ANALYSIS AND SIMULATION OF AN ANALOG GUITAR COMPRESSOR

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ABSTRACT

The digital modeling of guitar effect units requires a high physical similarity between the model and the analog reference. The famous MXR DynaComp is used to sustain the guitar sound. In this work its complex circuit is analyzed and simulated by using state-space representations. The equations for the calculation of important parameters within the circuit are derived in detail and a mathematical description of the operational transconductance amplifier is given. In addition the digital model is compared to the original unit.

1. INTRODUCTION

In the field of guitar technology, certain products enjoy cultic status because of their unique auditory characteristics, like the MXR DynaComp. This guitar compressor pedal, created by MXR in the 1970's, was a very popular tool of achieving a fattened up sound with noticeable more sustain to lead guitar lines. The compression effect of the DynaComp is used to smooth out differences in volume between notes. Thereby it is a kind of volume controller that varies its internal gain to sustain the guitar sound.

In recent years a new trend has won recognition in music technology - the digital modeling and simulation of analog audio circuits. The advantage is the independence of cost-intensive, unreliable and often impractical analog technologies.

The state-space model has turned out to be a practicable tool to simulate non-linear audio systems with parametric components.

2. CIRCUIT ANALYSIS

The circuit of the MXR DynaComp is depicted in Fig. 1. For the sake of clarity we split this complex circuit into the four adequate blocks: (1) input circuit, (2) output circuit, (3) power supply and (4) the heart of the DynaComp, the operational transconductance amplifier (OTA). The simplified structure is given in Fig. 2 and shows the coupling between each stage.

The power supply block feeds the other blocks with two constant voltages $V_{batt} = 9\,\text{V}$ and $V_{bias} = 2.93\,\text{V}$. This subcircuit has no further effect on the audio signals and is neglected in the following considerations.

2.1. Input Stage

The input stage buffers the input signal and provides two signals to the differential inputs of the OTA, which are commensurate to the input signal.

Input capacitance C_3 isolates the internal biased DC-level from the 0 V DC-level of the guitar. Transistor Q_1 is used as a buffer

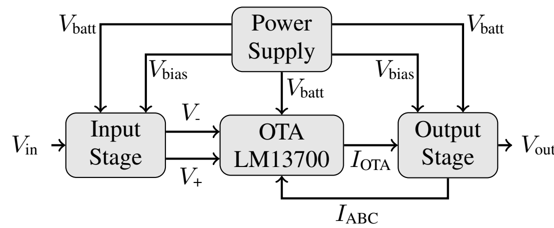


Figure 2: *Simplified block diagram of the circuit.*

and provides a low-impedance signal at its emitter. This buffered signal is set by capacitance C_2 to the DC-level of the OTA and is then routed to the inverting input. The signal is also routed to the non-inverting input with a potential drop at the potentiometer causing a signal-dependent difference between differential inputs.

2.2. OTA

The operational transconductance amplifier LM13700 is depicted in Fig. 3. It has a pair of differential inputs, a single output and one controllable gain input. The chip is completely composed of transistors and diodes. The task of the OTA is to produce an amplified output current depending on the differential input voltages. The gain of the OTA is variably controlled by the amplifier bias current I_{ABC}. Detailed information can be found in [1] and [2].

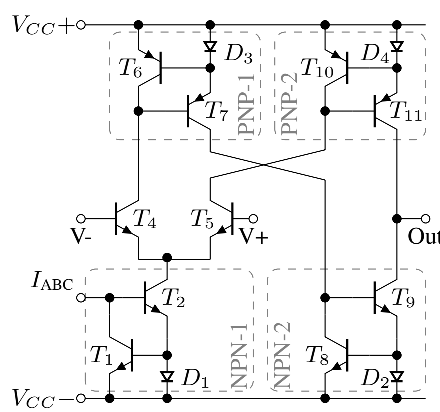
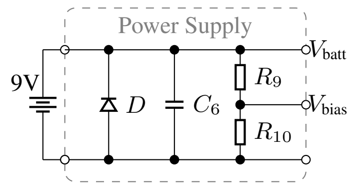


Figure 3: *Simplified circuit of the OTA LM13700.*

To find an analytical description of the OTA, it is necessary to simplify the integrated circuit. In addition to the differential amplifier, the integrated circuit can be expressed as an subtraction circuit



R_1	10 kΩ	R_8	150 kΩ	R_{15}	10 kΩ	C_1	10 nF	C_8	10 nF
R_2	470 kΩ	R_9	56 kΩ	R_{16}	1 MΩ	C_2	1 µF	C_9	50 nF
R_3	1 MΩ	R_{10}	27 kΩ	R_{17}	1 MΩ	C_3	10 nF	C_{10}	10 µF
R_4	1 MΩ	R_{11}	27 kΩ	R_{18}	10 kΩ	C_4	1 nF	D	1N914
R_5	15 kΩ	R_{12}	10 kΩ	R_S, sens	470 kΩ	C_5	1 µF	Q_i	2N3904
R_6	10 kΩ	R_{13}	10 kΩ	R_V, vol	47 kΩ	C_6	1 µF		(BJT)
R_7	1 MΩ	R_{14}	150 kΩ	P	2 kΩ	C_7	10 nF		

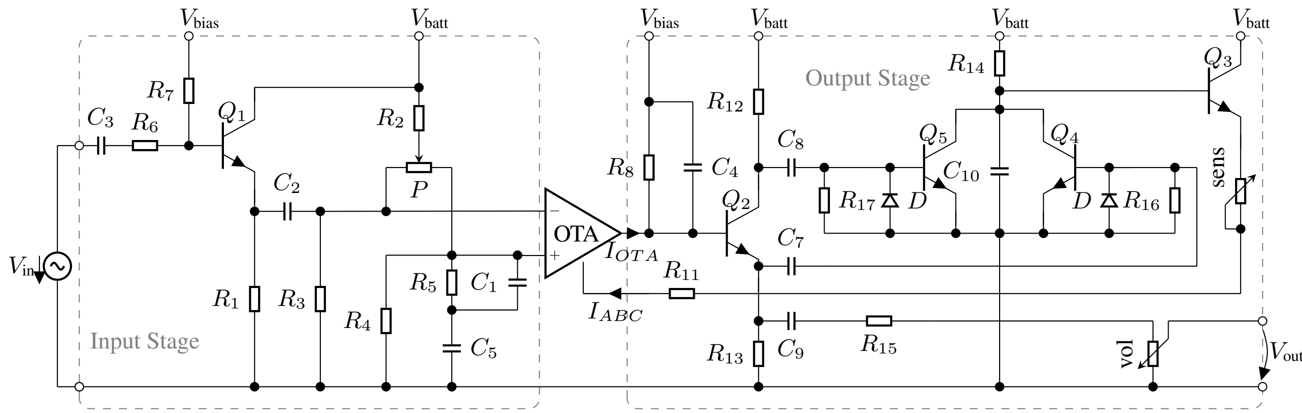


Figure 1: *Schematic of the MXR DynaComp and component values.*

composed of two NPN-current mirrors and two PNP-current mirrors. In the following we assume that the reference current and the output current are equal [3].

Since I_{ABC} is the reference current of the first NPN-current mirror, the same current has to be pulled out of the differential amplifier. Disregarding the base currents we obtain

$$I_{ABC} = I_{C,T4} + I_{C,T5} \tag{1}$$

with the collector currents of T_4 and T_5 composing the differential amplifier. Expressing these currents by the approximation

$$I_C = I_S \cdot e^{\frac{V_{BE}}{V_T}}, \tag{2}$$

with thermal voltage $V_T \approx 26\,\text{mV}$, we consider the ratio

$$\frac{I_{C,T4}}{I_{C,T5}} = \frac{e^{\frac{V_{BE,T4}}{V_T}}}{e^{\frac{V_{BE,T5}}{V_T}}} = e^{\frac{V_{BE,T4}-V_{BE,T5}}{V_T}} \tag{3}$$

assuming that the saturation currents I_S of T_4 and T_5 are equal. With $V_D = V_{BE,T4} - V_{BE,T5} = V_- - V_+$ and equations (1) and (3) we obtain two expressions for the collector currents

$$I_{C,T4} = \frac{I_{ABC}}{e^{-\frac{V_D}{V_T}} + 1}, \qquad I_{C,T5} = \frac{I_{ABC}}{e^{\frac{V_D}{V_T}} + 1}. \tag{4}$$

Transforming these terms using [4]

$$\frac{2}{1+e^{-x}} = 1 + \frac{1-e^{-x}}{1+e^{-x}} = 1 + \tanh\frac{x}{2} \tag{5}$$

the collector currents result in

$$I_{C,T4} = \frac{I_{ABC}}{2}\left(1 + \tanh\frac{V_D}{2V_T}\right) \tag{6}$$

$$I_{C,T5} = \frac{I_{ABC}}{2}\left(1 - \tanh\frac{V_D}{2V_T}\right). \tag{7}$$

Tracking the collector currents through the current mirrors, the output current of the OTA is

$$\begin{aligned} I_{OTA} = I_{C,T5} - I_{C,T4} &= -I_{ABC} \cdot \tanh\frac{V_D}{2V_T} \\ &= -I_{ABC} \cdot \tanh\frac{V_- - V_+}{2V_T}. \end{aligned} \tag{8}$$

2.3. Output Stage

The output current of the OTA is the input variable of the output stage. This stage is a circuit controlled by the signal level of I_{OTA} applying the required gain to the OTA by feeding back the amplifier bias current I_{ABC}. Another task is to derive the output voltage V_{out} as the output signal of the DynaComp circuit.

The output of the OTA is attached to a high frequency roll-off composed of R_8 and C_4 and to transistor Q_2. Q_2 performs two tasks - firstly it buffers the output signal and secondly it inverts the phase to follow the envelope of the signal. Both the emitter and the collector current, provide out-of-phase signals to a rectifier-filter arrangement. The negative parts of both signals are earthed by the diodes attached to the base of Q_5 and Q_4. The base currents of these two transistors, derived from the emitter and collector currents of Q_2, follow the envelope by controlling the voltage across capacitance C_{10}. This voltage represents an inversion of the input signal, i.e. higher signals of I_{OTA} causing higher base currents of Q_4 and Q_5. Thus the voltage across C_{10} is pulled down because of the current flowing out of R_{14} divides into the collector currents of Q_4 and Q_5. If there are small signals at the input, voltage V_{C10} rises - with no signal at the input it rises nearly to $V_{Batt} = 9\,\text{V}$. Voltage V_{C10} controls I_{ABC}, which is adjustable by the 470 kΩ-potentiometer, as depicted in Fig. 4.

As a result, the amplifier bias current rises and amplifies the gain of the OTA in case of a decreasing input signal. The output

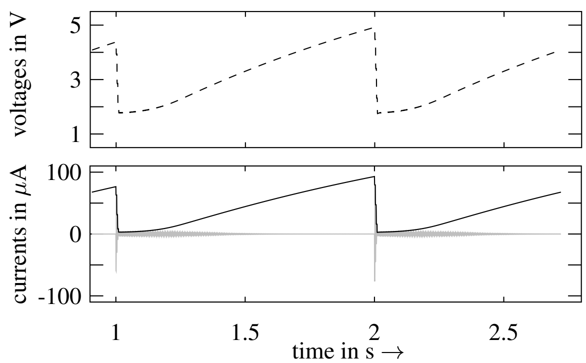


Figure 4: *Stimulation with exponentially decreased sine burst. Capacitor state V_{C10} (dashed), currents I_{OTA} (gray) and I_{ABC} (black).*

voltage of the circuit is tapped from the emitter of Q_2. It is also adjustable by a $47\,\mathrm{k}\Omega$-potentiometer.

3. STATE-SPACE MODELS

The state-space representation is a common tool to describe physical systems, especially to simulate non-linear systems with changeable parameters. This method models the system as a set of input, $\boldsymbol{u}$, output, $\boldsymbol{y}$, and state variables, $\boldsymbol{x}$ and $\dot{\boldsymbol{x}}$. The relation between them are based on network theory basics and formed into first-order differential equations. The equations used in this paper and the discretization are derived in detail in [5]. The differential equations describing continous non-linear systems are

$$\dot{\boldsymbol{x}}(t) = \mathbf{A}\cdot\boldsymbol{x}(t) + \mathbf{B}\cdot\boldsymbol{u}(t) + \mathbf{C}\cdot\boldsymbol{i}(\boldsymbol{v}(t)) \quad (9)$$
$$\boldsymbol{y}(t) = \mathbf{D}\cdot\boldsymbol{x}(t) + \mathbf{E}\cdot\boldsymbol{u}(t) + \mathbf{F}\cdot\boldsymbol{i}(\boldsymbol{v}(t)) \quad (10)$$
$$\boldsymbol{v}(t) = \mathbf{G}\cdot\boldsymbol{x}(t) + \mathbf{H}\cdot\boldsymbol{u}(t) + \mathbf{K}\cdot\boldsymbol{i}(\boldsymbol{v}(t)). \quad (11)$$

The number of independent energy storage elements defines the number of state variables in vector $\boldsymbol{x}$ and $\dot{\boldsymbol{x}}$, typically the voltages across capacitors. The discrete-time system is obtained by using the trapezoidal rule with sampling interval T. The substitution $\boldsymbol{x_c}(n) = \frac{T}{2}\left(\left(\frac{2}{T}\mathbf{I} + \mathbf{A}\right)\boldsymbol{x}(n) + \mathbf{B}\boldsymbol{u}(n)\right)$ serves the discrete state-space model with

$$\boldsymbol{x_c}(n) = \overline{\mathbf{A}}\cdot\boldsymbol{x_c}(n-1) + \overline{\mathbf{B}}\cdot\boldsymbol{u}(n) + \overline{\mathbf{C}}\cdot\boldsymbol{i}(\boldsymbol{v}(n)) \quad (12)$$
$$\boldsymbol{y}(n) = \overline{\mathbf{D}}\cdot\boldsymbol{x_c}(n-1) + \overline{\mathbf{E}}\cdot\boldsymbol{u}(n) + \overline{\mathbf{F}}\cdot\boldsymbol{i}(\boldsymbol{v}(n)) \quad (13)$$
$$\boldsymbol{v}(n) = \overline{\mathbf{G}}\cdot\boldsymbol{x_c}(n-1) + \overline{\mathbf{H}}\cdot\boldsymbol{u}(n) + \overline{\mathbf{K}}\cdot\boldsymbol{i}(\boldsymbol{v}(n)). \quad (14)$$

To solve the non-linear equations given in (14) we use the damped Newton algorithm. Function $\boldsymbol{F}(\boldsymbol{v}(n))$ and its Jacobian $\boldsymbol{J}(\boldsymbol{v}(n))$ are required in this algorithm. To obtain a non-linear relation between the transistor currents $\boldsymbol{i} = (I_B, I_E)$ and the transistor voltages $\boldsymbol{v} = (v_{BE}, v_{BC})$ we use the Ebers-Moll equations

$$I_B = I_{ES}\frac{1}{1+\beta_F}\left(e^{\frac{V_{BE}}{V_t}} - 1\right) + I_{CS}\frac{1}{1+\beta_R}\left(e^{\frac{V_{BC}}{V_t}} - 1\right) \quad (15)$$

$$I_E = -I_{ES}\left(e^{\frac{V_{BE}}{V_t}} - 1\right) + I_{CS}\frac{\beta_R}{1+\beta_R}\left(e^{\frac{V_{BC}}{V_t}} - 1\right). \quad (16)$$

4. SIMULATION

4.1. Input Stage

To simulate the input stage, we just have to use the algorithm of the discrete state-space model. Because there are no variations from the procedure explained in Section 3, we deal briefly with this part. With the conventions for currents through capacitances and transistors $i_C = C\cdot\dot{u}_C$ and $I_C + I_B + I_E = 0$ we obtain the system matrices by using Kirchhoff's circuit laws. Firstly all voltages across resistors have to be expressed by non-linear transistor elements, known values and/or capacitor states. Afterwards a mesh analysis has to be accomplished to find adequate meshes to express the transistor voltages, the output and the whole system. We find the mesh equations

$$\begin{aligned}
0 =& V_{in} - V_{R_1} - V_{BE_1} + V_{R_6} + V_{C_3}\\
0 =& V_{R_4} - V_{C_5} - V_{C_1}\\
0 =& V_{R_3} - V_{R_4} + V_{P_b} + V_{P_a}\\
0 =& V_{R_2} + V_{P_a} + V_{C_2} - V_{BE_1} + V_{BC_1}.
\end{aligned}$$

To gain the dependencies on the different parameters and to build the system matrices we have to solve these equations for $\dot{v}_{C_i}$. After this the discretization has to be operated and non-linear equations have to be solved. Since there are two output voltages, V_- and V_+, the discrete output $\boldsymbol{y}(n)$ consists of two entries.

4.2. Output Stage

The simulation of the output stage is more complicated, because of the feedback of the amplifier bias current. This feedback causes linear dependencies in system matrix $\overline{\boldsymbol{K}}$, which has to be inverted to calculate the non-linear transistor currents. This problem is known as a delay-free loop which can be eliminated by using, for example, the K-method [6]. In our case we decided to simplify the feedback loop as a one sample delay.

A method to solve this problem is to modify the output stage by neglecting the base current of Q_3. In addition Q_3 is replaced by a voltage-controlled voltage-source V_{C10} and a current-controlled current source with a non-linear internal resistance R_{ABC}. V_{C10} serves the same voltage as the voltage drop at capacitance C_{10} to control I_{ABC} as depicted in Fig. 4. Another problem is the unknown voltage drop at the ABC-input of the OTA, V_{ABC}, which is necessary to set up a mesh equation for the calculation of I_{ABC}. We obtain an expression for V_{ABC} by observing current mirror NPN-1 in Fig. 3 with

$$V_{ABC} = V_{BE,T2} + V_{D_1}. \quad (17)$$

Knowing the electrical properties of the disposed elements within the integrated circuit, we can use the Shockley-equations with approximation (2) for the collector current of T_2 and diode current of D_1 to express I_{ABC} by

$$I_{ABC} \approx I_{C,T2} = I_{S,T2}\cdot e^{\frac{V_{BE,T2}}{V_T}} \approx I_{D1} = I_{S,D1}\cdot e^{\frac{V_{D1}}{V_T}} \quad (18)$$

with the saturation currents $I_{S,T2}$ and $I_{S,D1}$. After transposing and inserting these equations in (17), we get

$$V_{ABC} = V_t\cdot\left(\ln\left(\frac{I_{ABC}}{I_{S,T2}}\right) + \ln\left(\frac{I_{ABC}}{I_{S,D1}}\right)\right). \quad (19)$$

The voltage drop across R_{ABC}, $\overline{V}$, is defined by V_{ABC} and the base-emitter voltage of the replaced transistor Q_3, V_{BE_3},

$$\overline{V} = V_t \cdot \left(\ln \left(\frac{I_{ABC}}{I_{S,T2}} \right) + \ln \left(\frac{I_{ABC}}{I_{S,D1}} \right) + \ln \left(\frac{I_{ABC}}{I_{S,Q3}} \right) \right) \quad (20)$$

With the mesh equation

$$\overline{V} = V_{C_{10}} - V_{R_S} - V_{R_{11}} = V_{C_{10}} - I_{ABC} \cdot (R_S - R_{11}), \quad (21)$$

I_{ABC} can be numerically calculated by solving

$$I_{ABC} = \sqrt[3]{I_{S,T2} \cdot I_{S,D1} \cdot I_{S,Q3} \cdot e^{\frac{V_{C_{10}} - I_{ABC} \cdot (R_S - R_{11})}{V_t}}}. \quad (22)$$

Thus the simulation of the output stage can be done straight forward by using the conventions made in section 4.1 and the general algorithm of the discrete state-space model. The mesh analysis for the system is set up by

$$\begin{aligned}
0 &= -V_{R_{13}} + V_{C_7} + V_{BE_4} \\
0 &= -V_{Batt} + V_{R_{12}} - V_{C_8} + V_{BE_5} \\
0 &= -V_{Batt} + V_{R_{14}} + V_{C_{10}} \\
0 &= -V_{bias} + V_{C_4} + V_{BC_2} - V_{C_8} + V_{BC_5} + V_{C_{10}} \\
0 &= V_{R_8} - V_{bias} + V_{BE_2} - V_{C_9} - V_{R_{15}} + V_{R_{vol_1}} + V_{R_{vol_2}}
\end{aligned}$$

and provides on the one hand the output voltage, $V_{out} = V_{R_{vol_2}}$, and on the other hand the voltage drop across C_{10} to calculate via (22) and (8) the new values of I_{ABC} and I_{OTA} [7]. The state-space vectors are

$$\dot{\boldsymbol{x}}(t) = \left[\dot{v}_{C_4}(t)\ \dot{v}_{C_7}(t)\ \dot{v}_{C_8}(t)\ \dot{v}_{C_9}(t)\ \dot{v}_{C_{10}}(t) \right]^T, \quad (23)$$

$$\boldsymbol{u}(t) = \left[I_{OTA}(t)\ V_{bias}(t)\ V_{Batt}(t) \right]^T, \quad (24)$$

$$\boldsymbol{i}(t) = \left[I_{B_2}\ I_{E_2}\ I_{B_4}\ I_{E_4}\ I_{B_5}\ I_{E_5} \right]^T. \quad (25)$$

To simplify the circuit and to reduce computational costs, it is practical to enhance the Ebers-Moll-Model by connecting the diodes attached to Q_4 and Q_5 in parallel to the base-emitter junction. Thus equation 15 has to be modified [7].

5. RESULTS AND DISCUSSION

To evaluate the performance of the state-space model, the output data has been compared to the original MXR DynaComp as reference by stimulating both with the same test signals.

Fig. 5 displays the dynamic characteristics of both models using the same settings - sensitivity: maximum, output: maximum.

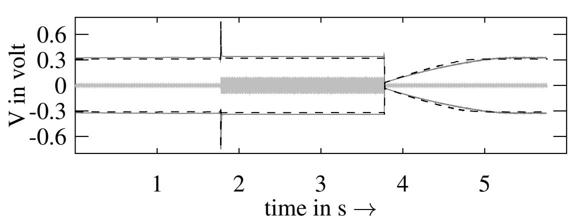


Figure 5: *Dynamic characteristic. Input signal with* $10/100\,\mathrm{mV}$ *(gray), envelopes of measurement (dashed) and simulation.*

For audio compressors it is useful to analyze the static behavior. Thus the static characteristics of the digital model is compared

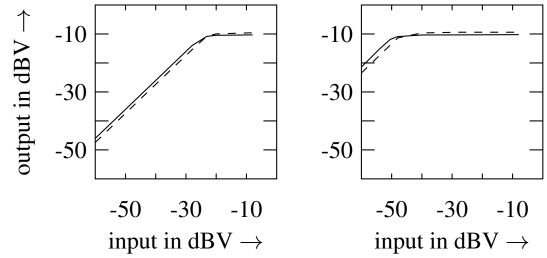


Figure 6: *Static curves of simulation (dashed) and measurement (solid). On the left: sensitivity =0, right: sensitivity =100.*

to the original pedal by showing the input-output-relation using different settings for sensitivity in Fig. 6. Both, the static and the dynamic characteristics, illustrate a good similarity. There are differences in the attack and release behavior which can be explained by component tolerances in the measured system and by approximations made during the derivation of the discrete system.

In addition some sound clips of electric guitar playing are available on our homepage

`http://ant.hsu-hh.de/dafx2011/compressor`

6. CONCLUSION

This paper presented a state-space model of the MXR DynaComp for a digital simulation of its sustaining and dynamic range controlling effect. The analog circuit was analyzed and equations for the calculation of the OTA output current and the amplifier bias current were derived in detail. The algorithm of the derivation of the discrete state-space matrices was introduced.

The comparison between the model and the reference showed a good match, too, although a lot of approximations had been made to develop a functional state-space description of the DynaComp.

7. REFERENCES

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