

SIMULATION OF A VACUUM-TUBE PUSH-PULL GUITAR POWER AMPLIFIER

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ABSTRACT

Power amplifiers play an important role in producing of guitar sound. Therefore, the modeling of guitar amplifiers must also include a power amplifier. In this paper, a push-pull guitar tube power amplifier, including an output transformer and influence of a loudspeaker, is simulated in different levels of complexity in order to find a simplified model of an amplifier with regards to accuracy and computational efficiency.

1. INTRODUCTION

Guitar power amplifiers have a big effect for guitar sound. Power vacuum-tubes have characteristic nonlinear distortion [1]. Furthermore, compared to semiconductor amplifiers, tube power amplifiers have different circuit topology that brings other effects, such as typical frequency response and signal compression when a large input signal is supplied [1]. Moreover, output transformers play an important role in the output signal generation. The output signal is distorted by nonlinear hysteresis of a transformer core and frequency response of the output transformer [2].

Two types of topologies are used in tube power amplifier construction. Single ended power amplifiers consist mainly of one power tube, an output transformer, and resistors and capacitors. They provide asymmetrical limiting of the output signal that creates foremost even order harmonic components in the output signal spectrum. Push-pull power amplifiers have more complex topology. They consist of a phase splitter, two or four power tubes that processes opposite half-waves of the signal and output transformer that sums contributions from opposite power tubes. The push-pull amplifiers offer a symmetrical transfer function that creates odd order harmonic components in the output signal spectrum [3].

A single-ended power amplifier is simulated in [4] using wave digital filters. The model of the power amplifier contains a triode, a linearized model of the output transformer and a linearized model of a loudspeaker recomputed to its electrical equivalent. This model was improved in [5] where the complex nonlinear WDF model of the output transformer is used. A single-ended power amplifier with a pentode tube is discussed in [6]. This model also contains a linearized model of the output transformer but the loudspeaker is replaced by a constant load. Nevertheless, guitar tube amplifiers mostly contain a push-pull power amplifiers with two or four tubes (pentodes EL34 or beam tetrodes 6L6) that work in class AB or B [2]. However, the circuit topology and the circuit equations are the same for both classes and they differ only in bias point. A simulation of a push-pull amplifier is described in [1]. The complete circuit is divided into a phase splitter, a pentode circuit and a feedback circuit and they are connected using a modified

blockwise method [1]. However, this simulation uses a model of the ideal output transformer and the constant load.

In this paper, the pentode circuit is simulated in different level of complexity. At first, the pentode circuit with an ideal transformer and a constant load is solved, then the circuit with an ideal transformer and a linearized model of a loudspeaker. Finally, the circuit with a nonlinear model of the output transformer is simulated.

2. BASE CIRCUIT AND DEVICE MODELS

A typical pentode push-pull power amplifier contains two or four pentodes, the output transformer with connected loudspeaker and some resistors and capacitors (see figure 1). The circuit schematic also contains input units built by resistors R_g , R_b , capacitors C_g and bias voltage V_b . The input units are independent from the rest of the circuit because there is no coupling via the tube's cathode in topology with fixed bias. Their simulation is described in [1], and therefore the simulation of this part is omitted and there is only a focus on the output part of the amplifier. The typical values for circuit elements are listed in table 1.

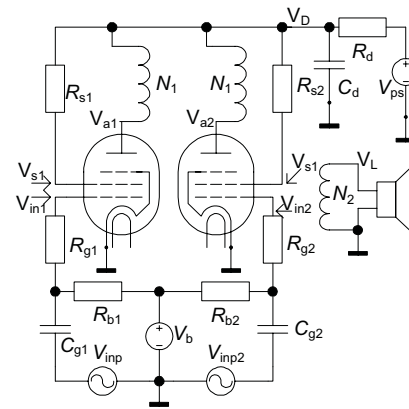


Figure 1: Circuit schematic of a push-pull tube amplifier.

Table 1: Values for circuit elements from figure 1.

R_s [Ω]	R_d [Ω]	C_d [μ F]	N_1	N_2	V_{ps} [V]	V_b [V]
500	500	100	1560	60	500	-50

2.1. Pentode Model

The Koren's model of the pentode [7] was chosen as the pentode model. The pentode plate current is in form $I_a(V_{ak}, V_{gk1}, V_{gk2})$ where V_{ak} is plate-to-cathode voltage, V_{gk1} is the grid-to-cathode voltage and V_{gk2} is the screen-to-cathode voltage. The screen current is given in form $I_s(V_{gk1}, V_{gk2})$. The description of functions I_a and I_s is omitted here and is available in [7]. Frequency properties of the tube (e.g. Miller capacitance) are not considered because it should be included in the simulation of input unit.

2.2. Output Transformer Model

An ideal output transformer is considered to be an impedance divider that transforms input voltages V_p and currents I_p to output V_s and I_s according to

$$\frac{V_s}{V_p} = \frac{N_2}{N_1} = \frac{I_p}{I_s} \quad (1)$$

where N_1, N_2 are numbers of windings of the transformer. However, a real transformer is far away from the ideal one. For an accurate simulation, losses caused by hysteresis and core saturation have to be considered. Nonlinear behavior of the real transformer is described in numerous literature, e.g. [8, 9]. According to Ampere's law, the magnetizing force H is

$$Hl_{\text{mag}} = N_1 I_p - N_2 I_s \quad (2)$$

where l_{mag} is the length of the induction path. The flux density B is computed from Faraday's law

$$\frac{\partial B}{\partial t} = \frac{V_s}{N_2 S} \quad (3)$$

where S is the transformer-core cross-section. The well-known nonlinear relation $B = \mu H$ can be implemented according to the Frolich equation [9] given by

$$B = \frac{H}{c + b|H|} \quad (4)$$

where c and b are constants derived from material properties. However, this model simulates only the core saturation. When simulating hysteresis, one can use e.g. Jiles-Atherton model [10] modified in [8] in order to remove nonphysical behavior of minor hysteresis loops. Magnetization of the core is obtained from

$$\frac{\partial M}{\partial H} = \delta_M \frac{M_{\text{an}} - M}{k\delta} + c \frac{\partial M_{\text{an}}}{\partial H} \quad (5)$$

where M_{an} is anhysteretic curve given by

$$M_{\text{an}} = M_s \left(\coth \left(\frac{H + \alpha M}{a} \right) - \frac{a}{H + \alpha M} \right) \quad (6)$$

and $\delta = \text{sign}(\partial H / \partial t)$. Parameters M_s, α, a, c and k are derived from material properties and their identification can be found e.g. in [11]. Parameter $\delta_M = 0$ when the nonphysical minor loop is going to be generated (anhysteretic magnezation has lower value than the irreversible magnezation) alternatively $\delta_M = 1$ [8]. Flux density is then obtained from

$$B = \mu_0 (M + H). \quad (7)$$

2.3. Loudspeaker Model

Loudspeakers play a very important role in the output signal generation via its frequency response. When considering linearized loudspeakers, one can model the frequency response with measured impulse responses with good results [1]. However, it is important to simulate the interaction between the tube amplifier and loudspeaker because the loudspeaker impedance is frequency dependent. The impedance can be modeled using the circuit schematic in figure 2 [12]. The values are derived from the added mass method and Thiele/Small parameters of a Celestion Vintage 30 loudspeaker placed in an Engl combo. The transformer leakage inductance and resistance can be modeled by modifying inductor L_{sp1} and resistor R_{sp1} values.

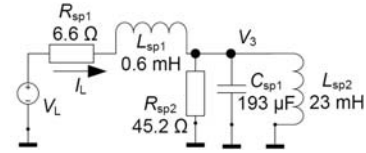


Figure 2: Simplified loudspeaker model – electric equivalent.

The loudspeaker impedance given by voltage V_L and current I_L can be expressed as the solution of the set of equations

$$\begin{aligned} I_L[n] &= I_L[n-1] + \frac{V_L[n] - I_L[n]R_{\text{sp1}} - V_3[n]}{L_{\text{sp1}}f_s} \\ V_3[n] &= V_3[n-1] + \frac{V_3[n]G_{\text{sp2}} - I_L[n] - I_{L2}[n]}{C_{\text{sp1}}f_s} \\ I_{L2}[n] &= I_{L2}[n-1] + \frac{V_3[n]}{L_{\text{sp2}}f_s} \end{aligned} \quad (8)$$

where $I_L[n-1], V_3[n-1], I_{L2}[n-1]$ are state variables and f_s is a sampling frequency. The equations were obtained using nodal analysis of the circuit in figure 2 and then discretized using Backward Euler formula.

Because the set of equations (8) is linear, it can be simplified into one linear equation

$$I_L[n] = -c_1 V_L[n] + I_{\text{tmp}} \quad (9)$$

where I_{tmp} is a linear combination of state variables given by

$$I_{\text{tmp}} = -c_2 I_L[n-1] + c_3 V_3[n-1] - c_4 I_{L2}[n-1]. \quad (10)$$

The new state variable values are then computed from

$$V_3[n] = -c_5 V_3[n-1] - c_6 I_L[n] + c_7 I_{L2}[n-1] \quad (11)$$

and

$$I_{L2}[n] = I_{L2}[n-1] + c_8 V_3[n]. \quad (12)$$

Coefficients c_{1-8} are derived from (8).

3. SIMULATION OF THE AMPLIFIER

In the simplest case, the load is considered to be constant. Using nodal analysis and discretization by Euler method, one can obtain the set of circuit equations

$$\begin{aligned}
 0 &= -\frac{V_L[n]}{R_L} + \frac{N_1}{N_2} (I_{a1} - I_{a2}) \\
 0 &= -V_{a1}[n] + V_D[n] - \frac{N_1}{N_2} V_L[n] \\
 0 &= -V_{a2}[n] + V_D[n] + \frac{N_1}{N_2} V_L[n] \\
 0 &= I_{s1} + \frac{V_D[n] - V_{s1}[n]}{R_{s1}} \\
 0 &= I_{s2} + \frac{V_D[n] - V_{s2}[n]}{R_{s2}},
 \end{aligned} \tag{13}$$

where V_D is voltage on the power supply, capacitor C_d is from the circuit schematic in figure 1 and $I_{a1} = I_a(V_{in1}[n], V_{a1}[n], V_{s1}[n])$, $I_s = I_s(V_{in1}[n], V_{s1}[n])$ and similarly $I_{s2} = I_s(V_{in2}[n], V_{s2}[n])$ and $I_{a2} = I_a(V_{in2}[n], V_{a2}[n], V_{s2}[n])$. The equations (13) are solvable for given value V_D and the solution can be implemented using a static waveshaper. It can be precomputed for different values of V_D voltage and stored in a look-up table. Then, during the simulation, the proper static waveshaper is chosen according to the V_D voltage. The V_D voltage is then actualized using

$$V_D[n] = \frac{-I_{a1} - I_{a2} - I_{s1} - I_{s2} + \frac{(V_{PS} - V_D[n-1])}{R_D}}{C_1 f_{fs}} + V_D[n-1]. \tag{14}$$

3.1. Circuit with Loudspeaker Model

If the loudspeaker is connected to the power amplifier, the load impedance is no longer given explicitly, but it is determined by V_L , I_L relation implicitly given by equations (8). In order to exclude the loudspeaker impedance R_L , the first equation from system (13) is modified to

$$I_L[n] = \frac{N_1}{N_2} (I_{a1} - I_{a2}) \tag{15}$$

and then solution of the modified equations (13) with (15) is expressed as a function $I_L[n] = f_{IL}(V_{in1}[n], V_{in2}[n], V_D[n], V_L[n])$. Finally, the solution of the whole system using (9) is given by

$$-c_1 V_L[n] + I_{tmp} = f_{IL} \tag{16}$$

for the unknown variable $V_L[n]$ and state variables I_{tmp} and $V_D[n]$.

3.2. Circuit with Nonlinear Transformer Model

The simulation of a circuit with a nonlinear transformer model is based on (2), (3) and the nonlinear core model. The system is described using

$$\begin{aligned}
 0 &= -H(B[n])l_{mag} - (c_1 V_L[n] + I_{tmp})N_2 + f_{IL}N_2 \\
 0 &= B[n-1] - B[n] + \frac{V_L}{N_2 S f_s}
 \end{aligned} \tag{17}$$

where term $f_{IL}N_2$ is recomputed current I_L to primary winding, $B[n-1]$ is the flux density in the previous sampling period and $H(B)$ is the core model derived from (4) or from (7) if the hysteresis is considered.

4. SIMULATION RESULTS

All the simulations from section 3 were implemented in Matlab environment using Mex files and C language. The Newton method was used for solving implicit nonlinear equations. The derivation of function or the Jacobian matrix were obtained using finite difference formula, the maximal number of iterations was 100 and the numerical error was chosen as 0.0001. The values for the transformer model were chosen experimentally: $M_s = 1.11 \times 10^6$, $3a = 8.56$, $\alpha = 8.82 \times 10^{-5}$, $c = 0.14$ and $k = 51.65$. However, they can be computed from a measured hysteresis loop data [11]. The parameters for the Frohlich core model were $c = 113.38$, $b = 0.71$ and the dimensions of the transformer were chosen as $S = 0.003 \text{ m}^2$ and $l_{mag} = 0.2 \text{ m}$.

The transformer-core cross-section S together with the number of windings determines the low cutoff frequency of the transformer. The simulation of the output part of the power amplifier was appended with a phase splitter, input pentode unit and feedback according to [1] and the results were compared to the measured Engl combo. The data was obtained from a measured voltage on a parallel loudspeaker output with connected soundcard. The frequency dependence of the first harmonic content of the voltage output signal, obtained using sweep sine signal, is shown in figure 3. The simulation with a constant load provides the worst results – there is no resonance around 45 Hz that appears in all of the other simulations and measured data. The simulation with hysteresis loop has similar behavior as a measured amplifier in the area of low frequencies due to the core losses. In the area of mid-frequencies, the simulation and measured data differ because of the other resonances and mechanical properties of the loudspeaker diaphragm, which are not considered in the simulation but can be included in the simulation by improving the loudspeaker model like in [13]. The nonlinear distortion was investigated as well. The output spectrum for an input sinewave signal with an amplitude of 2 V is shown in figures 4. The individual spectra are measured for the same frequency but they are shifted in the graph. All the algorithms excluding the variant with constant load give very similar results. The nonlinear distortion caused by transformer hysteresis manifests very slightly and only at frequencies below cca 150 Hz and it is very dependent on transformer parameters. The spectrogram of simulation using JA-model is shown in figure 5.

The table 2 shows the hypothetical computational complexity that was determined using measurement of time duration of simulation for input guitar riff signal with a length of 18 s and an amplitude of 30 V. The results showed that the algorithm is capable of working in real-time. Sound examples of all the algorithm variants and detailed graphs and other spectrograms are available on the web page www.utko.feec.vutbr.cz/~macak/DAFx11/.

Table 2: Normalized computational complexity.

Constant load	Loudspeaker	Frohlich	J-A model
0.04 %	0.05 %	0.12 %	0.27 %

5. CONCLUSIONS

The simulation of a push-pull tube amplifier was discussed in this paper. The impact of the loudspeaker and output transformer to

the amplifier properties was investigated. The loudspeaker was modeled using simplified electric circuit and Frohlich and Jiles-Atherton transformer models were used. The results showed that the output transformer had a minor impact on the simulation results. It only manifested at very low frequencies while the computational complexity was increased significantly. Both transformer models provided very similar simulation results and therefore, the Frohlich model is more efficient for real-time simulations.

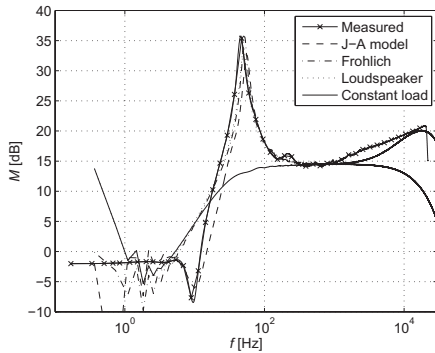


Figure 3: Frequency dependence of the first harmonic content of the output signal.

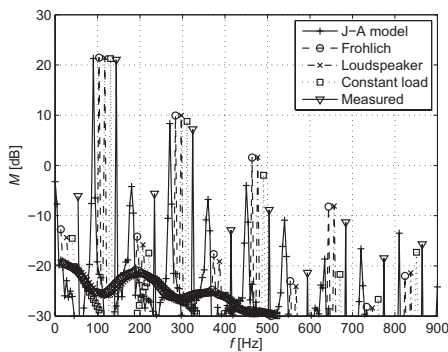


Figure 4: Output spectrum for input 90 Hz sinewave.

6. ACKNOWLEDGMENTS

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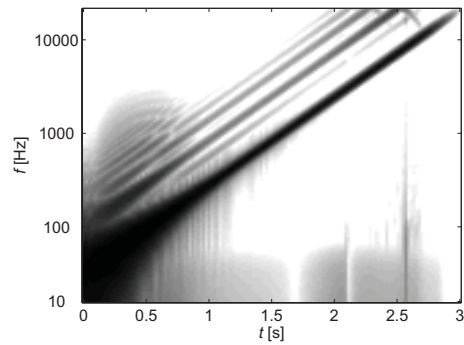


Figure 5: Spectrogram of output signal for simulation with JA model of transformer.

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