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Formal Aspects of Iannis Xenakis' "Symbolic Music": A Computer-Aided Exploration of Compositional Processes

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Abstract

We present computer models of two works for solo instrument by Iannis Xenakis: *Herma* for piano (1962) and *Nomos Alpha* for cello (1965). Both works were described by the composer (in his book *Formalized Music*) as examples of "symbolic music." Xenakis' detailed description of formal aspects in his compositional process makes it possible to implement computer models that recreate essential elements of the final scores. For our implementations, we use OPENMUSIC, a visual programming language based on CommonLisp/CLOS developed at Ircam. Based on the experiences gathered from developing and exploring these computer models, we discuss the theoretical concepts used by Xenakis in his creative process. Further, we examine how the algebraic organization of *Nomos Alpha* can be considered as an *abstraction* of the set-theoretical one used in *Herma*. We finally suggest to extend Xenakis' *outside-of-time/in-time* dichotomy by means of a third conceptual category: the "logical time."

1. Introduction

Iannis Xenakis is known as an extremely original and prolific composer, the author of masterpieces that had a profound impact on the music of his time, and the proponent of a number of novel approaches to musical composition. A trained engineer, having closely collaborated with the architect Le Corbusier, Xenakis was particularly fascinated by science and, more specifically, by mathematics. His lifelong interest in these matters deeply influenced his approach to musical composition, so much so that formal/mathematical considerations represent an integral part of his creative process. In the middle of the 1950s, he started with *stochastic music*, using probability distributions to shape large masses of sounds, and later applied aspects of game theory to music. By the end of the 1950s, he turned to algebra and logic. He called *symbolic music* the body of musical works that resulted from the latter approach.

Herma, for piano, and *Nomos Alpha*, for cello, are two such examples. Both are nowadays considered masterpieces and have become part of the repertoire of many pianists and cellists. These works were chosen as the subject of our study because of the wealth of details provided by Xenakis in his *a-priori* theoretical description of these compositions.

Xenakis has published numerous commentaries on his own musical output. For him, each one of his works "poses a logical or philosophical thesis" (Bois, 1966, p. 14). With *Herma* and *Nomos Alpha*, we have two such "theses" of equal relevance, who can only be analyzed within a more general discussion on Xenakis' symbolic music (Andreatta, 1997). As pointed out by the composer in his long interview with Balint Varga (Varga, 1996), there is a thread linking *Herma* to *Nomos Alpha*, having mainly to do with their formal organization. However, while in *Herma* the composer used amorphous sets, the relationships between them only depending on external set-theoretical operations (inclusion, intersection, union, complement, etc.), in the cello piece he used structured sets, i.e. collections of elements together with a binary operation such that the group axioms are satisfied.[1] Most of

[1] Let us recall that a *group* is a set G of elements together with a binary operation (written as $\cdot$) such that the four following properties are satisfied: (1) *closure*: $a \cdot b$ belongs to G for all a and b in G; (2) *associativity*: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ for all a, b, c belonging to G; (3) *identity*: there exists a unique element e in G such that $a \cdot e = e \cdot a = a$ for all a in G; (4) *inversion*: for each element a in G there exists a unique element a' in G such that $a \cdot a' = a' \cdot a = e$.

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Xenakis' theoretical constructions are documented in his book *Formalized Music* (Xenakis, 1992), where the discussion of *Herma* and *Nomos Alpha* (Chapters 5 and 6, respectively) ranges among the most comprehensive and detailed ones. Indeed, Xenakis' own description of these two pieces is close to providing formal *models* of their scores. Thus, based on his discussion, one can attempt to elaborate a computer program that re-creates essential parts of the musical score. The present paper is the account of such an attempt, and provides a discussion of the results thus obtained.

In order to clarify our object of study, a few comments should be made concerning the nature itself of Xenakis' theoretical descriptions. An important difference exists between the reconstruction of a score by implementing the composer's model for that score, and what is usually meant by "music analysis." While a strict definition of the latter cannot be provided here, let us just emphasize that the two do not necessarily meet. The composer can, for instance, introduce layers of analyzable and pertinent structure without necessarily being aware of it. Conversely, what the composer considers as pertinent might very well be entirely lost in the final result, either at the score level or at the level of the reception of the work. The latter is particularly the case with Xenakis, as will become evident later. To scrutinize his formal models is to analyze a specific – and important – part of his creative process. We believe that such considerations are pertinent in an actual analysis, yet such a claim is not the object of the present article. Rather, it is an insight into the composer's creative process that is primarily sought here.

To transcribe the compositional process into computer models is to adopt a particular perspective on Xenakis' theoretical discussion and its implications. Indeed, greater attention must be given to the strictly operational level, the one at which the actual transposition from the composer's speculative considerations about *symbolic music* into actual musical output is realized. The broader implications of these "mechanisms" and their musical relevance will be the focus of the next two sections, bearing on the composition of *Herma*, and subsequently on *Nomos Alpha*. In a later section, the processes relative to those two works will be brought together into a common perspective. Despite some crucial differences between the two pieces, the respective formal aspects reveal a number of common features. In addition, the compositional process behind *Nomos Alpha* seems to feature a higher level of abstraction compared with *Herma*. We also discuss the importance of the implementation with respect to this point, and suggest further developments of our computational approach.

2. Herma

2.1. The theoretical framework

2.1.1. The basic material

Xenakis describes *Herma* as a presentation in "sonorous symbols" – instead of "graphic symbols" – of three pitch sets together with the basic set operations, namely *union* (denoted as "+"), *intersection* (denoted as "·") and *complementation* (denoted by the superscript "⁻"). First, the composer considers a "referential set" that he calls *R*, "consisting of all the sounds of a piano" (Xenakis, 1992, p. 170). He then selects three pitch sets *A*, *B* and *C* among the elements of *R*. Now, these three sets, plus the reference set and their combinatorial potential constitute the basic material of the piece. With the well-known terminology introduced by Xenakis himself, this material clearly belongs to the *outside-of-time* domain.[2] The way this material will unfold in time is not yet specified. To avoid too much arbitrariness, Xenakis introduces what he calls a "knot of interest" (Xenakis, 1992, p. 173) and adds two organizational elements to his construction: The first, is a "finality," i.e. the aim towards which the entire piece should be oriented and that will constitute the final section. This set – that Xenakis calls *F* – is denoted as the black area in the Venn diagram of Figure 1.

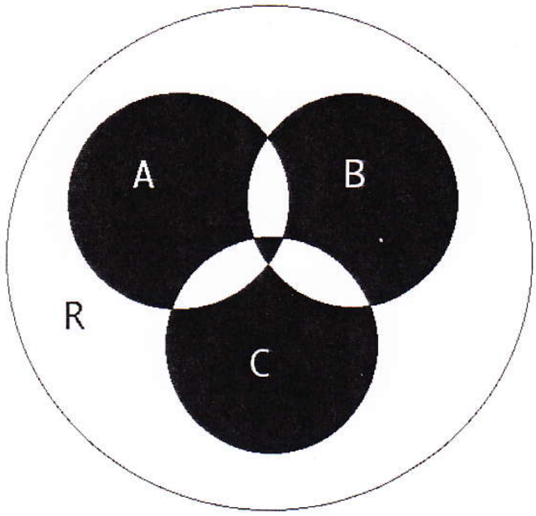

Fig. 1. The Venn diagram of the set *F*

The second element is a selection principle, such that among all sets that could possibly result, only a limited number is selected, together with a first embryo of its *in-time* organization. To this end, Xenakis notes that the set *F* can be algebraically expressed in two ways, as follows:

$$F = (A \cdot B + \overline{A} \cdot \overline{B}) \cdot C + \overline{(A \cdot B + \overline{A} \cdot \overline{B})} \cdot \overline{C}$$

and

$$F = \overline{A} \cdot B \cdot C + A \cdot \overline{B} \cdot \overline{C} + \overline{A} \cdot B \cdot \overline{C} + \overline{A} \cdot \overline{B} \cdot C$$

[2] The *outside-of-time* refers to any aspect of a work of music that can be formalized independently of time. Any other aspect, particularly if dependent on the time flow, belongs to the *in-time* domain. A 12-tone row, considered in its pre-compositional, theoretical state, is *outside-of-time* (although the composer seems to suggest the opposite), while a particular instance of this series in a score is *in-time*.

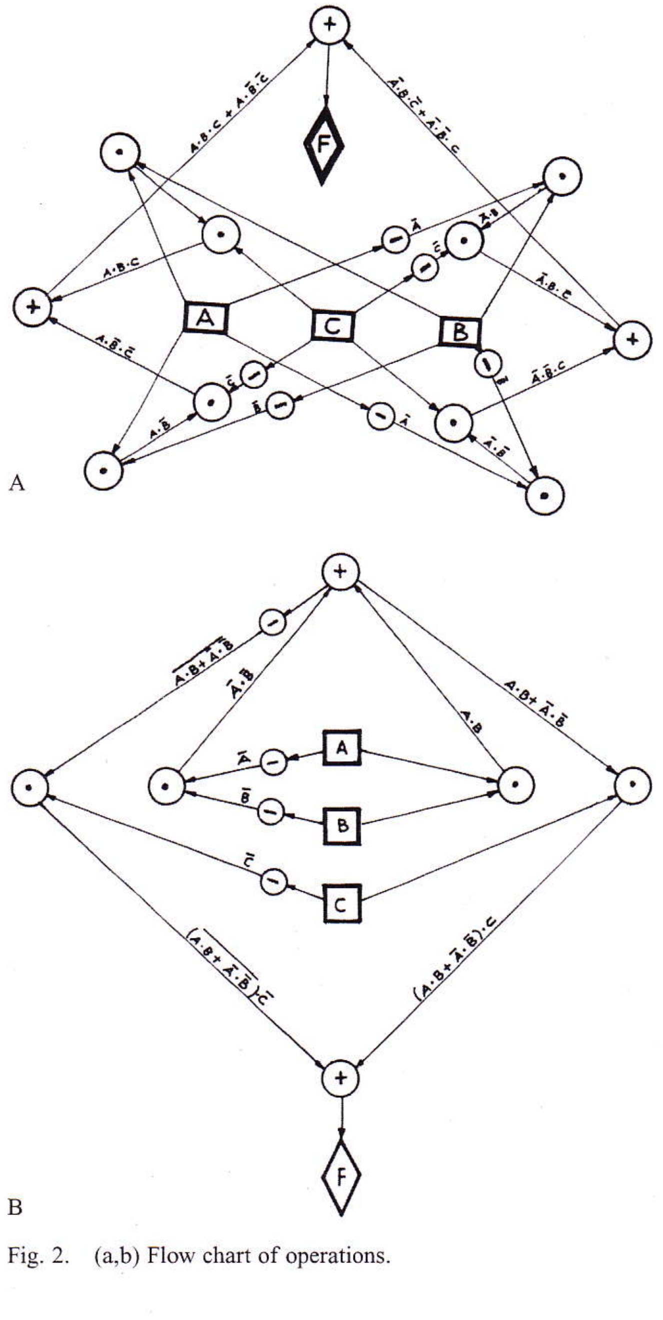

Fig. 2. (a,b) Flow chart of operations.

For each representation he draws a diagram of operations which, starting from sets *A*, *B* and *C*, outlines the sequence of the set-theoretic operations to apply in order to obtain the set *F* (Figure 2a and 2b). These diagrams can be equivalently described as determining specific series of sets. Each element of a given series is the result of one, and only one, set-theoretic operation either applied on sets that have appeared previously in the series, or applied directly on sets *A*, *B* or *C*.

2.1.2. *The "construction" of the piece*

From this point on, Xenakis enters what can be called the *construction* of the work, in the sense that he does not impose on himself any constraint of a "formal" nature anymore. Indeed, he observes that the first diagram is "more economical" while the second presents "more elegant symmetries," and finally takes the decision to work out his composition based on a "confrontation" between the two (Xenakis, 1992, p. 175).

To this aim, he defines two layers of different dynamics: *fff* and *f* for the one diagram; *ff* and *ppp* for the other. Each layer will carry a different sequence unfolding in parallel (Figure 3) and reaching set *F* "simultaneously" at the end of the piece. Each layer is itself divided into two sub-layers, again according to dynamics. One sub-layer (*fff* and *ff*) carries mutually disjoint sets whose union results in set *F*. In mathematical terminology, these sub-layers constitute a *partition* of the set *F*. The other sub-layer (*f* and *ppp*) carries all the remaining, intermediary sets, imposed by the sequence of operations illustrated by the diagram in Figure 3.

With this procedure, Xenakis has already left the *outside-of-time* domain in its strictest sense. The sequences thus obtained, however, are not yet sufficient to concretely determine the elements of his score. In order to achieve a transition he introduces two new elements, each corresponding to techniques he used extensively in previous works: the organization of the overall form by means of a graph, and the stochastic selection of elements.

The graph, with the four layers on the y-axis and time on the x-axis, specifies the relative position together with a time span during which a particular set is deployed. A short examination reveals three clearly distinguishable sections. In the first, the sets *R*, *A*, *Ac*, *B*, *Bc*, *C* and *Cc* are presented, in that particular order, providing an "exposition" of the main elements. In a second section, the two parallel series of sets are presented, as determined by the graph in Figure 2. This can be considered as the "development" section of the piece. The last part functions as a conclusive musical formula, and features the elements of the final set *F*.

2.1.3. *The stochastic elements*

Xenakis also describes how the sets are "transcribed" from their amorphous outside-of-time state into a specific in-time succession of pitches by stating that "there exists a stochastic correspondence between the pitch components and moments of occurrence" (Xenakis, 1992, p. 175). Although this could resemble a return to the techniques of *stochastic music*, Xenakis insists on distinguishing it. He explains that the stochastic elements in the composition of *Herma* solely serve the purpose of "demonstrating the elements of the sets" (Varga, 1996, p. 85) as opposed to being a means to "sculpting" sound masses. In other words, the introduction of randomness in the composition of *Herma* enables the transfer of the sets into the *in-time* domain, and at the same time rules out the emergence of any audible regularity that would contradict their amorphous (i.e. unstructured) nature..

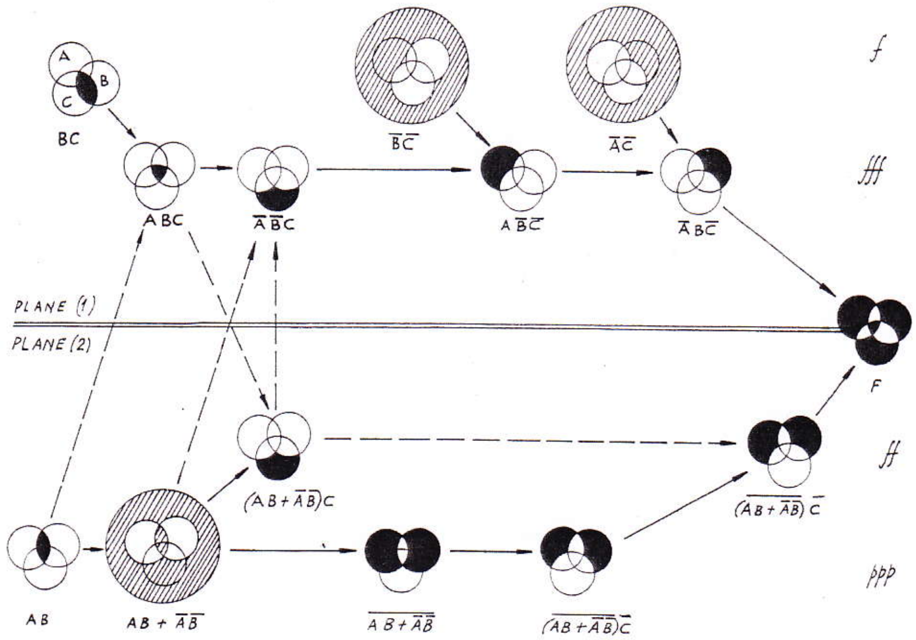

Fig. 3. Flow chart of the layered series.

In conjunction with this stochastic element, Xenakis also introduces contrasting "densities."[3] For each occurrence of a given set, its density is stipulated in the *in-time flow chart* (Figure 4). In addition, Xenakis also distinguishes between two modes in the sonic manifestation of sets. He names them "cloud" and "linear," but does not clarify how exactly these terms should be interpreted. Another even more important point he does not clarify is the one concerning the type of probability distribution used for the duration values as well as for the pitch selection.

In reading Xenakis' description of *Herma*, one gets the impression that his aim is to describe the piece entirely in theoretical terms, disregarding the actual preparation of the score. The next section focuses on the transcription of the composer's theory into a computer program. Several hypotheses had to be made and tested concerning the details of the stochastic aspects, before an "optimal" solution could be found. The resulting computer-generated "sonorous equivalent" of the theory provides a basis for discussing the extent to which *Herma* can be considered as a direct outcome of Xenakis' theoretical framework.

2.2. The computer implementation

2.2.1. Some general aspects

Models of sequentiality in music are generally partial orders represented by lattices showing temporal logic relations between musical units. Let us call *logical time* (Assayag, 2000) this particular representation level in the composition process. In *Herma*, the logical time structure is partly revealed by the chart in Figure 3. The in-time structure shown in Figure 4 is one of the many possible in-time manifestations. It is interesting to note that Xenakis finds it necessary to exhibit a logical time chart, clearly an intermediate between the *outside-of-time* and the *in-time* instances, while not explicitly identifying this *logical time* category in his paper. Nevertheless, it is precisely on this dichotomy between *logical time* and *in-time* instances that we have built our implementation of the model.

2.2.2. The "maquette" and the temporal blocks in OPENMUSIC

The implementation has been realized in the OPENMUSIC environment, a visual programming language for composers and musicologists developed at Ircam. For the encoding of *Herma*, a *maquette* object was used, that is, a container that displays information concerning different levels of temporal organization. At any of these levels, visual encoding of instructions is available. A recursive container structure, enabling the embedding of substructures into larger musical forms, is also possible, but this feature was not necessary for our purposes, here. Figure 5 shows the maquette corresponding to *Herma*. In essence, this maquette reproduces the *temporal flow chart* provided by the composer (however, set R – the beginning of the piece – was omitted). The rectangle blocks (called *temporal blocks* in OPENMUSIC) represent the

[3] In Xenakis' terminology, a *density* corresponds to a mean value, that is, an average number of events per unit of time, and not to the more technical notion of "probability density function."

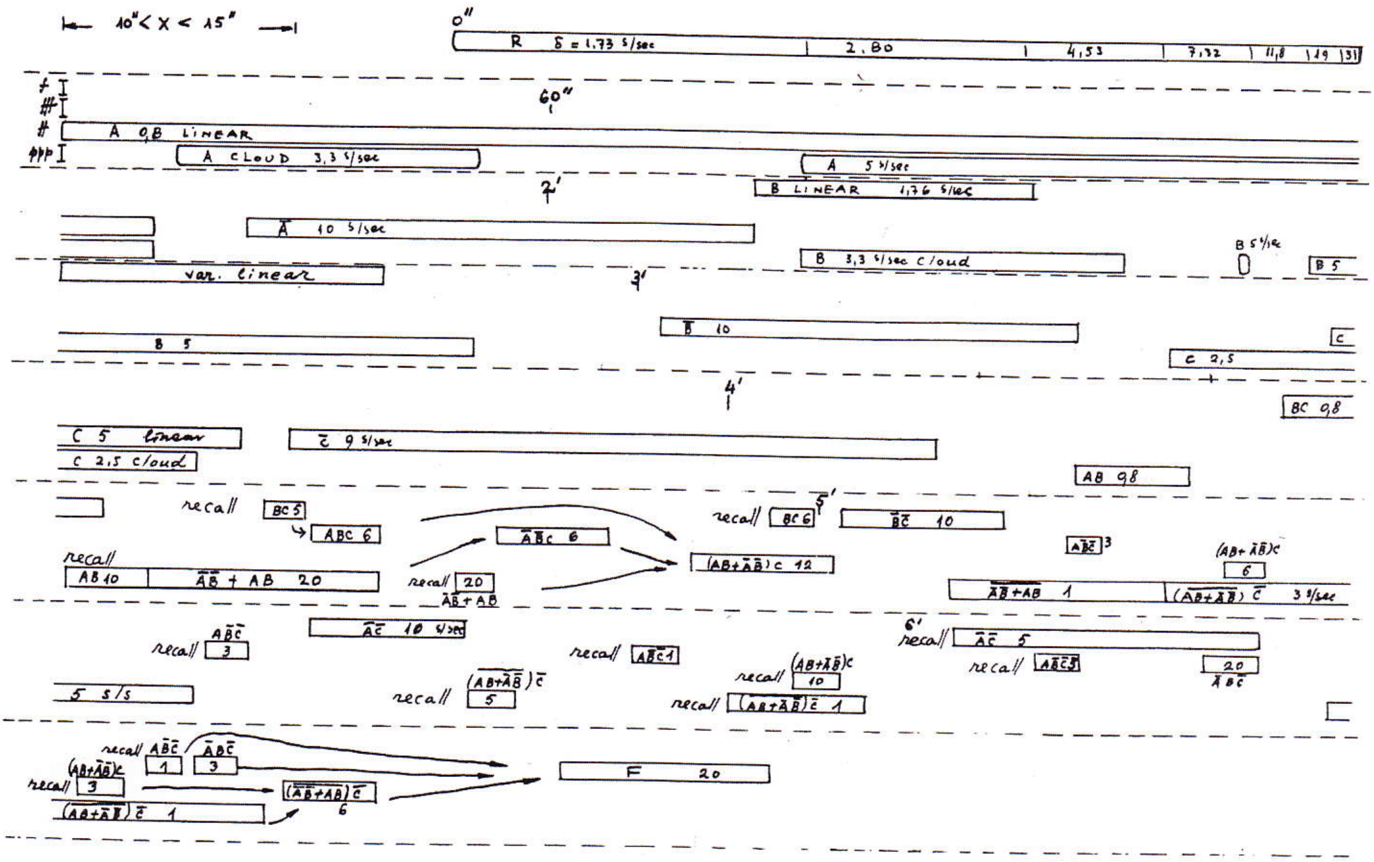

Fig. 4. *In-time* domain chart.

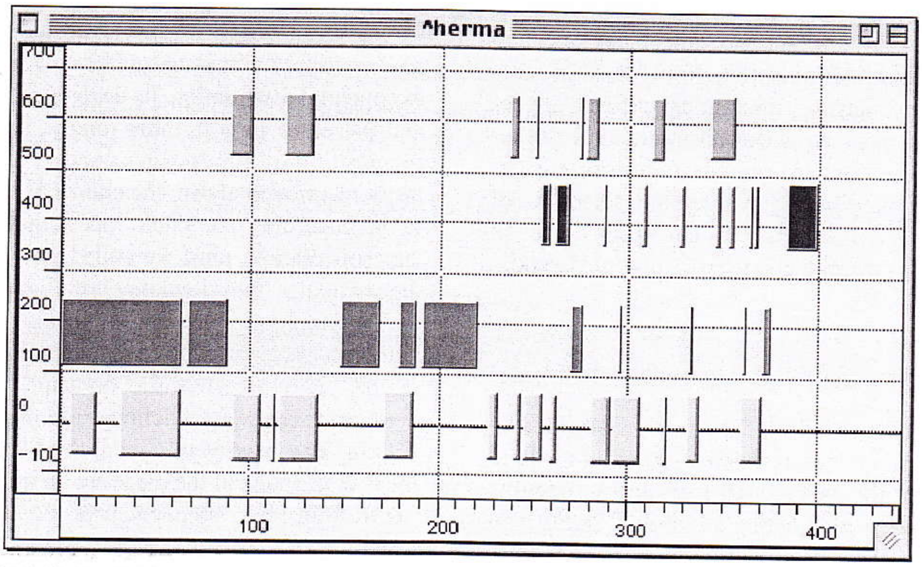

Fig. 5. The maquette of *Herma*.

sets, and the four layers correspond to those chosen by Xenakis.

A deeper look into the construction reveals the connections between the various blocks (see Figure 6). Here, the pattern of threads reflects the flow of information inherent in Xenakis' construction. Sets built earlier in the process are used again later for the computation of new sets. This idea is summarized in the flow charts shown in Figure 2, which are

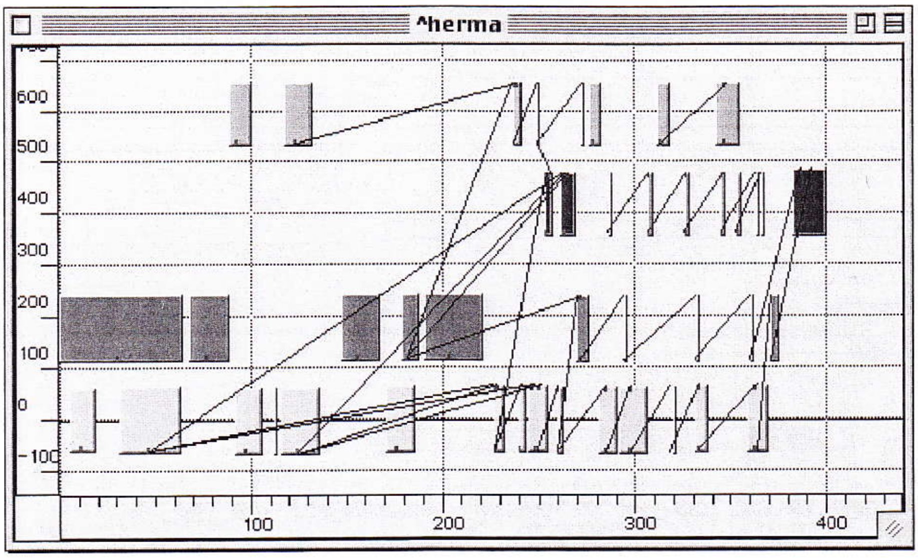

Fig. 6. Connection patterns in the maquette.

then only shaped as a logical time structure in Figure 3.[4] However, the pattern of threads also reflects a purely pragmatic goal: it allows us to avoid the redundancy of representing two separate series leading to the final set F, and that saves computation time.[5]

Figure 7 shows the typical content of a *temporal block.* This construct, called a *patch* in OpenMusic, is actually the equivalent of a computer program. It is encoded using visual rather than text code. The input arrow in the upper right corner represents the input data. In the case of *Herma*, the input data include the sets built up in previous blocks, needed to determine the set that will be active throughout the temporal block. The particular set operations utilized appear next. The other patch icons represent abstractions for other visual subprograms (not shown) aimed at determining, in accordance with a given density value, the selection of pitches within the considered set and their onset times. In short the subprograms govern the stochastic part of Xenakis' construction.

2.2.3. *The selection of probability distribution*

As pointed out earlier, there is little information provided by the composer concerning the stochastic aspects. For durations, we have chosen the exponential probability function, which is commonly used in modeling all kind of events happening at random time intervals with an average density (a typical example is the average time at which customers arrive at a counter). Considering that Xenakis had frequently utilized the exponential probability function for some of his earlier compositions, this lends itself quite naturally. The raw output of this probability function, however, requires an additional computation step. Indeed, events close together need to be aligned into "chords," and one should adopt a time granularity fitting with the complexity of the rhythms found in the music while eliminating the strictly continuous character of the raw output.

Our choice of the *ArcSin* distribution for the selection of pitches was of a more speculative nature, compared with the exponential distribution. Its density curve is flat in the middle and increases in both outer ranges, thus roughly emulating the position of a musician's hands. As with the time granularity mentioned above, the choice of the probability distribution governing the pitch was a matter of trial and error experimentation, until we ended up with a result reflecting the particular "physiognomy" of *Herma*.

The stochastic element in Xenakis' construction implies that each evaluation of the model yields a new manifestation in the pitch/intensity/time domain. It is safe to say that every instance bears a definite resemblance with the original *Herma*. The masses of sound, the long stretches of silence, the overall shape of the piece are all quite faithfully rendered. Yet the "impetus" so peculiar of the original work is clearly lost in the computer model. This leads to the question of whether any objective element can be found to corroborate this inherently subjective observation.

2.3. The "gap" between theoretical construction and musical realization

Two assumptions can be made, concerning the qualitative difference observed above. First, the "human touch" is of

[4] Some of the arrows in Xenakis' chart depicted in Figure 3 cannot be explained as precedence relations in a logical time structure. Their meaning is actually a mystery. Any suggestions would be welcome!

[5] Accordingly, the particular set of connections used in the implementation is not the only possible one.

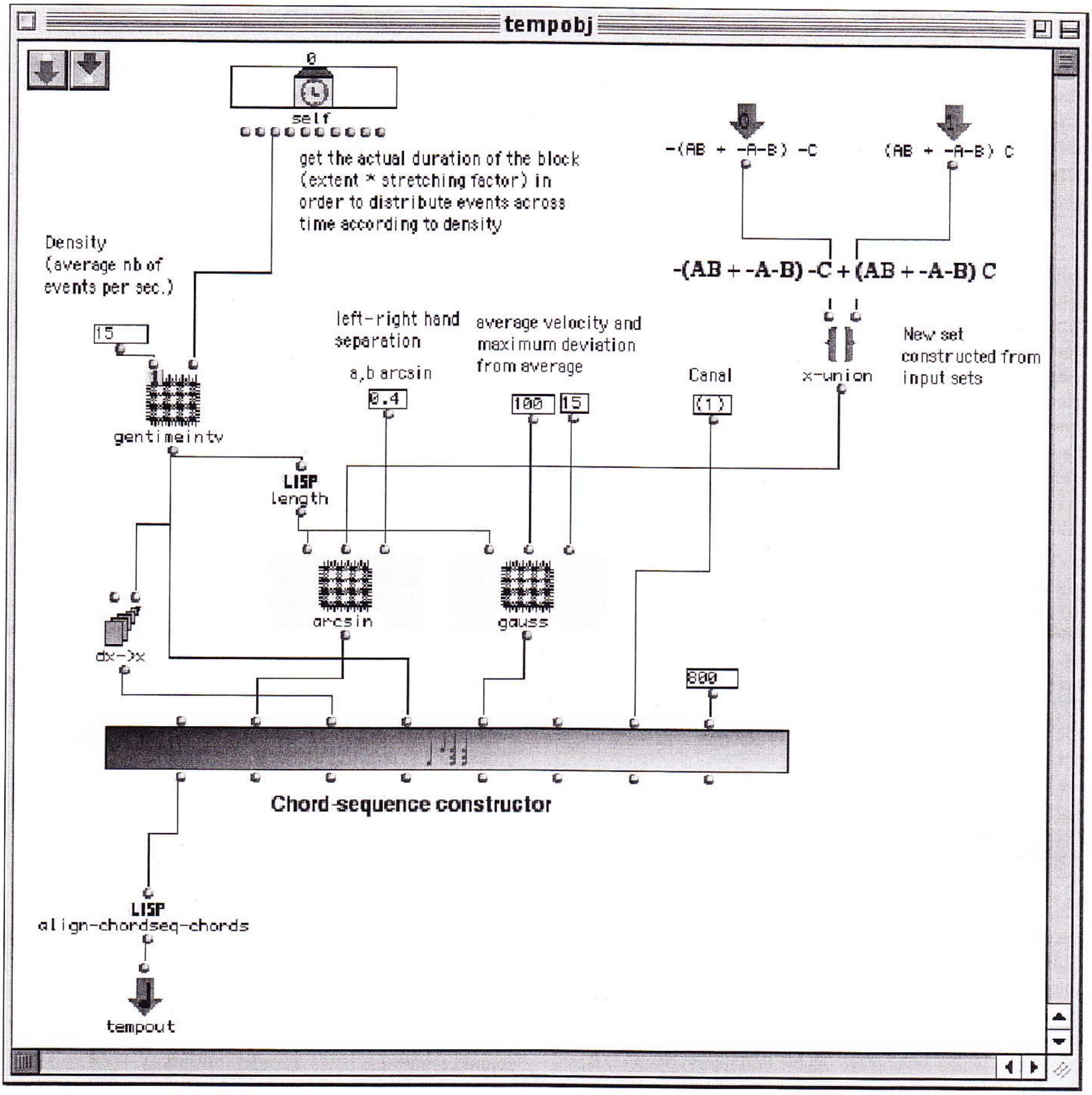

Fig. 7. Inside a temporal block.

course missing in the computer version, even though – in an attempt at humanizing the final result – we had the intensity levels modulated by a normal (Gaussian) distribution centered around the main intensity. Second, despite the effort put in carefully choosing the probability functions, the chosen functions might not be close enough to Xenakis' own. Both assumptions remain, of course, open to further discussion. However, we should also address ourselves to another point: a closer look at the score reveals that Xenakis made "corrections," deviating from the theory at several levels. In particular, we focus on two occurrences of these.

2.3.1. Interventions in the stochastic process

The passages at bars 30–31 and 136–138 are overtly in sharp contrast to the rest of the score. The one has almost a diatonic character. The other is a pianissimo buildup of held notes leading into the harmony of bar 138. In both cases, there is a sort of respite in the music's activity that contradicts the stochastic deployment of pitches. The former passage marks the end in the "exposition" of the reference set (bars 1–29). The latter marks the end of the complete first section in the piece (bars 1–132). Thus, both passages appear at special transition points in the development of the overall musical form. It seems quite clear that Xenakis has allowed himself to intervene in the stochastic mechanism in order to emphasize these articulations in the musical form.

Two more details add to this evidence. In the passage eventually leading to bar 136, the amount of pitches per unit of time increases, yet Xenakis' temporal flow chart does not stipulate such a "density crescendo." Moreover, Xenakis gives a clear direction to the pitch sequence: starting with bar 124, notes are first comprised within a relatively narrow pitch range, less than two octaves; then pitches shift towards the lower range, then move back upwards and finally broaden in their range before bar 136. This overall gesture, clearly "sculpting" the flow of pitches, is reminiscent of passages found in Xenakis' stochastic works, an aspect which is not accounted for in the theoretical description of *Herma*.

2.3.2. *Interventions in the selection of pitches*

Several commentators have pointed out that some of the pitches found in the score are foreign to the sets stipulated by the composer (Bayer, 1981; Gibson, 1994; Montague, 1995; Schaub, 2001; Wanamaker, 2001). In his detailed analysis, Bayer (1981) was the first to make such findings public. He suggested that, in some cases, such "corrections" are due to the fact that certain sets prescribed by the theory are too poor to "sustain" the musical flow. Indeed, for a set to be presented over an extended time span, with a relatively high density, it should include a large number of pitches as to avoid an undesired "stall" in the unfolding of the music. This is not enough to explain all of the exceptions found in the score, but it certainly applies to some passages.

For instance, a passage featuring quite a large number of such exceptions can be found in bars 167–172. According to the theory, about 80 pitches are supposed to be deployed here, but the set stipulated in the theory contains only 18 pitches, of which several are octave doublings (pitch classes C, C#, E*b*, B*b*, and B are not featured at all). To strictly obey the theory would imply numerous pitch repetitions naturally leading to the "stall" effect mentioned above.

Such observations provide a plausible, albeit partial, explanation of the shortfalls of the computer model in rendering the original score. Clearly, no refinement of the distribution could *by itself* reproduce the above mentioned "corrections." However, it would not be impossible to return to the computer model with the aim to "correct" it. The OPENMUSIC environment could easily allow us to introduce the additional constraints required. To do so, though, we would need a change of perspective, as the computer model would no longer be a reflection of Xenakis' theoretical description.

3. *Nomos Alpha*

3.1. The formal compositional process

Nomos Alpha is probably one of the most analyzed works in contemporary music. At the present there are at least four lengthy analyses (DeLio, 1980; Solomos, 1993; Vandenbogaerde, 1968; Vriend, 1981). To this list we may also add the composer's detailed description, in his book *Formalized Music* (Chapter 8, "Toward a Philosophy of Music"). The special length of these analyses can be viewed as a symptom of the difficulties found in the attempt to summarize the questions raised by this piece in few pages. Nevertheless, it seems not useless to offer the reader a concise discussion of mathematical aspects not developed in previous studies, and to draw some conclusions based on our implementation of the compositional process. We are aware that, in doing so, we disregard important aspects that resist formalization, as the sieve-theoretical pitch organization, and the question of the so-called "kinematic diagrams" by which Xenakis supposedly determined pitch-regions and playing techniques (*pizzicati*, *battuto col legno*, *pizzicati glissandi*, etc.).[6]

As already mentioned, we are interested in a more general issue, namely the process of abstraction leading from the amorphous sets of *Herma* to the more complex algebraic structures of *Nomos Alpha*. The group-theoretical conception behind the latter work utilizes mathematical group structures in two ways: (1) as an organizing principle for what Xenakis calls "sound complexes;" and (2) as the theoretical background in the construction of musical scales by means of the so-called "sieves."

3.1.1. *Abstract (or outside-of-time) sound complexes*

Figure 8 illustrates the eight prototypical "sound complexes" as described and graphically represented by Xenakis himself. The order of sound complexes is provided by means of a mathematical group, in this case the 24 rotations that transform a cube into itself. The eight sound complexes are attached to the eight vertices of the cube, such that every single rotation determines a permutation of the order of the sound complexes.[7]

[6] See Solomos (1993) for a discussion of these issues.

[7] We quickly discuss the way in which an element of the group of rotations determines a specific sequence of sound complexes. This is made possible by taking a reference cube (which is, in fact, the unitary element of the group) that provides the initial association between sound complexes and vertices. A rotation induces a permutation of the vertices of the reference cube, hence a particular sequence of sound complexes. Moreover, a given group element may be affected by a parameter (α, β or γ) which changes its function along the piece. The previous labeling of sound complexes, for example, only concerns the so-called β sections of the piece. We will come back to the structural role of the parameters in the discussion of the *in-time* process.

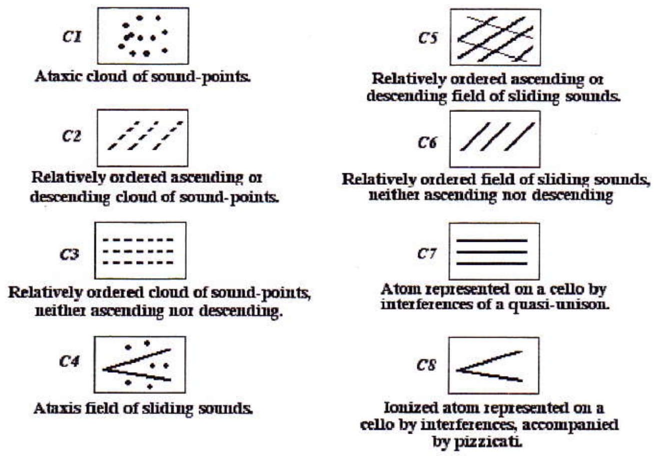

Fig. 8. The eight basic sound complexes.

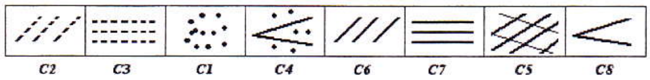

Fig. 9. The first sequence of eight sound complexes in *Nomos Alpha.*

Figure 9 shows the sequence of eight abstract sound complexes attached to the group transformation *D* that has been chosen as the starting point for the piece. Thanks to the group property, any combination of two elements remains in the set of rotations. In other words, the product of two rotations is still a rotation, as shown in the group table illustrated in Figure 10.

3.1.2. *The generalized Fibonacci process*

The closure axiom, together with the fact that the group of rotations is finite, enables the construction of Fibonacci sequences of rotations (Xenakis did not call them such, but we prefer to remain consistent with the mathematical terminology). The latter turn out to have a cyclic character. Indeed, selecting two given elements x_1 and x_2 of the group and applying the group operation "·" we obtain a sequence of terms $x_3, x_4, \ldots, x_i$ where $x_3 = x_2 \cdot x_1$, $x_4 = x_3 \cdot x_2, \ldots, x_{i+1} = x_i \cdot x_{i-1}$. That is, each element in the sequence (each rotation) is the *product* of the two previous ones, just as each term in a Fibonacci sequence of integers is the *sum* of the two previous terms.

	I	A	B	C	D	D2	E	E2	G	G2	L	L2	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12
I	I	A	B	C	D	D2	E	D2	G	G2	L	L2	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12
A	A	I	C	B	G	L	G2	L2	D	E	D2	E2	Q7	Q4	Q5	Q2	Q3	Q12	Q1	Q10	Q11	Q8	Q9	Q6
B	B	C	I	A	L2	E	D2	G	E2	L	G2	D	Q6	Q9	Q8	Q11	Q10	Q1	Q12	Q3	Q2	Q5	Q4	Q7
C	C	B	A	I	E2	G2	L	D	L2	D2	E	G	Q12	Q11	Q10	Q9	Q8	Q7	Q6	Q5	Q4	Q3	Q2	Q1
D	D	L2	E2	G	D2	I	C	L	E	A	B	G2	Q3	Q6	Q4	Q1	Q11	Q10	Q8	Q9	Q7	Q2	Q12	Q5
D2	D2	G2	L	E	I	D	G	B	C	L2	E2	A	Q4	Q10	Q1	Q3	Q12	Q2	Q9	Q7	Q8	Q6	Q5	Q11
E	E	L	G2	D2	B	L2	E2	I	A	D	G	C	Q11	Q5	Q6	Q8	Q7	Q9	Q2	Q12	Q3	Q1	Q10	Q4
E2	E2	G	D	L2	G2	C	I	E	L	B	A	D2	Q10	Q7	Q9	Q12	Q2	Q3	Q5	Q4	Q6	Q11	Q1	Q8
G	G	E2	L2	D	L	A	B	D2	G2	I	C	E	Q5	Q12	Q2	Q7	Q9	Q8	Q10	Q11	Q1	Q4	Q6	Q3
G2	G2	D2	E	L	C	E2	L2	A	I	G	D	B	Q9	Q3	Q12	Q10	Q1	Q11	Q4	Q6	Q5	Q7	Q8	Q2
L	L	E	D2	G2	A	G	D	C	B	E2	L2	I	Q2	Q8	Q7	Q5	Q6	Q4	Q11	Q1	Q10	Q12	Q3	Q9
L2	L2	D	G	E2	E	B	A	G2	D2	C	I	L	Q8	Q1	Q11	Q6	Q4	Q5	Q3	Q2	Q12	Q9	Q7	Q10
Q1	Q1	Q7	Q12	Q6	Q9	Q5	Q8	Q2	Q11	Q10	Q3	Q4	A	L2	D2	E2	L	B	I	G2	G	E	D	C
Q2	Q2	Q11	Q9	Q4	Q10	Q6	Q1	Q8	Q3	Q12	Q7	Q5	E	I	G	C	L2	D2	L	E2	B	D	A	G2
Q3	Q3	Q8	Q5	Q10	Q7	Q11	Q9	Q6	Q12	Q2	Q4	Q1	L2	G2	I	L	B	E2	D	A	E	C	D2	G
Q4	Q4	Q9	Q11	Q2	Q8	Q12	Q7	Q10	Q5	Q6	Q1	Q3	G2	A	D	B	E2	L	D2	L2	C	G	I	E
Q5	Q5	Q10	Q3	Q8	Q1	Q9	Q11	Q12	Q6	Q4	Q2	Q7	E2	E	A	D2	C	L2	G	I	G2	B	L	D
Q6	Q6	Q12	Q7	Q1	Q2	Q10	Q3	Q9	Q4	Q5	Q8	Q11	C	D	E	G	G2	I	B	L	E2	D2	L2	A
Q7	Q7	Q1	Q6	Q12	Q11	Q3	Q10	Q4	Q9	Q8	Q5	Q2	I	E2	L	L2	D2	C	A	E	D	G2	G	B
Q8	Q8	Q3	Q10	Q5	Q12	Q4	Q2	Q1	Q7	Q9	Q11	Q6	D	L	B	G2	I	G	L2	C	D2	A	E	E2
Q9	Q9	Q4	Q2	Q11	Q5	Q1	Q6	Q3	Q8	Q7	Q12	Q10	D2	B	E2	A	D	E	G2	G	I	L2	C	L
Q10	Q10	Q5	Q8	Q3	Q6	Q2	Q4	Q7	Q1	Q11	Q9	Q12	G	D2	C	E	A	D	E2	B	L	I	G2	L2
Q11	Q11	Q2	Q4	Q9	Q3	Q7	Q12	Q5	Q10	Q1	Q6	Q8	L	C	L2	I	G	G2	E	D	A	E2	B	D2
Q12	Q12	Q6	Q1	Q7	Q4	Q8	Q5	Q11	Q2	Q3	Q10	Q9	B	G	G2	D	E	A	C	D2	L2	L	E2	I

Fig. 10. Table of 24 rotations of the cube into itself (Xenakis' own notation).

Some mathematical properties of this generalized Fibonacci process are of real interest, and may shed some light on apparently arbitrary decisions made by the composer. Firstly it turns out that this process always ends with a *loop*. In other words, starting with two group elements *X*, *Y* and constructing a sequence with the Fibonacci method, we necessarily find the same elements *X*, *Y* in the same order after a *finite* number of steps. This shows the inherently cyclic nature of the process, which strictly depends on the character of the given group. Secondly, loops may have different lengths, where *length* means the total number of iterations in the Fibonacci process that are necessary to end up with a loop. It can be shown mathematically that the Fibonacci process can never cover all 24 elements of the group: the maximal length is 18 and the largest number of different elements inside a loop is 13. We will call this number the *degree* of the loop. In other words, only 13 of the 24 group elements may be selected by a Fibonacci process giving loops of maximal length (18 iterations).

Xenakis makes use of the following loop, obtained by 18 iterations of the Fibonacci process (starting with elements *D* and *Q12*): *D* → *Q12* → *Q4* → *E* → *Q8* → *Q2* → *E2* → *Q7* → *Q4* → *D2* → *Q3* → *Q4* → *L2* → *Q7* → *Q2* → *L* → *Q8* → *Q11* → . . .[8]

The overall structure of the piece can now be easily summarized, taking the Fibonacci loop as the main skeleton, and inserting non-structured sections ("intermezzi") every third loop element. We will not take into account the "intermezzi," for all authors agree that they are completely independent from the group-theoretic mechanism (on the interrelationship between these two layers in *Nomos Alpha*, see DeLio, 1985). Denoting the group transformations with X_i (i = 1, . . . , 18), and the "intermezzi" with I_j, the piece can be segmented in the following way:

$$[X_1 \to X_2 \to X_3 \to I_1] \to [X_4 \to X_5 \to X_6 \to I_2] \to \ldots \to [X_{16} \to X_{17} \to X_{18} \to I_6]$$

3.1.3. Temporal (or in-time) sound complexes

It should be observed that the sound complexes associated to the group elements have to be considered as *outside-of-time* musical structures. They become *in-time* objects only when three further parameters (density, intensity and duration) are taken into account. Densities, intensities and durations in each sound complex are determined by the group of rotations of an auxiliary cube. As in the case of the abstract sound complexes, Xenakis makes use of an additional parameter (α, β or γ) in order to increase the variability of densities, intensities and durations of the sound complexes. In other words, there are eight musical objects (i.e., in-time sound complexes) for each parameter, which gives 8 × 3 = 24 *different* musical objects. Following Xenakis' notation, we will denote the latter with K_i. As in the case of abstract sound complexes, Xenakis changes the parameter every third rotation, and does so in a cyclic pattern: $[\beta \to \gamma \to \alpha \to \beta \to \gamma \to \alpha]$. Observe that this operation, too, can be considered in the light of group theory. In fact, α, β and γ can be associated to the vertices of a triangle; six successive rotations of 120° around the center produce the cycle that provides the order of the different sections of the piece.

The table in Figure 11 lists the characteristics of the musical objects K_i, as dependent on parameters α, β and γ (see Xenakis, 1992, p. 227). For example, if we consider the first sound complex in the score of *Nomos Alpha*, which is K_2 with parameter β, the table has these values for it: density = 0.5 (events/sec), intensity = *fff* and duration = 4.5 sec. The process of attaching an abstract sound complex C_i to the physical characteristics provided by a given K_j is governed, once again, by the group of rotations of the cube. Each rotation induces a permutation of the eight vertices of the cube, hence a given ordered sequence of elements K_j. Using the same Fibonacci process that we have described above, a new loop is constructed, this time providing the logical temporal ordering of the different concrete musical objects K_j. Note that this second loop has the same characteristics as the first one, i.e. it has maximal length (18) and maximal degree (13).

3.1.4. Sieve theory and Fibonacci process in the construction of musical scales

Before turning to our computer-aided model, we should mention a third Fibonacci process in *Nomos Alpha*. It was used for the pitch selection by means of the so-called "sieve-theory." According to Xenakis, the latter "annexes the congruences modulo z and is the result of an axiomatic theory of the universal structure of music" (Xenakis, 1965). In *Nomos Alpha*, Xenakis makes use of the group Z^*_{18} which consists of the set of integers smaller than 18 and relatively prime to 18, together with the multiplication (modulo 18). As with the group of rotations of the cube, it is possible to create loops starting with two given elements *a* and *b*. The third element in the loop, *c*, is the product of *a* and *b*, the fourth is the product of *b* and *c* and so on. By taking the starting elements *a* = 11 and *b* = 13 we have the following loop of period 23:

11, 13, 17, 5, 13, 11, 17, 7, 11, 5, 1, 5, 7, 17, 11, 7, 5, 17, 13, 5, 11, 1, 11, . . .

These numbers are used by Xenakis as modules for a sieve representing a musical scale which is, in the composer's mind, "not too symmetric (regular) nor too empty" (Vriend,

[8] Xenakis seems to be interested in other loops as well, as he offers their graphical representations (Xenakis, 1992, p. 225). This raises the question concerning the relevance of a parametrized model of this compositional process, where one may change the initial conditions and explore the potentialities hidden in the system. We will stress this point in the final section, discussing musicological implications of our model for *Nomos Alpha*.

$\kappa^{\alpha}_{1} = 1 . mf . 2 = 2.mf$	$\kappa^{\beta}_{1} = 0.5 . mf . 2 = 1.mf$	$\kappa^{\gamma}_{1} = 1 . mf . 2 = 2.mf$
$\kappa^{\alpha}_{2} = 1 . fff . 4.5 = 4.5. fff$	$\kappa^{\beta}_{2} = 0.5 . fff . 4.5 = 2.25. fff$	$\kappa^{\gamma}_{2} = 1 . fff . 2 = 2. fff$
$\kappa^{\alpha}_{3} = 2.5 . fff . 4.5 = 11.25. fff$	$\kappa^{\beta}_{3} = 5 . fff . 4.5 = 22.5. fff$	$\kappa^{\gamma}_{3} = 4.0 . fff . 4.5 = 18.0. fff$
$\kappa^{\alpha}_{4} = 2.5 . mf . 2 = 5.mf$	$\kappa^{\beta}_{4} = 5.0 . mf . 2 = 10.0.mf$	$\kappa^{\gamma}_{4} = 4.0 . mf . 2.0 = 8.0.mf$
$\kappa^{\alpha}_{5} = 1.5 . f . 2.62 = 3.93.f$	$\kappa^{\beta}_{5} = 1.08 . f . 2.62 = 2.83.f$	$\kappa^{\gamma}_{5} = 2.0 . f . 2.62 = 5.24.f$
$\kappa^{\alpha}_{6} = 1.5 . ff . 3.44 = 5.15.ff$	$\kappa^{\beta}_{6} = 1.08 . ff . 3.44 = 3.72.ff$	$\kappa^{\gamma}_{6} = 2.0 . ff . 3.44 = 6.88.ff$
$\kappa^{\alpha}_{7} = 2.0 . ff . 3.44 = 6.88.ff$	$\kappa^{\beta}_{7} = 2.32 . ff . 3.44 = 7.98.ff$	$\kappa^{\gamma}_{7} = 3.0 . ff . 3.44 = 10.32.ff$
$\kappa^{\alpha}_{8} = 2.0 . f . 2.62 = 5.24.f$	$\kappa^{\beta}_{8} = 2.32 . f . 2.62 = 6.08.f$	$\kappa^{\gamma}_{8} = 3.0 . f . 2.62 = 7.86.f$

Fig. 11. Table of temporal musical complexes K_i.

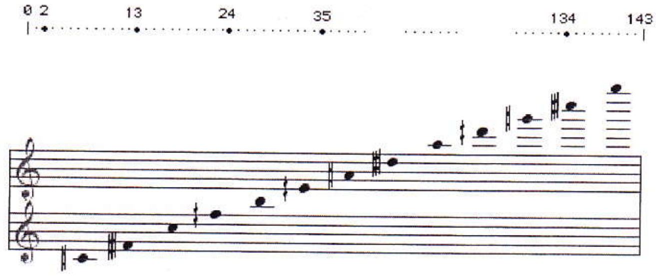

Fig. 12. Musical equivalent of the sieve B.

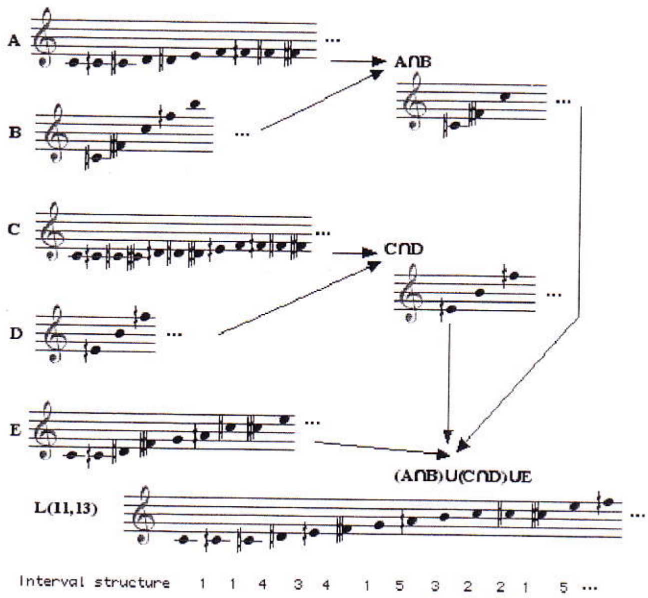

Fig. 13. The full sieve-theory process, with one possible musical realization.

1981, p. 78). The initial sieve is provided by the following set-theoretical expression:[9]

$$L(11,13) = (\mathrm{A} \cap \mathrm{B}) \cup (\mathrm{C} \cap \mathrm{D}) \cup \mathrm{E}$$

where:

$\mathrm{A} = (13_3 \cup 13_5 \cup 13_7 \cup 13_9)^c$
$\mathrm{B} = 11_2$
$\mathrm{C} = (11_4 \cup 11_8)^c$
$\mathrm{D} = 13_9$
$\mathrm{E} = 13_0 \cup 13_1 \cup 13_6$.

The symbol a_b means that we take the set consisting of the elements b, $b + a$, $b + 2a$, etc. (modulo a given integer, n). For example the set B gives the numbers 2, 13, 25, etc. A sieve defines a musical scale once a beginning note is associated with the number 0 and once the unitary step is replaced by a given (tempered) interval. Figure 12 shows a musical transcription of the sieve B (modulo 143) with origin 0 = middle C, and unit step = quarter-tone. The full process leading to the construction of the set-theoretic expression $L(11, 13)$ is detailed in Figure 13.[10] The intervallic structure clearly shows how set-theoretical operations operate on locally periodic structures in order to break the symmetric character of the final musical scale. Note that the result is crucially dependent on the order of set-theoretic operations. In this respect, the composer's original sieve-expression (Xenakis, 1990, p. 230) has no order specification, and that may engender some confusion.

3.2. Implementation of the compositional process

One of the main characteristics of our implementation model of *Nomos Alpha* is the graphical representation of the group-theory process, together with a greater emphasis on interactivity. As with *Herma*, the implementation was realized in OPENMUSIC. In this case, however, we developed a special, three-dimensional representation, helping us visualize all possible group rotations. This enables the transformation of Xenakis' static group table into a highly dynamic object, where one may see, for each element, the corresponding rotation of the cube (with respect to a particular axis of symmetry) as well as the permutation induced by such a rotation.

[9] Note that basic set operations are now differently written, i.e., "∪" (union), "∩" (intersection) and "c" (complement).
[10] We do not discuss how Xenakis practically attaches musical scales to the abstract sieve expression L(m,n). We can only mention that he used many *metabolae* (i.e., transformations), by means of which he would, for example, attribute "different notes to the origins of the sieves constituting the function" (Xenakis, 1992, p. 230). The reader will find an excellent discussion of Xenakis' sieve-theory, as utilized in *Nomos Alpha*, in (Solomos, 1993).

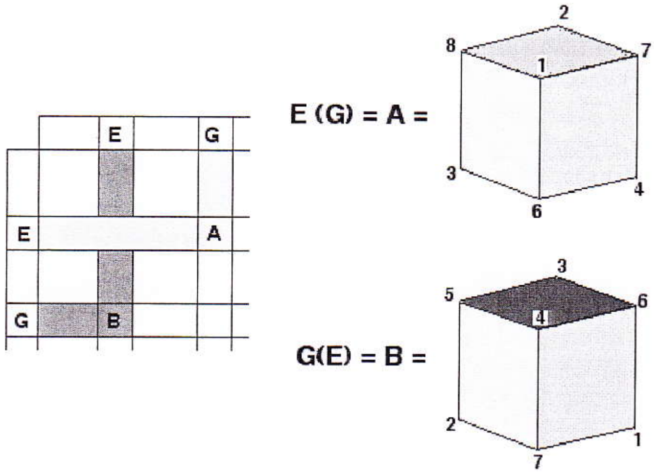

Fig. 14. Two elements of the non commutative group.

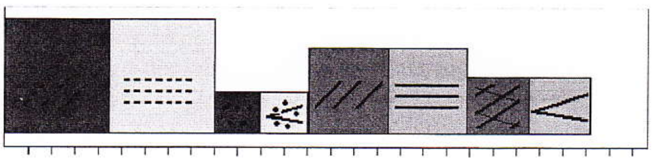

Fig. 15. Abstract sound complexes associated with the group element D.

3.2.1. *Group of rotations and outside-of-time/in-time sound complexes*

Figure 14 shows an example of one of the 24 possible rotations of a cube into itself, namely rotation A (180° around the vertical axis of symmetry). In this case, that rotation is obtained as the product of the two transformations E and G (respectively 120° around the axis passing through the vertices 7 and 3 of the unitary cube and 120° around the axis passing through the vertices 2 and 6). To be noted that the group is not commutative, in other words the product of E and G is different from the product of G and E (which is in fact B).

Let us now briefly examine the beginning of the piece, to see how abstract sound complexes are transformed into temporal musical objects by means of a given group element. Consider rotation D. This induces a permutation which affects the abstract sound complexes in the way illustrated in Figure 15.

Note that there are some differences with the *Herma* implementation. In particular, the shade of each block now depends on the density value. Darker blocks correspond to a lower density in the sound-object. The intensity is represented by the height of a block. Of course the duration of the sound-object is represented by the length of the block. Similar to what happens with the sound complex itself, here too the choice of a different parameter gives very different results in terms of density, intensity and durations. Figure 16

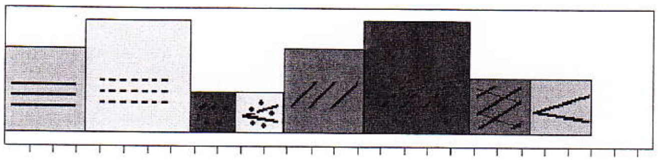

Fig. 16. In-time sound complexes associated to D with parameter α.

shows the result of group operation D using α instead of β (the latter was Xenakis' choice).

It must be stressed that the change of parameters at every third group operation, may be seen as one of the additional strategies used by the composer in order to compensate for the impossibility of using all 24 different permutations provided by the group of cube rotations. The cycle of those parameters may be easily changed in order to test to what extent the musical results are affected within any given loop structure. For example, despite Xenakis' efforts in keeping the system under control, it turns out that one and the same group element is associated to both complexes C_i and K_j for all score sections labeled with the parameter β. This means that during the piece, there will be two sequences of eight musical objects having the same properties in terms of density, intensity and durations (this is easily checked by means of our implementation).

3.2.2. *Generality and singularity of the Fibonacci process*

From a more analytical perspective, the OpenMusic implementation offers a general parametrized model of the compositional process with strong connections between macro- and micro-structures. This interplay between different abstractions of the process is one of the most interesting aspects of a piece that, surprisingly enough for a contemporary musical work, poses no problems of segmentation: blocks are easily recognizable, in the score and in sound analysis alike.[11] But what is pointed out by our implementation is the great generality of the Fibonacci process, operating at many different levels in this music, from the logical organization of *outside-of-time* sound complexes to their

[11] We do believe that in *Nomos Alpha*, like in other music based on algebraic methods, group transformations also have a cognitive and perceptual relevance that demands to be studied more accurately. In fact, Xenakis insisted many times on the relevance of the group structure for music not just from an operational point of view, but also from a cognitive perspective. In an unpublished article where he retraced the evolution of his compositional ideas since the stochastic music period, Xenakis stressed the necessity for a composer to delve more deeply into the mental processes of music: "music, as our universe indeed, is plunged into the idea of recursion, of more or less faithful repetition, of symmetry, as well as in-time and outside-of-time. For that reasons one finds group structures almost everywhere" (Xenakis, 1983). We thank *Les Amis de Xenakis* for making this text available.

practical realization into temporal musical objects. We already mentioned the question of other possible loop solutions for *Nomos Alpha*, a problem that was of high interest to Xenakis, although sometimes he could not control the sheer complexity hidden in a generalized Fibonacci process. Our implementation enables one to exhaustively study the range of possibilities inherent to the system, comparing all of them with Xenakis' own solutions. Concerning the system loops, we could see that their lengths and degrees are strongly limited by the group type that the composer used for his piece. As we said, maximal length and maximal degree of the loop are strictly connected with Xenakis' variation principle, aiming at avoiding repetitions in the type and order of the sound complexes. Under a mathematical perspective, these characteristics are statistically relevant, considering the full range of all possible loop solutions. In other words, there are 216 loops of length 18 and degree 13 over a universe of 576 possible loops, which means that almost half of all possible loops are of the same type as those used by the composer. This leads to the question of whether the formal structure of the piece, clearly divided into 18 sections (each consisting of eight sound complexes), was an *a-priori* compositional decision, or, as we suggest, a direct consequence of the underlying Fibonacci process.

4. Towards a unifying perspective of the formal compositional process: the notion of abstraction

So far, we have discussed the techniques involved in the composition of *Herma* and *Nomos Alpha* separately. At the operational level (that has so far been emphasized, in the present article), the connection between the two pieces seems to be not really straightforward. Indeed, whereas the composition of *Herma* involved unstructured sets and stochastic procedures, the composition of *Nomos Alpha* involved "sound complexes" distributed according to the permutations induced by a group of transformations. One could be tempted to say that both pieces constitute, at the very least, two quite contrasting instances of symbolic music. We would like to address, now, a notion hopefully capable of offering a unifying perspective.

There are many ways of defining this concept in mathematical as well as more general, philosophical terms. However, we consider that modern mathematics can provide an excellent theoretical perspective for discussing this notion with respect to works such as *Herma* and *Nomos Alpha*. According to the French mathematician Jean Dieudonné, the twentieth-century notion of "mathematical structure" stems from the fact that *relations* between objects have dramatically become more prominent and finally replaced considerations as to the *nature* of the objects (Dieudonné, 1987). The main concept involved is that of *abstraction*. In fact, most of the techniques used in composing *Nomos Alpha* can be considered as abstractions of strategies used in composing *Herma*.

4.1. From amorphous to structured sets

A first common element is the *a-priori* combinatorial potential of basic material: sets in the case of *Herma*, "sound complexes" in the case of *Nomos Alpha*. In both cases, a mathematical process helps reduce their proliferation, but in two slightly different ways. In *Herma*, privileged set-theoretic relations exist that operate on musical objects through the so-called "knot of interest." By linking the boolean expression of the set F to the flow-charts of set operations, the composer obtains two series of sets of manageable length. Note that the "knot of interest" only affects the external relations between musical sets, which in themselves remain unstructured and amorphous. In *Nomos Alpha*, a different process takes place. At one level, by introducing the group of the cube rotations as a means to generate permutations, the composer reduces the number of possible rearrangements of eight elements, from a staggering $8! = 40320$ to a small collection of 24 possibilities. At a second level, the group process is applied to the sound complexes themselves, in such a way that the musical objects become structured collections of elements together with inner relations. This is also the case of the sieve-theoretical constructions although the algebraic group is of a different nature. Nevertheless, a sieve is nothing but a family of set-theoretical operations with the additional property that the resulting object naturally exists in a conceptual universe which turns out to be a *structure* in the strict mathematical sense.[12] There are no sieves in *Herma*, which is why we can speak of an abstraction process taking place from the set-theoretical universe of *Herma* to the algebraic one of *Nomos Alpha*.

4.2. From the golden section to generalized Fibonacci sequences

In *Herma* the combining of a set-theoretical expression with a flow-chart of operations ("knot of interest") imposes a given sequence of sets. In principle there are several possible sequences, but once the starting operation is chosen the sequence is completely determined. The same phenomenon takes place in *Nomos Alpha*, but in more abstract terms. Here the composer develops a group-theoretical mechanism that imposes a specific ordering within the family of all possible group transformations. This ordering acts in a similar way as above, in the sense that the initial conditions completely determine the sequence of abstract events. In this case, however, Xenakis utilizes a generalized notion of the classi-

[12] This cyclic group structure is now commonly used by musicologists working in the field of music representation and formalization. It must be stressed that Xenakis has been historically one of the first composers to analyze the relevance of the concept of cyclic groups in music. Note that this idea has been developed independently by the American theorist and composer Milton Babbitt, who had a great influence on the *set-theory* approaches on music analysis and composition.

cal Fibonacci process, which is applied to group elements instead of integers.

We have seen that Xenakis creates sequences of rotations simply choosing two elements of the group and combining them according to a Fibonacci law. This process affects the *in-time* domain when the same operation is applied on abstract sound complexes featuring specific sonic characteristics, like intensity, density and, notably, duration. The very abstract character of the Fibonacci process in *Nomos Alpha* raises the problem of what would mean, in such a context, the concept of *golden section* that has a more concrete meaning in the case of *Herma*. In fact, the ratio between two successive density values appearing in the opening of the piece (1.73, 2.80, 4.53, 7.32, etc.) all correspond to the golden mean. We should note that the golden section can be found in numerous works by Xenakis.[13]

4.3. Abstraction levels in the outside-of-time/in-time dichotomy: the "logical time"

As in the case of the set-theoretical flow-chart organization used in *Herma*, the use of Fibonacci sequences in *Nomos Alpha* actually leaves the *outside-of-time* domain, and naturally belongs to a third, intermediate, temporal category that we called the *logical time*.

To use Xenakis' terminology, sets as well as "sound complexes" belong to the *outside-of-time* domain. Their transition into the *in-time* domain is actually not obtained directly, but through an intermediary domain that we have called, above, *logical time*. Xenakis does not directly discuss such a category – except perhaps in passing, in a remark concerning temporal succession (Xenakis, 1992, pp. 157, 160).[14] A final examination of the compositional mechanisms reveals that, in *Herma* just as in *Nomos Alpha*, Xenakis first introduces a logical order of succession, before rendering it in the *in-time* domain. The algebraic nature of the logical temporal process in *Nomos Alpha*, that operates in two different levels (sound complexes organization and sieve-theoretical structuring of pitch materials) clearly represents an instance of the abstraction that links the two compositional processes.

5. Conclusion

As mentioned in the Introduction, our computational models cannot be considered as an *analysis* of these musical works, at least not in the meaning usually attached to that word in traditional musicology. Nevertheless, the analytical relevance of the computer-aided model is evident if we agree that a musical work is a "field of potentialities," only a small part of which comes to be actually realized in a given piece. This is particularly clear in *Nomos Alpha*. The implementation makes evident that the special loops Xenakis chose as the skeleton for the macro-form of the piece, are not only the most interesting ones, in terms of length and degree, but they also are the most frequent ones built into the system he had set up for himself. This suggests that the macro-form of *Nomos Alpha*, often considered as a degree of freedom of the composer, is probably one of the greatest constraints imposed by the system itself. Concerning *Herma*, we emphasized the gap between Xenakis' theoretical description and the final score. The fact that the composer frequently used stochastic distributions for selecting the musical material shows that the implementation is a necessary step toward a deeper discussion on the possibilities of the system.

Despite some practical differences in the implementation process of *Herma* and *Nomos Alpha*, our approach involved the use of the computer in both cases, not as a mean of confirming or refuting Xenakis' theories, but as a basically heuristic tool.[15] Of course, it could be argued that our conclusions concerning the abstraction process from *Herma* to *Nomos Alpha* could have been proposed without computer experiments. Nonetheless, the modeling gave us a chance to explore a number of common points at different levels of abstraction.

The fact that no other work in Xenakis' repertoire can be as straightforwardly linked to a theoretical description of its genesis does not mean, we believe, that the approach we propose could not be applied to, say, *stochastic* or *strategic* music. In other words, we consider that computer-aided analyses are extremely useful heuristic tools that can provide a new approach in musicological research. They make it easier to discuss more objectively the theoretical aspects of the formal compositional process together with its actual musical realization.

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[13] See, e.g., Baltensperger (1995).

[14] Xenakis also discusses the idea of a third category, besides *outside-of-time* and *in-time*: that of the *temporal*. The latter is however distinct from our *logical time*.

[15] Nonetheless, we could use the model with different goals in mind, provided that the implementation takes into account all the pertinent parameters. Solomos (1993, p. 504) suggests that a detailed analysis of *Nomos Alpha* may follow several orientations. For example, it could be possible to rewrite the piece replacing Xenakis' own deviations from the theory with the "correct" data. It could be possible to compare two versions of the piece, the first having the same structure but a different material, and the second having the same material but a different structure. For us, a more interesting experiment would be to produce several instances of *Nomos Alpha* adopting *loop* solutions other than Xenakis' but having the same general characteristics as those that he proposed.

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