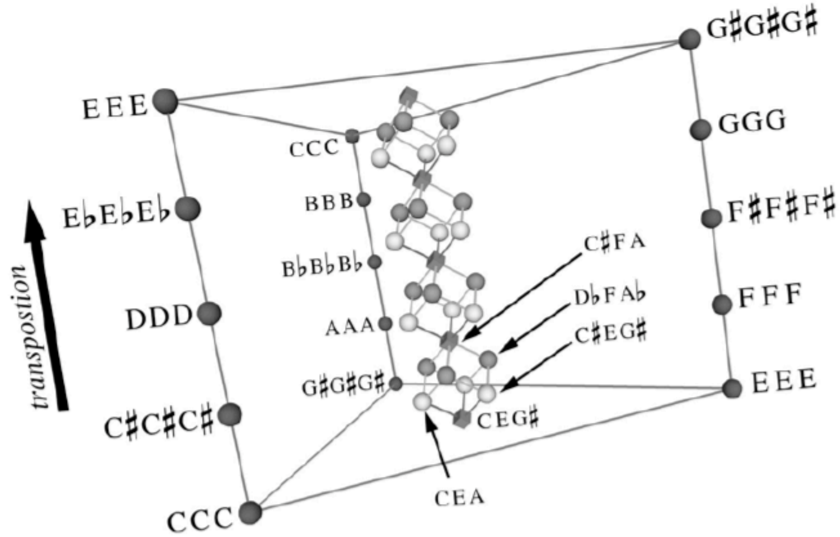
Dmitri Tymoczko, « The Geometry of Musical Chords », *Science*, 313, 2006

$$T^2 = (\mathbf{R}/\mathbf{12Z})^{\mathbf{2}} \longrightarrow T^2/S_2$$



Teoria degli orbifolds in musica

$T^3 = (\mathbf{R}/\mathbf{12Z})^\mathbf{3} \longrightarrow T^3 / S_3$ $T^2 = (\mathbf{R}/\mathbf{12Z})^\mathbf{2} \longrightarrow T^2 / S_3$



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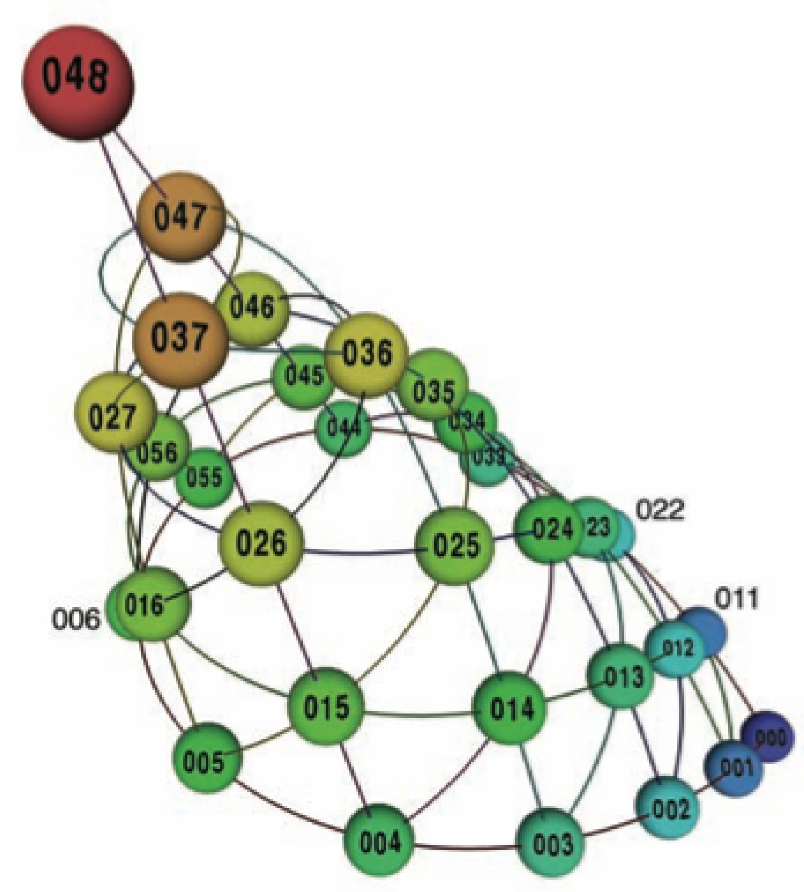
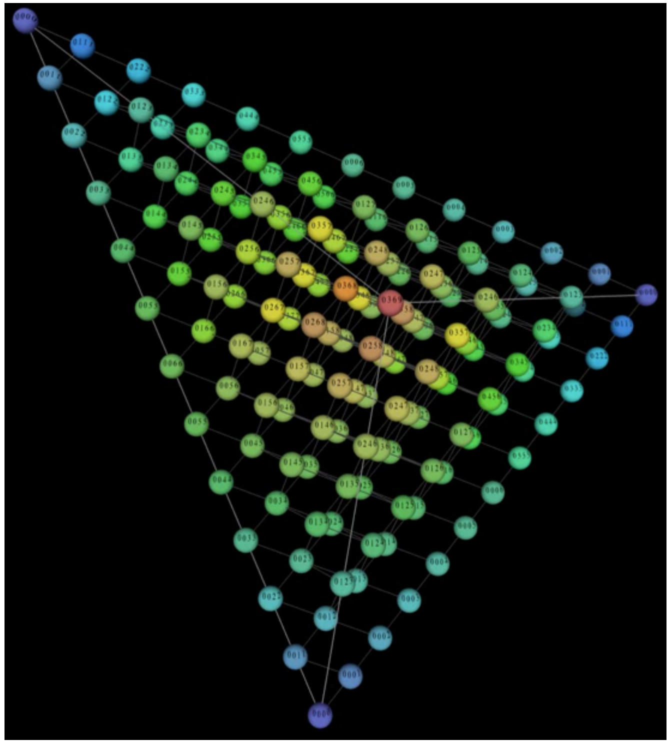


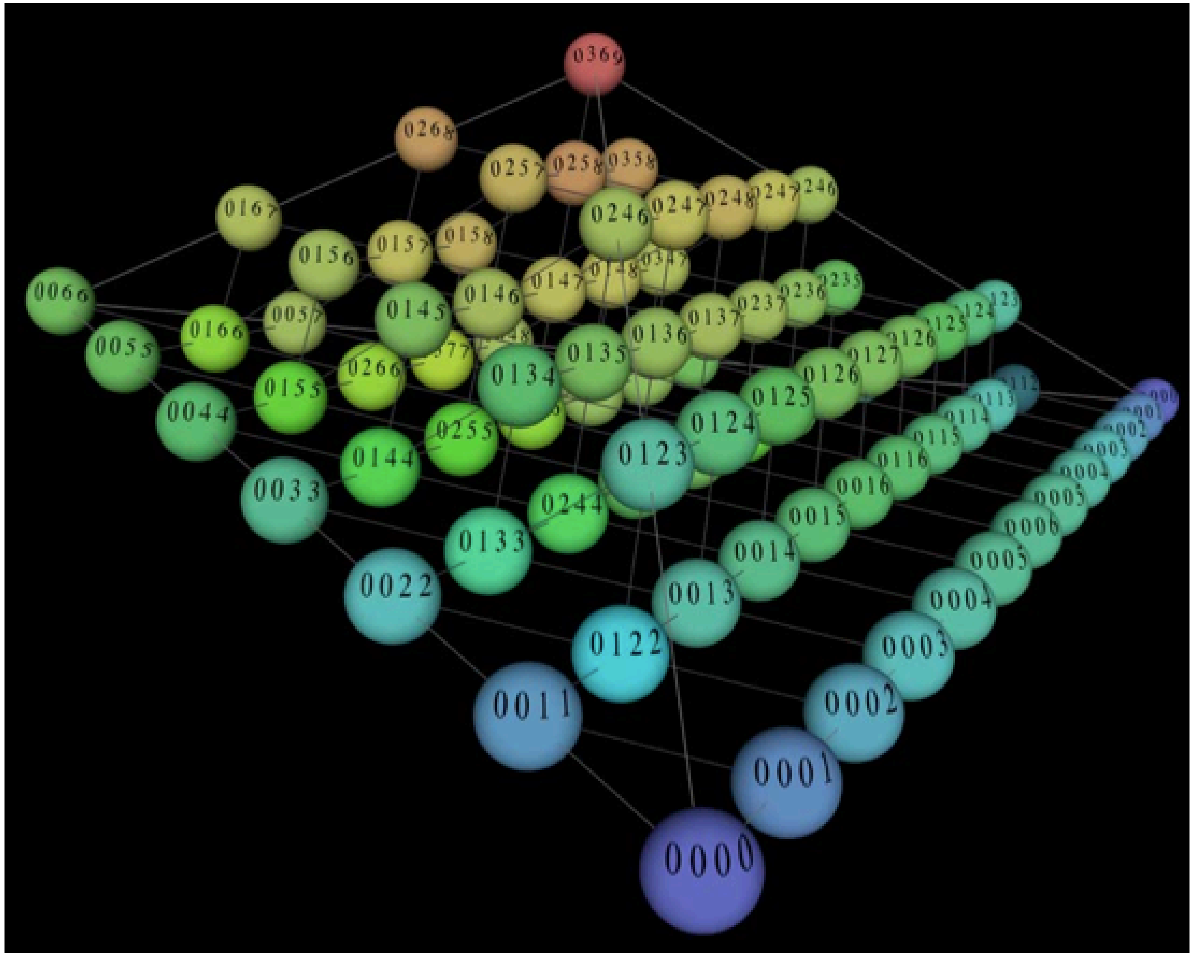
C. Callender, I. Quinn & D. Tymoczko, « Generalized Voice-Leading Spaces », *Science*, 320, 2008

Teoria degli orbifolds in musica

T^3/S_4



$T^3/(S_4 \times Z_2)$



C. Callender, I. Quinn & D. Tymoczko, « Generalized Voice-Leading Spaces », *Science*, 320, 2008

Enumerazione e classificazione delle strutture musicali

- Lemma di Burnside e teoria dell'enumerazione di Polya
  - Classificazione paradigmatica degli accordi musicali (azioni del gruppo ciclico, diedrale e affine sul sistema temperato tradizionale)
    - **Dimostrare le formule di enumerazione**
  - Modi di Messiaen a trasposizione limitata
    - **Dimostrare la formula**
  - Serie dodecafoniche e serie omni-intervallari
    - **Sviluppare l'approccio topologico basato sulla teoria dei nodi**
  - Asimmetria ritmica
  - Spazi microtonali

- La *Set Theory* d'Allen Forte
  - Il vettore intervallare
  - Teorema dell'esacordo (Milton Babbitt)
  - **La relazione Z**
    - **Sviluppare il legame fra relazione Z e insiemi omometrici**

Teorie trasformazionali, diatoniche e neo–riemanniane (1)

- Il sistema d'intervalli generalizzati (GIS) di David Lewin
  - La funzione intervallare IFUNC e la trasformata di Fourier discreta
    - **Dimostrare l'equivalenza fra GIS e azione semplicemente transitiva**
    - **Studiare il caso particolare IFUNC(A,B)$(i)=k\ \forall i$**

- Reticoli di Klumpenhouwer (*K-nets)*
  - Isografie forti
    - **Dimostrare il risultato sull'enumerazione delle isografie forti con il teorema cinese dei resti**
  - Isografie positive e isografie negative
    - **Isomorfismi interni e esterni**

Teorie trasformazionali, diatoniche e neo–riemanniane (2)

- Teorie diatoniche
  - Insiemi ripartiti in maniera massimale (*Maximally Even Sets*)
    - **Sviluppare la parte sul legame fra ME sets e modello di Ising**
    - **Esercizi:**

      a) Un insieme è periodico se e solo se il valore assoluto della FT si annulla tranne in un sottogruppo
      b) Dimostrare il teorema di Babbitt nel caso di due sottoinsiemi aventi diversa cardinalità
      c) Principio di Heisenberg per il numero degli zeri della DFT
      d) Polinomio caratteristico di un sottoinsieme di Z/nZ e coefficienti della trasformata di Fourier
      e) A è un Cluster se e solo se il modulo del primo coefficiente della FT è massimo
      f) Il modulo del d esimo coefficiente di Fourier di A è massimo se e solo se il modulo del primo coefficiente di Fourier di dA è massimo
- Teorie neo-riemanniane
  - **Teorema delle azioni duali (fra PRL e D_{24})**

Tassellazioni musicali: la costruzione dei canoni a mosaico

- Fattorizzazione di gruppi ciclici
  - Gruppi di Hajos e gruppi non-Hajos
  - Teorema di Hajos
  - Teorema di Redei

- Fattorizzazioni polinomiali (polinomi ciclotomici)
  - Condizioni di Coven-Meyerowitz

- Congetture geometrico-algebriche
  - Congettura di Minkowski
  - Congettura di Keller (i.e. Minkowski senza ricoprimento reticolare)
  - Congettura di Fuglede (congettura spettrale)

- **Tassellazioni e trasformata di Fourier**
  - **Legame fra fattorizzazione e insiemi degli zeri della FT**
  - **Periodicità di un sottoinsieme di Z/nZ**
  - **Proprietà di compatibilità della relazione Z rispetto alla tassellazione**

Sequenze periodiche e calcolo delle differenze finite

- Sequenze riducibili, riproducibili e teorema di fattorizzazione
  - **Teorema di fattorizzazione e Lemma di Fitting**
  - **Studiare il problema della proliferazione nel caso del processo di addizioni successive**